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THE STRAINS

IN

FRAMED STRUCTURES,

WITH NUMEROUS PRACTICAL APPLICATIONS

TO

CRANES—BRIDGE, ROOF AND SUSPENSION TRUSSES—BRACED ARCHES —PIVOT AND DRAW SPANS—CONTINUOUS GIRDERS, ETC.

ALSO,

DETERMINATION OF DIMENSIONS AND DESIGNING OF DETAILS—SPECIFICATIONS AND CONTRACTS—COMPLETE DESIGNS AND WORKING DRAWINGS.

BY

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PREFACE.

To those who make use of this work, whether teacher or student, a few words as to its plan may be of service.

The work is divided into two PARTS. PART I. consists of two SECTIONS. SECTION I. gives, in four short chapters, the principles which lie at the bottom of all methods of calculation. In these chapters all unnecessary detail has been avoided, the object being to familiarize the student first with the fundamental principles. Each chapter contains the ground-work of a separate method of calculation, and the same illustrative example is used in each. The student who has thus familiarized himself with the four fundamental methods of calculation, can afterwards combine these methods in the solution of any particular case, as may seem best. It is believed that every method in use will be found on analysis to be a combination of two or more of the methods set forth in these first four chapters, and, so far as known to the writer, the present work is the only one in which such division has been made, and each method given clearly by itself independently of the others.

In SECTION II. the application of these methods to the solution of various structures is given with all necessary detail. It has been the aim of the writer to make this section very complete in full solutions of every existing form of bridge. The student, already familiar with the four fundamental methods, is now in no danger of being confused by detail, and can easily devise for himself other methods of solution for individual cases, as good, or even better, than those given. In Chapter I. of this section will be found a more complete treatment of Roof Trusses than has been thus far given in any work known to the writer.

In Chapters III. and IV. a simple bridge girder is taken, and calculated fully, first by each of our four fundamental methods, and lastly by that combination of methods which seems best adapted to the case in hand. The remainder of the section gives the complete calculation of every form of bridge known, each case illustrated by an example carefully worked out. In Chapters VI. to VIII. upon the continuous girder, pivot or swing bridge and braced arch, much new matter will be found, and it is thought that the methods given will commend themselves as practical and easy of application. Whatever may be thought of the comparative advantages or disadvantages of these forms of bridges, they cannot well be omitted from a work which aims at any degree of completeness. For the average student, perhaps, so full a course is not desirable, at least at first, and therefore the attempt has been made by means of finer print to mark out two courses of study. In any case the intelligent teacher will know what to omit, and it is no disadvantage to a student to be possessed of a Text Book which includes more than he has been able to read, and which may, therefore, be of future benefit instead of being laid on the shelf when finished.

In Chapter IX. the suspension system is given at considerable length, perhaps more than its importance demands. The entire chapter is believed to be new, and so far as the writer knows, it is the only solution of this construction which is free from assumptions known to be false.* It is, therefore, given here for what it is worth as a contribution to the science of Bridge Calculation. In the Supplement to Chapter IX. the ordinary theory is also given with all requisite fulness.

Everywhere it has been the aim of the writer to keep mathematical demonstrations out of the body of the work, so far as possible to do so. In the APPENDIX to PART I. will be found, in connected form, all the mathematical deductions referred to and made use of in the Text. The chapter upon the Theory of Flexure is especially full, and in that upon the Continuous Girder new matter will be found.

The mere calculation of strains alone is but one part of the general problem of design, and by no means the most important. It is quite as necessary to properly proportion a structure for the stresses it has to sustain, as to know beforehand what these stresses are.

In PART II., therefore, will be found as full a treatment of the important topics of cross-sectioning and designing of details and connections as the writer has been able to give. That practical bridge builders will find here much to criticize, goes without saying. The writer will be glad of any and all such criticisms, and will avail himself of them in the improvement of this very important part of the work. He can only ask for the present attempt the indulgence which, perhaps, it can claim, as among the first in a work of this character to combine "practice" with "theory," and hopes that portions of it may not be without interest and value even to the practical engineer.

NEW HAVEN, *June*, 1883.

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* Since writing the above, I have learned that precisely the same method of discussion was first applied by *Charles Bender, C. E.*, and published in Van Nostrand's *Engineering Magazine* for November, 1881. It will also be found, extended in treatment, in "Principles of Economy in the Design of Metallic Bridges," by the same author (Wiley & Sons).

PREFACE TO THE FIFTH EDITION.

IN the present edition many important and considerable changes and additions have been made.

The strains due to concentrated load systems, and the diagram method now so widely used, have been fully discussed and presented, in connection with a large number of examples (pages 87–95 and 215–233). Among these examples will be found the full calculation of a skew span. While thus presenting fully and in detail the applications of this method. the method by locomotive excess loads is still retained. The author believes that this method will yet be found the most satisfactory, and that practice will sooner or later return to it. Meanwhile both methods will be found in the present work, given with all desirable fulness and detail.

The chapter upon Theory of Flexure has been extended since the third edition, by a complete Table, giving the strength and properties of various materials, and by a discussion of the deflection of framed girders. There has also been added a discussion of the various column formulæ; and the subject of combined tension or compression and flexure is fully treated. This chapter has now been further enlarged by the deduction of the general formulæ for the continuous girder (page 291). Here and always, numerous examples have been introduced. It is believed that throughout the work no single point of importance is without an example fully illustrating and explaining the text.

The Second Part, treating of the determination of dimensions and designing, has been entirely re-written and greatly extended. It will be found more complete and systematic in arrangement, and better adapted to the requirements of the student. Every rule and point of importance is fully illustrated by an example, and the sequence is such that the student follows the natural order of designing, and can easily refer, at any stage, to that portion which bears upon his work.

Tension and Compression Members, Pins and Eye Bars, Riveting, Wind Bracing and Miscellaneous Details, Floor System, Dead Weight and Proportions, Specifications, Complete Design for a Bridge, Shop Drawings, Order Book, Inspection, Shipping, and Erection follow in regular order, and each subject receives careful practical treatment, and is thoroughly illustrated by Plates and Examples.

The "straight-line formula" for struts is given and used, together with the older formulæ (page 330). The chapters upon Pins and Eye Bars, and upon Riveting, have much new and valuable matter. In the chapter upon Miscellaneous Details (page 395) the subjects of wind bracing, lateral and sway bracing, knee-braces, portals, and portal bracing, have been fully treated and illustrated by examples, as also the effect of centrifugal force on curves. The wind strains in Trestle Bents and Braced Piers are given very completely, and new formulæ for camber are given. We have also specially treated the subject of Skew Portals, and inserted new Tables in many places.

In the chapter upon Floor System, the Solid Iron and Ballast Bridge Floor, as recently built by the N. Y. C. & H. R. R. R., finds appropriate mention (page 433).

The subject of Dead Weight and Economic Proportions receives more extended and satisfactory treatment (page 438).

In addition to these changes, and others too numerous for special mention, we have given, by permission, the *General Specifications for Iron and Steel Railroad Bridges and Viaducts*, of Theodore Cooper, C.E. (page 455). Mr. Cooper's specifications are widely known and used. While giving the specifications *verbatim*, we have given, in the shape of foot-notes on each page, such remarks as may be of use to the reader, by calling attention to the reasons of each article of the specifications, or to differences of practice, with other explanatory remarks.

In Chapter X. (page 484) will be found the complete design of an Iron Railway Bridge, in accordance with these specifications. Each step is carefully detailed and referred to its proper article in the specifications and in the work, so that the example forms a summary of the application of all the principles previously explained.

Chapter XI., upon *Shop Drawings*, has been kindly contributed by Morgan Walcott, C.E., formerly with the Phœnix Bridge Company. Mr. Walcott is well fitted, both by practical experience and by his experience as a teacher, to give the student valuable hints upon this important topic.

In Chapter XII. (page 500) we have given various forms for the Order Book, such as have been tested in practice and found suitable and serviceable. We have also added remarks upon Shipping and Inspection.

Finally, by the courtesy of John Sterling Deans, C.E., Member Am. Soc. C. E., we are enabled to give the student, in Chapter XIII., the first full and complete treatment of the important subject of *Erection* yet published. The numerous and elaborate plates which illustrate the subject would alone give this chapter a practical value for the constructor as well as student.

The author is, perhaps, justified in assuming that the plan of the present work is now well understood both by teachers and students. It aims to be full and complete, even beyond the needs of class-room work. It comprises a complete work on Strains, on Theory of Flexure and Strength of Materials, and upon Designing, Construction, and Erection. In such a work many courses are possible, according to the time at disposal. To the student without guidance, we should recommend, on first reading, the omission of pages 139 to 215, and 278 to 306. He would do well to begin the Second Part as early as possible, and to bear in mind that this is, after all, the most important and most practical part of the subject. His ability in finding strains should be taken advantage of as soon as possible in the actual designing of members to resist those strains, and upon his familiarity with the principles of the Second Part will depend the value to him of all the rest.

As this Part is therefore by far the more important, and has thus far been generally the most inadequately treated, the author has labored to make it, in this edition, specially full and complete. The labor and cost of the additions and changes in this direction have been equivalent to a new work. It is hoped that it will not be found lacking in the treatment of any important point. In minor details there must always be differences of expert opinion, but the author has earnestly labored to incorporate the latest and best practice as he understands it. As previous editions have been so well received by the Engineering Profession, and by teachers and students, in respect to the Second Part, as among the first serious treatments of this most important branch, it is hoped that this fresh attempt will be found worthy of acceptance as among the most complete.

NEW HAVEN, *April*, 1890.

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PART I.

SECTION I.

DIFFERENT METHODS OF CALCULATION.

I. GENERAL PRINCIPLES.

INTRODUCTORY.

DEFINITION OF FRAMED STRUCTURES.—A framed structure, or "*truss*," is a collection of parts or "pieces," either of wood or metal, or both combined, so joined together by pins or rivets as to form a rigid frame-work. The office of such a structure may be either to transmit or transform motion or work, in which case it may form part or whole of a mechanism or machine; or to resist the action of outer forces tending to cause motion, in which case it is called a structure of stability or statical construction. The principles which govern the discussion of the first case are therefore dynamical, and belong to the science of mechanics; while in the second they are those of statical equilibrium, and belong to the science of statics. The latter class of structures alone are discussed in this work. The simplest kind of truss is a triangle, because that is the only figure whose shape cannot alter without changing the length of its sides. The triangle is thus the truss element, and all framed structures, no matter how complicated, which contain no superfluous pieces, may be considered as assemblages of triangles.

OUTER AND INNER FORCES—STRESS AND STRAIN.—We have to distinguish between the outer forces which may act at various portions of a framed structure, tending to cause motion of its parts, and the inner forces which prevent this motion, and cause equilibrium. The first we may call *stresses*, and the second *strains*. We, therefore, speak of the "stresses" *upon* a framed structure or assemblage of pieces, and the "strains" *in* such a structure. The first comprise all the exterior or applied forces, such as weights or loads, shocks, concussions, force of the wind, weight of snow, etc.; and the second include simply the forces (generally of pure tension or compression) caused by these outer forces in the various pieces of the structure. Thus the "stresses" or outer forces give rise to certain inner forces or "strains" in the pieces composing the structure.* The stresses are always given, and the investigation of these resulting inner forces or strains is the object of this work.

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* It will be observed that we use above the words stress and strain to denote *forces*, in each case, and only seek to distinguish between the forces exterior to the structure and those acting upon the pieces of the structure, caused by these exterior forces.

Many writers use the word *stress* to denote the "various forces which are exerted between contiguous bodies or parts of bodies, and which are distributed over the surface of contact of the masses between which they act," while they use the word *strain* to denote the alteration of volume and figure due to such stresses. However valuable such a distinction may be in the discussion of general mechanical problems, we prefer, for the purposes of the present work, the distinction above given. The word "strain," as applied to the *forces* by which the pieces of a structure are extended or compressed, is now so widely used by engineers that we have no wish to try to change its signification.

VARIOUS KINDS OF STRAINS.—The stresses or outer forces may produce, according to circumstances, different inner forces or strains in the various pieces of the structure. We may classify all these strains with which we have to do, as follows:

Tensile strain; Compressive strain; Shearing strain; and strain of Torsion.

In pure tension the forces causing it must act truly in the axis of the piece in question, so as to tend to pull its particles asunder in parallel lines. In pure compression they must act in similar manner to force its particles together. A shearing strain is caused by two stresses acting parallel to each other and at right angles to the axis of the piece, but in opposite directions and at two immediately consecutive points. The tendency is to cause two lines or planes of adjacent particles to slide on each other. The term is derived from the similar action of the blades of a pair of shears in the act of cutting. Torsion may be produced by a single transverse force applied at a distance from the axis.

These strains are very often found acting in combination, giving compound strains. Thus, in the case of a beam, one end of which is built into a wall horizontally, and sustaining a weight at the other end, we have tensile strains in the upper side, compressive strains in the lower side, and shearing strains between any two adjacent lines of particles, taken at right angles to the length. As similar cases are of frequent occurrence, we may for convenience call such a combination "*Bending strain.*"

STRUT, TIE, BRACE, COUNTERBRACE, etc.—The word "piece," which we have used already so many times, signifies a body whose length is generally great in comparison to its other dimensions. It is nearly always straight, but may be either straight or curved. By the union of such pieces the structure is formed, and the whole combination is termed a *framework*, The piece has different names according to the strain it is designed to resist. When it resists a compressive strain in general, it is called a *Strut*, and when the strut is vertical it becomes a *Post*. When the strain is tensile, the piece is called a *Tie*. The term *Brace* is used to denote both struts and ties. When a brace is rendered capable of acting either as a strut or as a tie indifferently, it is said to be *counterbraced.*

BEAM, GIRDER.—In the case of a bending strain, the piece is called a *Beam*. When the beam is of considerable length and subjected to *transverse stresses only*, it is called a *Girder*, and may be either *solid* or *flanged*. The cross-section of a solid girder is either rectangular, triangular, or round, or some modification of these forms. The flanged girder consists of one or two flanges of any desirable cross-section united to a thin vertical *web.** The office of the flanges is to resist the compressive and tensile strains. That of the web is to resist the shearing strain. The web may be continuous as in *plate girders,** or open-work as in *framed girders*. It is with the latter only that we have to do in this work. The intersection of a brace with a flange is called an *Apex.** That portion of a flange between two adjacent apices is called a *Bay* or *Panel.*

FUNDAMENTAL PRINCIPLES.—All the various methods of investigating the conditions of stability of framed structures are based upon one of two principles—the so-called "principles of statical equilibrium."

The first of these is as follows:

If any number of forces, all in the same plane, and acting at a common point of application, or at different points of the same rigid body, are in equilibrium, the algebraic sum of all their components in any given direction is zero. That is, the sum of all the components tending to cause

* For illustration of a flange cross-section with web, see page 320, Fig. 201. For illustration of a plate girder, see Fig. 206, page 345, and for a framed girder, see Fig. 283, page 418. For illustration of bay and apex, see Fig. 7, page 11.

motion in any one given direction is exactly equal to the sum of all those tending to cause motion in the precisely opposite direction.

This we shall call the "principle of the resolution of forces."

The second principle is as follows:

If any number of forces, all in the same plane, and acting at a common point of application, or at different points of the same rigid body, are in equilibrium, the algebraic sum of the moments of these forces, taken with reference to any point whatever in the plane of the forces, is zero. That is, the sum of the moments tending to cause rotation in one direction is balanced by the sum of the moments tending to cause rotation in the other direction.

This we shall call the "principle of the equality of moments."

DEFINITION OF "MOMENT."—The "moment" of a force is the product of the force into its "*lever arm.*" The lever arm of a force with respect to any point, which is called the "centre of moments," is the shortest distance of that point from the direction of the force, that is, it is the length of the perpendicular let fall from the point upon the force, prolonged in direction if necessary.*

UNNECESSARY PIECES.—*If any framed structure be conceived as cut entirely through so as to divide it into two parts, it is evident that if it held the outer forces in equilibrium before it was cut, that the strains in the cut pieces must have formerly held in equilibrium all the outer forces acting upon each of the parts into which the structure is divided.*

This principle is evident and does not need demonstration. Now we may resolve each of the outer forces, whatever their direction, and also the strains in the cut pieces, into vertical and horizontal components respectively.

We then have, according to our fundamental principles:

1st. The algebraic sum of all the vertical components is zero.

2d. The algebraic sum of all the horizontal components is zero.

3d. The algebraic sum of the moments with reference to any point in the plane of the forces is zero.

Here, then, are three conditions, which furnish us in general with three equations between the acting forces. If only three of these forces are unknown, they can therefore be determined. But if more than three are unknown, they cannot be determined, because there are more forces to be found than there are equations of condition. Now in general all the outer forces acting upon a framed structure are known. It follows, therefore, *that if it is impossible to divide the structure in any direction without cutting more than three strained pieces, the strains in which are necessarily unknown, the problem is indeterminate, and the structure has unnecessary or superfluous pieces.*

The frame should therefore be altered so as to dispense with one or more of these pieces, when the problem becomes determinate.

We can easily deduce a criterion for determining whether any frame has superfluous pieces. Assume, in general, the position of one side, thus fixing the position of two apices. Now, from these two apices we can locate another by two new sides. Then we can locate another by two sides from two previously located, and so on. If, then, m is the number of sides necessary for stability, and n is the number of apices, we have $m = 2(n-2) + 1$, or $m = 2n - 3$, for the number of necessary sides. If the number of sides in any case exceeds $2n - 3$, the extra number are unnecessary for rigidity. If the number of sides in any case is less than $2n - 3$, the frame can change its shape without changing the length of its sides, and is therefore not rigid.

METHODS OF CALCULATION.—The two fundamental principles already given, give rise to two methods of calculation: the method by "resolution of forces," or the "method of

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* See page 23, Fig. 11, for illustration.

sections," as it is often called, and the "method of moments." Each of these may be applied graphically or analytically. We may therefore draw up the following scheme, which includes all the methods of solution of framed structures of equilibrium:

$$\text{I. Resolution of forces} \begin{cases} (a) \text{ Graphic method of solution.} \\ (b) \text{ Algebraic " " "} \end{cases}$$

$$\text{II. Method of moments} \begin{cases} (c) \text{ Algebraic method of solution.} \\ (d) \text{ Graphic " " "} \end{cases}$$

Any one of these methods may be used in the solution of any given case, but in general there will be one, the employment of which in any special case will be found preferable in point of ease and simplicity to the others. Or, it may be, a combination of two or more of these methods furnishes a readier solution. It is therefore desirable that the engineer should be familiar with the principles and application of all, in order to proceed in the best manner in any special case.

The presentation and illustration of these four methods, in the order named, will therefore constitute the first Section of this work.

POSTULATES.—There are certain postulates which we require shall be understood and agreed to, before we can proceed to the application of our fundamental principles.

As the structures which we are to discuss are all of them structures of stability, that is, must oppose the action of outer forces and hold these forces in a state of rest, we assert:

1st. That all the forces which act upon any apex of a framed structure must constitute a system of forces in equilibrium, for which, therefore, the fundamental principles of equilibrium just stated hold good.

If, therefore, the outer forces or stresses at any apex are not in equilibrium already, they must be held in equilibrium by the strains which they cause in the pieces meeting at that apex.

2d. If the entire structure or frame-work is required to remain at rest, it follows *that all the outer forces acting upon it must also constitute a system of forces in equilibrium.*

3d. A uniformly distributed load may, without sensible error, be assumed to be grouped into weights resting upon the apices, each apex supporting a weight equal to the load resting upon the adjoining half bays.

This is evidently correct in the case of pin joints, and in the case of riveted joints the influence of continuity can be disregarded. In practice, moreover, cross-girders* occur generally only at the apices, so that no bay is subject to transverse strain except from its own weight.

4th. The strain in each bay or brace is uniform throughout its length, and acts in the direction of the length only.

This must evidently be the case for any assemblage of straight pieces connected by pin joints. In riveted structures there may be a slight wrenching at the joints if the pieces are not accurately in the direction of the lines of stress, which can be neglected.

5th. A brace cannot undergo tension and compression simultaneously.

6th. The effect of several stresses acting at once upon any brace is the same as the algebraic sum of the effects produced by each stress when acting separately.

Thus, if the stresses are all tensile or all compressive, the combined effect is equal to the sum of the effects produced by each. If some are tensile and some compressive, the difference between the sum of the tensile and the sum of the compressive will be the resultant stress.

* For illustration of cross-girder, see Fig. 206, page 345.

UNIT-STRAIN—INCH-STRAIN.—The strain in any piece per unit of area of its cross-section is called the *Unit-strain.* If this unit is the square inch, then the strain per square inch of cross-section is the inch-strain. The entire strain upon a piece is then equal to its area of cross-section multiplied by its unit-strain.

SIGNS FOR TENSION AND COMPRESSION.—Different writers vary in the signs which they adopt to denote tensile and compressive stresses or strains. Some denote tensile strains by a plus sign, and some the reverse. As whichever we adopt we shall be in good company, we shall denote compressive strains by a plus sign, and tensile by a minus sign. The notation may be recommended to memory from the fact that a tension piece, being only a rod or even a string between two points, is fitly represented by the minus sign ($-$), while a compression piece of some length, being liable to bending, is made of various forms of cross-section of which the "cruciform" ($+$) is a common shape.

QUESTIONS FOR EXAMINATION.

Define a framed structure. What structures are discussed in this book? What do you understand by outer forces? what by inner? What is the simplest kind of truss, and why? Distinguish between *stress* and *strain*. Name the different kinds of strains. What is a tensile strain? A compressive? A shearing? How is torsion produced? What do you understand by a bending strain? What do you understand by the word "piece"? What is a strut? a post? a tie? a brace? When is a brace counterbraced? What is a beam? What is a girder? What kinds of girders are there? What is the "*web*" of a girder? What is an "apex"? a "bay"? a "flange"?

State the principle of the resolution of forces. Of the equality of moments. What methods of calculation do these give rise to? State the postulates which are asserted to hold good for all framed structures. What is unit-strain? inch-strain? What is the relation between the entire strain, the unit strain, and the area of cross-section? What is a "moment"? A "lever arm"? Give the principle by which it may be determined whether any structure has unnecessary or superfluous pieces.

CHAPTER I.

GRAPHIC RESOLUTION OF FORCES.

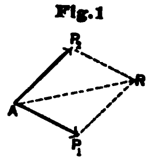
A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, COMMON POINT OF APPLICATION.

GRAPHIC REPRESENTATION OF A FORCE.—Three things are necessary to be known in order that a force may be completely given—its point of application, its direction, and its magnitude. All three may be at once represented by a straight line. Thus the length of the line to any convenient scale, may represent the magnitude of the force; one end of this line then gives the point of application, and the direction of the line from this point gives the direction in which the force acts.

All forces of which we shall have occasion to speak will be considered as lying and acting in the same plane.

TWO FORCES—COMMON POINT OF APPLICATION.—If two forces P_1 and P_2, given in direction and magnitude by AP_1 and AP_2 have a common point of application A, Fig. 1, we may find the resultant R according to well known principles, by completing the parallelogram, as indicated by the dotted lines and drawing the diagonal AR. AR is the resultant in direction and intensity. If then we apply to the point A, a force AR it will have the same effect upon the point as the two forces P_1 and P_2 had when acting together, that is, it will *replace* P_1 and P_2. If, however, the resultant R acts in the direction RA, it will produce a precisely opposite effect from P_1 and P_2 acting together. If, therefore, we let P_1, P_2, and R all act upon the point A simultaneously, and suppose R to act in the direction from R to A, then these three forces *will be in equilibrium.*

Fig. 1



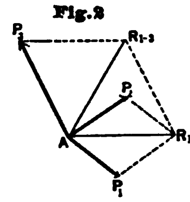
Now we wish to call attention to the fact that it is unnecessary to complete the parallelogram fully. Thus it would have been sufficient to have drawn a line as P_1R parallel and equal to AP_2, or a line P_2R parallel and equal to AP_1. In either case we should have found the point R, and would have found, therefore, the magnitude of the resultant.

Next, as to the direction of the resultant, notice that if it acts in the direction from R to A it holds the forces in equilibrium. If it should act in the direction AR it would replace the forces.

If then the resultant is supposed to act in the direction obtained by following round either triangle AP_1R or AP_2R, *in the direction of the forces*, as from A to P_1 and P_1 to R and R to A, or from A to P_2 and P_2 to R and R to A, the direction RA thus obtained is the direction for equilibrium. The opposite direction is that in which the resultant must act when it *replaces* the forces.

THREE FORCES—COMMON POINT OF APPLICATION.—Suppose we have three forces

Fig. 2



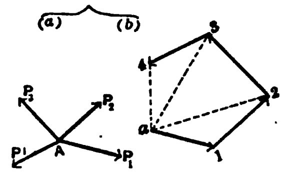
acting at A, as in Fig. 2. Then from the preceding, $R_{1\text{-}2}$ is the resultant of the forces P_1 and P_2. If we suppose it to act in the direction from A to $R_{1\text{-}2}$, it will replace P_1 and P_2 completely. We have then only to find the resultant of $R_{1\text{-}2}$ and P_3, by completing the parallelogram upon these forces, and we find $R_{1\text{-}3}$ the resultant of the forces P_1, P_2 and P_3.

Again we see it is unnecessary to complete all the parallelograms. It would have been sufficient to draw $P_1R_{1\text{-}2}$ parallel and equal to P_2, and then $R_{1\text{-}2}R_{1\text{-}3}$ parallel and equal to P_3, and we should have found the resultant $R_{1\text{-}3}$.

Again, if we go around *in the direction of the forces*, from A to P_1 and P_1 to $R_{1\text{-}2}$, then to $R_{1\text{-}3}$, and then back to A, the direction thus obtained is, as before, *the direction of the resultant for equilibrium.*

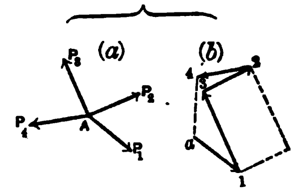
It is not necessary, or even desirable, to go through the construction upon the diagram of the forces. It is better to keep the two constructions separate.

Fig. 3



FOUR FORCES.—Let us apply these remarks to four forces P_1, P_2, P_3, P_4, acting at the point A, Fig. 3. The diagram (a) we call the *force diagram.* Now parallel to every force in the force diagram, we draw a line equal by scale to the magnitude of the force to which it is parallel. We thus obtain the polygon a 1 2 3 4. Thus $a1$ is parallel to P_1, and equal by scale to the magnitude of P_1. Then from the end of $a1$, we draw 1 2 parallel and equal to P_2, then 2 3 parallel and equal to P_3, then 3 4 parallel and equal to P_4. The polygon we thus obtain is called the *force polygon.* As we see, it is precisely the outline we should have obtained had we completed all the parallelograms directly upon the diagram (a) as in the last case, Fig. 2. Thus the diagonal $a2$ is the resultant of 1 and 2, $a3$ is the resultant of 1, 2 and 3, and $a4$ is the resultant of 1, 2, 3 and 4.

Fig. 4



ORDER OF FORCES IMMATERIAL.—The order in which the forces are laid off in the force polygon is immaterial. Thus in Fig. 4, it is evidently a matter of indifference whether we lay off the forces in the order 1, 3, 2, 4, or in the order 1, 2, 3, 4. In both cases we obtain the same resultant $a4$, and the same direction and magnitude, for the resultant. But by the same change of two and two we can produce any order we please.

GENERAL PRINCIPLE.—We see, in Figs. 3 and 4, that the direction obtained for the resultant by following around the force polygon *in the direction of the forces* as laid off, is the direction *necessary for equilibrium.* The opposite direction is that which *replaces* the forces. Thus $a2$, Fig. 3 (b), is the resultant of forces P_1 and P_2, just as in Fig. 1, and if it is conceived as acting at A in the force diagram (a) in the direction given by $a2$, it will replace forces P_1 and P_2. We have then $a3$ as the resultant of $a2$ and 3 or of the forces 1, 2, and 3, and acting at the common point of application A in the direction from a to 3 it will replace forces 1, 2, and 3. Finally then, $a4$ is the resultant of forces 1, 2, 3, and 4, and acting in the direction from a to 4 will replace these forces, or will have the same effect upon the point of application A, as all the forces when acting together. Of course the opposite direction, or the direction from 4 to a, obtained by following round the force polygon in the direction of the forces, is the direction necessary for equilibrium. If then we conceive a force applied at A in the force diagram (a) equal and parallel to $4a$ and acting in the direction from 4 to a, as given by the force polygon (b), we should have a system of five forces all acting at the same point, *in equilibrium.* We have then the following general principle:

If any number of forces in the same plane having a common point of application are in

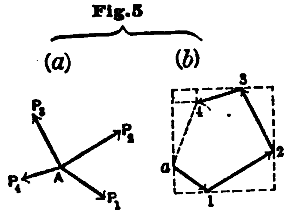
equilibrium, the force polygon is closed. If the force polygon does not close, the line necessary to close it is the resultant. If this resultant acts upon the point of application in the direction obtained by following around the force polygon with the forces, it will hold the forces in equilibrium. If taken as acting in the opposite direction, it will replace the forces.

We see also that any diagonal of the force polygon, as shown by the dotted lines in Fig. 3 (*b*), is the resultant of the forces on each side, and replaces those upon one side, and holds in equilibrium those upon the other, or *vice versa*, according to the direction in which we let it act.

Thus, in Fig. 3 (*b*), we have 5 forces in equilibrium, because the polygon is closed by $4a$. If these are in equilibrium, then any two, as $a1$, 1 2, must hold the others in equilibrium, but the resultant of $a1$ and 1 2 is $a2$, and acting in the direction from a to 2, replaces these two forces. It would therefore hold the other forces in equilibrium if acting in this direction.

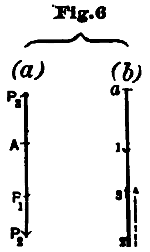
FIRST FUNDAMENTAL PRINCIPLE OF EQUILIBRIUM.—The general principle just enunciated is nothing more than a statement in other words of our first fundamental principle of equilibrium given on page 4. For if we resolve each force represented by a line of the polygon, into a horizontal and vertical component, for instance, as shown in Fig. 5 (*b*), it is evident, that if the algebraic sum of all the vertical components is zero, and the algebraic sum of all the horizontal components is zero, the polygon must be closed. Hence when the force polygon closes, the forces must be in equilibrium. Thus, starting from the point a, we see that three of the forces give downward vertical components, viz. 1, 4, and the resultant $4a$, and the sum of these, since the polygon closes, must be equal to the upward vertical components. So also for the horizontal components, 1 and 2 give components acting from left to right. Their sum is the horizontal distance from a to 2. But 3, 4 and the resultant give horizontal components acting from right to left, and if the polygon closes, their sum must be equal to the horizontal distance from 2 to a.

Fig. 5



FORCES ALL PARALLEL.—If the forces are all parallel, the force diagram will be a straight line as in Fig. 6 (*a*), where we have three forces P_1, P_2, P_3, all vertical and acting at the same point A.

Fig. 6



If we lay off these forces in the order given we have the force polygon (*b*), which in this case is also a straight line. Thus $a1$ is laid off downwards, equal to P_1, then 1 2 equal to P_2, then 2 3 upwards equal to P_3. The line $3a$ then closes the polygon, and hence the resultant is the algebraic sum of the forces, or $P_1+P_2-P_3$. The line a 1 2 3 in Fig. 6 (*b*) should be regarded still as a polygon or double line. Thus following round in the direction of the forces we go from a to 1, 1 to 2, 2 to 3, and hence the resultant $3a$, which closes, must act upwards for equilibrium.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

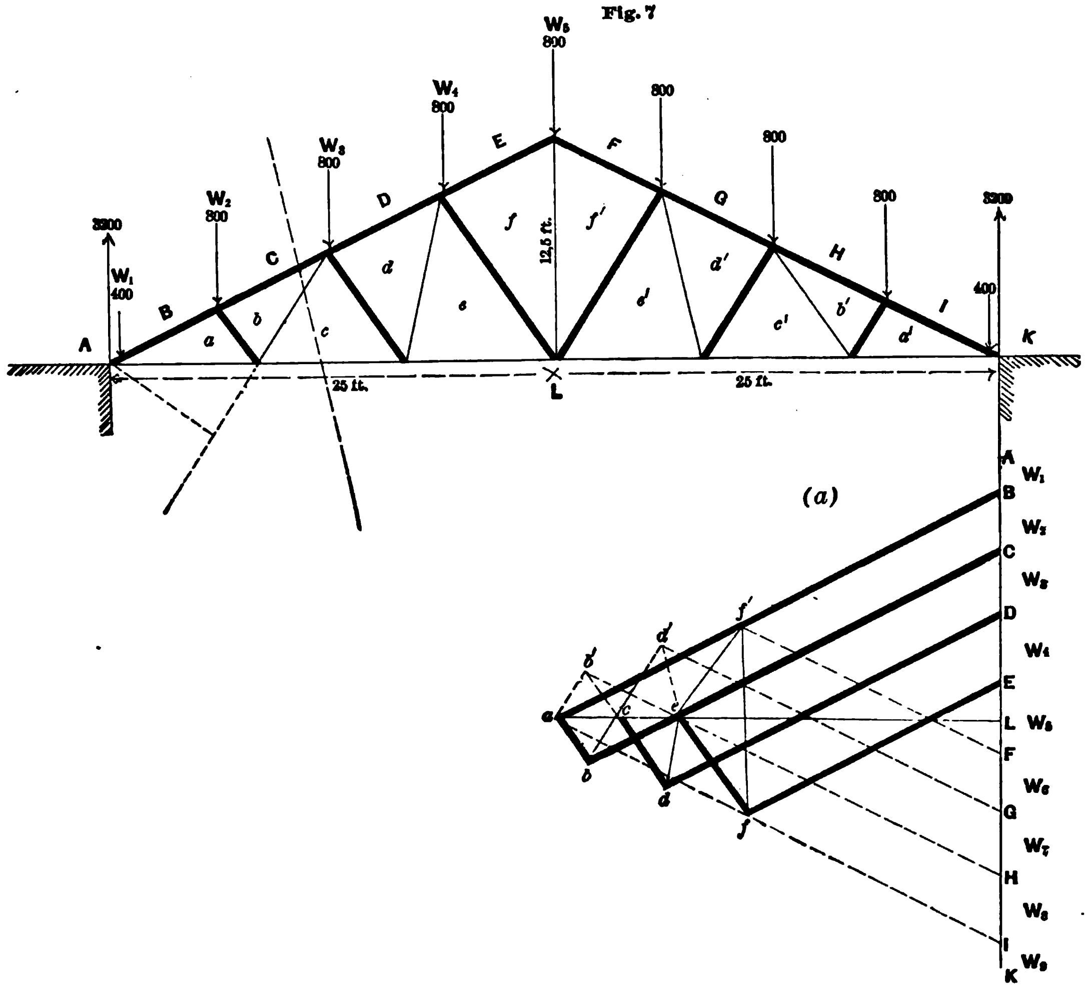
The foregoing principles, simple as they are, furnish us with the means of finding the strains in any framed structure, however complicated, which the civil engineer can legitimately be called upon to erect, *provided only all the stresses or outer forces are known.* They will be applied in detail to many different kinds of structures hereafter (see p. 68), so that the reader may obtain complete mastery of the method. We shall content ourselves here with a single example, merely to illustrate the method of application. For this pur pose we select a very simple structure.

APPLICATION TO A ROOF TRUSS.

DIMENSIONS OF TRUSS.—FRAME DIAGRAM.—The truss shown in Fig. 7 is 50 feet span, and 12.5 feet high. Each rafter is divided into four equal bays, and the lower horizontal tie is divided into six equal bays. The bracing is as shown in the Figure. Each half of the frame is perfectly symmetrical with the other half. The Fig. 7 we call the *frame diagram*. It may be drawn to any convenient scale, ***the larger the better.***

LOADING OF THE TRUSS.—According to our postulate 3, page 6, we suppose all that portion of the weight of roof covering which extends from the centre of one bay to the

Fig. 7



centre of the next, including weight of cross-pieces, planking, shingles, etc., to be concentrated at each apex. Let us assume that we thus have a weight of 800 lbs. acting at each upper apex, except the two end ones, where the weight is one-half of this, or 400 lbs. Since the truss is symmetrical, with respect to the centre, and symmetrically loaded, the upward reaction or pressure upon the wall at each end will be one half the sum of all the weights, or 3,200 lbs. at each end. These constitute all the outer forces or *stresses* which act upon the frame-work.

NOTATION.—The notation which we adopt in order to conveniently designate any piece or stress is as follows. We letter the triangular spaces into which the truss is divided

by the braces, also the spaces between the stresses. The letter L refers to all the space below the truss. Thus, the bays into which the rafter is divided are Ba, Cb, Dd, Ef, etc. The bays into which the lower horizontal tie is divided are La, Lc, Le, etc. In general any piece is denoted by the letters upon each side of it. Thus ab is the first brace, bc the next, and so on. In like manner AB is the first weight, BC the second, etc.

STRESS POLYGON.—We can now proceed to form the "*stress polygon.*" Thus in (a), Fig. 7, we lay off the weights to any convenient scale, in regular order one after the other, and thus obtain the line A, B, C, D, ... K. Then the two reactions are laid off upwards from K to L and L to A, thus closing the polygon, as should be the case, since if the truss is not to move bodily, the stresses must form a system in equilibrium. This is in accordance with our postulate 2, page 6. The stress polygon in this case is therefore a straight line, or rather a double line, from A to K and K back to A again. This is evidently because all the stresses or outer forces are parallel. [Fig. 6, p. 10].

STRAIN DIAGRAM.—We may now proceed to form the "*strain diagram,*" or find the strain in each piece caused by these stresses. According to our postulate 1, page 6, the strains in all the pieces which meet at any apex, together with all the stresses at that apex, must form a system of forces in equilibrium. Hence the polygon obtained by drawing lines parallel to these forces, and equal by scale to their magnitude, must close. Wherever then in general we know all the forces acting at any apex except two, we can easily find these two, if their directions are only given, by drawing lines parallel to these given directions, and prolonging them until they close the incomplete polygon formed by the known forces. These remarks will be evident from the construction. Thus at the left end, Fig. 7, we have two known forces, *viz.*, the half weight (400 lbs.) acting down, and the reaction (3,200 lbs.) acting up. We have also the unknown strains in the pieces Ba and La, and these four forces are all which act at the apex A. If equilibrium exists they must therefore form a closed polygon.

But the reaction LA and weight AB are already laid off in order in the stress polygon (a), the one up, the other down. We have therefore only to unite the points B and L by lines parallel to Ba and La, and we shall have the strains in these pieces respectively, *to the same scale as that chosen for the stress polygon.*

Now that we know the strain in the piece Ba, we can pass to the next upper apex. Because of the four forces acting there, we know already Ba and the weight BC, and hence there are only two unknown, *viz.*, the strains in ab and Cb. But in the strain diagram (a) now commenced, we have already Ba and BC laid off, and we have therefore only to join the points C and a by lines parallel respectively to ab and Cb above.

We next proceed to the first lower apex, where La and ab are known, and bc and Lc are to be found. We therefore join L and b in the strain diagram by lines parallel to Lc and bc above, and we obtain the strains in these pieces. Thus the polygon $LabcL$ is made to close.

We then proceed to the next upper apex, where we have Cb, bc and the weight CD now known. Hence we join D and c below by lines parallel to these pieces, and thus complete the polygon $DCbcdD$.

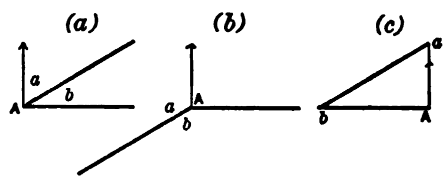
It is unnecessary to follow out the method of procedure further. The reader, however ought to do it for himself carefully and thoroughly.

THE SYMMETRY OF THE FIGURE A CHECK UPON THE ACCURACY OF THE WORK.—Proceeding in the method indicated, we have found the strains in every piece of the frame. The broken lines give the strains in the right hand half. It will be at once seen that they should be precisely equal to the strains in the corresponding pieces of the left half. This affords several excellent checks upon the accuracy of our work. Thus the two halves of

the Fig. should be perfectly symmetrical, and the broken half should unite with the full half exactly at the points e, c and a.

CHARACTER OF THE STRAINS.—The determination of the character of the strains is second only in importance to the determination of the strains themselves. Suppose we have a force Aa acting at any point as A, upwards, as shown by the arrow in Fig. 8, and that this force is held in equilibrium by the strains in the two pieces ab and Ab, which also act upon the same point A. Then, as we know, these forces must make a closed polygon as given at (c). Now follow round this polygon in the direction given by Aa, and we find that for equilibrium the strain in ab must act upon the point A in the direction from a to b given in Fig. (c). As this force can only act upon the point A *by means of the piece ab* which conveys it there, if ab is on the right of the point A, as in Fig. (a), the piece ab must be in *compression.* If ab is on the left of A, the strain in it must be tension. So for the piece Ab. We find from Fig. (c) its equilibrium direction from b to A or from left to right. Transferring this direction to the Figs. (a) and (b) we see that in (a) the strain in Ab must be tension, and in (b) tension also. This is sufficient to furnish us with a general rule for finding the character of the strain in any piece, as well as to illustrate the reason of the rule.

Fig. 8



If we take any apex of the frame and consider the forces acting upon that apex as a system of forces in equilibrium, the rule is:

Follow round the polygon formed by these forces, in the direction indicated by those forces which are already known in direction, and transfer the directions thus obtained for the forces to the apex under consideration. If the strain in any piece is thus found acting away from the apex, the corresponding piece is in tension; if towards the apex, it is in compression.

An application of this to Fig. 7 will make it plain. Thus take the first apex. Here we have the reaction known to act up, and the weight AB acting down, in equilibrium with Ba and La. Following round the polygon in (a), therefore, we go *up* from L to A, then *down* from A to B, then continuing round, we go in order from B to a, and then from a to L. We thus find the direction for the strains in Ba and aL, viz.: Ba from right to left and aL from left to right. Referring now to the frame itself, and transferring these directions to the corresponding pieces, we see that the direction for Ba gives us the strain in that piece acting towards the apex; it is therefore in compression. In like manner we have the strain in aL acting away from the apex, or tension.

Once more; take the next apex. Here the weight BC acts down. We follow round the polygon in (a), then, from B to C, then to b, then to a, and then back to B. We thus find the direction for aB from a to B. Referring back to the frame we find that this gives us the strain in this piece acting towards the apex we are now considering, and therefore compressive, just as we have already found it.* The direction Cb gives us the strain in Cb acting towards the apex, hence compression. The direction ba gives us the strain in ba acting towards the apex, and therefore compression.

Again; take the first lower apex. Here we have already found La to be in tension, hence the strain in that piece must act away from the apex we are now considering. With this to guide us we refer to Fig. (a) and follow round from L to a, then from a to b, b to c, and c back to L. We thus find ab acting towards the apex, and therefore compression, just as we have already found it. Also bc acting away from apex, or tension, and cL away, and therefore tension also.

* Observe that by changing the apex we have the strain in Ba opposite in direction to what it was before, but *in each case it is towards the apex considered,* and therefore in each case compression.

This is enough to indicate the application of our rule. The reader will do well to apply it carefully to every apex until thoroughly familiar with it. We have denoted compression in Fig. 7 (*a*) by heavy lines and tension by light lines.

It is well, when solving any problem, to avoid confusion in following round the various polygons, to determine the character of the strains by our rule ***as we go along, and not to wait until the strain polygon (a) is completed.***

REMARKS UPON THE METHOD.—The truth of the principle enunciated upon page 5, viz., that if the truss be cut entirely in two at any point, the strains in the pieces cut will hold the outer forces in equilibrium, is also evident from Fig. 7.

Thus suppose a section cutting *Dd*, *de* and *Le*, then the strains in these pieces ought to be in equilibrium with the algebraic sum of the weights and reaction. We see from the strain diagram below that this is the case, because the strains *Dd*, *de* and *eL* make a closed polygon with $LD = LA - AB - BC - CD =$ algebraic sum of weights and reaction.

The Figure also shows other relations not evident from any principles and peculiar to the frame of the truss. Thus we see that the strain in *ab* will be the least possible when it is perpendicular to the rafter. We can see, also, at a glance how the strains would be affected by altering the inclination of any piece.

Finally, the application of the method is equally simple and easy of execution, no matter how irregular the frame-work of the truss.

CHOICE OF SCALES.—In general the larger the frame is drawn the better, as it gives us more accurately the direction of the pieces composing it. The stress polygon should be taken to as small a scale as possible consistently with reading off the forces conveniently to as great a degree of accuracy as is required—so as to avoid the intersection of very long lines, where a slight deviation from true direction multiplies the error. When the strain polygon is completely finished, the strains may be read off according to scale, and written down upon the frame if required. Thus a good scale, dividers, triangle, straight-edge, and hard fine-pointed pencil are all the tools required. The work should be done with care, all lines drawn light with a hard pencil, and points of intersection carefully located, and lettered properly to correspond with the frame. Care should be exercised to secure perfect parallelism in the lines of the strain and frame diagrams. Thus in Fig. 7, since the piece *ab* is very short, its direction is better given by the piece *ef*, which is parallel to it and longer. ALWAYS OBSERVE THE NOTATION GIVEN.

The student will find in SECTION II. many examples for practice, and details of construction for various cases. He would do well to refer now to the examples there given. Some practice is necessary in order to obtain always reliable results. It should be remembered finally, that careful habits of intelligent manipulation, while they tend to give constantly increased skill and more accurate results, affect very slightly the rapidity and ease with which these results are obtained.

NUMERICAL DETERMINATION OF STRAINS.—In the case of Fig. 7, we have drawn the frame to a scale of 12 feet to an inch, and taken as our scale of force, 3,200 lbs. to an inch. Scaling off the strains in (*a*), we have, calling compression plus (+) and tension minus (−), the strains in the various pieces as follows:

For the rafters,

$$Ba = +\ 6280, \quad Cb = +\ 5816, \quad Dd = +\ 4700, \quad Ef = +\ 3580 \text{ lbs.}$$

For the lower bays,

$$La = -\ 5624, \quad Lc = -\ 4832, \quad Le = -\ 4024 \text{ lbs.}$$

For the diagonals,

$ab = +720,\quad bc = -720,\quad cd = +1060,\quad de = -928,\quad ef = +1452,\quad ff' = -2410$ lbs.

The checking of both halves of the Figure gives assurance of the substantial correctness of the result. The scale actually adopted for the frame by which the above results were found, was 10 feet to an inch, and for the strains 800 lbs. to an inch. As the error of the author working rapidly does not exceed $\frac{3}{100}$ ths of an inch, these strains may be depended upon within about 24 or 25 pounds.

Questions for Examination.

Show how to represent a force graphically. Show how to find the resultant of two forces graphically, and point out why it is unnecessary to complete the parallelogram. Point out the direction of the resultant necessary for equilibrium. The direction necessary to replace the forces. Do the same for three forces. For four forces. Distinguish between the force diagram and the force polygon. Show the significance of any diagonal in the force polygon. Show that the order of the forces is immaterial. State the general principle we thus arrive at. Prove that this is the same as our first fundamental principle of equilibrium. Show what the force polygon becomes when the forces are all parallel.

Illustrate these principles by application to a Roof Truss. What is the frame diagram? Stress polygon? Strain diagram. What notation is adopted? How is the loading supposed to be applied? Show how to form the strain diagram. Point out what checks you have upon the accuracy of the construction. Explain how to determine the character of the strains. Give the rule. Illustrate by the Figure. What can you say about the choice of scales?

CHAPTER II.

ANALYTIC RESOLUTION OF FORCES.

A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, COMMON POINT OF APPLICATION.

FUNDAMENTAL PRINCIPLE.—The principle upon which the method of solution by means of the analytic resolution of forces depends, is the same as that upon which the graphic method of the preceding chapter is based, viz.:

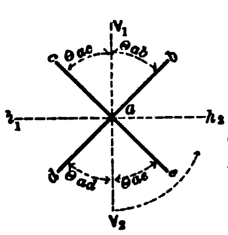
If any number of forces, in the same plane and acting upon the same point, are in equilibrium, the algebraic sums of their vertical and horizontal components must be respectively zero.

The two methods are therefore identical, and the present is only the algebraic representation of the preceding graphical construction.

If then, at any apex of a framed structure, which is the point of application for a system of forces in equilibrium, we know the directions of all the acting forces and the magnitude of all but two, we can at once write down two equations of condition between these two unknown forces, by means of which their magnitude may be determined.

NOTATION.—We always measure the angle of inclination of any piece *from the vertical* through the apex. This angle we denote in general by θ, and denote by subscripts the piece to which it refers. Thus, Fig. 9, let *ab*, *ac*, *ad* and *ae* be four pieces meeting at the apex *a*. Then the angles of inclination of these pieces are measured from the vertical line V_1 V_2 through the apex. Thus θ_{ab} is numerically the angle baV_1. θ_{ac} is numerically the angle caV_1. θ_{ad} is numerically the angle daV_2. θ_{ae} is numerically the angle caV_2.

Fig. 9



It is, however, necessary that we should always introduce the sines and cosines of these angles with their proper signs in the expression for the algebraic sum of the vertical and horizontal components. For this purpose we adopt the following conventions:

A compressive strain in a piece is always plus, a tensile strain minus. This convention we have already introduced in the preceding chapter.

Any force acting vertically *upwards*, as for instance, a reaction, is plus; when it acts *downwards*, as for instance, a weight, it is minus.

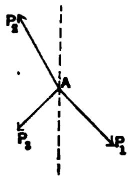
The cosine of θ is plus when the piece in question lies *below* the horizontal through the apex. Thus, Fig. 9, cos θ_{ae} is plus and cos θ_{ad} is plus. Similarly cos θ_{ab} and cos θ_{ac} are minus.

The sine of θ is plus when the piece lies to the *right* of the vertical through the apex. Thus, Fig. 9, sin θ_{ae} and sin θ_{ab} are plus, while sin θ_{ad} and sin θ_{ac} are minus.

The reader will observe that *this is equivalent to always reckoning the angle θ from aV_2*

around towards the right, as indicated in Fig 9. Thus V_2ah_2 is the *first quadrant*, for which sine and cosine are both positive. The second quadrant is h_2aV_1, for which sine is positive and cosine negative. The third quadrant is V_1ah_1, for which cosine is negative and sine negative. The fourth quadrant is h_1aV_2, for which cosine is positive and sine negative. Hence our rule just given. If we adhere strictly to this notation, we shall always be able to write down the various terms in the algebraic sum of the vertical and horizontal components with their proper signs. If then we find any strain minus, it will denote tension ; if plus, compression.

Fig. 10



GENERAL FORMULÆ.—Suppose we have three forces, P_1, P_2, P_3, acting at the point A, Fig. 10, in equilibrium. Then if we resolve each of these forces into a vertical and horizontal component, the algebraic sum of the vertical components must be zero, and the algebraic sum of the horizontal components must be zero. Adhering to the notation just described, the signs of these components in any particular case will take care of themselves, and we can write down the general equations:

For the vertical components,

$$P_1 \cos \theta_1 + P_2 \cos \theta_2 + P_3 \cos \theta_3 + \text{etc.} = 0.$$

For the horizontal components,

$$P_1 \sin \theta_1 + P_2 \sin \theta_2 + P_3 \sin \theta_3 + \text{etc.} = 0.$$

If now P_1 is known, we have two equations containing two unknown quantities, P_2 and P_3, and hence these forces can be easily found.

It is evident, then, that the method is applicable to any apex of any framed structure, where all the acting forces at that apex are known, *except two only*.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

Let us apply the foregoing principles to the same example, as in the preceding chapter, and thus check the results there obtained by the graphic method of resolution of forces.

APPLICATION TO A ROOF TRUSS.

DIMENSIONS.—We take the same dimensions as before, page 11, and refer to Fig. 7, p. 11. The angle, then, which the upper bays make with the vertical through any apex is about 63° 26'. The angle for any bay of the horizontal tie is 90°. The angle for all the parallel braces *ab*, *cd*, *ef*, Fig. 7, is 33° 41. The angle for the brace *bc* is also 33° 41'. The angle for the brace *de* is 12° 31'.

For the apex *BC*, for instance, we have for the bay *Cb*, $\theta_{cb} = 63° 26'$, and according to our convention, cos θ_{cb} is minus, because the piece *Cb* lies in the second quadrant, and sin θ_{cb} is plus for the same reason.

CALCULATION.—Remembering, then, always to take the sines and cosines with their proper signs in the general formulæ for the algebraic sum of the vertical and horizontal components, and also recollecting that upward forces are positive and downward forces negative, we can proceed to the calculation.

The numerical values of the sines and cosines are easily found to be as follows:

For the upper bays, $\begin{cases} \cos \theta = 0.44724 \\ \sin \theta = 0.89441 \end{cases}$ lower bays, $\begin{cases} \cos \theta = 0 \\ \sin \theta = 1 \end{cases}$

braces parallel to *ab*, $\begin{cases} \cos \theta = 0.83212 \\ \sin \theta = 0.55460 \end{cases}$

for $bc \begin{cases} \cos\theta = 0.83212 \\ \sin\theta = 0.55460 \end{cases}$ $de \begin{cases} \cos\theta = 0.97623 \\ \sin\theta = 0.21672 \end{cases}$

We are now ready to apply our principles.

Take the left hand apex, Fig. 7. Here we have the reaction R, the weight W_1, and the strains in Ba and La, forming a system of forces in equilibrium. We have, then, for the algebraic sum of the vertical forces,

$$R + W_1 + Ba \cos\theta_{Ba} + La \cos\theta_{La} = 0 \quad \ldots \quad (a)$$

and for the algebraic sum of the horizontal components,

$$La \sin\theta_{La} + Ba \sin\theta_{Ba} = 0 \quad \ldots \quad (b)$$

From (a) we find, since La is horizontal, and hence $\cos\theta_{La} = 0$,

$$\text{strain in } Ba = \frac{-R - W_1}{\cos\theta_{Ba}} \quad \ldots \quad (1)$$

From (b) we find

$$\text{strain in } La = \frac{-Ba \sin\theta_{Ba}}{\sin\theta_{La}} \quad \ldots \quad (2)$$

Inserting numerical values, and observing our notation and conventions, we have, since $R = +3200$, $W_1 = -400$, $\cos\theta_{Ba} = -0.44724$, and $\sin\theta_{La} = 1$,

$$\text{strain in } Ba = \frac{-3200 + 400}{-0.44724} = +6260 \text{ lbs.}$$

Hence Ba is in compression.

$$\text{strain in } La = \frac{-6260 \times 0.89441}{1} = -5600 \text{ lbs.}$$

Hence La is in tension.

Let us pass to the next apex. Here we have for the algebraic sum of the vertical components,

$$W_2 + Ba \cos\theta_{Ba} + Cb \cos\theta_{Cb} + ab \cos\theta_{ab} = 0 \quad \ldots \quad (c)$$

and for the horizontal components,

$$Ba \sin\theta_{Ba} + Cb \sin\theta_{Cb} + ab \sin\theta_{ab} = 0 \quad \ldots \quad (d)$$

Inserting in equation (c) the value of $Ba \cos\theta_{Ba}$, as found from equation (a), we have, after substituting value of Cb from (d) and reducing,

$$\text{strain in } ab = \frac{W_2 \sin\theta_{Cb}}{\sin\theta_{ab}\cos\theta_{Cb} - \cos\theta_{ab}\sin\theta_{Cb}} = \frac{W_2 \sin\theta_{Cb}}{\sin(\theta_{ab} - \theta_{Cb})} \quad \ldots \quad (3).$$

In the same way we find from equation (d),

$$\text{strain in } Cb = -\frac{Ba \sin\theta_{Ba}}{\sin\theta_{Cb}} - \frac{W_2 \sin\theta_{ab}}{\sin(\theta_{ab} - \theta_{Cb})} \quad \ldots \quad (4).$$

Inserting numerical values, we have

$$\text{strain in } ab = \frac{-800 \times 0.89441}{\sin(33^\circ 41' - 116^\circ 34')} = \frac{-715{,}528}{-0.99230} = +720 \text{ lbs.}$$

Hence ab is in compression. (Observe, that the angles θ_{ab} and θ_{Cb} are reckoned as shown in Fig. 9, page 16. Thus $\theta_{Cb} = 116^\circ 34'$.)

We have in like manner

$$\text{strain in } Cb = +6260 - \frac{-800 \times 0.55460}{-0.99230} = +6260 - 447 = +5813 \text{ lbs.}$$

Hence Cb is in compression.

At the first lower apex, Fig. 7, we have for the vertical components,

$$ab \cos \theta_{ab} + bc \cos \theta_{bc} = 0 \quad \ldots \quad (e)$$

and for the horizontal components,

$$La \sin \theta_{La} + ab \sin \theta_{ab} + bc \sin \theta_{bc} + Lc \sin \theta_{Lc} = 0 \quad \ldots \quad (f)$$

From the first of these equations we obtain

$$\text{strain in } bc = -\frac{ab \cos \theta_{ab}}{\cos \theta_{bc}} \quad \ldots \quad (5)$$

and for the second,

$$\text{strain in } Lc = \frac{-La \sin \theta_{La} - ab \sin \theta_{ab} - bc \sin \theta_{bc}}{\sin \theta_{Lc}} \quad \ldots \quad (6)$$

Inserting numerical values, we have

$$\text{strain in } bc = -\frac{720 \times -0.83212}{-0.83212} = -720 \text{ lbs.}$$

Hence bc is in tension.

Also,

$$\text{strain in } Lc = \frac{5600 \times -1 - 720 \times -0.55460 + 720 \times 0.55460}{+1}$$

$$= -5600 + 399 + 399 = -4802 \text{ lbs.}$$

Hence Lc is in tension.

At the apex CD, Fig. 7, we have for the vertical components,

$$W_3 + Cb \cos \theta_{Cb} + bc \cos \theta_{bc} + cd \cos \theta_{cd} + Dd \cos \theta_{Dd} = 0 \quad \ldots \quad (g)$$

and for the horizontal components,

$$Cb \sin \theta_{Cb} + bc \sin \theta_{bc} + cd \sin \theta_{cd} + Dd \sin \theta_{Dd} = 0 \quad \ldots \quad (h)$$

From these two equations we have, after reduction,

$$\text{strain in } cd = \frac{W_3 \sin \theta_{Dd} + bc \sin (\theta_{Dd} - \theta_{bc})}{-\sin (\theta_{Dd} - \theta_{cd})} \quad \ldots \quad (7)$$

Also,

$$\text{strain in } Dd = \frac{Cb \sin \theta_{Cb} + bc \sin \theta_{bc} + cd \sin \theta_{cd}}{-\sin \theta_{Dd}} \quad \ldots \quad (8)$$

Inserting numerical values, we have

$$\text{strain in } cd = \frac{-800 \times 0.89441 - 720 \sin (116°34' - 326°19')}{-\sin (116°34 - 33°41}$$

$$= \frac{-716 - 720}{-0.99230} = +1081$$

Hence cd is in compression.

Also,

$$\text{strain in } Dd = \frac{5813 \times -0.89441 - 720 \times -0.55460 + 1081 \times 0.55460}{-0.89441}$$

$$= \frac{-5199.205 + 399 + 599}{-0.89441} = +4696 \text{ lbs.}$$

Hence Dd is in compression.

By comparison with the formulæ already found, we can now write down the formulæ for the remaining pieces at once, without first writing down the equations of condition. Thus at the second lower apex, Fig. 7, we have at once, by simply referring to the formulæ already found for bc and making the proper changes in the subscripts,

$$\text{strain in } de = -\frac{cd \cos \theta_{cd}}{\cos \theta_{de}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In like manner, referring to equation (6),

$$\text{strain in } Le = \frac{-Lc \sin \theta_{Lc} - cd \sin \theta_{cd} - de \sin \theta_{de}}{\sin \theta_{Le}} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Inserting numerical values, we have

$$\text{strain in } de = \frac{-1081 \times -0.83212}{-0.97623} = -924 \text{ lbs.}$$

$$\text{strain in } Le = \frac{-4802 - 1081 \times -0.55460 + 924 \times 0.21672}{+1}$$

$$= -4802 + 599 + 200 = -4003 \text{ lbs.}$$

In similar manner, for the apex DE, Fig. 7, referring to equations (7) and (8), we can write down at once,

$$\text{strain in } ef = \frac{W_4 \sin \theta_{Ef} + de \sin (\theta_{Ef} - \theta_{de})}{-\sin (\theta_{Ef} - \theta_{ef})} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\text{strain in } Ef = \frac{Dd \sin \theta_{Dd} + de \sin \theta_{de} + ef \sin \theta_{ef}}{-\sin \theta_{Ef}} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Inserting numerical values, we have

$$\text{strain in } ef = \frac{-800 \times 0.89441 - 924 \sin (116°34' - 347°29')}{-\sin (116°34' - 33°41')}$$

$$= \frac{-715 - 717}{-0.99230} = +1443 \text{ lbs.}$$

$$\text{strain in } Ef = \frac{4696 \times -0.89441 - 924 \times -0.21672 + 1443 \times 0.55460}{-0.89541}$$

$$= \frac{-4200 + 200 + 800}{-0.89441} = +3577 \text{ lbs.}$$

At the centre apex in the lower flange, Fig. 7, we have, since by reason of the symmetry of the frame and the symmetrical loading the strains in all the pieces of the right half are equal to those in the left,

$$2\, fe \cos \theta_{fe} + ff' \cos \theta_{ff'} = 0.$$

Or

$$\text{strain in } ff' = \frac{-2\, fe \cos \theta_{fe}}{\cos \theta_{ff'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Inserting numerical values

$$\text{strain in } ff' = \frac{-2 \times 1443 \times -0.83212}{-1} = -2401 \text{ lbs.}$$

A comparison with the strains found for the same case in Chapter I. shows a satisfactory agreement in the results of the two methods. Thus :

	Ba	*Cb*	*Dd*	*Ef*	*La*	*Lc*	*Le*	*ab*	*bc*	*cd*	*de*	*ef*
Method of Chapter I.........	+6280	+5816	+4700	+3580	−5624	−4832	−4024	+720	−720	+1060	−928	+1452
Method of Chapter II........	+6260	+5813	+4696	+3577	−5600	−4802	−4003	+720	−720	+1081	−924	+1443

COMPARISON WITH PRECEDING METHOD.—We see that the application of the present method to the case chosen is much more difficult than the graphic method of Chap. I., in that it involves much calculation and requires very careful attention to avoid errors. The present method, therefore, does not adapt itself readily to cases where the various pieces have different inclinations, although, as we shall see hereafter in the applications of Section II., p. 55, there are many cases of frequent occurrence in practice where the application of the method is quick and easy. When the calculations are performed with proper care, the results are more accurate than by the graphic method. This latter, however, by the proper choice of scales, gives results practically correct.

One important point of difference we may note here, however, which holds good for all analytic methods as compared with graphic—that is, the graphic method gives indeed a *general method* of solution, but, in any case, only *particular results*, while the analytic method gives general results or formulæ which hold good *for all similar cases*. Thus the formulæ we have just obtained hold good for *all* trusses of the pattern of Fig. 7, no matter what their dimensions. We have, in any case, only to insert the special numerical values, and the formulæ give us at once the strains for that case.

In solving, then, any particular case, we solve at the same time all others like it, while the graphic method must be applied anew for every fresh case. This is generally true of all graphic methods.

If it were required to compute a large number of trusses, therefore, of different dimensions but same type, the present method would possess perhaps practical advantages superior to the graphic. Each method has thus its particular advantages, and the engineer should be able to choose in any case, that which leads most directly and easily to the required results. Illustrations of the use of this method will occur in Section II., wherever it is advantageous to make use of it.

THE METHOD IDENTICAL WITH THE METHOD OF SECTIONS.—We have stated at page 5 the principle that if the truss is conceived as cut in two at any point, the strains in the cut pieces are in equilibrium with the outer forces acting upon each portion into which the truss is divided. We can therefore write down two equations of condition for the cut pieces, expressing the condition that the sums of all the horizontal and vertical components are zero, and thus if only the strains in two cut pieces are unknown, we can find them. The formulæ thus obtained would be precisely identical with those already found, and we can therefore, if we choose, call the present method the analytic method of sections, instead of the analytic method of resolution of forces.

Thus by the application of our principle we have for the apex BC, Fig. 7, the equation (c), page 18, viz.:

$$W_2 + Ba \cos \theta_{Ba} + Cb \cos \theta_{Cb} + ab \cos \theta_{ab} = 0.$$

But we have already found for the preceding apex,

$$R + W_1 + Ba \cos \theta_{Ba} + La \cos \theta_{La} = 0.$$

If we find the value of $Ba \cos \theta_{Ba}$ from this, and insert in the first equation we have, since for the second apex $\cos \theta_{Ba}$ is minus,

$$W_2 + R + W_1 + La \cos \theta_{La} + Cb \cos \theta_{Cb} + ab \cos \theta_{ab} = 0,$$

which is precisely the same equation as we should obtain by the method of sections, and expresses the condition that the vertical components of the cut pieces *La*, *Cb*, and *ab*, are in equilibrium with the outer forces. The two methods are therefore identical, and whether we had proceeded from the principle that all the forces at any apex are in equilibrium or from the principle just stated of sections, we would have obtained in either case precisely the same results and equations.

ALGEBRAIC REPRESENTATION OF THE STRAIN DIAGRAM.—We can write down all the formulæ obtained for the various pieces directly from the strain diagram Fig. 7 (*a*), without stating the equations of condition at all. Since the present method and the graphic method of Chapter I. are both based upon precisely the same principle, Fig. 7 (*a*) is simply the graphic interpretation of our algebraic work. The simple trigonometrical solution of the various lines in the strain diagram Fig. 7 (*a*), will therefore give us at once the formulæ of this chapter. Thus a little inspection of the strain diagram will suffice to make evident that

$$ab \sin (\theta_{ab} - \theta_{Cb}) = W \sin \theta_{Cb}.$$

This is the same expression as equation (3) page 18. So for the other pieces.

Any one therefore familiar with the graphic method of Chapter I., can readily deduce from the strain diagram itself the trigonometrical formulæ for the strains in the various pieces.

QUESTIONS FOR EXAMINATION.

What is the fundamental principle upon which the method of the preceding chapter depends? Give and explain the notation adopted. How are angles of inclination measured? What sign has a force when acting upwards? downwards? from left to right horizontally? from right to left? What sign denotes compression? tension? Show how to apply the principle. Write down the general equations of condition for three forces in equilibrium.

Describe the notation of Fig. 7. How do you designate any piece? any apex? any apex weight? In equation (1) page 18, show how to insert the numerical values with proper signs. Do the same for equation (4), page 18. For equation (6), page 19. Show how to deduce equation (7) from equations (*g*) and (*h*), page 19, and go through the process of reduction. Show how equations (9) and (10) may at once be written down by reference to equations (5) and (6). Do the same for (11) and (12), page 20. Show how to write down equation (13), page 20. What can you say of the method of this chapter as compared with that of the preceding? What is the principle of the method of sections? Show that the present method is identical. Show how to obtain from the strain diagram, Fig. 7 (*a*), the trigonometrical results of the present chapter.

CHAPTER III.

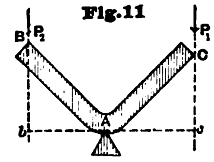
METHOD OF MOMENTS—ALGEBRAIC SOLUTION.

A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, DIFFERENT POINTS OF APPLICATION.

MOMENT, LEVER ARM, CENTRE OF MOMENTS.—The "*moment*" of a force with reference to any point is the product of the force into its "*lever arm.*" The point with reference to which the moment is taken is called the "*centre of moments.*" The lever arm of a force is the length of the perpendicular let fall from the centre of moments upon the direction of the force. For this purpose the force must be considered as prolonged in direction if necessary.

Thus in Fig. 11, if we have a bent lever BAC, with its fulcrum at A, acted upon at C by the force P_1 and at B by the force P_2, the lever arm of P_1 with reference to A is Ac, the perpendicular to the direction of P_1 prolonged, and the moment of P_1 with reference to A is $P_1 \times Ac$. In like manner the lever arm of P_2 is Ab and its moment $P_2 \times Ab$.

Fig.11



FUNDAMENTAL PRINCIPLE.—The methods of solution of the two preceding chapters are based upon the first two fundamental principles of equilibrium, viz.: that if any number of forces acting upon a rigid body are in equilibrium, the algebraic sum of the vertical components must be zero, and the algebraic sum of the horizontal components must be zero. That is, all the forces tending to raise the body vertically must be balanced by those tending to move the body downwards, and all those tending to shove it horizontally in one direction must be balanced by all those tending to shove it horizontally in the other.

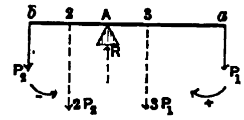
The method of solution of the present chapter is based upon the *third* fundamental principle of equilibrium, viz.:

If any number of forces, in the same plane and acting upon the same point or at different points of the same rigid body, are in equilibrium, the algebraic sum of the moments must be zero.

We may therefore call the present method the "method of moments." As the solution is algebraic, it is the "algebraic method of moments."

SIGNIFICANCE OF MOMENT.—In Fig. 12 suppose we have a lever ab resting upon a fulcrum at A, and acted upon by the forces P_1 and P_2. Suppose that there is equilibrium. Then, according to our principle, we must have the algebraic sum of the moments of the forces equal to zero.

Fig.12



Now when any force, acting alone, would tend to cause rotation in the direction of the hands of a watch from left to right, thus, we call its moment positive. The opposite direction is negative. In general when one force tends to cause rotation in one direction and the other in the opposite direction, we

must add their moments algebraically with opposite signs. Thus in the case of Fig. 12 we should have

$$P_1 \times Aa - P_2 \times Ab = 0.$$

$$\text{or, } P_1 \times Aa = P_2 \times Ab.$$

Suppose that Ab is 2 feet and Aa 3 feet. Then we could, by our principles, ***replace*** P_2 by a parallel force at a distance from A of only 1 foot, provided the new force is twice as great, or equal to $2P_2$, because its moment $2P_2 \times 1$ would then be equal to the moment of the old force, $P_2 \times 2$, and our equation above would still hold good. In like manner we can replace P_1 by a force $3P_1$ at a distance from A of 1 foot. Suppose P_1 and P_2 thus replaced by forces $2P_2$ and $3P_1$, as shown in the Figure. Since now the lever arms are unity, *the new forces themselves are the former moments.*

We see then the significance of a moment. ***The moment of a force, with reference to any point, is the magnitude of an equivalent parallel force at a unit's distance from that point.***

We can thus reduce any number of forces to their equivalents at a unit's distance, and then the algebraic sum gives the resultant force ***at that distance.*** Our principle simply says that for equilibrium this resultant should be zero.

From our equation above, we have

$$P_1 = P_2 \frac{Ab}{Aa}, \text{ or } \frac{P_1}{P_2} = \frac{Ab}{Aa}.$$

That is, when any number of forces are in equilibrium the forces are to each other *inversely as their lever arms.*

POSITION OF CENTRE OF MOMENTS INDIFFERENT.—It makes no difference whereabouts in the plane of the forces we take the centre of moments. The principle holds good for any point in the plane.

Thus in Fig. 12 we have really three forces in equilibrium, viz.: P_1, P_2, and the upward reaction of the fulcrum R. R did not appear in our equation above, because we took the centre of moments at A, and therefore its lever arm was zero.

Let us, however, take the centre of moments at b. Then the moment of P_2 disappears, and we have

$$P_1 \times ab - R \times Ab = 0, \text{ or } R = P_1 \frac{ab}{Ab}.$$

We have also, taking the centre of moments at a,

$$R \times Aa - P_2 \times ab = 0, \text{ or } R = P_2 \frac{ab}{Aa}.$$

From the last of these we have, since $ab = Aa + Ab$,

$$R = P_2 \frac{Aa + Ab}{Aa}.$$

But we have already found, for the centre of moments at A,

$$P_1 = P_2 \frac{Ab}{Aa}.$$

We shall have then

$$P_1 + P_2 = P_2 + P_2 \frac{Ab}{Aa} = P_2 \frac{Aa + Ab}{Aa}.$$

This is the same as we have just found for R when the centre of moments was at a. In the same way we can show that

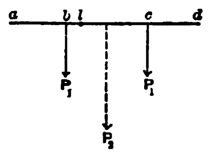
$$P_1 + P_2 = P_1 + P_1 \frac{Aa}{Ab} = P_1 \frac{Aa + Ab}{Ab},$$

which is the same as found for R when the centre of moments was at b. In both cases, then, we have the same value for R, viz.: $P_1 + P_2$, or the reaction is equal to the sum of the weights, as should be the case.

We can therefore take the centre of moments anywhere in the plane of the forces we choose.

PAIR.—Two forces having different points of application, but in the same plane, equal in magnitude and parallel, and having the same direction, are called a *pair*. Thus in Fig. 13, the two equal and parallel forces, P_1, P_1, are called a pair. Suppose we take any point to the left of b, as for instance a, as a centre of moments, then we shall have for the combined moment,

Fig. 13



$$P_1 \times ac + P_1 \times ab = P_1(ac + ab) = P_1(ab + bc + ab)$$

$$= P_1(2ab + bc) = 2P_1\left(ab + \frac{bc}{2}\right).$$

If we take any point as d, to the right, as the centre of moments, we have for the combined moment

$$-P_1(cd + bd) = -P_1(2cd + bc) = -2P_1\left(cd + \frac{bc}{2}\right).$$

If we take any point as l between the forces as the centre of moments, we have for the resultant moment

$$P_1 \times cl - P_1 \times bl = P_1(cl - bl) = P_1(bc - bl - bl)$$

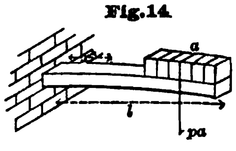
$$= P_1(bc - 2bl) = 2P_1\left(\frac{bc}{2} - bl\right).$$

We see, therefore, that wherever the centre of moments is taken, the moment of a pair is equal to the moment of the sum of the forces $2P_1 = P_2$ acting at a point midway betweeen them. A pair can therefore be replaced by a single force, P_2, equal to the sum of the two forces and parallel to them, acting at a point midway between them.

UNIFORM LOAD.—Any uniformly distributed load can be regarded as a system of pairs, symmetrically placed with reference to the centre of the load.

Thus let Fig. 14 represent a beam fixed horizontally in the wall at the left, whose length is l, and let a load of p pounds per unit of length be distributed over a distance of a units from the right end. This load is then composed of a number a of unit loads, each of which is equal to p. Consider the two extreme ones, right and left. These form a pair, and can therefore be replaced by a weight of $2p$ acting at the centre of the loaded portion. The same holds true for the next pair right and left, and so on. *The whole load can then always be replaced by the sum of all the unit loads, or the whole load, pa, applied at the centre of the loaded portion.* The moment of this force with reference to any point *not covered by the load* is the same as the moment of the load itself. Thus the moment with reference to a point distant x from the left end is, from the Fig., if the point is not covered by the load,

Fig. 14



$$pa \times \left(l - x - \frac{a}{2}\right).$$

If the point is at the left end of the load, we have $x = l - a$, and the moment is

$$pa \times \frac{a}{2} = \frac{pa^2}{2}.$$

If the point is covered by the load, or $x > l - a$, we have the loaded portion on the right equal to $l - x$, and hence the load on the right of the point equal to $p\,(l - x)$. Its lever arm is $\frac{l - x}{2}$, hence the moment of all the right hand unit weights is

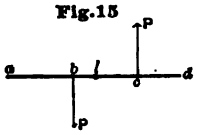
$$\frac{p\,(l - x)^2}{2}.$$

In any case, then, wherever the centre of moments, the moment of any system of uniform loads is equal to the moment of the sum of these loads when concentrated at the centre of the system.

COUPLE.—Two forces in the same plane, having different points of application, parallel and equal in magnitude, but having opposite directions, are called a couple.

Thus the two forces P, P, in Fig. 15, form a couple. If we take any point to the left, as a, as a centre of moments, we have for the resultant moment

Fig. 15



$$P \times ab - P \times ac = -P\,(ac - ab) = -P \times bc.$$

If we take any point to the right, as d, as a centre of moments, we have

$$P \times cd - P \times bd = -P\,(bd - cd) = -P \times bc.$$

If we take any point between the forces, as l, we have

$$-P \times cl - P \times bl = -P\,(cl + bl) = -P \times bc.$$

The moment, therefore, of a couple is constant, wherever the centre of moments is chosen, and equal to the product of either force into the distance between the forces.

A couple, then, can only be replaced or balanced by another couple in the same plane. The forces of the new couple may have any magnitude, provided the distance between them is so chosen that the product of either force into this distance is constant and equal to the moment of the first couple.

METHOD OF APPLICATION OF PRINCIPLES.—We have already seen (page 5) that if a truss is properly braced, and has no superfluous pieces, it is always possible to divide it at some point in some direction, such that not more than three strained pieces whose strains are necessarily unknown shall be cut. Also that the strains in the pieces cut must hold in equilibrium the outer forces acting upon either portion of the truss. According to our principle, then, the algebraic sum of the moments of the strains in the pieces and of the outer forces must be zero. Now, in any case the outer forces are always given, or they must first be found before we can attempt to determine the strains. There are, then, at most, only three unknown strains to be determined, *viz.*, the strains in the pieces cut by the section. Now as we can take the centre of moments anywhere we please, we have only to take it at the intersection of two of the pieces, and we shall have at once the moment of the strain in the other, balanced by the sum of the moments of the outer forces, because the lever arms, and therefore the moments of the other two cut pieces, will be zero.

We have thus the following rule:

Conceive at any point a section completely through the truss, cutting not more than three

pieces the strains in which are unknown. In order to find the moment of the strain in any one of these pieces, take the centre of moments at the intersection of the other two.

The moment of the strain in this piece must be equal to the algebraic sum of the moments of the outer forces acting upon either portion into which the truss is divided.

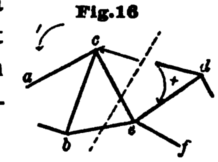
The resultant moment of rotation of the outer forces, divided by the lever arm of the piece, gives the strain in the piece.

It is evident that the section may cut more than three pieces whose strains are unknown, in fact any number, *provided all but that one in which the strain is required meet at a common point.* We have only to take this point as the centre of moments.

NOTATION.—We denote the lever arm for any piece in general by the letter l with subscripts denoting the piece. Rotation with the hands of a watch is positive, the contrary direction of rotation is negative. A compressive strain is denoted by a plus sign, a tensile by a minus sign.

Since now, the outer forces are all known, both in direction, magnitude and points of application, we can easily write down their moments in any given case with the proper signs according to the direction of rotation which they severally tend to cause. It remains only to give a rule for determining the proper sign to give to the lever arm of the piece the strain in which is required, in order that a minus sign in the result may indicate tension, and a plus sign compression.

Fig.16



PROPER SIGN FOR LEVER ARM.—Thus let Fig. 16 represent a portion of any truss subjected to the action of known outer forces, not shown in the Figure. Suppose we wish the strain in *cd*. Taking a section through *cd*, *ce* and *be*, we find the centre of moments for *cd* at *e*, the intersection of the other pieces cut.

Unless otherwise stated, we shall always consider the state of equilibrium of the *left hand one* of the two portions into which the truss is divided, and write down the algebraic sum of the moments of the outer forces acting upon *this left hand portion.*

We have, then,

$$cd \times \text{lever arm for } cd + \left\{ \begin{array}{l} \text{Algebraic sum of moments of the} \\ \text{outer forces acting upon the left} \\ \text{hand portion} \end{array} \right\} = 0.$$

Now in order to always insert the lever arm with its proper sign, we have the following rule:

Whatever the inclination of the cut piece the strain in which is desired, consider yourself standing upon the piece at the cut end, facing the apex of the left hand portion from whence the piece radiates. If then the centre of moments lies upon the left hand, the lever arm is minus. If upon the right hand, the sign of the lever arm is plus.

Or, put an arrow on the cut piece pointing away from the section and towards the left hand portion of the truss. The moment of the strain in the piece will have the same sign as the rotation indicated by this arrow.

Thus in Fig. 16, for *cd* we have the lever arm l_{cd} minus, because the centre of moments e lies on the left when we face the apex c; hence,

$$cd \times -l_{cd} + \left\{ \begin{array}{l} \text{Algebraic sum of moments of outer} \\ \text{forces acting upon the left hand por-} \\ \text{tion} \end{array} \right\} = 0.$$

Let, for instance, the resultant moment of the outer forces upon the left hand portion be minus. Then it acts to cause negative rotation, as shown by the arrow in Fig. 16, from right to left. Now we may consider the left hand portion, $a\ c\ e\ b$, as a bent lever with its fulcrum at e. The outer forces cause revolution from right to left, or negative rotation.

This rotation is resisted by the strain in cd acting at c. This strain must, therefore, act *away* from the fixed apex c, as shown by the arrow in the Figure, and must, therefore, be tensile.

This is precisely the same result as we should get by our rule, and the sign of the strain in cd would come out minus.

Thus,

$$cd \times - l_{cd} + \text{algebraic sum of moments} = 0.$$

But in the case supposed, the algebraic sum of the moments of the outer forces acting on the left hand portion is minus; hence,

$$- cd \times l_{cd} = + \text{sum of moments},$$

where the right side of the equation is essentially plus. We have, therefore,

$$cd = - \frac{\text{moment}}{l_{cd}}.$$

As the result is minus it denotes tension. If we observe, therefore, the above rule and notation, the signs of the strains will take care of themselves, and will denote the character of the strains.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

Let us choose as an example to illustrate these points, the same truss which we have already become familiar with in the preceding chapters, represented in Fig. 7.*

APPLICATION TO A ROOF TRUSS.

LEVER ARMS.—It is necessary first to find the lever arms of the various pieces. This in any case is a simple question of trigonometry. The lever arms for the upper bays, Fig. 7, are evidently the perpendiculars drawn to those bays from each opposite lower apex. For the lower bays we have the perpendiculars let fall upon these bays from each opposite upper apex. For each brace, the lever arm is the perpendicular to the direction of the brace drawn through the left end A, where rafter and tie intersect. This will be evident by considering sections through the truss and applying our rule.

Thus suppose a section cutting Cb, bc and Lc, as indicated by the broken line, or Dd, dc and Lc. Then by our rule, the point of moments for Lc is the apex CD, the point of intersection of the other two pieces. For Cb, it is the second lower apex. For bc, it is the apex AB, or the left end of the truss. The bays Ba and Cb have evidently the same lever arm.

If we pass a section through Ef, ff', $f'e'$ and Le' it cuts, to be sure, more than three braces. The strain in $f'e'$ can, however, be easily found, since it is equal to ef by reason of the symmetry of the frame and loading. If this were not the case we could easily find it by working toward it from the right end. The intersection of the unknown pieces Ef and Le' is at the left end, and this is therefore the centre of moments for ff'.

We can easily find, then, the lever arms for the various pieces by simple trigonometrical computation.

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* The student will find the method of moments of this chapter applied in detail to Bridges and Roofs of various kinds in "*Dach und Brücken-Constructionen*," by *A. Ritter*, Hanover, 1873, a translation of which, under the title of "*Elementary Theory and Calculation of Iron Bridges and Roofs*," by *H. R. Sankey*, has been published by E. & F. N. Spon.

It is unnecessary to explain this work in detail. The lever arms thus computed are as follows:

For the lower bays,

	La	Lc	Le
lever arms =	3.125	6.25	9.375 ft.

For the upper bays,

	Ba	Cb	Dd	Ef
lever arms =	3.727	3.727	7.454	11.151 ft.

For the braces,

	ab	bc	cd	dc	ef	ff'
lever arms =	6.934	6.934	13.869	16.2	20.803	25 ft.

Length of each lower bay = $8\frac{1}{3}$ ft.

Horizontal projection of each upper bay = 6.25 ft.

CALCULATION.—Let us first calculate the lower bays. Conceive La cut.* The centre of moments is then at the apex BC, Fig. 7. Let R be the reaction at the left end.

Then,

$$R \times l_R + La \times l_{La} = 0 \quad \ldots \quad (1)$$

Inserting numerical values, and having regard to our notation and rule for sign of lever arm, we have, since the rotation due to R is positive and the lever arm of La is positive according to our rule, because the centre of moments is on the right hand as we look towards the apex A,

$$+ 2800 \times 6.25 + La \times 3.125 = 0.$$

Hence,

$$La = -\frac{2800 \times 6.25}{3.125} = -5600 \text{ lbs.}$$

La is therefore in tension.

For Lc we have by our rule, page 27, the centre of moments at the apex CD, whether we pass a section cutting Cb, bc and Lc, or Dd, cd and Lc.

We have for the general equation of equilibrium,

$$R \times l_R + W_2 \times l_{W_2} + Lc \times l_{Lc} = 0. \quad \ldots \quad (2)$$

As the centre of moments is on the right of Lc, according to our rule, page 27, it is plus.

Inserting numerical values, and having regard to the signs for positive and negative rotation, we have, since R causes positive rotation and W_2 negative,

$$2800 \times 12.5 - 800 + 6.25 + Lc \times 6.25 = 0.$$

Hence,

$$Lc = \frac{-2800 \times 12.5 + 800 \times 6.25}{6.25} = -4800 \text{ lbs.}$$

Lc is therefore also in tension.

---

* Let the section cut La, ab and Cb. Of these three pieces the two not desired meet at the apex BC. This, therefore, is our centre of moments for La. For Ba, in like manner, take a section through Ba, ab, bc and Lc.

For Le we have, in like manner,

$$R \times l_R + W_2 \times l_{W_2} + W_3 \times l_{W_3} + Le \times l_{Le} = 0. \quad \ldots \quad (3)$$

Inserting numerical values,

$$2800 \times 18.75 - 800 \times 12.5 - 800 \times 6.25 + Le \times 9.375 = 0.$$

Hence,

$$Le = \frac{-2800 \times 18.75 + 800 \times 12.5 + 800 \times 6.25}{9.375} = -4000 \text{ lbs.}$$

Let us now calculate the upper bays.

For the bay Ba, the centre of moments is at the first lower apex. The general equation is

$$R \times l_R + Ba \times l_{Ba} = 0. \quad \ldots \quad (4)$$

According to our rule, the lever arm for Ba is minus, because looking towards the apex A, the centre of moments lies on our left.

Inserting numerical values,

$$2800 \times 8.33 - Ba \times 3.727 = 0.$$

Hence,

$$Ba = \frac{2800 \times 8.33}{3.727} = +6260 \text{ lbs.}$$

Ba is therefore in compression.

For the bay Cb, we have the same point of moments; but when we pass a section through Cb, ab and La, the weight W_2 acts upon the left hand portion also, as well as R, Hence,

$$R \times l_R + W_2 \times l_{W_2} + Cb \times l_{Cb} = 0. \quad \ldots \quad (5)$$

Inserting numerical values, we have, since R causes positive rotation and W_2 negative, and since the lever arm for Cb is negative,

$$2800 \times 8.33 - 800 \times 2.08 - Cb \times 3.727 = 0.$$

Hence,

$$Cb = \frac{2800 \times 8.33 - 800 \times 2.08}{3.727} = +5813 \text{ lbs.}$$

For the bay Dd, the centre of moments is at the second lower apex. The lever arm, according to our rule, is minus. The general equation is

$$R \times l_R + W_2 \times l_{W_2} + W_3 \times l_{W_3} + Dd \times l_{Dd} = 0. \quad \ldots \quad (6)$$

Inserting numerical values,

$$2800 \times 16.66 - 800 \times 10.416 - 800 \times 4.166 - Dd \times 7.454 = 0.$$

Hence,

$$Dd = \frac{2800 \times 16.66 - 800 \times 10.416 - 800 \times 4.166}{7.454} = +4695 \text{ lbs.}$$

For the bay Ef, we have the centre of moments at the centre of the lower tie. The lever arm is minus according to rule. We have, then,

$$R \times l_R + W_2 \times l_{W_2} + W_3 \times l_{W_3} + W_4 \times l_{W_4} + Ef \times l_{Ef} = 0. \quad \ldots \quad (7)$$

Inserting numerical values,

$$2800 \times 25 - 800 \times 18.75 - 800 \times 12.5 - 800 \times 6.25 - Ef \times 11.151 = 0.$$

Hence,

$$Ef = \frac{+2800 \times 25 - 800 \times 18.75 - 800 \times 12.5 - 800 \times 6.25}{11.151} = +3587.$$

Let us now calculate the strains in the braces. For the brace *ab*, and indeed for all the braces, the centre of moments, according to our rule, is at the left end. The lever arm for *ab* is minus according to rule. The general formula is

$$W_2 \times l_{W_2} + ab \times l_{ab} = 0. \qquad (8)$$

Inserting numerical values, we have, since W_2 tends to cause positive rotation, and the lever arm of *ab* is minus,

$$800 \times 6.25 - ab \times 6.934 = 0.$$

Hence,

$$ab = \frac{800 \times 6.25}{6.934} = +721 \text{ lbs.}$$

For the brace *bc*, we have, according to rule, the lever arm plus, because it lies on the right when we face the apex on the left from which it radiates. We have, for the general formula,

$$W_2 \times l_{W_2} + bc \times l_{bc} = 0. \qquad (9)$$

Inserting numerical values,

$$+800 \times 6.25 + bc \times 6.934 = 0.$$

Hence,

$$bc = \frac{-800 \times 6.25}{6.934} = -721 \text{ lbs.}$$

For the brace *cd*, the lever arm is minus, and we have,

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + cd \times l_{cd} = 0. \qquad (10)$$

Inserting numerical values,

$$+800 \times 6.25 + 800 \times 12.5 - cd \times 13.869 = 0.$$

Hence,

$$cd = \frac{+800 \times 6.25 + 800 \times 12.5}{13.869} = +1081 \text{ lbs.}$$

For the brace *de*, in like manner, the lever arm is plus. We have then,

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + de \times l_{de} = 0. \qquad (11)$$

Inserting numerical values,

$$+800 \times 6.25 + 800 \times 12.5 + de \times 16.2 = 0.$$

Hence,

$$de = \frac{-800 \times 6.25 - 800 \times 12.5}{16.2} = -926 \text{ lbs.}$$

For the brace *ef*, the lever arm according to rule is minus. We have,

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + W_4 \times l_{W_4} + ef \times l_{ef} = 0. \qquad (12)$$

Inserting numerical values,

$$+ 800 \times 6.25 + 800 \times 12.5 + 800 \times 18.75 - ef \times 20.803 = 0.$$

Hence

$$ef = \frac{800 \times 6.25 + 800 \times 12.5 + 800 \times 18.75}{20.803} = + 1442.$$

For the brace ff' we pass a section cutting Ef, ff', $f'e'$, and Le'. Since the point of moments is on the right, according to our rule, the lever arm for ff' is plus. The lever arm for $f'e'$ is also plus. The strain in $f'e'$ is, by reason of the symmetry of frame and loading, equal to that already found for ef. We have then

$$W_2 \times l_{W_2} + W_3 \times l_{W_3} + W_4 \times l_{W_4} + f'e' \times l_{f'e'} + ff' \times l_{ff'} = 0 \quad . \quad . \quad . \quad . \quad (13)$$

Inserting numerical values,

$$800 \times 6.25 + 800 \times 12.5 + 800 \times 18.75 + 1442 \times 20.803 + ff' \times 25 = 0$$

Hence,

$$ff' = -\frac{800 \times 6.25 + 800 \times 12.5 + 800 \times 18.75 + 1442 \times 20.803}{25} = - 2400 \text{ lbs.}$$

REMARKS.—These results compare favorably with those found for the same case in the two preceding chapters. The student will do well to select another example and compute it thoroughly, according to our method, paying special attention to the rules for determining the centres of moments and the signs for the lever arms, and checking his results by the method of Chapter I. Only in such way can he obtain mastery of the method. He would do well also to remember that time cannot be better spent than in getting familiar with the *principles* in these first four chapters. When we pass to applications in the second section, he will then find no difficulty in following the text, and will not be confused by the special details peculiar to different structures.

COMPARISON OF METHODS.—Much use will be made of the present method in this work. We shall call it hereafter the "*method of sections.*" We see that it is general in its application to all properly braced structures—that is, all framed structures which have no superfluous pieces. As compared with the analytic method by resolution of forces, of the preceding chapter, it will be seen that its application in the case chosen is much simpler and involves much less calculation. Still, for trusses in which the pieces have various inclinations, all different, the computation of the lever arm is tedious, and the graphic method of Chapter I. commends itself as specially adapted to such cases. Indeed it is the special advantage of the graphic method, that it is entirely unaffected by irregularities of form and loading which necessitate much calculation by the other methods.

The present method can, however, in all cases, be used as a *check* upon the accuracy of the results obtained by the graphic method, to great advantage, inasmuch as it gives the strain in any piece without reference to any others of the frame.

Thus in the example Fig. 7, after having found all the strains by the graphic method, as shown in Fig. 7 (*a*), we can compute the strain *in the last piece* of that Figure, *Le*, by the present method of moments. If this is found to agree with the strain given by the graphic method, we may have confidence in the accuracy of all the others, because any error would have been carried along from piece to piece, and would have showed itself in the last.

QUESTIONS FOR EXAMINATION.

What is meant by the moment of a force? Define lever arm. Centre of moments. Illustrate. What is the fundamental principle upon which the method of sections is based? What is the significance of a moment? Illustrate. Prove that for equilibrium the moments are inversely as their lever arms. Show that the position of the centre of moments is indifferent for equilibrium. Illustrate the application of the principle to a frame. Give the rule for finding in such case the centre of moments for any piece. If the moment of a piece is known, how can you find the strain? What sign denotes compression? Tension? In what direction is positive rotation? Negative? Illustrate. Give the rule for determining the proper sign for the lever arm of a piece. Illustrate. In Fig. 7, point out the lever arm for the various pieces. Show how they may be calculated. State the signs which these lever arms ought to have for the various pieces. Write down the general equation for equilibrium of any upper bay, and point out what sign each term should have when numerical values are introduced. Do the same for any lower bay. For any brace. Show how to find the strain in the brace ff'. Write the general equations for *Ba* and *Cb*. For *La*. Show what signs each term ought to have when numerical values are introduced. What can you say of the present method as compared with those of the two preceding chapters? What is a *pair?* A couple? By what can a pair be replaced? A couple? Show how to find the moment at any point for a uniform loading.

3

CHAPTER IV.

METHOD OF MOMENTS.—GRAPHIC SOLUTION.

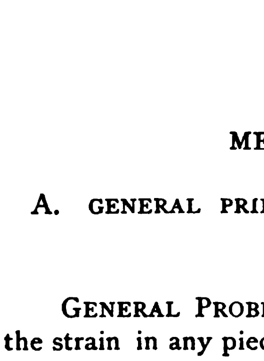
A. GENERAL PRINCIPLES. FORCES IN THE SAME PLANE, DIFFERENT POINTS OF APPLICATION.

GENERAL PROBLEM.—We have seen in the preceding chapter that in order to find the strain in any piece of a framed structure we have simply to divide the algebraic sum of the moments of all the outer forces by the lever arm for this piece. The centre of moments for both piece and outer forces is at the intersection of the other pieces cut by an imaginary section which completely divides the truss into two portions and cuts the piece the strain in which is required. The outer forces acting upon ***the left hand portion*** of the truss are alone considered.

We see, then, that in any case, the problem to be solved is,—What are the moments of these outer forces? If the algebraic sum of these is once found, we have only to divide by the lever arm of the piece in order to find its strain. The object of the present chapter, therefore, is to deduce a ***graphic method for finding the algebraic sum of the moments of the outer forces.***

POSITION OF RESULTANT.—Suppose we have any number of forces, Fig. 17, given in direction and magnitude, and acting at different points of application in the same plane.

Fig. 17



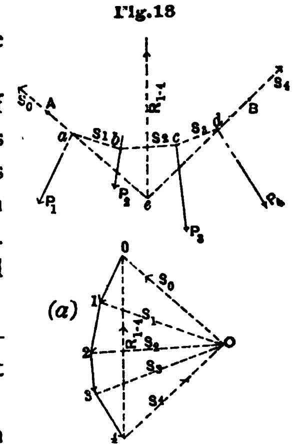
If we lay these forces off to scale, the one after the other, and thus form the force polygon (*a*), the line necessary to close this polygon will be, as in Chapter I, the resultant to scale, and given in direction. But we do not know whereabouts in the plane of the forces, in Fig. 17, this resultant should act.

In the present case a ready method suggests itself at once. Thus we can consider P_1 as acting at any point in its line of direction, and so also for P_2. The resultant of P_1 and P_2, then, we can consider as acting at the intersection *a* of P_1 and P_2, prolonged if necessary. But the resultant of P_1 and P_2 is given in the force polygon (*a*) in direction and magnitude by the diagonal 02, because that diagonal closes the polygon commenced by the forces P_1 and P_2. At *a* then, parallel to 02 below, we can draw a line representing the direction of the resultant of P_1 and P_2, and produce it till it meets P_3, prolonged if necessary, at *b*. At *b* we can consider the resultant of $R_{1\text{-}2}$ and P_3 acting, or the resultant of P_1, P_2 and P_3. But 03 in the force polygon (*a*) below gives this resultant in direction and magnitude. Parallel to 03 then draw a line through *b* till it meets P_4, prolonged if necessary, at *c*.

Thus *c* is a point in the plane of the forces through which the direction of the resultant passes. In the force polygon (*a*), 04 is this resultant in direction and magnitude.

Parallel to 04 draw a line through c, and it will represent the resultant in proper position and direction. This resultant, taken as acting in the direction obtained by following around the force polygon in the direction of the forces, or in the direction from 4 to 0, as shown by the arrow, will hold the forces in equilibrium.

Fig. 18



If in the opposite direction, it will replace the forces (see page 10).

THE PRECEDING METHOD NOT GENERAL.—This method of finding the position of the resultant, though sufficiently obvious, is evidently not general in its application. Thus, suppose the forces were all parallel or inclined so slightly as not to intersect within the limits of the drawing. In such case the method would fail. It is necessary, therefore to find some method which shall avoid this difficulty.

GENERAL METHOD FOR FINDING POSITION OF RESULTANT.—In Fig. 18 we have four forces given: required to find the resultant in direction, magnitude and position.

We shall evidently find the first two from the force polygon as before. Thus lay off the forces to scale in Fig. 18 (a), one after the other, and we shall have the resultant given in magnitude and direction by the closing line 04. If this resultant acts in the direction from 4 to 0, obtained by following around the polygon in the direction of the forces, it will hold the forces in equilibrium. In the opposite direction it will replace the forces.

It remains to determine the *position* of the resultant in the plane of the forces.

For this purpose we choose a point O at any convenient point, and draw the lines O0 and O4. This point thus chosen we shall hereafter call a "*pole.*" Now, since every line in the force polygon represents a force, by thus choosing a pole and drawing lines to the extremities of the resultant, *we have resolved the resultant into* the *two forces*, O0 *and* O4. This is evident from the fact that these two lines close the polygon, and hence, taken as acting from 4 to O and O to 0, as shown by the arrows, hold the forces P_1, P_2, P_3, P_4 in equilibrium. But these same lines make a closed polygon with 04, and taken in the direction shown by the arrows, *replace* the resultant when acting in the direction necessary for equilibrium. As the pole O can be taken anywhere, we can thus resolve the resultant into any two directions we wish.

Let us then consider the resultant as *replaced* by the two forces O0 and O4. Anywhere in the plane of the forces above, Fig. 18, draw a line S_0 parallel to O0, and produce it till it meets P_1, produced if necessary, at a. The resultant of S_0 and P_1 will pass through a and be parallel to S_1, in the force polygon, since S_1 in the force polygon is the resultant of P_1 and S_0, given in direction and magnitude. Through a then draw a line parallel to S_1 and produce to intersection b with P_2. The line S_2 in the force polygon is the resultant of S_0, P_1, and P_2. Parallel to this line draw S_2 through b above, and produce to intersection c with P_3. The point c will be the point where the resultant of S_0, P_1, P_2, and P_3 should act. The force polygon gives the direction of this resultant as well as its magnitude. It is S_3. Parallel to this draw S_3 above, and produce to intersection d with P_4. Finally through d, draw a line S_4 parallel with S_4 in the force polygon.

Proceeding in this manner, we thus find for any assumed position of S_0 in the plane of the forces, the proper corresponding position for S_4. Since now, S_0 and S_4 are components of the resultant, and each may be considered as acting at any point in its line of direction, we have only to prolong them and *their intersection gives a point through which the resultant must act.* Through the point e, therefore, draw a line parallel to the

line 40 in the force polygon, and it will represent the resultant in proper direction and position. Acting as shown by the arrow it causes equilibrium. The magnitude of the resultant is given to scale in the force polygon.

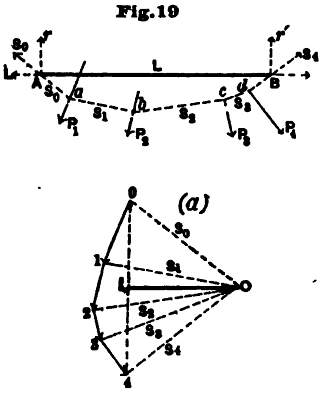
POSITION OF POLE AND OF S_0 INDIFFERENT.—A little inspection will make it apparent that our method is general, no matter where in the plane of the forces we take S_0 as acting, that is no matter where the point a is taken. Thus if a had any other position upon the direction of P_1, if everything else remained unchanged, we should evidently obtain a polygon every side of which would be parallel to that shown in Fig. 18. The new S_4 would then be parallel to that line in the present Figure as also S_0. Their intersection would, therefore, lie in a point upon the direction of the resultant as drawn.

Also any other position of pole would give different directions for the lines S_0, S_1, S_2, etc, but the intersection e of the end lines would still lie in the same line.

POLE, EQUILIBRIUM POLYGON, CLOSING LINE, STRINGS.—The point O we call the "*pole*" in the force polygon. It may be taken where we please. The polygon *abcd* above we call the "*equilibrium polygon.*" In the present case it is evidently the shape a string would take if suspended at any two points, as A and B, in S_0 and S_4 respectively. The strains in the string would be tensile. We denote these strains $O0$, $O1$, $O2$, etc., by S_0, S_1, S_2, etc., and call them "*strings.*" In general, forces may act up as well as down, in which case some of the strings might represent compressive strains, and our polygon above would contain struts as well as strings.

Let us suppose, in Fig. 19, that we take any two points, as A and B, upon the strings S_0 and S_4, and suppose them to be made fixed. The force S_0 acting at A we shall then have to *replace* by two forces, one parallel to the resultant, and one through AB. So also for S_4. The sum of the two components parallel to the resultant must be equal and opposite to the resultant, and the component in the direction AB must be resisted in the present case by a strut or compressive piece AB. This resolution we can make at once, by drawing through O in the force polygon a line parallel to AB. The line AB we call the "*closing line.*" Thus we see from Fig. 19 (a) that the sum of the components $4L$ and Lo equals the resultant $o4$.

Fig. 19



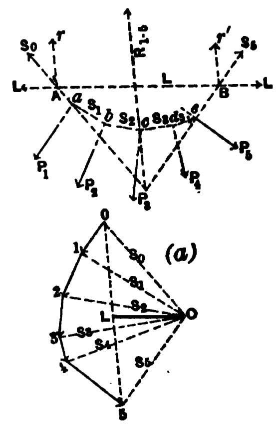
In any case then we can fix any two points of the polygon, as A, B, by drawing the closing line. A line through O parallel to this in the force polygon gives the components into which S_0 and S_4 are resolved. We must consider, then, the entire polygon $AabcdB$, with its closing line, as a *frame in equilibrium*, and can apply to it the principles of Chapter I. Thus take the apex A. Here we have the force r in equilibrium with the strains in AB and Aa. Following round in the force polygon from L to o and so around, we find by our rule, page 13, Aa in tension and L in compression. So also for the other end B, we find Bd in tension and L in compression. The components r and r' act opposed to the resultant which replaces the forces, and the forces at A and B parallel to L are equal and opposite, hence there is no motion of the entire frame in any direction.

RECAPITULATION; FORCE AND EQUILIBRIUM POLYGON FOR ANY NUMBER OF FORCES IN A PLANE.—Suppose then we have any number of forces, as $P_1 \ldots P_5$, Fig. 20. Our method is as follows:

1st. Form the force polygon, Fig. 20 (a), by laying off the forces to scale, one after the other in any order. The line 05 which closes the polygon is the resultant in magnitude

and direction. When it acts in the direction from 5 to 0, obtained by following round in the direction of the forces, it will cause equilibrium. In the opposite direction it will replace the forces.

Fig. 20



2d. Choose a pole O at any convenient point, and draw the strings S_0, S_1 . . . S_5.

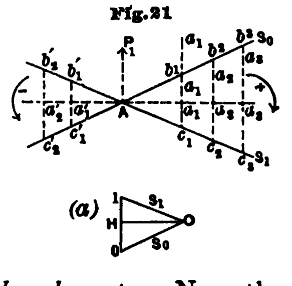
3d. Form the equilibrium polygon by drawing anywhere in the plane of the forces a line parallel to S_0 until it meets P_1, prolonged if necessary, at a. From a a line parallel to S_1 till it meets P_2 at b. From b, a line parallel to S_2 till it meets P_3 at c. From c a line parallel to S_3 till it meets P_4 at d. From d a line parallel to S_4 till it meets P_5 at e. From e a line parallel to S_5. The first and last strings of this polygon intersect at a point upon the resultant. Moreover, *any two strings, as ab and cd, intersect at a point upon the resultant for the forces P_2 and P_3 acting between these strings.* The intersection of ab and de gives thus a point upon the resultant of P_2, P_3 and P_4. These resultants may be found in magnitude and direction from the force polygon (a).

4th. Fix any two points in the extreme strings of the polygon by drawing the closing line AB. Resolve S_0 and S_5 into forces r and L, and r' and L, respectively parallel to the direction of the resultant and closing line. This is at once done by drawing the line OL in the force polygon parallel to AB. Then OL is the force to scale, acting at each end of the closing line AB, and Lo is the component r, and $5L$ the component r'. If these forces are to *replace* S_0 and S_5, they must act as shown by the arrows, in directions opposite to those obtained by following round in the direction of the forces in the force polygon. Thus S_0 acts from O to 0 for equilibrium. Following round, we obtain, then, r acting from L to 0, and L acting from O to L, as the directions necessary to *replace* S_0. In the same way we find, since S_5 acts from 5 to O for equilibrium, $5L$ and LO as the directions for r' and L at the right end of the closing line.

5th. Conceive now the forces S_0 and S_5 removed, and *replace* them at the points A and B, by r, L, and r' and L, and we have a frame-work, $AabcdeB$, which, acted upon by the forces P_1 . . . P_5 and r, L, r' and L, is in equilibrium. Applying the principles of Chapter I. to the apex A, where we have r, L, and the strain in Aa in equilibrium, we find the strain in AB in this case, compression. So also for the apex B. The strains in all the strings are tensile in this case. The magnitude of these strains can be found to scale from the force polygon (a).

CULMANN'S PRINCIPLE.—Suppose we have a single force P_1, Fig. 21. The force polygon (a) becomes a straight line equal by scale to P_1. Let us choose a pole O anywhere, and draw the strings S_0 and S_1. This is the same thing as resolving the force P_1 into two components parallel to S_0 and S_1. These components are given in direction and magnitude in the force polygon (a). Parallel to them draw lines S_0, S_1, through the point of application A of the force P_1.

Fig. 21



Now draw from the pole O a line OH perpendicular to 01. This distance OH, we call the "*pole distance.*"

In the plane of the forces take any point whatever having any position, as a_1, or a_2, and draw through this point the ordinates b_1c_1, b_2c_2, etc. Now the moment of P_1 with reference to any point, as a_1, is $P_1 \times Aa_1$. But referring to the force polygon (a), we have by similar triangles,

$$P_1 : H :: b_1c_1 : Aa_1.$$

Hence,

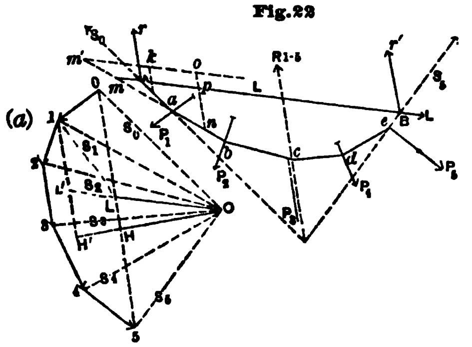
$$P_1 \times Aa_1 = H \times b_1c_1.$$

That is, *the moment of the force P_1, with reference to any point, is equal to the ordinate through this point, parallel to P_1, included by the two components into which P_1 is resolved, multiplied by the pole distance in the force polygon.*

This principle we call Culmann's principle. It has, as we shall see, direct application to the equilibrium polygon. For any point situated to the left of P_1, the moment is by convention minus. For any point to the right it is plus, provided the force P_1 acts upward as represented. If it acted downward, the moments to the right would be minus, and those to the left plus.

GRAPHIC REPRESENTATION OF MOMENTS OF ANY NUMBER OF GIVEN FORCES.—SIGNIFICANCE OF EQUILIBRIUM POLYGON.*—We are now able to solve the problem proposed in the beginning of this chapter, and find graphically the moments of any number of forces. Thus, suppose we have any number of forces, P_1 . . . P_5, Fig. 22.

Fig. 22



1st. Form the force polygon (a); choose a pole O, and draw the strings, S_0, S_1 . . . S_5.

2d. Form the equilibrium polygon, produce the two end strings, draw the direction of the resultant, and draw a closing line L anywhere, as AB.

3d. Parallel to AB draw OL in the force polygon (a). It will be to scale the strain in the closing line, and the segments into which it divides the resultant, viz., Lo, and $5L$ will be the forces r and r' at AB, parallel to the resultant. Draw OH perpendicular to the resultant.

Let us suppose the upward force r resolved in the directions AB and S_0. The force polygon gives at once this resolution. Referring then to Culmann's principle, preceding, we see the moment of r for any point k in the ordinate parallel to the resultant is equal to the pole distance OH multiplied by the ordinate through k, included by AB and S_0. Since the pole distance OH is constant, *this ordinate is itself proportional to the moment.*

The pole distance must of course be measured to the scale of force in the force polygon, since every line there represents a force. The ordinate must be measured to the scale of distance assumed in Fig. 22.

The ordinate, then, included between AB and S_0, is proportional to the moment of r for any point whatever, through which this ordinate passes. This ordinate to the scale of length, multiplied by the projection of S_0 in a direction perpendicular to the resultant, or OH taken to the scale of force, gives the moment of r for any point through which the ordinate passes.

Suppose we wish the *combined* moment at a point p upon the closing line of all the forces left or right of this point. Since we may regard the polygon $AabcdeB$ as a frame, the strains in which hold the outer forces in equilibrium, then according to the principles of the preceding chapter, if we suppose a section cutting AB and ab, and take the point of moments at the intersection m, the moments of the strains in AB and ab with reference to this point will be zero. Hence the moment of the outer forces r and P_1, with respect to this point, must be zero also, since there is equilibrium. But since the combined moment of any number of forces is equal to the moment of the resultant, the moment of the resultant of r and P_1 with respect to m must be zero. The resultant of r and P_1 must therefore pass through m. This is in accordance with the property of the equilibrium polygon already

* The student may here omit to "Application to Parallel Forces," page 40, if he finds what follows to be at first difficult of comprehension.

enunciated, that the intersection of any two of its lines is the point of application for the resultant of the forces at the apices between those lines. Thus the intersection *m* of *AB* and *ab* is the point of application for the resultant of *r* and P_1. This resultant is given in the force polygon by $L1$. (The student should join L and 1 by a line.)

Suppose this force acting at *m* to be resolved into a force parallel to the resultant and a force along the closing line. These components are given in the force polygon by $L'1$ and $L'L$, respectively. The moment of the latter is zero, because it passes through the point *p*. The moment of the former is, by Culmann's principle, equal to the ordinate *pn* multiplied by the pole distance, or the perpendicular from O upon $L'1$. That is, by the projection of S_1 in a direction perpendicular to the resultant, or by the distance OH'.

If we should take the point *o* as a centre of moments, the moment of $L'L$ would not be zero, unless we should prolong *ab* to intersection m' with a line through *o* parallel to L. If we consider the force $L1$ acting at *this* point, and resolve it as before, we should have the moment equal to the ordinate *on* multiplied by the same distance, OH', in the force polygon.

We can proceed in similar manner for any other point upon the closing line *AB*. We have thus learned two important properties of the equilibrium polygon:

1*st*. *The intersection of any two of its lines gives a point upon the resultant of the intermediate forces.*

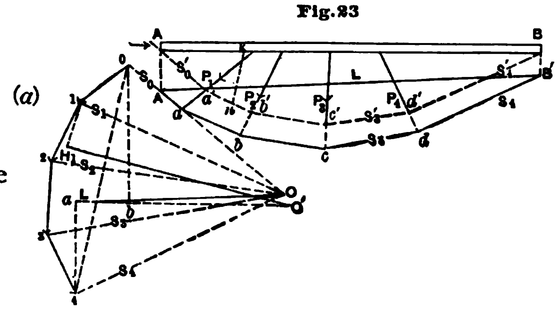
2*d*. *The ordinate parallel to the resultant of all the forces, multiplied by the projection in the force polygon perpendicular to this resultant of the string cut by this ordinate, gives the combined moment of all the outer forces, right or left, with respect to that point on the closing line through which the ordinate passes.**

Ordinates must always be measured to the scale of length, and the projection in the force polygon to the scale of force.

POSITION OF POLE A MATTER OF INDIFFERENCE.—It evidently makes no difference where the pole is taken. If, for instance, in Fig. 22, the pole were in the same line *OH*, but at a distance of $2OH$, the ordinates in the equilibrium polygon would only be one half as great, and hence the product would remain unchanged.

APPLICATION TO A BEAM.—Let *AB*, Fig. 23, be a beam or other rigid body, acted upon by forces P_1 . . P_4, and holding these forces in equilibrium. Required to find the reactions at the supports and the combined moment of all the forces right or left of any point, with reference to that point.

Fig. 23



1st. Form the force polygon (*a*). Choose a pole O and draw $S_0, \ . \ . \ . \ S_4$.

2d. Form the equilibrium polygon *AabcdB*, and draw the closing line $A'B'$, thus fixing the points *vertically beneath the ends of the beam*. Then *Lo* is the reaction at the left end *A*, and $4L$ is the reaction at the right end *B*. If the beam rests upon supports upon which the pressure is vertical only, then *bo* and $4a$ are these vertical reactions, and *aL* and *Lb* are the horizontal forces of displacement which must be resisted by a horizontal outer force *ab* equal to their sum, at the right end.

Now to find the moment at any point of the beam we must first make the closing line parallel to the beam, so that any point on the closing line will be a point on the beam also.

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* There are many other interesting and important properties of the equilibrium polygon, which may be found in "Elements of Graphical Statics," DuBois—Wiley & Sons. Upon these properties the entire science of graphic statics is based. The above are, however, all of which we shall need to make use in this work.

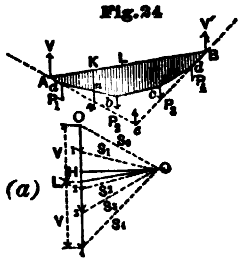
This we can easily do. Thus in the force polygon (*a*), the reactions Lo and $4L$ being known, take the pole anywhere in the line LO' parallel to the beam, and draw S'_0, S'_1, etc. With this new pole O', we can now form a new equilibrium polygon a', b', c', etc., the closing line of which will correspond with the beam if we draw the first string S'_0 through the left end A.

Then according to our principles, the moment at any point k of the beam is equal to the ordinate kn parallel to the resultant, multiplied by $O'H'$ or by the component perpendicular to the resultant of the string S'_1. So for any other point.

APPLICATION TO PARALLEL FORCES.—The outer forces acting upon framed structures are generally weights and the reactions of supports due to these weights, and therefore in a majority of practical cases, it is required to investigate a system of parallel forces.

Suppose we have a number of parallel forces, $P_1 \,.\,.\, P_4$, Fig. 24.

Fig. 24



1st. Form the force polygon (*a*). This becomes in this case a straight line 04.

2d. Choose a pole O and draw S_0, $S_1 \,.\,.\, S_4$.

3d. Form the equilibrium polygon *abcd*.

4th. Fix any two points, as A and B, by drawing the closing line AB. Parallel to AB in the force polygon draw the line L. Then Lo and $4L$ are the upward reactions which must be applied at A and B to produce equilibrium. Their sum is equal to the resultant, as should be.

Now the resultant will act at *e*, and be parallel to the forces and equal to their sum. The pole distance is the perpendicular from the pole O upon the direction of the forces. The projection of each of the strings S_0, S_1, etc., in this direction is constant and equal to the pole distance OH.

From Culmann's principle, the moment at any point K of the reaction V, is therefore the ordinate Km measured parallel to the resultant, multiplied by the pole distance H. But according to the same principle, the moment of the force P_1 is equal to the ordinate $nm \times H$. Hence, the combined moment, since V acts up and P_1 down, is $H\,(Km - nm) = H \times Kn$.

For parallel forces then, any *ordinate of the equilibrium polygon parallel to the resultant is directly proportional to the algebraic sum of the moments of the forces on one side of the section with reference to any point in that ordinate, whether on the line AB or not.*

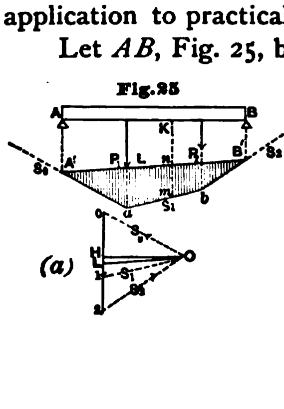
The moment itself is equal to the ordinate to the scale of length, multiplied by the pole distance to the scale of force.

We see, also, that *the ordinate included between any two lines of the equilibrium polygon prolonged, gives in the same way the moment of the force at their intersection, with reference to any point on that ordinate.* Thus mn multiplied by H, gives the moment of P_1 with reference to any point on Km, as K, or n, or m.

EXAMPLE 1.—A few examples will make the above principles clear and show their application to practical problems.

Let AB, Fig. 25, be a beam subjected to two unequal weights P_1 and P_2 applied at any two points. Required the reaction at the supports A and B, also the moment at any point of all the forces right or left of that point, when equilibrium exists.

Fig. 25



1st. Form the force polygon (*a*).

2d. Choose a pole O, and draw the strings S_0, S_1 and S_2, and the pole distance H.

3d. Construct the equilibrium polygon by drawing a line parallel to S_0 till it meets P_1, produced if necessary, at *a*. From *a*

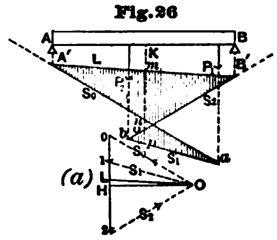
a line parallel to S_1 till it meets P_2 at b. From b draw a parallel to S_2, and prolong it indefinitely. Drop verticals from the ends A and B of the beam, and draw the closing line $A'B'$. Parallel to $A'B'$ draw OL in the force polygon.

Then Lo and $2L$ are the reactions at the ends A and B, and acting upwards they hold the weights in equilibrium. The supports should, therefore, be below the beam at each end.

The moment at any point K of the beam is equal to the ordinate nm, multiplied by the pole distance H.

EXAMPLE 2.—It is well to observe that the order in which the forces are taken, makes no difference as to the results, although the Figure obtained may be very different.

Fig. 26



Thus let us take the same example as before, but number the forces in inverse order.

We form the force polygon as before, choose a pole and draw S_0, S_1 and S_2. Now parallel to S_0 we must draw a line *till it meets* P_1 at a. [Note that S_0 must *always* be prolonged to intersection with P_1.] Then from a a parallel to S_1 till it meets P_2 at b. Then from b a parallel to S_2. Draw the closing line $A'B'$. A parallel to it in (a) gives the reactions Lo and $2L$ as before. Since S_0 acts from O to o for equilibrium, Lo must act up to *replace* it. Hence the support at A is below. So also for $2L$. Since S_2 acts from 2 towards O for equilibrium, $2L$ must act up to *replace* it, and the support at B should be below also.

In general, always take the directions of the reactions *opposed* to the directions of S_0 and S_n for equilibrium, obtained by following round the force polygon.

As to the moments, we see that the moment of the left reaction with reference to any point, as K, is $mn \times H$. But the moment of P_2 with reference to the same point, is $op \times H$. The difference then of mn and op, gives us the same ordinate as in the first example. The lower ordinates subtracted from the upper, will give us the same Figure as before.

We see, therefore, that whenever we obtain a double Figure, as in the present case, it shows simply that we have taken the forces in inconvenient order. We have only to change the order, to obtain the moments directly from the polygon.

In Fig. 25, we have a comprehensive picture of the way in which the moments change for every point of the beam from end to end.

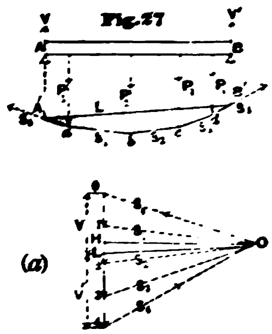
CLOSING LINE PARALLEL TO BEAM; CHOICE OF POLE DISTANCE.—It makes no difference what inclination the closing line may have, because, as we have seen, the ordinate in the equilibrium polygon parallel to the resultant, multiplied by the pole distance, gives the combined moment *with reference to any point on that ordinate*, of all the forces right or left.

We can, however, if we wish, always render the closing line parallel to the beam itself, and this it is sometimes desirable to do. We have only first to find by preliminary construction, the reactions, or the point L where the parallel to the closing line in the preliminary force polygon intersects the force line (Figs. 25 and 26). If then we take a new pole anywhere upon a line through this point, *parallel to the beam*, the closing line will be parallel to the beam.

As to choice of pole distance, we have only to so choose the position of the pole as to give good intersections for the polygon. The multiplication may be directly performed by properly changing the scale in the equilibrium polygon. The ordinate to the new scale will then give the moment at once. Thus if our scale of length in Fig. 25 is five feet to an inch, and the pole distance in the force polygon measured to the scale adopted

for forces is, say, ten pounds, we have only to take fifty moment units to the inch as the scale for the ordinates, and they will then give the moments directly.

EXAMPLE 3.—BEAM WITH ANY NUMBER OF WEIGHTS.—Suppose we have any number of weights as P_1 . . P_4, Fig. 27.



The method of procedure is as follows:

1st. Construct the force polygon (*a*). Choose a pole O and draw the strings S_0 . . . S_4.

2d. Construct the equilibrium polygon.

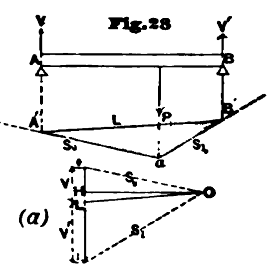
3d. Draw the closing line through the points A', B', *vertically beneath the supports.*

A parallel in the force polygon gives the reactions at the end, Lo and $4L$. These reactions must always act so as to *replace* the strains in those lines of the equilibrium polygon which meet at A and B. Thus at A', Lo must replace the strains in $A'B'$ and $A'a$. In the force polygon below, we see that $A'a$ or S_0, for equilibrium, acts from O to o. Hence Lo must act opposed, or up. In same way, strain in dB' or S_4 acts from 4 to O for equilibrium, hence $4L$ must act opposed, or up also. The supports at A and B must then be below.

Knowing the reactions, we can now make the closing line parallel to the beam if we choose, by simply taking a new pole anywhere upon a line through L parallel to the beam, and making a new equilibrium polygon. No advantage would be gained by such construction in this case.

The moment at any point is given by the ordinate, in the equilibrium polygon, parallel to the resultant or to the forces, multiplied by the pole distance H.

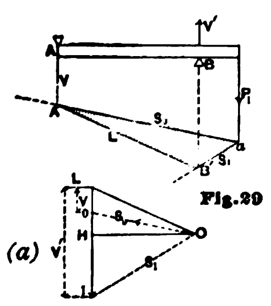
EXAMPLE 4.—BEAM WITH A SINGLE WEIGHT.—Let the weight P_1, Fig. 28, act at any point of the beam AB. Then the equilibrium polygon is $A'aB'$. The vertical reaction at the ends of the beam are Lo and $1L$, both acting up, and hence the supports must be below the beam.



We see at once that the moment is greatest at the weight, and decreases both ways to zero at the ends.

EXAMPLE 5.—BEAM WITH WEIGHTS BEYOND BOTH SUPPORTS.—Observe in the construction of the equilibrium polygon that S_0 is *always* prolonged *till it meets* P_1. Also that the closing line $A'B'$ always unites the two points vertically below the supports. The equilibrium polygon $A'aB'$ is then easily drawn.

The reactions require special notice. Thus, the reaction at B is the resultant of the strains in the lines S_1 and L, which meet at B'. This, as shown by the force polygon, is $1L$. Since S_1 has the direction from 1 to O for equilibrium, the reaction $1L$ to *replace* S_1 and L must act up. The support at B is therefore below the beam. Again, the reaction at A is the resultant of the strains in S_0 and L which meet at A'. This is given in the force polygon by Lo. But S_0 acts from O to o for equilibrium. The reaction, then, in order to *replace* S_0 and L, must act opposed to this direction, or down. Hence the support at A must be above the beam. The reaction at B is then greater than the weight P_1 by the amount of the reaction V at A, just as should be the case.

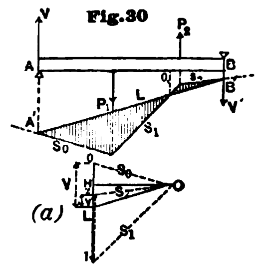


The moment at any point is given, as always, by multiplying the ordinate in the equilibrium polygon into the pole distance H.

EXAMPLE 6.—BEAM WITH ONE DOWNWARD AND ONE UPWARD FORCE BETWEEN

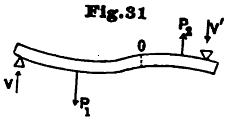
THE SUPPORTS.—Here we need only call special attention to the fact that as P_2 acts up, and is less than P_1, S_2 in the force polygon lies between S_0 and S_1. The reactions are Lo and $2L$, and obtaining the directions of S_0 and S_2 for equilibrium, we see that one of the reactions must act up in this case and the other down.

Fig. 30



We see also that if P_2 should be taken less, so that 2 falls below L in the force polygon, the reaction at B would be upward also, and the support there would have to be below the beam. The student would do well to sketch the construction for P_2 greater than P_1.

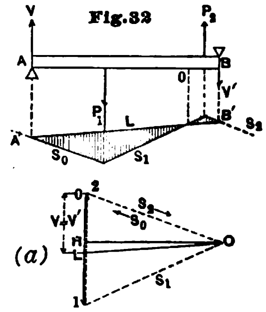
Fig. 31



At the point o we see that the moment is zero. At this point the moment of V is equal and opposite to the moment of P_1. At o, then, we would have a "point of inflection," or the beam would be concave upward as far as o, and from o on convex upward, as shown in Fig. 31. At o the two curves would have a common tangent.

EXAMPLE 7.—BEAM SAME AS BEFORE, BOTH FORCES EQUAL.—Laying off the force polygon, the first force extends from o to 1, Fig. 32 (*a*), and the second from 1 back to o again. Choosing, therefore, a pole O and drawing S_0, S_1, S_2, we find that S_0 and S_2 fall together. The directions of S_0 and S_2 for equilibrium are shown by the arrows.

Fig. 32

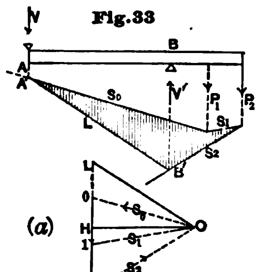


Constructing the equilibrium polygon, and drawing the closing line $A'B'$ and its parallel L in the force polygon, we see that the reaction at A, or the resultant of S_0 and L is Lo, and the reaction at B, or the resultant of S_2 and L, is also Lo. The reactions V and V' are therefore equal. This is in accordance with our principle, page 26, that a couple can only be held in equilibrium by another couple. As to the direction of these reactions, taking S_0 as acting as shown by the arrow for equilibrium, V, in order to replace, must act up. In like manner V' must act down. The support at A should be below and B above. At o we have the moment zero. Here then is a point of inflection, and the beam has the deflected shape shown in Fig. 31.

The moments at any point are, as always, given by the ordinates multiplied by the pole distance H. We see that the moment is greatest at each force, and zero at o and the two ends.

EXAMPLE 8.—BEAM WITH TWO EQUAL WEIGHTS BEYOND THE SUPPORTS.—Fig. 33 needs no explanation, except to call attention to the reactions.

Fig. 33

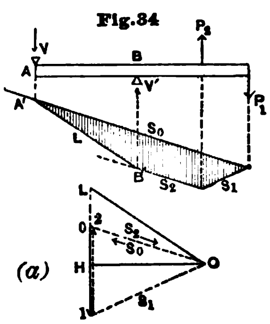


Thus the reaction at A is oL acting down. At B, it is the resultant of S_2 and L, or $2L$, acting up.

We see from the ordinates in the equilibrium polygon, how the moments vary from point to point.

We repeat here, that the order in which the forces are taken in all these examples, is indifferent, as also the position of the pole. The student will do well to work out cases to scale and satisfy himself that this is so.

Fig. 34

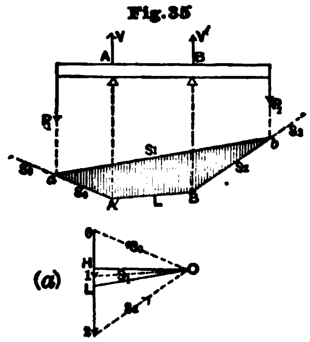


EXAMPLE 9.—BEAM WITH A COUPLE BEYOND THE SUPPORTS.—Observe that S_0, Fig. 34, is produced till it intersects P_1 in the equilibrium polygon. Then S_1 to P_2, then S_2 parallel to S_0. The closing line $A'B'$ is then drawn. A parallel to it in the force polygon (*a*) gives Lo acting down as the reaction at A and oL acting up as the reaction at B. Between B and P_2 we see that the moment is constant, because S_0 and S_2 are parallel. This is the graphical interpretation of our principle, page 26, that the moment of a couple is constant

for any point in the plane. We see, also, that since S_0 and S_2 intersect at an infinite distance and the resultant of S_0 and S_2 is zero in the force polygon, that ***the resultant of a couple is an infinitely small force situated at an infinite distance.***

EXAMPLE 10.—BEAM WITH A VERTICAL WEIGHT BEYOND EACH SUPPORT.—Here we would call attention, Fig 35, to the construction of the equilibrium polygon. We draw a parallel to S_0 in the force polygon till it meets P_1 at a. From a a parallel to S_1 till it meets P_2 at b. Through b a parallel to S_2. Prolong S_2 and S_0 till they meet the verticals through B and A at B' and A'. $B'A'$ is the closing line.

Fig. 35



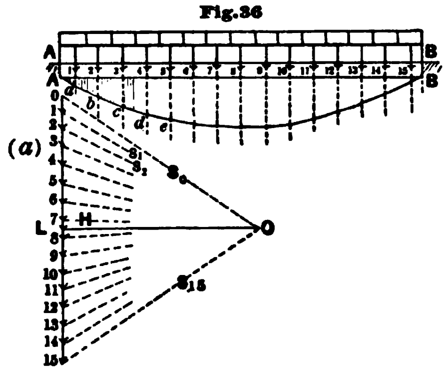
This gives us in (a) Lo acting up for reaction V at A, and $2L$ acting up for reaction V' at B.

If we had taken the forces in inverse order, we should have got a double figure, as in Fig. 26.

EXAMPLE 11.—UNIFORM LOADING.—A uniform load may be considered as a system of equal and equidistant weights very close together.

Thus, in Fig. 36, the load area, which is a rectangle whose height is the load per unit of length, and whose length is the length of the beam, may be divided into any number of equal parts. The area of each of these parts we may consider as the weight which acts at its centre of gravity, and lay it off to any assumed scale in the force polygon. Since the reaction at A and B must be equal, we take the pole in a horizontal through the centre of the force line. The closing line will then be parallel to the beam, and $OH = OL$.

Fig. 36

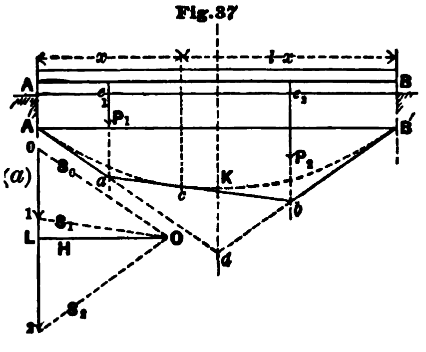


Now draw the strings $S_0\ S_1 \ldots S_{15}$, and then construct the equilibrium polygon. It is evident that the points *abcde*, etc., of the polygon will enclose a curve tangent to *ab*, *bc*, *cd*, *de*, etc., at the points midway between, that is, where the lines of division of the load area meet the sides of the polygon.

The ordinates to this curve, multiplied by the pole distance H, give the moment at any point of the beam.

It will be seen, however, that this method is deficient in accuracy, because the lines *ab*, *bc*, etc., are so short, and there are so many of them. Any error in direction is thus carried on, and even the most extreme care would fail to produce accurate results.

Fig. 37



We may avoid this objection by taking fewer lines of division.

Thus, suppose, Fig. 37, we divide the load into only two portions, x and l-x. The entire weight over the portion x can be considered as acting at the centre e_1 of the load-area x, page 25. The same holds good for the load over l-x. We thus have two forces, P_1 and P_2. Taking the pole as before, so that the closing line shall be parallel to the beam, construct the equilibrium polygon $A'abB'$. The curve of moments required will be tangent at A', c and B', as shown by the dotted curve, and may therefore be sketched in. We may thus determine as many points of tangency as we wish, and sketch the curve with considerable accuracy.

We may, however, devise a still better method. It is evident that the curve required is symmetrical with respect to the vertical through the centre of the beam. If we can determine what this curve is, it may be possible to construct it directly without using the force polygon at all.

Now we see that no matter where the load area is supposed to be divided, we shall have always, Fig. 37,

$$e_1e_2 = \frac{1}{2}x + \frac{1}{2}(l-x) = \frac{1}{2}l.$$

That is, no matter where the line of division, the horizontal projection of the line *ab* of the equilibrium polygon is constant and equal to $\frac{1}{2}l$. But the line *ab* is a tangent to the curve required. But if from any point on the line *A'd* we draw a line *ab*, limited by the line *B'd*, in such a way that its horizontal projection is constant, the line *ab will envelop a parabola.*

This may easily be proved analytically as follows :—Let the load per unit of length be p. Then the entire load is pl and the reaction at each end is $\frac{pl}{2}$.

The moment at any point distant x, Fig. 37, from the left end, is then

$$y = \frac{pl}{2}x - P_1\frac{x}{2},$$

but since P' is always equal to px,

$$y = \frac{pl}{2}x - \frac{px^2}{2}.$$

This is the equation of the curve of moments when the origin is at A'. For the centre of the beam, $x = \frac{l}{2}$, and we have, therefore, the ordinate to the curve at the centre, $\frac{pl^2}{8}$.

If we take the origin at K, we have,

$$x = \frac{l}{2} + x' \quad y = \frac{pl^2}{8} - y',$$

hence,

$$\frac{pl^2}{8} - y' = \frac{pl}{2}\left(\frac{l}{2} + x'\right) - \frac{p}{2}\left(\frac{l}{2} + x'\right)^2,$$

or

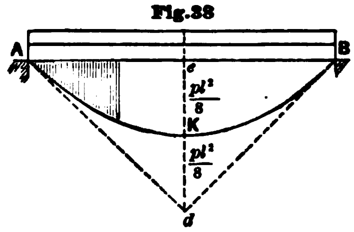
$$x'^2 = \frac{2}{p}y',$$

which is the equation of a parabola referred to its vertex.

We have, therefore, the following construction :

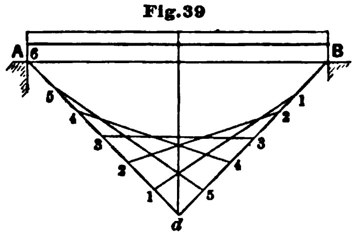
In Fig. 38, lay off a perpendicular *eK* to the beam at its centre, and make it equal by scale to $\frac{pl^2}{8}$. Through A, B, and K, construct a parabola, having its vertex at K. The ordinate to this parabola at any point of the beam will give the moment at that point, to the same scale as that by which *eK* was laid off. The distance *Kd* is equal also to $\frac{pl^2}{8}$, because *ed*, or the moment of the reaction is $\frac{pl}{2} \times \frac{l}{2} = \frac{pl^2}{4}$, and $ed - eK = Kd = \frac{pl^2}{4} - \frac{pl^2}{8} = \frac{pl^2}{8}$. The distance *ed* then is equal to $\frac{pl^2}{4}$.

Fig. 38



HOW TO DRAW A PARABOLA.—Since in any case we know, then, the distance ed, we can always draw the lines Ad and Bd, Fig. 39. If then we divide Ad into any number of equal parts, as say, six, and Bd into the same number of equal parts, and number these parts in the one case away from d, and in the other case towards d, we have only to draw lines joining any two points having the same number, and these lines will all have the same horizontal projection equal to $\frac{l}{2}$. They will, therefore, enclose the parabola required. Tangent to these lines we may sketch the curve.

Fig.39



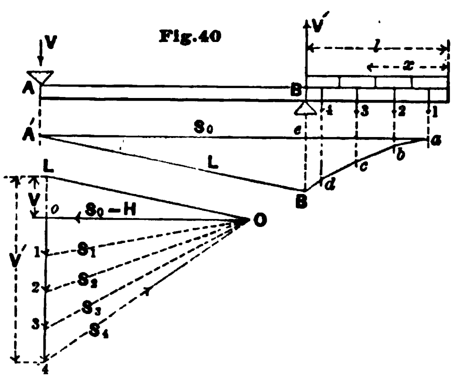
A better method, because more accurate, is to plot the ordinates to the curve, from its equation,

$$y = \frac{pl}{2}x - \frac{px^2}{2}$$

by inserting for x, measured from the left end different values, as $\frac{1}{10}l$, $\frac{2}{10}l$, $\frac{3}{10}l$, etc., and finding the corresponding values for the ordinate y.

EXAMPLE 12.—BEAM LOADED UNIFORMLY BEYOND THE SUPPORTS.—Let the beam, Fig. 40, be loaded uniformly beyond the support B. If we divide the total load into say four equal parts, and consider each weight acting at the point midway between the points of division, we may form the force polygon and then draw the equilibrium polygon as shown.

Fig.40



Take the pole in a horizontal through o. Then S_0 will be parallel to the beam and be equal to H. We obtain the equilibrium polygon $A'abcdB'$, and $A'B'$ is the closing line. This gives us Lo for the reaction at A, acting down, and $4L$ for the reaction at B, acting up. The ordinate at any point, multiplied by H, gives the moment.

Again, we see that the polygon $abcdB'$ is properly a curve, tangent to ab, bc, etc, at the points where the lines of division prolonged meet these sides. The polygon approaches this curve more nearly, the greater the number of parts into which we divide the load.

It is better, to insure accuracy, to plot this curve, which is, as before, a parabola, by points.

Thus, if the loaded portion is l, we have for the moment at any point distant x from the right end the moment $y = \frac{px^2}{2}$.

Inserting different values for x, we can find the corresponding moment and lay it off to scale. The moment at B is then $\frac{pl^2}{2}$. Laying this off from e to B' by scale, we can join $B'A'$, and thus obtain the equilibrium polygon. The ordinate at any point taken to the scale adopted gives then at once the moment at that point, *i. e.*, the pole distance is unity.

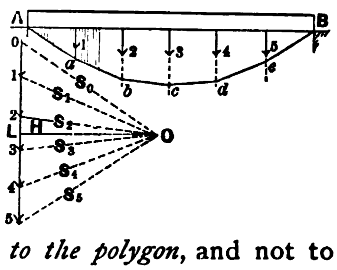
EXAMPLE 13.—BEAM LOADED WITH CONCENTRATED EQUAL WEIGHTS, EQUI-DISTANT.—Let the distance from the ends to the nearest weight be equal to the distance between the weights.

Take the pole as before, Fig. 38, so that the closing line shall be horizontal. We can then construct the polygon $abcde$, Fig. 41, the ordinates to which, multiplied by the pole

distance, give the moments. As the weights are concentrated, we have not in this case a curve, but a true polygon. We meet, however, the same practical difficulties of construction as in Fig. 36.

These difficulties may be overcome, as in that case, by constructing the parabola for an equal uniform load, and then remembering that the polygon required *is inscribed in this parabola*, that is, has its angles upon the curve.

Fig. 41



We can then construct the parabola for an equal uniform load, as directed, page 44, and where the weights intersect the curve, we have the points *abcd*, etc. The polygon can then be drawn. The moment for the point of application of any weight then is given by the ordinate to the curve. The moment for a point between any weight *is given by the ordinate to the polygon*, and not to the curve.

To construct the parabola, we take the total weight upon the beam and divide by the length. This gives us the equivalent uniform load per unit of length p. We can then plot the parabola by points from its equation

$$y = \frac{pl}{2}x - \frac{px^2}{2}$$

by inserting for x the distance of each weight from the left end in terms of l. The moment is then given directly by y. We can lay off the values for y thus found, to scale, and the force polygon is unnecessary, since the pole distance is thus assumed as unity.

The above is sufficient to enable any careful student to thoroughly master the method. We see that in any case we can easily find, by a graphical construction, the moment of all the outer forces acting upon any rigid body, right or left of any point, and this was the problem proposed for solution at the beginning of this chapter. The student should at first draw all the examples with parallel ruler. Afterwards he can sketch merely by eye for purposes of elucidation only.

B. ILLUSTRATION OF GENERAL PRINCIPLES.

Let us choose as an example to illustrate the application of these principles the same truss as that already discussed in the preceding chapter.

APPLICATION TO A ROOF TRUSS.

Let Fig. 42 represent the truss. The end weights can evidently be disregarded in the force polygon, since they act directly upon the supports. This is also shown in Fig. 7 (a), where the end weights have no effect upon the strains, and the Figure is the same as though they were left out, provided the reaction is taken at 2,800 lbs. instead of 3,200 lbs. In the methods of Chapter II. and Chapter III. also, the same is the case. In general, a weight upon the support has no effect upon the truss, and can be disregarded.

Numbering the weights then as in Fig. 42, we can construct a force polygon (a), and then the equilibrium polygon, as shown. This, however, is not advisable, for reasons already given. It will be more accurate to assume the pole distance as unity, thus discarding the force polygon altogether, and construct the parabola from its equation

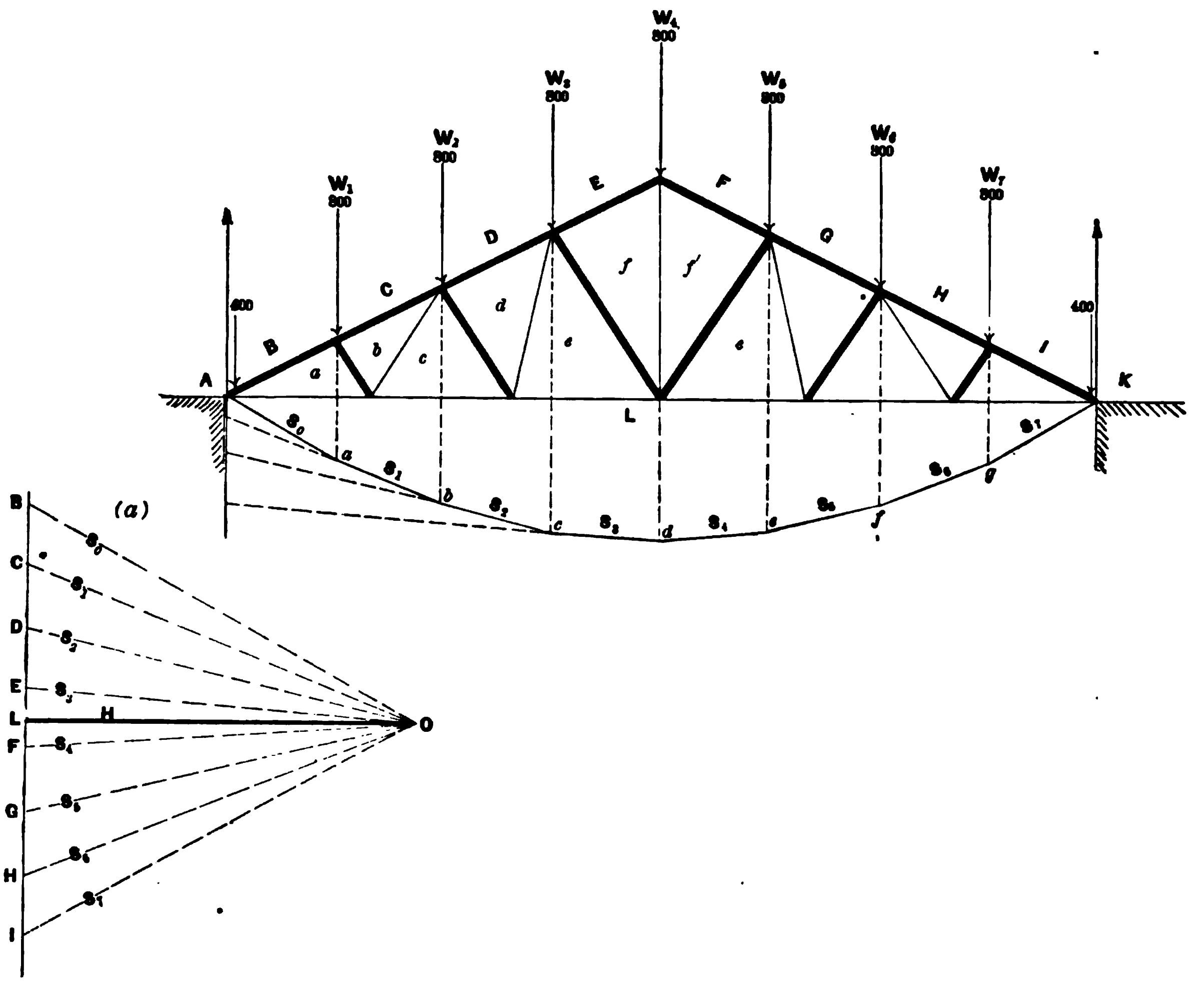
$$y = \frac{pl}{2}x - \frac{px_2}{2},$$

as directed in Example 13.

Putting then $x = \frac{1}{8}l, \frac{2}{8}l, \frac{3}{8}l, \frac{4}{8}l$, etc., we obtain for the moments at the points of application of the weights, and, therefore, for the apices *a, b, c, d*, etc., upon the curve, the ordinates,

$$y = \frac{7}{128}pl^2, \frac{12}{128}pl^2, \frac{15}{128}pl^2, \frac{16}{128}pl^2, \text{ etc.}$$

Fig. 42



The total weight acting upon the truss in the present case, ***including the end weights***, is 6,400 lbs.* The length of the span $l = 50$ feet. Hence $p = \frac{6400}{50} = 128$ lbs.

We have, then, for

$$x = \frac{1}{8}l \qquad \frac{2}{8}l \qquad \frac{3}{8}l \qquad \frac{4}{8}l$$

$$y = 17500 \qquad 30000 \qquad 37500 \qquad 40000 \text{ moment units.}$$

Laying these off to any convenient scale, we determine very accurately the points *abcd*, Fig. 42. The other half of the polygon is precisely similar.

* Observe that in calculating p, or the equivalent load per unit of length, *all* the weights must be taken.

The ordinates to this polygon will give, to the scale adopted for moment units, the moment for any point of the truss, of the outer forces left or right of this *point*. Thus the moment with reference to k, of all the forces right or left, is km, Fig. 42. We find by scale $km = 21666\frac{2}{3}$ moment units. In the same way, the moment for the next lower apex is 35000 moment units to scale. The moment at the next lower apex, or for the centre of the span, is 40000 moment units, since it is vertically beneath the weight W_4.

Now our rule is, as before, Chapter III., page 27, for any piece,

$$\text{Strain} \times \text{lever arm} + \Sigma \text{ moments of outer forces} = 0.$$

The second term is given by our ordinates to the polygon to scale. We have then only to divide these by the lever arm for any piece in order to obtain the strain.

As regards the centre of moments for any piece, we must observe the rule, Chapter III., page 27, *viz.*: Cut the truss entirely through by a section cutting only three pieces, the strains in which are unknown. For any one of these pieces take the point of moments at the intersection of the other two cut.

For the proper sign of the lever arm, we have, as before, the rule of Chapter III., page 27, *viz.*: Considering only the left hand portion of the truss, thus divided in two, imagine yourself standing at the cut end of the piece in question, and facing that apex of the left hand portion from whence it proceeds. Then if the centre of moments is on the right hand, the lever arm is plus. If on the left, it is minus.

The lever arms for this case have been calculated for each piece, and are given in Chapter III., page 29.

As always, a moment causing rotation in the direction of the hands of a watch is positive, in the reverse direction, negative.

Observing these conventions, a minus sign in the result will indicate tension in a piece; a plus sign, compression.

Let us first find the strain in the lower bays. For La, the centre of moments is at the first upper apex BC, according to rule. The moment for this point is given by the ordinate na, or is 17500 moment units. Considering always the left portion, this moment is positive, because the reaction—the only force acting on that portion—acts up. The lever arm is, according to the rule, plus, because the centre of moments lies on the right of the piece when we look towards A. This lever arm has been found to be 3.125 feet (page 29).

We have, then,

$$La \times 3.125 + 17500 = 0,$$

Or

$$La = -\frac{17500}{3.125} = -5600 \text{ lbs.}$$

In similar manner we have

$$Lc \times 6.25 + 30000 = 0,$$

Or

$$Lc = -\frac{30000}{6.25} = -4800 \text{ lbs.}$$

For Le, we have

$$Le \times 9.375 + 37500 = 0,$$

Or,

$$Le = -\frac{37500}{9.375} = -4000 \text{ lbs.}$$

Let us now find the strains in the upper bays. For the bay *Ba*, the centre of moments is at *k*. Since, when we cut *Ba* and *La*, the only force acting on the left hand portion of the truss is the reaction, the moment at *k* is the moment of this reaction. That is, it is the ordinate from *k* to the line *Aa of the polygon produced*. It is therefore positive, and larger than *km*, which gives the combined moment of the reaction and first weight. We find it by scale to be $23333\frac{1}{3}$ moment units. The lever arm of *Ba* is negative, because the centre of moments *k* lies on the left as we look along the piece facing *A*.

We have then

$$-Ba \times 3.727 + 23333\frac{1}{3} = 0,$$

Or,

$$Ba = \frac{23333}{3.727} = +\ 6260 \text{ lbs.}$$

In like manner, for *Cb* we have the moment $km = 21666\frac{2}{3}$. Hence,

$$-Cb \times 3.727 + 21666\frac{2}{3} = 0,$$

Or,

$$Cb = \frac{21666}{3.737} = +\ 5813 \text{ lbs.}$$

In the same way we have

$$-Dd \times 7.454 + 35000 = 0,$$

Or,

$$Dd = \frac{35000}{7.454} = +\ 4691 \text{ lbs.}$$

Also,

$$-Ef \times 11.151 + 40000 = 0,$$

Or,

$$Ef = \frac{40000}{11.151} = +\ 3587 \text{ lbs.}$$

For the braces, the point of moments is at *A*. Taking a section through *Cb*, *ab* and *La*, we have acting on the left hand portion only the weight at *BC*, which causes a moment about *A*. But the moment of this weight with reference to *A* is, by our principles, the ordinate *through A, which meets ab produced*. This moment is positive. We take it off to scale = 5000 moment units. The lever arm for *ab* is given on page 29. It is negative by our rule. We have, then,

$$-ab \times 6.934 + 5000 = 0,$$

Or,

$$ab = \frac{5000}{6.934} = +\ 721.$$

In like manner, for *bc* the moment is positive and the same as for *ab*, but the lever arm is positive, and equal to the lever arm for *ab*.

We have, then,

$$bc = -\ 721 \text{ lbs.}$$

Again, for the brace *cd*, the moment is the sum of the moments of the weights at *BC* and *CD* with reference to *A*, because when we cut *Dd*, *cd*, and *Lc*, both of these weights act upon the left hand portion. This moment is given to scale by the ordinate through *A* which meets the line *bc* in the equilibrium polygon produced. It is to scale 15000.

We have, then, since the lever arm is minus,

$$-\,cd \times 13{,}869 + 15000 = 0,$$

Or,

$$cd = \frac{15000}{13{,}869} = +\ 1081 \text{ lbs.}$$

For the brace de we have the same moment, because only the same weights act upon the left hand portion, but the lever arm is positive, and equal to 16.2 feet.

We have, then,

$$de \times 16.2 + 15000 = 0,$$

Or,

$$de = -\ \frac{15000}{16.2} = -\ 926 \text{ lbs.}$$

For the brace ef, in like manner, the moment is positive and equal to the ordinate through A, limited by the line cd of the equilibrium polygon, produced. This ordinate to scale is 30000 moment units. The lever arm is negative. We have then

$$-\,ef \times 20.803 + 30000 = 0,$$

Or,

$$ef = \frac{30000}{20.803} = +\ 1442 \text{ lbs.}$$

For the brace ff' we have the same moment, but the lever arm is positive, and equal to 25. But the piece, $f'e'$, which is also cut, has also a moment with respect to A which must be taken into account. Since, by reason of the symmetry of frame and loading, the strain in $f'e'$ is the same as that already found for ef, and its lever arm is the same; its moment is also + 30000.

We have, then,

$$ff' \times 25 + 60000 = 0,$$

Or,

$$ff' = -\ 2400 \text{ lbs.}$$

These values are precisely the same as those already found for the roof truss in the preceding chapters.

REMARKS UPON THE METHOD.—The present method is convenient for finding the strains in the upper and lower bays, *but it should never be used for the braces.* We see from Fig. 42 that in prolonging the sides ab, bc, etc., of the equilibrium polygon till they meet the vertical through A, which is necessary in order to find the moments for the braces, a little variation in direction will make considerable difference. As the sides ab, bc, etc., are short, they do not give direction accurately enough.

In fact, of all our four methods, none are so well adapted to the case of Fig. 42 as the method of Chapter I., checked in one or two of the last pieces by the method of Chapter III. The more irregular the frame the more advantageous is the graphic method of Chapter I. For girders with parallel flanges, like most bridge trusses, however, the method of the present chapter is very extensively used for the upper and lower flanges, and is in such cases very easy of application.

QUESTIONS FOR EXAMINATION.

State the general problem which it is the object of the present chapter to solve. Show how to find the position of the resultant for any number of forces applied at different points, without the aid of the equilibrium polygon. Explain why the method is not general. Explain the general method, Fig. 18. What is the "pole" in the force polygon? Show that the position of the pole is a matter of indifference. What is the equilibrium polygon, Fig. 19? Show how to fix any two points A, B. What is the closing line? Apply

these principles to any number of forces, Fig. 20. What is Culmann's principle? Deduce it. Apply it to the equilibrium polygon, Fig 22. What is the pole distance? To what scale must it be always taken? In what direction should the ordinate in the equilibrium polygon be taken? To what scale? What are the two important properties of the equilibrium polygon? Does it make any difference where the pole is?

Apply these principles to a beam, Fig. 23. Show their application to parallel forces, Fig 24. Sketch and explain Example 1. Show that the order of the forces makes no difference. Show how to make the closing line parallel to the beam. Sketch and explain Example 3. Also Examples 4, 5. Point out the reactions in Example 5, in what direction they act, and why. Do the same for Examples 7, 8. In Example 9, show that the moment of a couple is constant. Explain how to find the reactions. In all the cases show how to find the moment at any point.

Explain the method as applied to uniform loading. Show how and why it is inaccurate. Point out how to overcome this objection. Prove that the curve of moments is a parabola, and deduce its equation. Show how to construct a parabola. Apply these principles to Example 12. Explain the reactions in this example. Apply the method to Example 13.

Show the application of our principles to the roof truss, Fig. 42. Why are the end weights disregarded? What is the equation for the parabola? How do you find the load p per unit of length? What is it in this case? Why are the end weights *not* disregarded in finding p? Point out the ordinates which give the moments for the lower bays. Show how to find the strains from these moments. What is the rule for determining the centre of moments for any piece? Illustrate. What is the rule for the proper sign for the lever arm? Illustrate. In what direction is positive rotation? Negative? What is the sign for a weight? For a reaction? Illustrate these rules in their application to find the upper bays.

What is the point of moments for the braces? Why? How is the ordinate found which gives the moment for the brace ab? Point out the ordinate for cd and de. Explain how to find the strain in ff'. What can you say as to this method compared to the others? Why is it not accurate as applied to the braces? What method is best for irregular structures and loading? Is the present method ever advantageous? In what cases?

State the fundamental principle of the graphic method by resolution of forces. Illustrate by an example. State the fundamental principle of the analytic method by resolution of forces. Illustrate its application, and give rules for properly determining the signs of the various terms in the equations resulting. State the principle of the algebraic method of moments. Illustrate and give rules for signs. Illustrate the application of the graphic method of moments.

TEXT-BOOKS ON GRAPHIC STATICS.

The student will find the graphical method of Chapters I. and IV., as well as many other applications and principles, explained and treated in the following works:

Culmann, K.—"Die Graphische Statik." With Atlas of 36 Plates. Zürich, Meyer & Zeller, 1866. [I. Part, 1864: Elements and Graphical Investigations of Structures. Also a second edition, first volume, 1875, with 17 Plates. General Principles, second volume, to follow shortly. This is the pioneer work on the subject, and also the most complete.]

Bauschinger.—"Elemente der Graphischen Statik." With Atlas of 20 Plates. München, 1871. [A more popular presentation of the subject, requiring less mathematical preparation to read.]

Ott, K. Von.—"Die Grundzüge des Graphischen Rechnens und der Graphischen Statik." Prag, 1872. [English translation by G. S. Clarke. A small elementary treatise.]

Favaro, Antonio.—"Lezioni di Statica Grafica." Padua, 1877. Pp. 650.

Levy.—"La Statique Graphique et ses Applications." Paris, 1874. With Atlas of 24 Plates. [Principles and numerous applications.]

Du Bois, A. J.—"The Elements of Graphic Statics, and their Application to Framed Structures." Pp. 400. With Atlas of 32 Plates. New York, John Wiley & Sons.

Clarke, G. S.—"The Principles of Graphic Statics." Pp. 138. With Atlas of 11 Plates and numerous illustrations in Text. E. & F. N. Spon, London.

Greene, Charles E.—"Trusses and Arches Analyzed and Discussed by Graphical Methods." John Wiley & Sons, New York.

Bow, R. H.—"Economics of Construction in Relation to Framed Structures." E. & F. N. Spon, London.

The literature of graphic statics is now quite extensive. Our space forbids mention of monographs and papers. A more complete list may be found in the author's treatise above, which was the first systematic presentation in English. The preceding list comprises all the text-books upon the subject proper known to the author.

PART I.

SECTION II.

PRACTICAL APPLICATIONS.

SECTION II.

PRACTICAL APPLICATION OF PRECEDING METHODS TO VARIOUS STRUCTURES.

INTRODUCTORY.—CLASSIFICATION OF STRUCTURES.

PLAN OF THIS SECTION.—The preceding section includes all the methods used in the solution of framed structures. They are, as we have seen, four in number—two graphic and two algebraic. Special cases lead sometimes to modifications of these general methods, which we shall point out in their proper place. In the present section we shall discuss more in detail the various forms of framed structures most frequently met with, and, in doing so, shall sufficiently indicate the application of our principles to enable the reader to easily solve any other case not specially treated. We shall choose for each form that method or modification, or that combination of methods, which in each case seems most advantageous. The student familiar with the principles of the preceding section can easily apply any other method or combination, which seems to him to offer superior advantages as to accuracy or facility.

The choice of any method for any special case is in some measure a matter of individual preference. While, therefore, we shall adopt those methods which seem to us the best suited to the case in hand, or which are most generally in use, the student will understand clearly that he is by no means confined to such method unless it commends itself to him as, on the whole, the best.

CLASSIFICATION OF STRUCTURES.—We may divide all those structures of which we shall treat into two classes: those which sustain the action of a permanent load, or unvarying forces, and those which are subject to stresses of variable amount. To the first class belong, *Roof Trusses*, *Cranes*, *Cantilevers*, and in general all those structures which have to sustain a "dead load," such as their own weight and exterior forces of constant amount, such as the weight of roofing, snow, etc. To the second class belong *bridges*, which have to sustain, besides a "dead load" proper, consisting of their own weight and outside forces of constant amount, also the action of a "live load," such as that of moving cars or vehicles, cattle, and men.

ROOF TRUSSES.—Roof trusses are of almost innumerable forms. It will be unnecessary to discuss each form. The principles which apply to one apply to all, without variation. The selection of a few well-chosen cases will suffice for all. Such cases will be found in the next chapter.

TRUSS ELEMENT.—The truss element is in all cases a triangle. No rigid frame-work can be made which does not consist of a repetition of the triangle. Any frame of three sides is rigid. Its shape cannot be altered without altering the lengths of its sides. Any frame-work of more than three pieces can thus alter its shape, unless divided into triangles by diagonals which constitute the bracing.

SUPERFLUOUS PIECES.—The conditions of equilibrium are three, viz.: 1st. The alge-

braic sum of the vertical components must be zero; 2d. The algebraic sum of the horizontal components must be zero; 3d. The algebraic sum of the moments of the forces must be zero. As in any framed structure, we know, or must first independently détermine, all the outer forces or stresses, it follows that these outer forces must be held in equilibrium at any point of the frame by the strains in the pieces cut by a section through the frame at that point, (p. 5). If there are only three such pieces the strains in which are necessarily unknown, we can always write down three equations of conditions between the strains in these pieces, and therefore determine them. If there are more, the problem is indeterminate; there are more unknown quantities than there are equations of conditions between them.

At any point, therefore, of any properly framed structure, it should be possible to make, in some direction, a section, cutting the structure entirely in two, which shall not cut more than three pieces, the strains in which are *necessarily* unknown. Of course it may cut any number of pieces, provided it is possible to find independently the strains in all but three. Any framed structure which violates this rule is improperly framed, and has superfluous pieces.

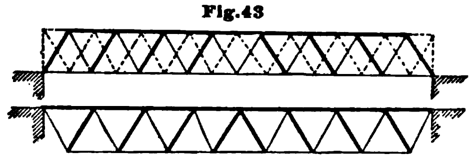
BRIDGE TRUSSES.—We may divide all bridge trusses into two classes, those in which the upper and lower members, or "flanges," are horizontal or parallel, and those in which the flanges are not parallel, and modifications of these.

I. GIRDERS WITH PARALLEL FLANGES.

In the first class there are two pure types which admit of many varieties. These are the *triangular* and the *quadrilateral* types, so called from the character of the bracing.

WARREN GIRDER.—The "*Warren*" girder, Fig. 43, is an example of the pure triangular type. Its bracing consists always of *equilateral triangles*. When the triangles are not equilateral, but isosceles, or have, indeed, any other shape, the truss is simply a "triangular truss." A common form is to make the height, or distance from centre to centre of flanges, half the length of bay, in which case the angles of the braces with the flanges are 45°. This truss is of more frequent occurrence in England than in this country.

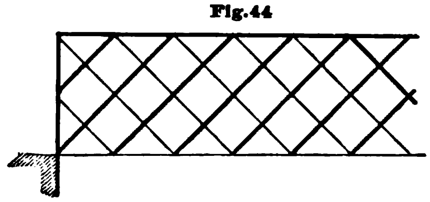
Fig. 43



DOUBLE TRIANGULAR-LATTICE TRUSS.—The triangular is the simplest form of truss, consisting simply of repetitions of the single truss element, or triangle. When, owing to the great length of bays, we have two or more systems of triangulation, as shown in Fig. 43, by the dotted lines, the truss becomes the "*double triangular*," or "*triple triangular*," as the case may be.

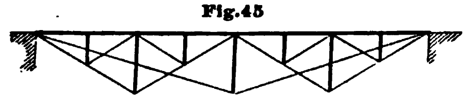
When there are in general more than three systems, and the braces are riveted to each other at their intersections, we have what is known as the "*lattice*" girder or truss, Fig. 44. A few lattice girders executed in wood are still to be found. With these exceptions, this style of truss may be said to be almost unknown in America.

Fig. 44



FINK TRUSS.—This is essentially a triangular truss with the lower chord left out, Fig. 45. The span is trussed or supported at the centre by a strut and ties from each end. Then the half spans, if sufficiently long to need it, are trussed as shown in the Figure. Again, the quarter spans may be trussed in similar manner, and so on. A number of these trusses are to be found in this country, but it is not now generally regarded with favor by bridge builders.

Fig. 45



These are, in general, all the modifications of the simple triangular type as applied to bridges.

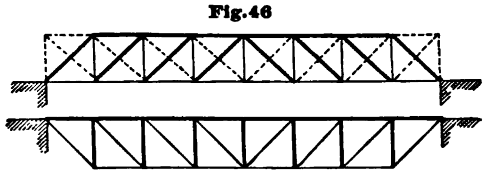
QUADRILATERAL TYPE.—It is evident that if such a structure as Fig. 43 is subjected to the action of a live load, some of the braces may be sometimes extended and sometimes compressed, according to the position of the moving load. It is not advisable, from a practical point of view, to subject the same piece to alternating strains of different character. Such action tends to deteriorate the material of which the piece is made, and shorten its life in the structure.

This has given rise to various constructions, in which each piece is required to sustain a strain of only one character, although this strain may indeed vary considerably in amount. In Fig. 43, the difficulty may be met by having each brace, when necessary, double, consisting of a hollow cylindrical piece for compression, enclosing a tie rod to take the tension.

Such considerations have led to the quadrilateral type of truss, in which each piece takes only stress of a certain character.

QUADRANGULAR TRUSS.—HOWE, PRATT, MURPHY-WHIPPLE.—A very common form is shown in Fig. 46. We may call it a single quadrangular, because it has but one system of bracing, and the panels are rectangular in form.

Fig. 46



When the vertical pieces sustain only compression, and the inclined pieces tension, it is known as the "*Pratt*" or "*Murphy Whipple*" system. In this shape it is often constructed of iron, and is then an advantageous form, because the shortest braces are compressed. As a long piece in compression always requires extra material to stiffen it and prevent it from doubling up or "buckling," this tends to save material. When there are two systems it is called the Whipple or double intersection truss.

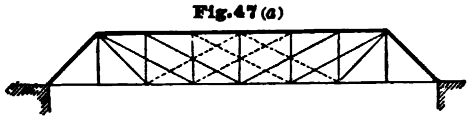
When the vertical pieces are in tension and the inclined braces in compression, the form is known as the "*Howe*" system. This is still often executed in wood and iron combined. The long braces are made of wood and the verticals of iron rods. This is again an advantageous use of material, as wood is comparatively cheap and best used in compression, while wrought iron is dearer and better adapted for tension.

COUNTER BRACES.—Where in any quadrangular system the action of the live load tends to cause in any inclined brace a strain opposite in character to that which it is designed to take, a brace in the direction of the other diagonal is inserted, as shown by the dotted braces in Fig. 46. Thus a load which tends to shorten one brace or diagonal cannot do so without elongating the other. If, for instance, the braces in full lines in Fig. 46 will take only tension, and buckle up under the action of a compressive stress, the dotted braces will be called into action. Such braces are called "*counter-braces.*" The strain in a counter-brace is, therefore, due entirely to the action of the live load. The dead load causes no strains in it whatever. The main braces, therefore, in any case, are those braces which are called into action by the dead load. The counter-braces, those which are called into action by the live load only.

SCREWING UP COUNTER-BRACES.—By properly screwing up the counters of such a truss as Fig. 46, the girder may be held down to that deflection which would be caused by the live load when it covers the whole span, and the girder thus rendered very rigid. The live load as it comes on would then act simply to relieve the strains in the counters without adding anything to those existing in the braces themselves. Under such circumstances, all the pieces sustain always a steady strain, except the counter-braces, and in these the strain, though fluctuating in amount, is always the same in character.

DOUBLE QUADRANGULAR—WHIPPLE TRUSS.—When the bays in Fig. 46 become very long, we may divide them up, and thus obtain the double quadrangular system of Fig. 47(*a*), or, as it is called sometimes, the "*Whipple*" truss. In the same way we may obtain triple quadrangular, etc. All such systems as Figs. 44 and 46 may be called *multiple systems.* The pure types are the triangular and quadrilateral, from which they are derived by multiplication of the system of bracing.

Fig. 47 (a)



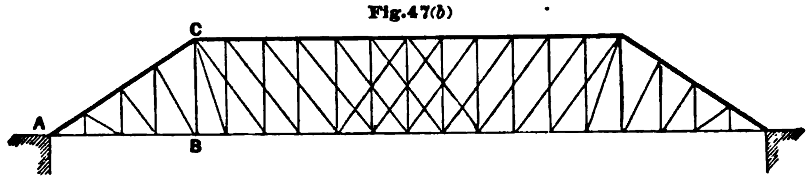
This is a very common form of truss. A modification of it of European origin is shown in Fig. 47(*b*).

If h represents the height and l the length of span, the best length a of the portion AB is given by

$$a = 0.006\, l + 1.08\, h.$$

The object of the variation is of course to effect a saving of material, but it may be doubted whether the design would compare favorably with the Whipple truss as executed in America with pin connections. Two such bridges are in existence in Vienna, over the Danube, each about 200 feet span.

Fig. 47(b)



Another modification, known in Germany as the *Schwedler* truss, consists in curving the ends AC, Fig. 47(*b*). In this truss the length of the portion AB is given by the formulæ

$$a = l\frac{g}{p}\left[\sqrt{1 + \frac{g}{p}} - 1\right],$$

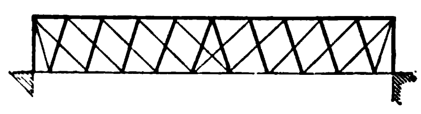
where l is the span, g the dead weight, and p the live load per unit of length. The height h at any distance x from the end of the curved portion is given by

$$h = \frac{h_0\, g}{l}\left[\sqrt{1 + \frac{p}{g}} + 1\right]^2 \frac{x\,(l-x)}{gl + px},$$

where h_0 is the height of the straight portion. All the pieces are, of course, straight, and only the apices of the portion AC lie in the curve given by the above equation. The height is so regulated by these equations that no counter-braces are required in the portions AB.

POST TRUSS.—A well-known form of double quadrilateral is that known as the "*Post*" truss, Fig. 48. In this truss the ties are made to slope at an angle of 45°, and the struts at an angle of 18° 26′ with the vertical. The dimensions, therefore, are taken so that the height being equal to one bay and a half, the ties extend across one bay and a half and the struts across one-half a bay. The apices in one chord are midway between those of the other.

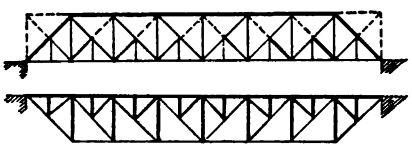
Fig. 48



BALTIMORE BRIDGE CO.'S TRUSS.—A modification of the single quadrangular system is shown in Fig. 49. It is known as the *Baltimore Bridge Co.'s Truss.*

Its peculiarity consists in the way in which a large bay is divided into two smaller ones by inserting half-braces and suspending ties.

Fig. 49



KELLOGG TRUSS.—This is another modification of the simple quadrangular. The object, as in all modifications, is to diminish the length of bay in a long span with the least material. The construction is shown in Fig. 50. For this purpose additional ties are run from the top of each post to the centre of the bay or panel. The counter-braces are shown by dotted lines.

BOLLMAN TRUSS.—A compound system consisting of a suspension system combined

Fig. 50

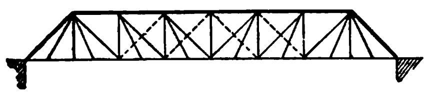
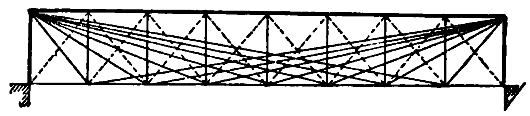


Fig. 51

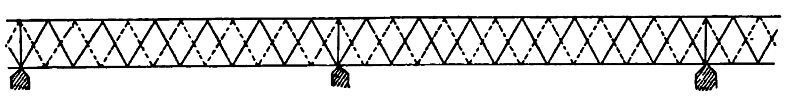


with a stiffening truss of the simple quadrangular type, is known as the Bollman truss, Fig. 51. A tie is run from each end directly to each loaded apex, thus forming a suspension system, which is stiffened by a quadrangular truss.

The above comprise all the best known varieties of quadrangular truss, as applied to bridges.

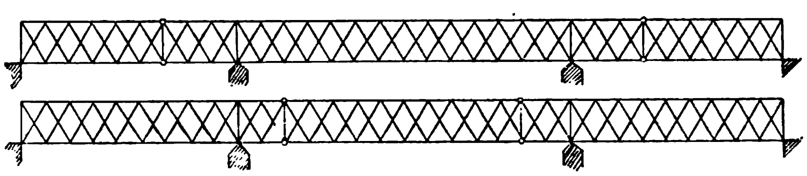
CONTINUOUS GIRDER.—When a girder with parallel flanges is extended over more than two supports it is called a continuous girder, Fig. 52 (*a*).

Fig. 52 (*a*)



The bracing may be of any character, either triangular or quadrangular, single or multiple, properly arranged so that each system shall transfer pressure directly to the supports. Thus if a double system is adopted in Fig. 52 (*a*), as shown by the dotted lines, we must introduce verticals over each support. A system shown in Fig. 52 (*b*), has been patented by Gerber, in Germany, in which the girder is continuous over the supports and *hinged* beyond the supports. The system is claimed to have all the advantages for long spans of the continuous girder, so far as saving in material is concerned, without the disadvantages of the latter system. It shows on strain sheet considerable gain over the simple girder in the parallel flanges, amounting to over 25 per cent., and is equally simple and certain in its calculation and construction. The distance of the hinges from the centre supports should be, for long spans of over 200 feet, about 0.2 of the centre span.

Fig. 52 (*b*)



In the case of a succession of long spans the system is worthy of more attention than it has heretofore received, as it offers some advantages over the discontinuous girder.

DECK AND THROUGH BRIDGE—LATERAL BRACING.—In all these forms, and in bridge trusses generally, the system may be so arranged as to allow the live load to traverse either the upper or the lower chord. A truss in which the live load traverses the lower or tension chord, is called a "*through*" truss. If the truss in this case is not high enough to admit of cross-bracing over head, it is called a "*pony*" truss. Such trusses are necessarily short. If over 100 feet in length they are apt to be deficient in lateral stability. If

the live load traverses the upper or compression chord, it is called a "deck bridge." A bridge consists essentially of two or more trusses placed side by side over the interval to be spanned, and connected together at either top or bottom, or both, by horizontal or lateral trussing, usually of the quadrangular type. The object of this bracing is to support the trusses and stiffen the structure against the action of the wind. The same principles apply to it as to the main trusses, and it is calculated in similar manner. From apex of one truss to apex of the other, floor beams are laid across, upon which the flooring is put.

II. GIRDERS WITH INCLINED FLANGES.

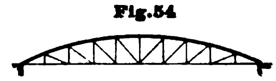
Girders whose flanges are not parallel, are named according to the general shape of truss, rather than the character of bracing adopted. They are used in general where, owing to the length of the span, the height of a girder with horizontal chords would be excessive.

Fig.53

Fig. 53 represents a girder with a curved upper flange and straight lower flange. The bracing is usually of the quadrilateral type. The well known Kuilenberg bridge in Holland is of this class. For long spans there is a saving of material over the girder with parallel flanges. The curve of the upper flange is usually that of a parabola.

BOWSTRING GIRDER.—The bowstring girder, Fig. 54, consists of a curved upper chord, usually parabolic or circular, and straight horizontal lower chord. The bracing may be of any character, generally quadrilateral. It is a very common form, and very well adapted to bridges of long span. It may sometimes be inverted, so that the bottom chord is arched, in which case we may call it the inverted bowstring.

Fig.54



DOUBLE BOW OR LENTICULAR.—The double bowstring or bowstring suspension, or lenticular truss, Fig. 55, consists of two arched flanges, so arranged that the thrust of the one outwards is balanced by the pull of the other inwards. The bracing may be of any sort. The roadway may pass through the centre or be above or below the truss. Of this class are the famous Saltash bridge, and the bridge over the Rhine at Mayence. In Germany, this shape is known as the *Pauli* truss.

Fig.55

The form of the *Pauli* truss is so arranged that the maximum strains in the flanges shall be constant. For this purpose the depth at any point distant x from the end is

$$h = 4h_0 \frac{x}{l}\left(1 - \frac{x}{l}\right)\left[1 + 2\frac{h_0^2}{l^2}\left(1 - 2\frac{x}{l}\right)^2\right],$$

where l is the span and h_0 the depth at centre.

This truss has all the advantages of the double bowstring, and is said to be from 4 to 17 per cent. more economical of material. It is often found in Germany, the most noteworthy example being the Mayence bridge, which consists of four spans of about 345 feet each, and 24 smaller ones of from 52 to 87 and 115 feet.

Fig.56

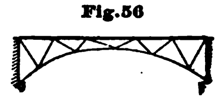


Fig.57

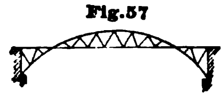


Fig.58

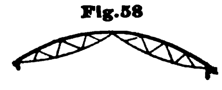
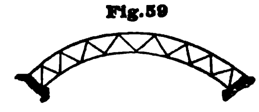


Fig.59



BRACED ARCH.—The braced arch, as its name implies, consists, Fig. 56, of an arched chord stiffened so as to resist the action of the live load by bracing in various ways. The

bracing maybe of either the triangular or quadrilateral types. The system admits of many modifications, as shown in Figs. 57, 58 and 59.

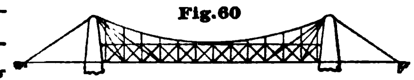
In Fig. 59 we have two parallel arches, braced together. This is the system of the braced arch over the Mississippi at St. Louis.

Many other modifications may be devised. The braced arch may be divided into three kinds in which the distribution of strains are entirely different.

Thus we may have, 1st, the arch hinged or free to turn at the crown and at both ends; 2d, the arch hinged at ends only; 3d, the arch without hinges. The St. Louis arch is of the latter kind.

SUSPENSION SYSTEM.—A very common form of suspension bridge is that shown in Fig. 60. A cable is stretched from towers at either end, over which it passes to anchorages where it is made fast. The office of this cable is to sustain the total load. The system is stiffened by a horizontal truss of ordinary form, and by stays extending out from each tower.

Fig. 60



This is the system of the suspension bridge at Niagara, of the East River bridge at New York, and of many others erected by Roebling. This system also admits of many modifications. Thus Figs. 56, 57, 58, 59, all become suspension systems when inverted.

DOUBLE SYSTEMS—LONG SPANS.—In general, all double systems of bracing are now avoided by good practice, owing to the indeterminate character of the strains, and the difficulty of ensuring that each system shall carry its own share, and no more or less. The Lattice Truss, Fig. 44; Fink Truss, Fig. 45; Post Truss, Fig. 48; Kellogg Truss, Fig. 50; Bollman Truss, Fig. 51, are also antiquated. No more will probably be built in America.

Of the forms remaining, only one system of bracing should be used. The tendency of modern practice is towards long panels, much longer than formerly. The Pratt Truss, Fig. 46, has thus become the standard form for horizontal chords. The Warren is less often used.

When, owing to length of span, the panels would become excessively long, the Baltimore Truss, Fig. 49, or some modification of it, either with or without inclined chords is used.

Thus Fig. 60 (a) is a sketch of one of the spans of the Cincinnati and Covington Bridge, span 484.5 feet; centre depth, 75 feet; depth at ends, 52 feet. It will be observed that the bracing is like the Baltimore Truss; the chords are inclined, and the long compression panels in the top chord are supported at the centre. The length of panel is 27 feet $9\frac{1}{2}$ inches at ends, and 26 feet 8 inches for the others. This is a good illustration of recent practice, long length of panel, large centre depth, inclined chords, and single-system bracing.

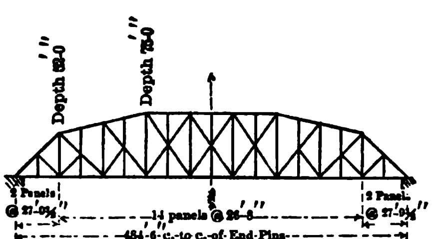


Fig. 60 (a).

CANTILEVER SYSTEM.—The cantilever system counts the longest spans of the day. The Forth Bridge, in Scotland, the longest existing clear span, is of this type. Its longest span is 1,710 feet, the central girder being 350 feet long, and the cantilever arms extending out 680 feet on each side.

The principle of this system is better illustrated by the subjoined cut than by any lengthy description.

This cut was given in the *Engineering News*, June 11, 1887, from the original photograph furnished by Tho. C. Clarke, C. E., and was used by Mr. Benjamin Baker in a lecture on the Forth Bridge, before the Royal Institution.

The sketch represents the Forth Bridge.

The four sticks which form the "compression members" simply abut against the chairs and are grasped by the "tension members" at each end. The "central span" is hung from the inner ends. The outer ends are anchored down.

Object of Section II.—It is not the place here to discuss the relative merits of these different forms, nor the conditions which lead to the adoption of one or the other in any special case. These will be alluded to as we discuss in turn each typical form. The object of the present section of this work, therefore, is to so apply the principles of Section I., to selected cases, as to enable the student to readily determine the strains in any of the preceding structures, or any modification of them. In doing this, we shall have occasion to make such comparisons as shall enable him to appreciate the special advantages of each form.

Questions for Examination.

What is the plan of Section II? Into what two classes can we divide framed structures? What structures belong to the first class? To the second? What is the truss element? Why? When has a structure superfluous pieces? Into what two classes may we divide bridge trusses? What two pure types compose the first class? Sketch the Warren girder. The double triangular. The lattice. The Fink truss. To what type do all these belong? What is the quadrilateral type? What object gives rise to it? What is a counter brace? What is the action of a counter brace? Sketch and distinguish between the Howe and Pratt truss. By what other name is the Pratt truss known? Why is the Pratt truss constructed of iron? Why is the Howe truss constructed of wood and iron? What can you say about screwing up of counter braces? Sketch the double quadrangular truss. Sketch and describe the arrangement of the Post truss. The Baltimore Bridge Co.'s truss. The Kellogg truss. The Bollman truss. What is a continuous girder? What is a deck bridge? A through bridge? Describe the office of lateral bracing. What is a multiple system?

Sketch the MacCallum truss. The bow string girder. The lenticular truss. By what other name is this known. Sketch different forms of braced arch. What three kinds are there? Sketch the suspension system. What modifications does it admit of? What are the objects of the present section of this work?

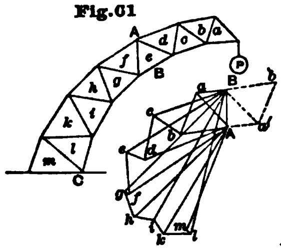
CHAPTER I.

STRUCTURES WHICH SUSTAIN A DEAD LOAD ONLY—ROOF TRUSSES.

THE method which we adopt for all structures of this class is the Graphic method of Section I., Chapter I., checking in every case our results by the calculation of the strains in one or two pieces, by the method of moments of Chapter III., Section I. The method is so simple and general in its application, that but little remains to be added to the remarks of Chapter I, Section I.

BENT CRANE.—In Fig. 61 we have given the frame diagram and strain diagram for a bent crane, bearing the load P at the peak. The notation is the same as on page 11. The student should follow out the strain diagram carefully with reference to determining the proper *character* of the strains in the various pieces. Thus he will observe that the strains in the braces alternate in character up to gh; at this point the strains in gh and hi are of the same character.

Fig. 61



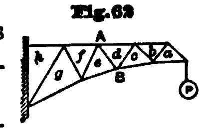
All the lower flanges radiate from B and are in compression. All the upper flanges radiate from A and are in tension. Observe also that the strain diagram could have been laid off equally well upon the right of the weight line, BA, in which case the letters B and A should be interchanged, and all the upper flanges would radiate from the top of the weight line, and the lower flanges from the bottom, as shown by the dotted lines. In general the strain diagram may thus be laid off upon either side of the weight line.

Observe, also, that to obtain accurate results in such a case, the frame should be drawn carefully to as large a scale as possible, as the braces Aa and the flanges Ba and Ab, etc., are very short. It may even be desirable to calculate the slope of these pieces from the given dimensions of the frame, and plot their directions by ordinates, so as to obtain longer lines of direction. The scale for the strain diagrams should, on the other hand, be taken as small as is consistent with reading off the strains to the desired degree of accuracy.

Finally, the strain in Am may be calculated by moments. For this purpose the lever arm of Am, with reference to C, may be calculated or measured directly from the frame to scale.

In all cases the student should determine the character of the strains in each piece as he makes the strain diagram, and not wait until it is completed. When it is all completed the strains may be taken off to scale.

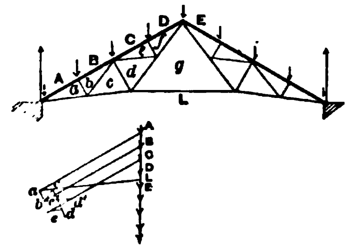
Fig. 62



CANTILEVER CRANE.—In Fig. 62 we have represented a cantilever crane. The same remarks apply as in the preceding case. It is given as an example for the student to solve, in accordance with the preceding remarks.

FRENCH ROOF TRUSS.—In Fig. 63 we have represented what is sometimes known as the "French Roof Truss." The bracing is formed by supporting each principal rafter at its middle point by a strut, *cd*, perpendicular to the rafter, and two tie rods, one from each end. If the half rafter is still too long, we insert the perpendicular struts *ab* and *ef* at the quarter points, as shown in the Figure, and join the ends to the centre by ties *bc* and *de*. This system of trussing is repeated as often as may be requisite. The form is a very common one, and we have chosen it as an illustration because there is in it at the apex *BC* an apparent indeterminacy and violation of our rule, page 5, which is apt to cause trouble to the beginner.

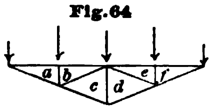
Fig. 63



Thus the student will find no difficulty in constructing the strain diagram and finding the strain as given in the Figure as far as apex *BC*. When we arrive at this apex, however, we have the strains in *Bb* and *bc* already determined, and the known apex weight at *BC*, in equilibrium with *three* unknown strains, *viz.: cd, de*, and *Ce*. The problem would, therefore, seem to be indeterminate. Such it would really be, had we not other conditions which must be satisfied.

Thus, from the symmetry of the truss and loading, the strains in *bc* and *de* must be *equal in amount and the same in character*. We therefore really know *de* also, and there are at the apex *BC* only two unknown forces, *viz.: cd* and *Ce*. This is evident at once when we consider that we may resolve each weight into a component along the rafter and a component perpendicular to the rafter. The first will cause no strains whatever in the bracing. Therefore, so far as the bracing is concerned, we have the rafter subjected to normal forces as shown in Fig. 64. Now all these normal forces are equidistant and symmetrically placed with respect to the centre. It is evident at once, then, that the pieces *bc* and *de*, which are equal in length, and slope equally in opposite directions, must be strained equally.

Fig. 64



If, in the strain diagram of Fig. 63 we draw a line through *c* parallel to *cd*, and a line through *C* parallel to *Ce*, and prolong both indefinitely, then a line parallel to *de* must close the polygon, and must, moreover, be so inserted as to be equal by scale to *bc* in the strain diagram already found. Moreover, it is evident that this line cannot be inserted as shown by the dotted line *e'd'*. Such a line, while it closes the polygon, and is equal by scale to *bc*, would give the strain in *de* of an *opposite character* from that found for *bc*. This the student will easily see by following round the polygon and referring the directions thus obtained to the apex *BC*, according to our rule, page 13.

There is, therefore, in the present case no real indeterminacy. We might also avoid the difficulty by going to the top apex *DE*. Here, since the weight is at the centre span, and the rafters and ties have equal inclinations in each direction, half the weight causes the strains in *Df* and *fg*, and the other half the strains in the corresponding pieces upon the other side.* We can thus begin at the top and work down to the apex *BC*, and thus find *Ce* and *de* directly. Every case of apparent indeterminacy should be susceptible of similar treatment.

CURVED PIECES.—Whenever curved pieces occur, as they sometimes do in roof trusses, the direction of the stress should be taken in the line connecting the two ends. Such pieces are subject to a bending strain as well as to direct compression or tension, and must be proportioned accordingly. The method of proportioning the cross-section of pieces to resist the strains to which they are subjected will be taken up in Part II. of this work. In all such cases as Figs. 61 and 62, the pieces in the curved flanges are always straight, and are chords of the curve in which the apices are situated.

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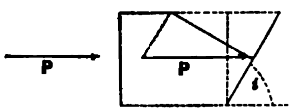
* Note that we should also have to calculate and take into the diagram the horizontal force at *DE*.

WIND FORCES.—Roof trusses have not only to sustain the weight of roofing, snow, etc., but also the pressure caused by wind. This is often, especially in the case of large trusses, placed at considerable intervals apart, very great. The action of the wind, moreover, may often be to cause in certain pieces strains opposite in character to those caused in the same pieces by the dead load. The calculation of the strains caused by wind forces is thus often of considerable importance, and ought never to be left out of account in designing iron roofs of large span.

When a horizontal current of wind strikes against an inclined surface, it is deviated from its original direction and causes a normal pressure upon that surface. This normal pressure, owing to the fluidity of the air, is found to be greater than the normal component of the pressure upon a surface at right angles to the wind.

Thus, Fig. 65, if P is the pressure of the wind in lbs. per sq. ft., upon a surface perpendicular to its direction, the normal component of this pressure upon a surface inclined at the angle i to the horizon, is not $P \sin i$ as it would be by the resolution of forces, but is found by experiment to be given by the experimental formula

Fig. 65



$$P_n = P \sin i^{1.84 \cos i - 1}. *$$

If we take the maximum pressure of the wind against a surface perpendicular to its direction, as 50 lbs. per square foot, we shall probably allow sufficient margin for the heaviest gales in our latitudes. The highest pressures, according to Unwin, do not exceed 55 lbs. per square foot, and the accuracy of these observations is stated by him as "doubtful."

Taking, then, the greatest pressure of wind to be anticipated at 50 lbs. per square foot we have, from our formula, the normal pressure per square foot upon surfaces inclined at various angles to the horizon, as follows:

ANGLE OF ROOF WITH HORIZON.	NORMAL PRESSURE PER LBS.	ANGLE OF ROOF WITH HORIZON.	NORMAL PRESSURE PER LBS.
5°	6.5	45°	45.1
10°	12.1	50°	47.6
15°	17.5	55°	49.5
20°	22.9	60°	50.0
25°	28.1	65°	50.7
30°	33.1	70°	51.5
35°	37.6	80°	50.5
40°	41.7	90°	50.0

From this Table we can find by interpolation the normal pressure upon a roof having any angle of inclination to the horizon, due to a gale of wind which would cause a pressure of 50 lbs. upon a square foot of surface perpendicular to its direction.

Again, in order to find the pressure from the velocity, let v be the velocity of the current in feet per second, and p be the pressure of the current in lbs. per square foot upon a surface perpendicular to its direction. Then, if w is the weight of the fluid in lbs. per cubic foot, we have, according to hydraulic principles,

$$p = 2hw,$$

where h is the "head" due to the velocity, or

$$p = \frac{v^2 w}{g}.$$

* This formula is given by Unwin, "*Iron Bridges and Roofs*," London, 1869, and is by him attributed to Hutton.

Since for ordinary atmospheric air, $w = 0.08$ lbs., approximately,

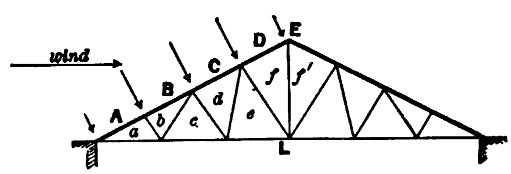
$$p = \frac{0.08}{32} v^2 = \left(\frac{v}{20}\right)^2.$$

If the wind impinges upon a surface oblique to its direction, the intensity of normal pressure is $\left(\frac{v \sin i}{20}\right)^2$; v being the velocity and i the angle of surface with direction of wind. From the formula $p = \left(\frac{v}{20}\right)^2$ we have the following Table:

VELOCITY IN FEET PER SECOND.	VELOCITY IN MILES PER HOUR.	PRESSURE IN LBS. PER SQUARE FOOT.
10	6.8	0.25
20	13.6	1.00
40	27.2	4.00
60	40.8	9.00
70	47.6	12.25
80	54.4	16.00
90	61 2	20.25
100	68.0	25.00
110	74.8	30.25
120	81.6	36.00
130	88.4	42.25
150	102.0	56.25

APPLICATION TO A ROOF TRUSS.—Let Fig. 66 represent a roof truss. Let the span be 50 feet and height 12.5 feet. The length of each rafter is then 27.95 feet. Suppose that the truss supports 8 feet of roofing, that is, that the main trusses, of which Fig. 66 is one, are placed 8 feet apart. Then the area of roof supported by one rafter is $27.95 \times 8 = 223.6$ square feet.

Fig. 66



Now the inclination of the rafter to the horizon is $i = 26° \ 34'$. From our Table, then, we have the normal wind pressure = 29.6 lbs. per square foot. The total normal wind pressure upon one side of the roof due to the wind is, then, $223.6 \times 29.6 = 6{,}619$ lbs. This pressure we divide into four equal parts of 1,655 lbs. each, or say, in round numbers, 1,600 lbs. Let us suppose the wind blowing upon the left side. Then we have a normal pressure of 1,600 lbs. at each of the apices AB, BC and CD, and a normal pressure of 800 lbs. at the left end and top apex. Since all the pressure from the centre of one bay to the centre of the next is supposed to be concentrated at the intermediate apex, we have at the top and bottom apex only half as much pressure as at the other apices.

It remains to determine the reactions. As soon as these are known, we shall know all the outer forces which act upon the truss, and can then proceed to diagram the strains.

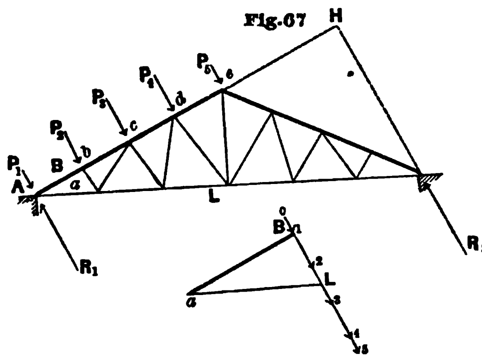
DETERMINATION OF REACTIONS.—The action of the wind, blowing horizontally upon the left, is to slide the entire truss off of its supports. The truss, if it is necessary, should, therefore, be fastened at the ends to the wall. In general, the weight of the truss and its roofing is sufficient to cause friction enough to keep it in place. If not, it can easily be fastened. Large iron trusses may sometimes be put upon friction rollers at one end, so as to allow for changes of temperature, in which case the other end must be fixed.

We have then two cases ; 1st, when both ends are fixed; 2d, when one end only is fixed and the other is upon rollers.

I. REACTIONS WHEN BOTH ENDS ARE FIXED.

In this case, the two reactions are parallel to the normal wind forces, and can easily be calculated. Thus if we take moments about the left end A, Fig. 67, we have the moment of the reaction R_2 balanced by the sum of the moments of the forces, or*

$$R_2 \times AH = P_2 \times Ab + P_3 \times Ac + P_4 \times Ad + P_5 \times Ae.$$



Or we may take the resultant of all the forces acting at the apex c, and therefore

$$R_2 \times AH = (P_1 + P_2 + P_3 + P_4 + P_5) \times Ac.$$

The reaction R_2 at the right end, is thus easily found. Of course, the reaction at the left end is found by subtracting R_2 from the sum of the forces. The reactions being thus known, we now know all the outer forces acting upon the truss. We can, therefore, form the force polygon and then proceed to construct the strain diagram. Thus the force polygon is the line 0 1 2 3 4 5, Fig. 67. The reaction R_2 is the distance $5L$, and the reaction R_1 is the distance $0L$.

II. REACTIONS WHEN ONE END IS FIXED AND THE OTHER ON ROLLERS.

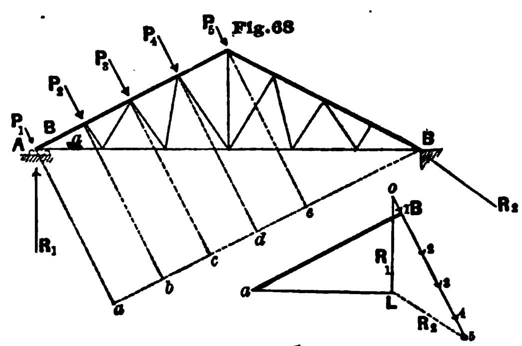
Suppose the left end is on rollers, Fig. 68. Then the reaction at the roller end must be vertical. Since, then, we know its direction, we can easily calculate it by taking moments about the right hand end. Thus

$$R_1 \times AB = P_1 \times Ba + P_2 \times Bb + P_3 \times Bc + P_4 \times Bd + P_5 \times Be.$$

The lever arms Ba, Bb, etc., can be easily found from the given dimensions of the Figure. Or we may consider the resultant of all the forces acting at the apex where P_3 acts. Hence

$$R_1 \times AB = (P_1 + P_2 + P_3 + P_4 + P_5) \times Bc.$$

Having thus found R_1, R_2 may be easily found. Thus, if we lay off the forces P_1, P_2, etc., to scale, as shown in Fig. 68 (a), and then lay off $0L$ vertically to scale equal to R_1 already found, the line $L5$ necessary to close the polygon is the resultant R_2 in magnitude and direction. The force polygon is then closed and we can proceed to form the strain diagram.



But the wind may blow upon the fixed end side, in which case the strains in the pieces may be very different. Instead of supposing the wind to blow on the right side, let us suppose the left end fixed and the right on rollers, Fig. 69. Then the reaction R_1 is inclined and R_2 must be vertical. We can, therefore, easily calculate R_2 by taking moments about the left end A. Thus

* The lever arms AH, Ab, Ac, etc., are understood, of course, to be the *perpendiculars* from A to R_2, P_2, P_3, etc.

$$R_2 \times BA = P_2 \times Ab + P_3 \times Ac + P_4 \times Ad + P_5 \times Ae,$$

Or,

$$R_2 \times BA = (P_1 + P_2 + P_3 + P_4 + P_5) \times Ac.$$

If, then, we lay off the wind forces to scale in Fig. 69(a), and lay off $5L$ vertical and equal to R_2 already calculated, the line Lo necessary to close the polygon is the reaction R_1 in magnitude and direction. We can now proceed to form the strain diagram.

Fig. 69

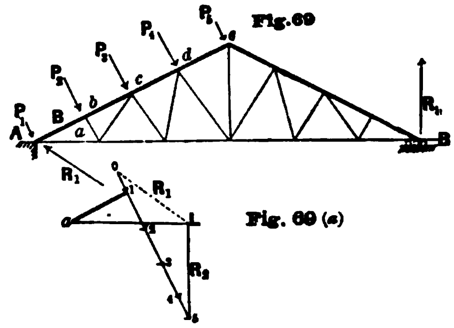


Fig. 69 (a)

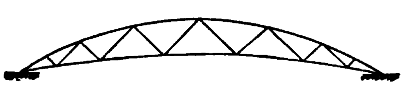
The student can now easily diagram the strains for the three cases of Figs. 67, 68 and 69. In each case we have given the strains in the first two pieces, Ba and La, in order to call attention to the fact that the first half weight P_1 has no influence upon the strains. Thus in Fig. 69 we have acting at the apex A, the reaction R_1 and the force P_1. We have, then, in the force diagram (a), to join 1 and L by lines parallel to Ba and La.

We see at once from Figs. 69 and 68 that the strain in La, for instance, is much greater when the wind blows on the fixed end side than when it blows on the roller end. This is evident, because when it blows on the roller end it tends to shut up the truss and thus relieve the tension in La. If the forces were great enough, we might even have compression in La, that is the intersection a, Fig 68(a), might fall to the right of L.

COMPLETE CALCULATION OF A ROOF TRUSS.—We see then that the complete calculation of a roof truss consists of two parts. 1st. We must find the strains due to the greatest dead load or weight of truss, together with roofing, snow load, etc. 2d. We must find the strains in each piece due to wind force, as already detailed. Here again, in case of rollers, we may have in any piece two strains, according as the wind comes on from right or left. If both these strains are of the same character as the dead load strain, we should add the *greatest* of them to the dead load strain to obtain the greatest strain in the piece. If one of these is of the same character as the dead load strain, we add it. As to the other, if it is less in amount than the dead load strain, it will only tend, when the wind blows, to relieve the strain due to dead load by that amount; but if it is *greater* than the dead load strain, it will cause a strain of opposite character, and the piece should be counterbraced for the difference. If both the wind strains are of different character from that caused by the dead load, we need only consider the greater of the two. If this is less than the dead load strain, it will produce no effect, except sometimes to diminish the strain in the piece. But if it is greater, it will cause strain of an opposite character, and the piece should be counterbraced for the difference. In all cases the wind has great influence upon the strains, and it should always be taken into account in the designing of large spans.

CURVED ROOFS.—For a curved roof, such as Fig. 70, the inclination of the surface exposed to the wind is different at every apex, and is always to be taken as tangent to the curve at each apex. In such a case as Fig. 70, it may often happen that the wind causes strains in certain pieces opposite in character to those caused by the dead load. We give in Plates 1 to 7 a large number of roofs of various kinds, with their strain diagrams.* For the sake of generality, acting forces and reactions are often taken as inclined. The student cannot do better than to follow out the strain diagrams for a number of cases, until he feels himself thoroughly master of the method, and can *determine the character of the strains* in each case.

Fig. 70



* These Plates are copied from "*Economics of Construction*," Bow, London, 1873.

PLATE I.

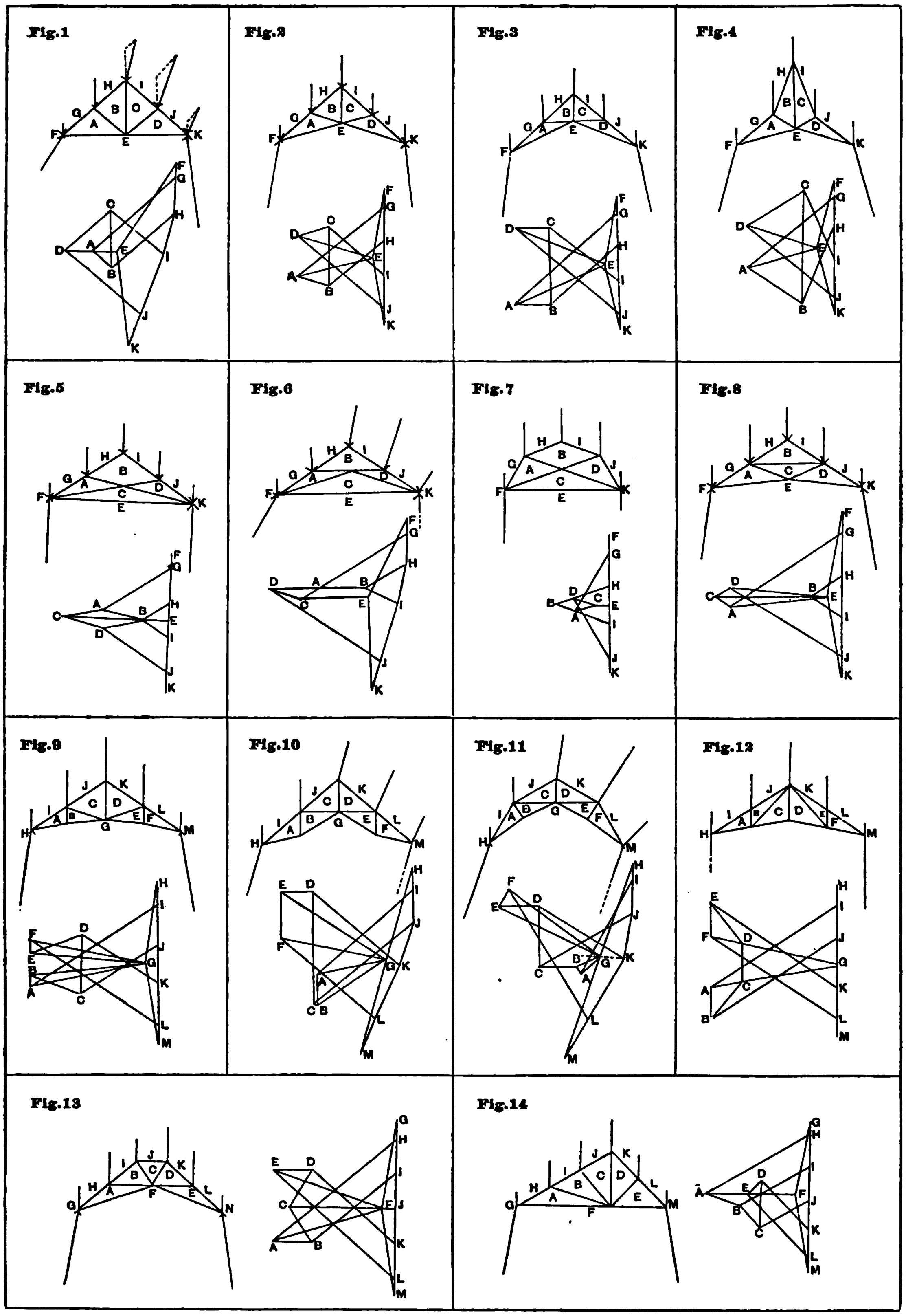


Fig.1
Fig.2
Fig.3
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PLATE 2.

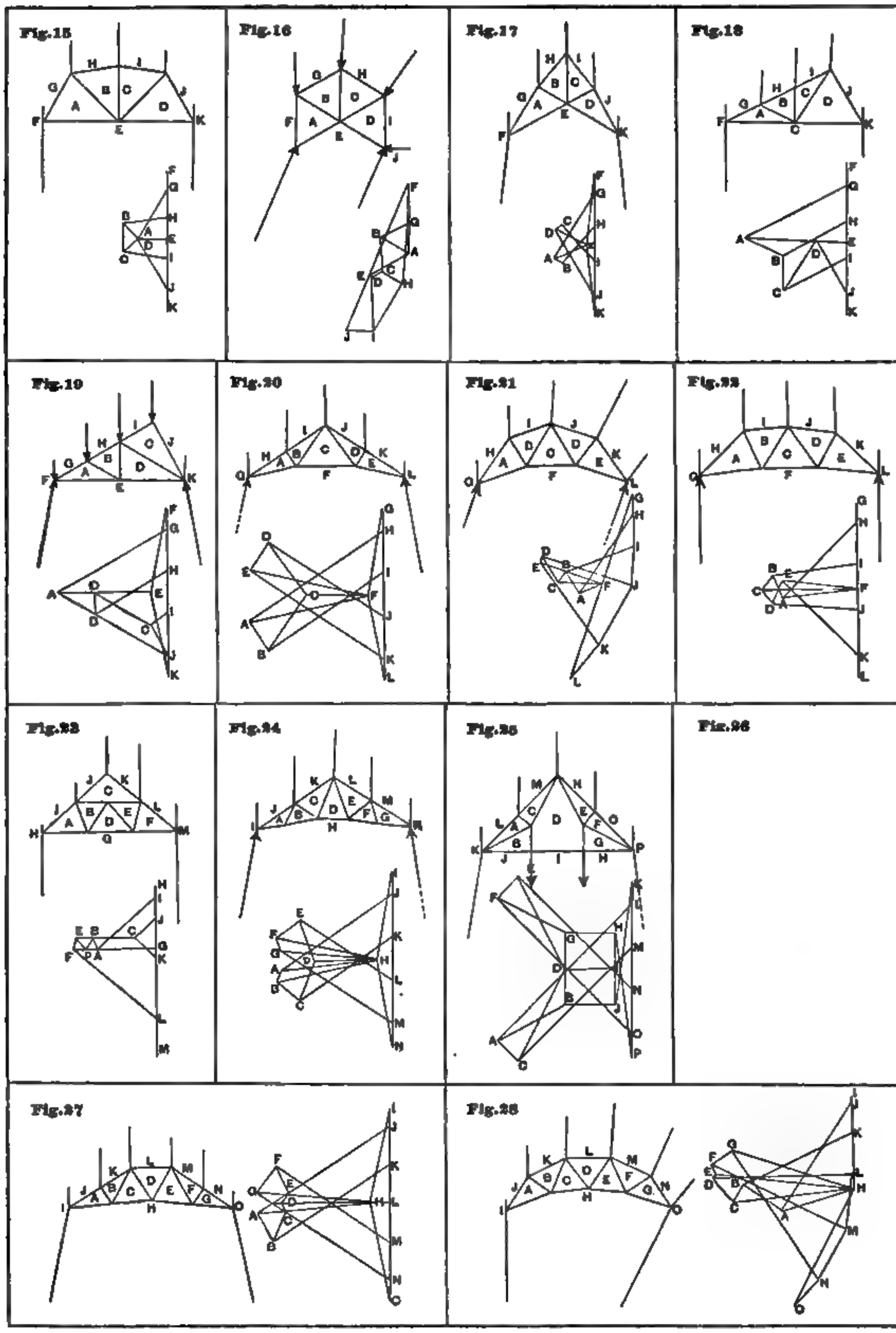
Fig.15
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Fig.28

PLATE 3.

Fig. 29

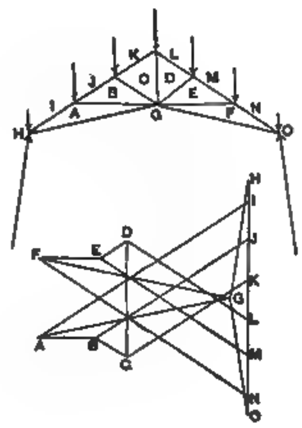


Fig. 30

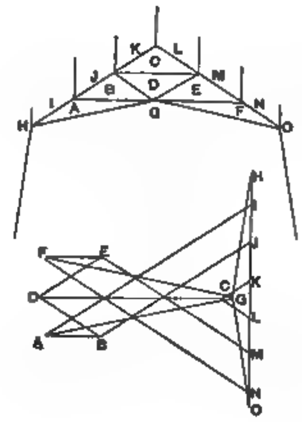


Fig. 31

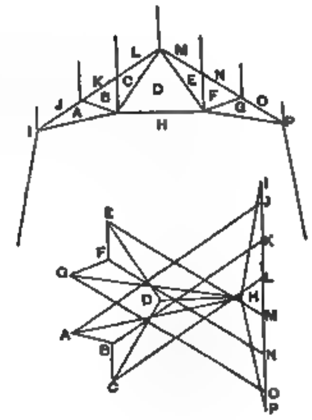


Fig. 32

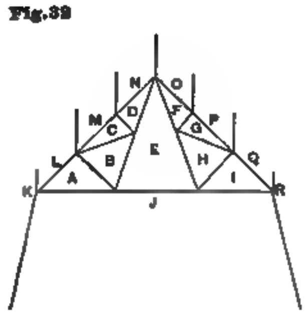


Fig. 33

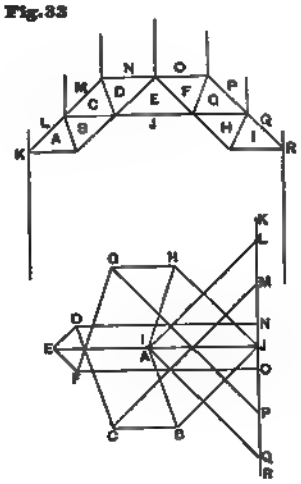


Fig. 34

Fig. 35

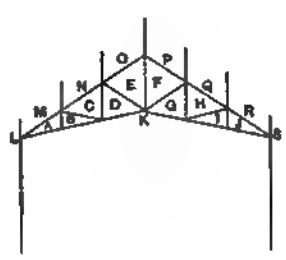


Fig. 36

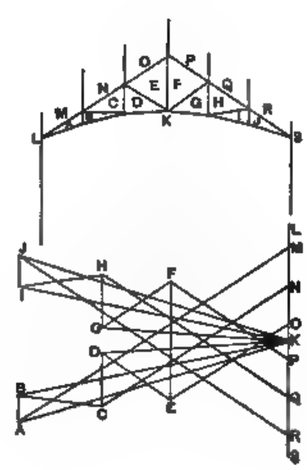


Fig. 37

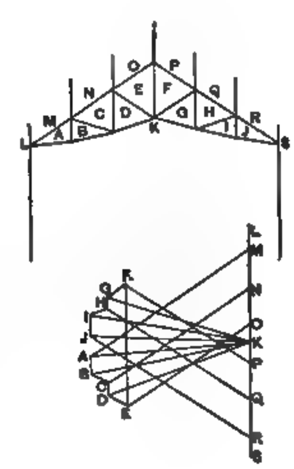


PLATE 4.

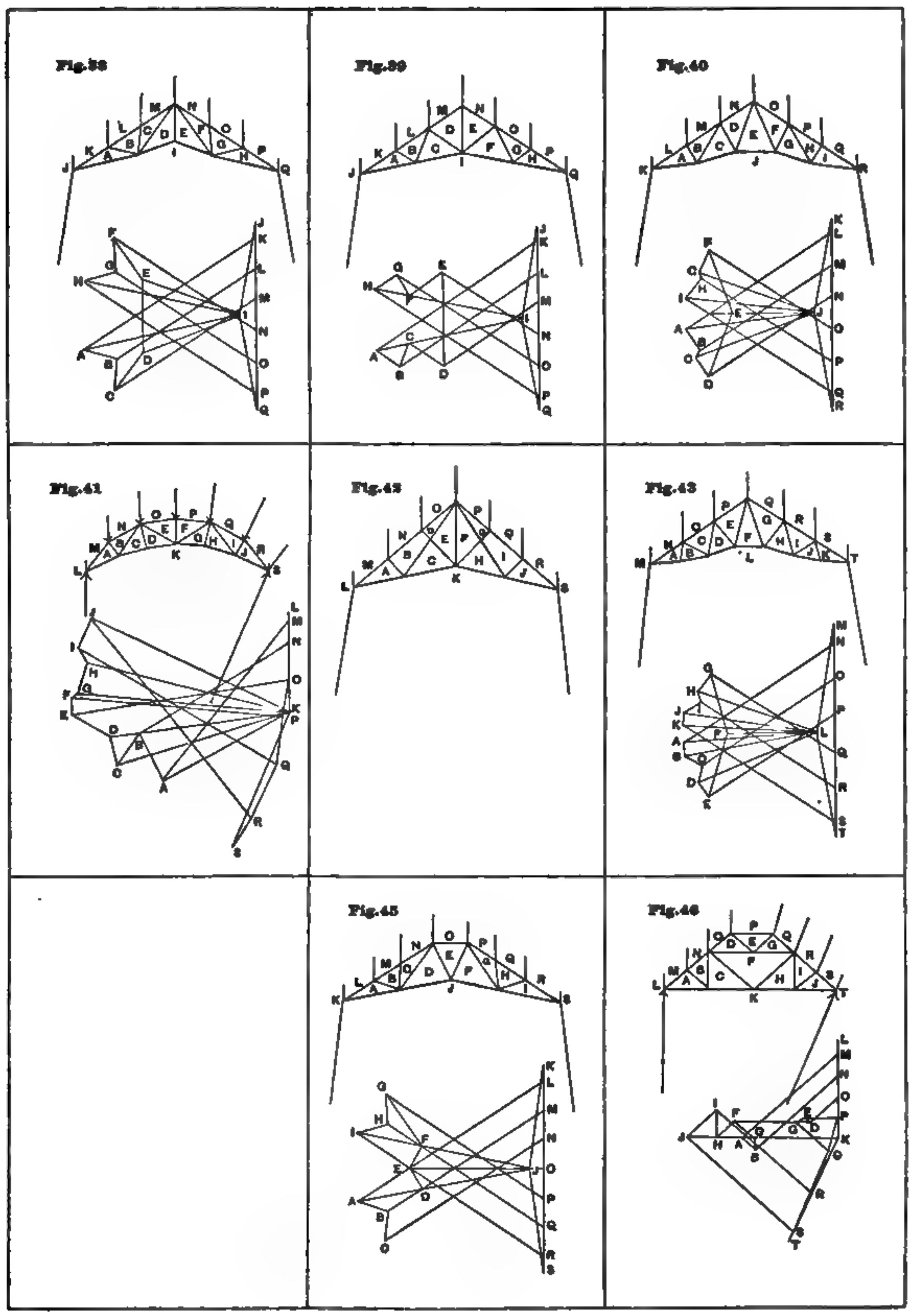


PLATE 5.

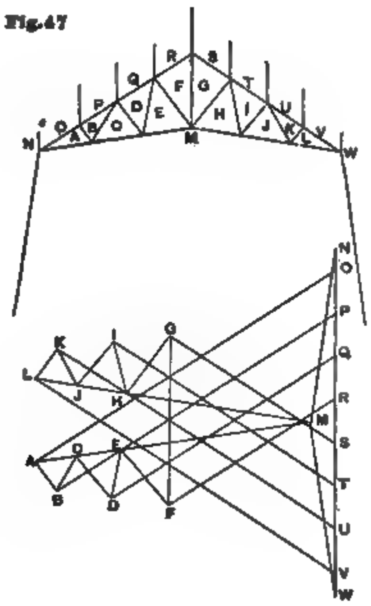


Fig.47

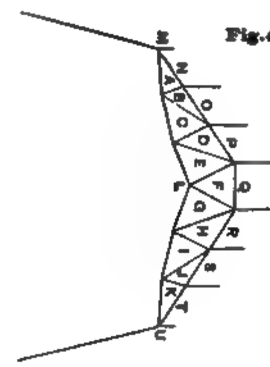


Fig.48

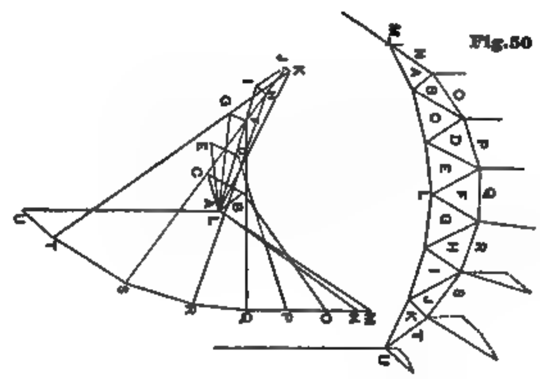


Fig.50

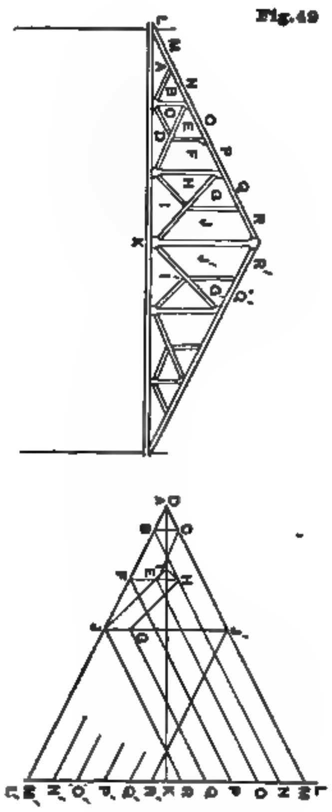


Fig.49

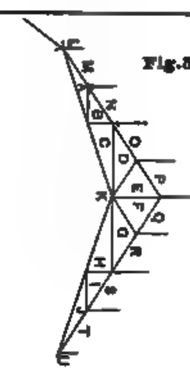


Fig.51

PLATE 6.

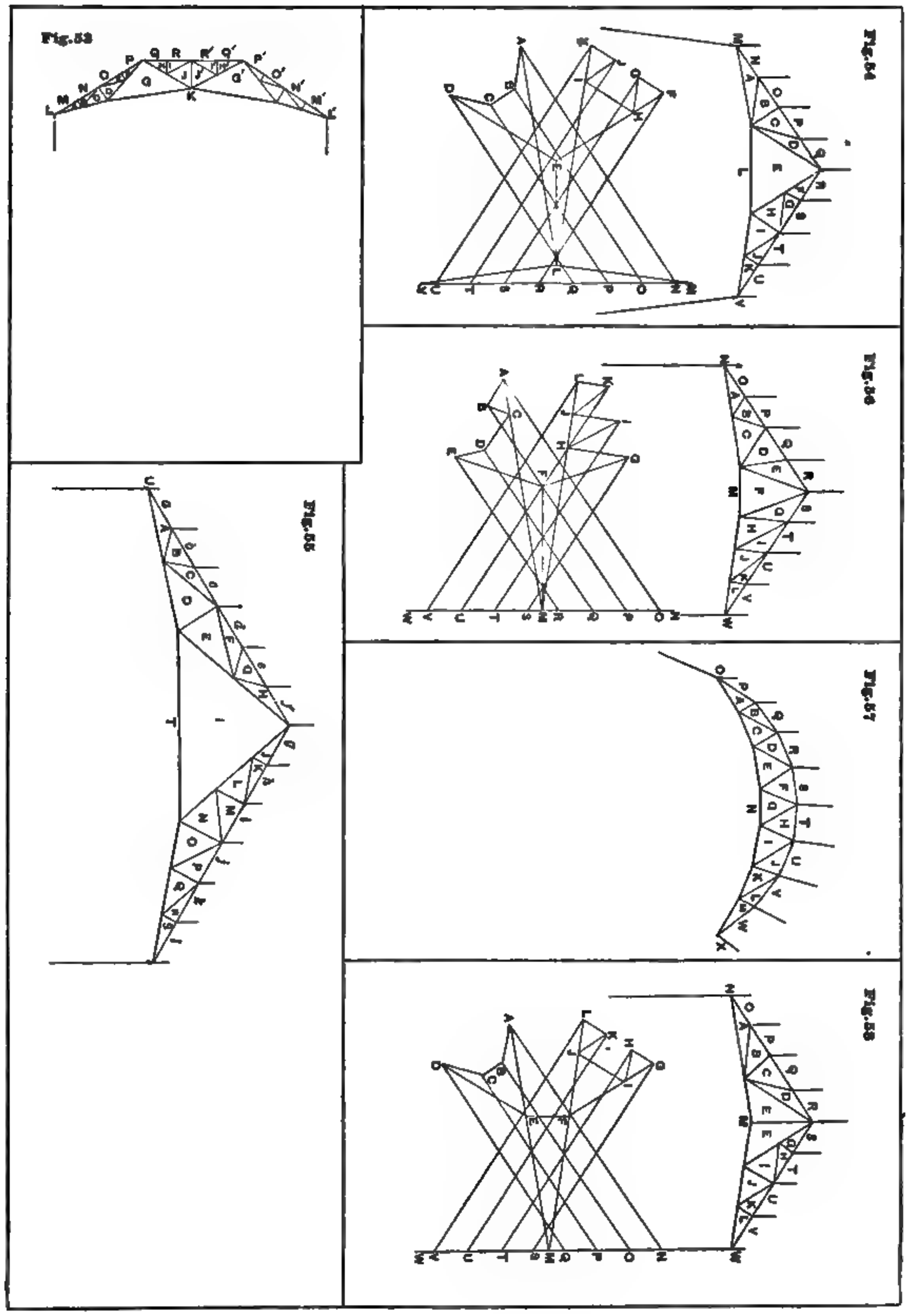


Fig.59

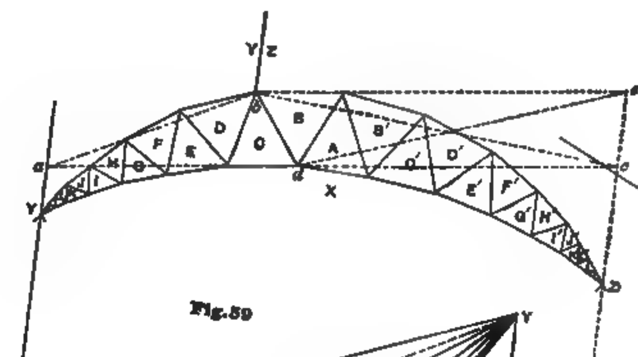


Fig.59

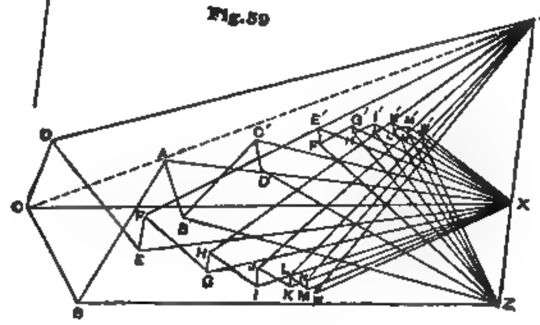


Fig.60

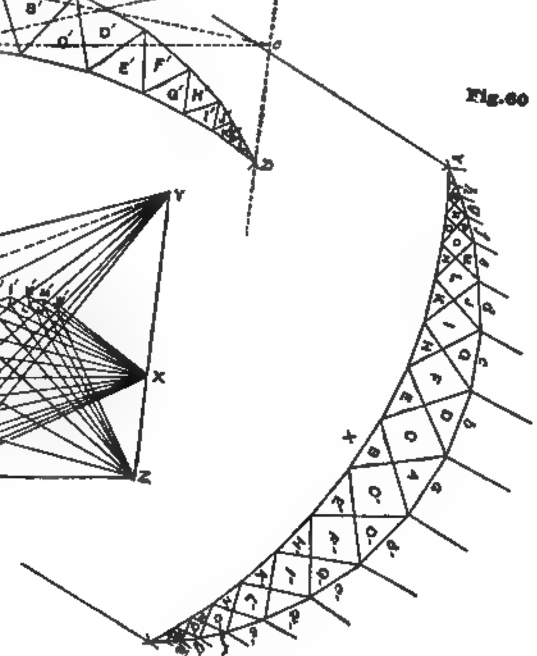


Fig.61

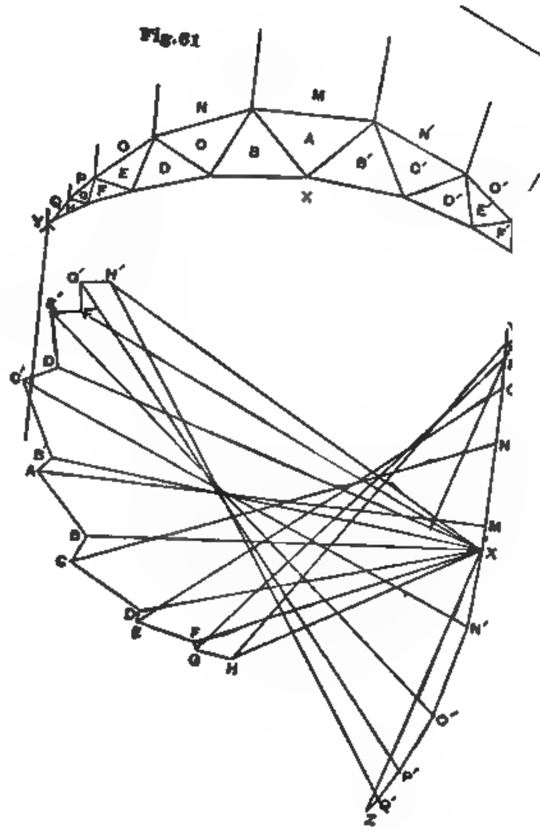


Fig.60

QUESTIONS FOR EXAMINATION.

What is the method adopted for the structures treated of in the preceding chapter ? What class of structures are there discussed? What points are illustrated by Fig. 61 ? What considerations should determine the choice of scales in such a case ? Show how to apply the method of moments. In Fig. 63, what apparent difficulty is there and how may it be overcome ? How are curved pieces to be treated ?

What is the effect and action of wind ? Show the application to a roof truss. What two cases may we distinguish ? Show how to determine the reactions for each case. Show how to lay off the force polygon and how to form the strain diagram for each case. State and illustrate the method for complete calculation of a roof truss. Show how the method is applied to curved roofs.

EXAMPLES.*

1. If the pole of an equilibrium polygon describe a straight line, show that the corresponding sides of the successive equilibrium polygons will intersect in a straight line which is parallel to the locus of the pole.

2. A system of heavy bars, freely articulated, is suspended from two fixed points ; determine the magnitudes and directions of the stresses at the joints. If the bars are all of equal weight and length, show that the tangents of the angles which successive bars make with the horizontal are in arithmetic progression.

3. If an even number of bars of equal length and weight rest in equilibrium in the form of an arch, and $a_1, a_2, \ldots \, a_n$, be the respective angles of inclination to the horizon of the 1st, 2nd, ... nth bars counting from the top, show that $\tan a_{n+1} = \frac{2n+1}{2n-1} \tan a_n$.

4. Four bars of equal weight and length, freely articulated at the extremities, form a square $ABCD$. The system rests in a vertical plane, the joint A being fixed, and the form of the square is preserved by means of a horizontal string connecting the points B and D. If W be the weight of each bar, show, 1st, that the stress at C is horizontal and $= \frac{W}{2}$; 2d. That the stress on BC at B is $W\frac{\sqrt{5}}{2}$ and makes with the vertical an angle $\tan^{-1}\frac{1}{2}$; 3d. That the stress on AB at B is $W\frac{\sqrt{13}}{2}$ and makes with the vertical an angle $\tan^{-1}\frac{3}{2}$. 4th. That the stress upon AB at A is $\frac{5}{2}W$; 5th. That the tension of the string is $2W$.

5. Five bars of equal length and weight, freely articulated at the extremities, form a regular pentagon $ABCDE$. The system rests in a vertical plane, the bar CD being fixed in a horizontal position, and the form of the pentagon being preserved by means of a string connecting the joints B and E. If the weight of each bar be W, show that the tension of the string is $\frac{W}{2}(\tan 54° + 3 \tan 18°)$, and find magnitudes and directions of the stresses at the joints.

6. Six bars of equal length and weight ($= W$), freely articulated at the extremities, form a regular hexagon.

First, if the system hang in a vertical plane, the bar AB being fixed in a horizontal position, and the form of the hexagon being preserved by means of a string connecting the middle points of AB and DE, show that, 1st, the tension of the string is $3W$; 2d., the stress at C is $\frac{W}{2\sqrt{3}}$ and horizontal ; 3d., the stress at D is $W\sqrt{\frac{13}{12}}$ and makes with the vertical an angle $\tan^{-1} 2\sqrt{3}$.

Second, if the system rest in a vertical plane, the bar DE being fixed in a horizontal position, and the form of the hexagon be preserved by means of a string connecting the joints C and F, show that, 1st., the tension of the string is $W\sqrt{3}$; 2d., the stress at C is $W\sqrt{\frac{31}{3}}$ and makes with CB an angle $\sin^{-1}\sqrt{\frac{3}{124}}$; 3d., the stress at B is $W\sqrt{\frac{7}{12}}$ and makes with CD an angle $\sin^{-1}\sqrt{\frac{3}{28}}$.

Third, if the system hang in a vertical plane, the joint A being fixed, and the form of the hexagon be preserved by strings connecting A with the joints E, D and C, show that, 1st., the tension of each of the

* The following Examples are taken from "*Applied Mechanics*," by Prof. Henry T. Bovey, Montreal, 1882. It is believed that the intelligent student can solve them by an independent application of preceding principles.

strings AE and AC is $W\sqrt{3}$; 2d., the tension of the string AD is $2W$, and determine the magnitudes and directions of the stresses at the joints, assuming that the strings are connected with pins distinct from the bars.

7. Show that the stresses at C and F, in the first case of Ex. 6, remain horizontal when the bars AF, FE, BC, CD, are replaced by any others, which are all equally inclined to the horizon.

8. An ordinary jib-crane is required to lift a weight of 10 tons at a horizontal distance of 6 ft. from the axis of the post. The post is a hollow cast-iron cylinder of 10 ins. external diam.; find its thickness, assuming the safe tensile and compressive stress to be 3 tons per sq. in.

The hanging part of the chain is in *four* falls; the jib is 15 ft. long, and the top of the post is 12 ft. above ground; find the stresses in the jib and tie when the chain passes, (1)—along the jib; (2)—along the tie.

The post turns round a vertical axis; find the direction and magnitude of the pressure at the tie, which is three feet below the ground.

9. If the post in the preceding question were replaced by a solid cylindrical wrought-iron post, what should its diam. be; the safe inch-stress being 3 tons as before?

10. The horizontal traces of the two back-stays of a derrick-crane are x and y feet in length, and the angle between them is a; show that the stress in the post is a maximum when $\frac{\cos(a-\theta)}{\cos\theta} = \frac{x}{y}$, θ being the angle between the trace x and the plane of the jib and tie.

11. The inner flange of a bent crane (Fig. 61, page 62,) forms a quadrant of a circle of 20 ft. radius, and is divided into *four* equal bays. The *outer* flange forms the segment of a circle of 23 ft. radius. The two flanges are 5 ft. apart at the foot, and are struck from centres in the same horizontal line. The bracing consists of a series of isosceles triangles, of which the bases are the equal bays of the inner flange. The crane is required to lift a weight of 10 tons. Determine the stresses in all the members.

12. A braced semi-arch is 10 ft. deep at the wall, and projects 40 ft. The upper flange is horizontal, is divided into *four* equal bays, and carries a uniformly distributed load of 40 tons. The lower flange forms the segment of a circle of 104 ft. radius. The bracing consists of a series of isosceles triangles, of which the bases are the equal bays of the upper flange. Determine the stresses on all the members.

13. Three bars, freely articulated, form the equilateral triangle ABC. The system rests in a vertical plane upon the supports B and C in the same horizontal line, and a weight, W, is suspended from A. Determine the stress in BC, neglecting the weight of the bars.

14. A triangular truss of white pine consists of two equal rafters, AB, AC, and a tie beam BC; the span of the truss is 30 ft., and its rise is $7\frac{1}{2}$ ft.; the uniformly distributed load upon each rafter is 8,400 lbs., and a weight of 10,000 lbs. is suspended from the centre of the tie-beam. Determine the dimensions of the rafters and tie-beam, assuming the safe tensile and compressive inch stresses to be 3,300 and 2,700 lbs., respectively.

15. A triangular truss consists of two equal rafters, AB, AC, and a tie beam BC, all of white pine; the centre D of the tie-beam is supported from A by a wrought-iron rod AD; the uniformly distributed load upon each rafter is 8,400 lbs., and upon the tie-beam is 36,000 lbs. Determine the dimensions of the different members, BC being 40 ft. and AD 20 ft.

What will be the effect upon the several members if the centre of the tie-beam be supported upon a wall, and if for the rod a post be substituted, against which the heads of the rafters can rest?

16. A triangular truss of white pine consists of a rafter AC, a vertical post AB, and a horizontal tie-beam AC; the load upon the rafter is 300 lbs. per lineal ft.; $AC = 30$ ft., $AB = 6$ ft. Find the resultant pressure at C.

How much strength will be gained if the centre of the rafter be supported by a strut from B?

17. The rafters of a roof are 20 ft. long, and inclined to the vertical at 60°; the feet of the rafters are tied by two rods, which meet under the vertex, and are tied to it by a rod 5 ft. long; the roof is loaded with a weight of 3,500 lbs. at the vertex. Determine the stresses in all the members.

18. The feet of the equal roof rafters AB, AC, are tied by rods BD, CD, which meet under the vertex and are joined to it by a rod AD. If W and W' are the distributed loads in lbs. upon the rafters, and if S is the span of the roof in feet, show that the weight of metal in the ties in lbs. is $\frac{5}{6}\frac{W+W'}{f} S \cot. \beta$, f being the safe inch stress in lbs., and β the angle ABD.

19. A roof truss consists of two equal rafters inclined at 60° to the vertical, of a horizontal tie-beam of length l, of a collar beam of length $\frac{l}{3}$, and of a queen-post at each end of the collar-beam; the truss is loaded with

a weight of 2,600 lbs. at the vertex, a weight of 4,000 lbs. at one collar-beam joint, a weight of 1,200 lbs. at the other, and a weight of 13,500 lbs. at the foot of each queen. Determine the stresses in the members.

20. A frame is composed of a horizontal top-beam 40 ft. long, two vertical struts 3 ft. long, and three tie-rods, of which the middle one is horizontal and 15 ft. long. Find the greatest stress produced in the several members when a single load of 12,000 lbs. passes over the truss.

21. An equilibrium polygon is drawn for a series of parallel loads at given distances. Show that, 1st. By properly drawing the closing line of the polygon a bending moment curve is obtained which corresponds to any position of the series of loads on any given beam; 2d. So long as the closing line lies on the same two polygon sides, its positions for any given beam form the envelope of a parabola; 3d. The centre of the beam corresponding to a given closing line bisects the distance between the verticals through the intersection of the polygon sides, and the point where the closing line touches the parabola.

22. Vertical loads of 4, 3, 7, and 2 tons are concentrated upon a horizontal beam of 20 ft. span, at distances of 3, 7, 12, and 15 ft., respectively, from the left support. Prove generally that the vertical ordinate intercepted between a point in the corresponding equilibrium polygon and a closing line whose horizontal projection is the span of the beam. represents on a certain determined scale the bending moment of a section at any point. Find its value by scale measurement for a section at 9 ft. from the left support, using the following scale: For *lengths*, $\frac{1}{4}$ inch = 1 foot; for forces, $\frac{1}{4}$ inch = 1 ton; the polar distance = 5 tons. Determine graphically, by means of the same diagram, the greatest bending moment that can be produced on the same section by the same series of loads traveling over the span at the stated distances apart.

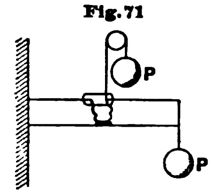
CHAPTER II.

STRUCTURES WHICH SUSTAIN A LIVE AS WELL AS A DEAD LOAD—BRIDGE TRUSSES.

GENERAL PRINCIPLES.

SHEAR—DEFINITION OF.—Let Fig. 71 represent a beam fixed horizontally at one end and sustaining a load P, at the other. Imagine the beam cut completely in two at any point, and then consider what forces are necessary in order that the separated portion may still retain its place and perform its duty. We know that before the section was made all the fibres above the neutral axis were extended, and all below were compressed. We can replace these forces by a link above and a strut or compression piece below, as shown in Fig. 71. But these alone are not sufficient. The link and strut prevent the right hand portion from turning about the section under the action of the weight P, and that is all. In order to prevent the right hand portion from falling vertically we must apply an upward force at the section equal and opposed to P, as shown in Fig. 71. The weight P, we call the "*shearing force*," and the equal and opposite force at the section, the "*resistance to shear*," or "shear."

Fig. 71

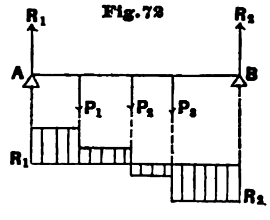


The shearing force is so called, because its action is to cause one section to slide upon the next, just as if the separation were effected by cutting with a pair of shears.

We may then define shearing force, as *that force which at any section tends to make that section slide upon the one immediately following.*

Thus, in Fig. 72, which represents a horizontal beam, AB, subjected to the action of outer forces R_1, P_1, P_2, P_3, R_2, the shear at any section between the left end and P_1 is constant and equal to the reaction R_1. R_1 acting up at the left end tends to slide each section upon the next, until we come to P_1. Here we have R_1 still tending to slide the section up, but P_1 tends to slide it down. The difference or algebraic sum is then the shear for any section between P_1 and P_2. Thus the ordinates to the shaded area below, give to scale the shear at any point of the beam AB.

Fig. 72



When, therefore, the section is vertical, and the outer forces all vertical, we may define the shearing force as *the algebraic sum of all the outer forces acting upon the beam, right or left of the section in question.*

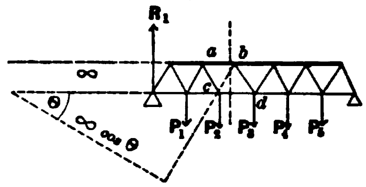
If any of the outer forces are not vertical, we must resolve them into components parallel and perpendicular to the vertical section, and take the former in the algebraic sum.

In general, to find the shear in a section taken in any direction, resolve all the outer

forces into components, perpendicular and parallel to the plane of the section, and the algebraic sum of all the latter, right or left of the section, will be the shear in that section, or the force tending to slide it upon the next consecutive section.

FRAMED GIRDER—HORIZONTAL FLANGES—SHEAR.—If in any framed structure we conceive a section dividing the structure into two portions, it is evident from the above that the strains in the cut pieces must hold the shear in equilibrium. This principle we have already proved in Chapter II., page 21. Thus in Fig. 73, conceive a section cutting *ab*, *bc* and *cd*. Then the strains in these three pieces must hold the shear in equilibrium.

Fig. 73



The shear in the present case is the algebraic sum of all the outer forces left or right of the section, because the section and forces are all vertical. In taking the algebraic sum we adhere to the conventions of Chapter II., page 16, and take, therefore, upward forces as positive, and downward forces as negative, and consider always only that portion of the truss *on the left of the section*. This must be carefully noted, for if we took the right hand portion, our conventions should be reversed. The shear, then, in the present case, is $+ R_1 - P_1 - P_2$, and the strains in the cut pieces *ab*, *bc*, and *cd*, must hold this shear in equilibrium.

But if the flanges are horizontal, as in Fig. 73, the vertical components of their strain is zero. That is, they cannot take any part in resisting the shear or tranverse action of the forces, and simply answer the purpose of the link and strut in Fig. 71. The brace *bc* must then resist the shear, and hence the vertical component of its strain must be equal and opposed to the shear.

Thus according to the conventions of Chapter II., page 16, that is, taking compression as plus, and tension as minus, and measuring the angle θ made by any brace with the vertical always around from right to left, as shown in Fig. 9, page 16, and considering always the left hand portion of the truss,

$$\text{strain in } bc \times - \cos \theta_{bc} + R_1 - P_1 - P_2 = 0,$$

or

$$\text{strain in } bc = \frac{-(+ R_1 - P_1 - P_2)}{- \cos \theta_{bc}} = \text{shear} \times \sec \theta_{bc}.$$

That is, for horizontal flanges and vertical loads, *the strain in any brace is equal to the shear multiplied by the secant of the angle which the brace makes with the vertical.*

The angle θ should always be measured, as directed in Fig. 9, page 16. There may arise some uncertainty as regards the proper sign for this angle θ. Thus, Fig. 73, if we measure the angle θ round from the vertical through c, the sec. of θ is minus, but if we measure from the vertical through b, the sec. of θ is plus. This uncertainty will be removed if we remember that since we are considering only the left hand portion of the truss, *we must always measure the angle θ for any brace, from the vertical through that end of the brace* BELONGING TO THE LEFT HAND PORTION.

Thus in the present case, for instance, the sec. of θ for *bc* is essentially minus, because measured as above it lies in the second quadrant. (See Fig. 9, page 16.) If all the weights are equal and equidistant, R_1 will be greater than $P_1 + P_2$, and the shear will be plus. We shall have then the strain in *bc*, essentially plus, denoting that *bc* is in compression.

In like manner, for the brace *bd*, the shear would be the same as for *bc*, but as the angle θ is to be measured from vertical through b the left hand end, the sec. of θ for *bd* will be plus.

We have then,

$$\text{strain in } bd \times \cos\theta_{bd} + R_1 - P_1 - P_2 = 0;$$

or

$$\text{strain in } bd = -\text{ shear} \times \text{sec. } \theta_{bc}.$$

We see at once that the strain in bd will be tension, and equal in amount to the compression just found for bc.

We can easily deduce the same result directly from the principle of moments, Chapter III., page 23. Thus the flanges ab and cd, Fig. 73, being parallel, their point of intersection is at an infinite distance. Taking this point as a centre of moments, we have for the lever arm of bc, $\infty \cos\theta$. The lever arms of R_1, P_1 and P_2 are each ∞. Then according to our rule, Chapter III., page 27,

$$\text{strain in } bc \times \infty \cos\theta_{bc} - R_1 \infty + P_1 \infty + P_2 \infty = 0;$$

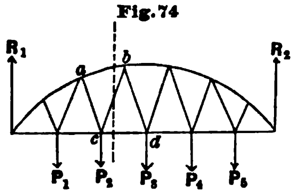
or

$$\text{strain in } bc = \frac{(+R_1 - P_1 - P_2)\,\infty}{\infty \cos\theta_{bc}} = \text{shear} \times \text{sec. } \theta_{bc}.$$

RESULTANT SHEAR.—If the flanges are *not* horizontal, they will themselves take some of the shear, and only what is left is to be resisted by the braces. This remainder we call the "*resultant shear.*"

Thus, for instance, Fig. 74, if we take a section cutting ab, bc and cd, the strains in these pieces are in equilibrium with the shear.

Fig. 74



The shear is from the preceding, $R_1 - P_1 - P_2$. But the flange ab resists a portion of this shear equal to the vertical component of the strain in it. The flange cd being in the present case horizontal, has no vertical component. The flange strains can always be found by moments independently of the braces. Let us suppose ab to be thus found, and to be compression. The vertical component of its strain is then,

$$\text{strain in } ab \times \cos\theta_{ab}.$$

The angle θ is to be measured as already noticed, always from the vertical *through the left end* of the piece, as directed in Fig. 9, page 16. We have then for the strain in cb,

$$\text{strain in } cb \times -\cos\theta_{cb} + R_1 - P_1 - P_2 - ab\cos\theta_{ab} = 0;$$

or

$$\text{strain in } cb = \frac{-(+R_1 - P_1 - P_2 - ab\cos\theta_{ab})}{-\cos\theta_{cb}} = \text{resultant shear} \times \text{sec. } \theta_{cb}.$$

That is, *the strain in any brace is equal to the* RESULTANT SHEAR *multiplied by the secant of the angle which the brace makes with the vertical.*

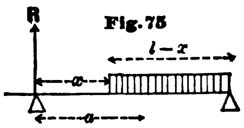
ACTION OF LIVE LOAD.—When a structure is designed to resist the action not only of a constant dead load, but also of a moving or live load, which may have various positions, it is evident that the strains in the pieces will vary according to the position of the live load. It is of great importance, therefore, to determine what position of the live load gives the greatest strain in any particular piece. Comparing, then, this greatest strain due to live load with the strain in the same piece due to dead load, if they are both of the same kind, the total maximum strain in the piece will be the sum of both. If they are of differ-

ent kinds, and the live load strain *exceeds* the dead load strain, the pieces must be counterbraced for the difference. But if the dead load strain is greatest, no counterbracing is necessary, because the action of the live load then is simply to relieve the strained piece of a certain amount of its dead load strain.

The live load may also often cause in the same piece strains of different kinds, sometimes compression and sometimes tension, according to its position.

DISTRIBUTION OF UNIFORM LIVE LOAD CAUSING MAXIMUM FLANGE STRAINS.—In any properly framed structure, such as we shall discuss hereafter, we can always divide the frame by a section at any point, cutting one brace and two flanges. Taking, then, the point of moments at the intersection of the other two pieces cut, we have the moment of the strain in the flange balanced by the sum of the moments of the outer forces right or left of the section. Since, then, the lever arm for the flange is constant, the strain in any flange will be greatest when the live load is so disposed as to give the greatest moment possible. It is required, then, to find that distribution of load which makes the moment at any point a maximum.

Fig. 75



This is easily found for a uniform load. Thus, in Fig. 75, suppose we have a uniformly distributed moving load of w lbs. per unit of length, coming on from the right. Let it cover the distance $l-x$, the end of the load being at a distance x from the left end. Then, for the reaction R at the left end, we have by moments,

$$Rl = w(l-x) \times \frac{l-x}{2},$$

because the weight $w\,(l-x)$ of the loaded portion can be considered as concentrated at the middle point of the loaded portion (Chapter III., page 25).

The reaction at the left end is, therefore,

$$R = \frac{w\,(l-x)^2}{2l}.$$

The moment at any point distant a from the left end, if a is greater than x, is

$$M_a = Ra - \frac{w\,(a-x)^2}{2}.$$

Substituting the value of R above,

$$M_a = \frac{wa\,(l-x)^2}{2l} - \frac{w\,(a-x)^2}{2}.$$

If we suppose x constant, and differentiate with respect to a, and put the first differential equal to zero, we have

$$R - w\,(a-x) = 0.$$

That is, *for any given position of the load, the moment is greatest at that point for which the shear is zero.*

But we can put the preceding equation after easy reduction in the form

$$M_a = \frac{wa\,(l-a)}{2} - \frac{wx^2\,(l-a)}{2l}.$$

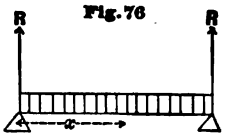
We see at once from this equation that for any given values of a and l, the moment

will be greatest when $x = 0$. That is, *the moment at any point is the greatest possible when the load covers the whole span.*

No special discussion, therefore, is necessary in order to find the methods of loading which give the greatest strains in the flanges for uniform load. We have only to suppose the live load to cover the whole span, just like the dead load. The greatest strains in the flanges will then be found when we suppose the girder fully loaded with both dead and live loads. *This holds good whether the flanges are parallel or inclined, provided the girder is a simple girder*, i. e., *not continuous over more than two supports, and whether the girder is framed or is a solid beam.*

GRAPHIC INTERPRETATION OF EQUATION FOR MAXIMUM MOMENTS.—From the preceding principle we can easily find the maximum moment at any point. Thus, let the moving load per unit of length be w and the dead load w'. Then the total load is $(w' + w) l$. The reaction at each end is, therefore, $\frac{(w' + w) l}{2}$, and the maximum moment at any point distant x from the left end is,

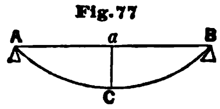
Fig. 76



$$M_{Max} = + \frac{(w' + w) l}{2} x - (w' + w) \frac{x^2}{2}.$$

This is the equation of a parabola, Fig. 77, whose middle ordinate at the centre of the span $aC = + (w' + w) \frac{l^2}{8}$, which passes through the ends of the girder A and B, and has its vertex at C. The same result has been already obtained in Chapter IV., page 45. If, therefore, we draw a parabola through A and B, whose middle ordinate aC is by scale $+ (w' + w) \frac{l^2}{8}$, the ordinates to this parabola will give the maximum moments at any other point of the beam.

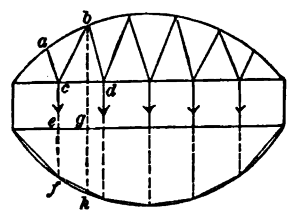
Fig. 77



APPLICATION TO A FRAMED GIRDER.—In the case of a framed girder, Fig. 78, the load consists of a succession of concentrated apex loads, and the parabola becomes a polygon whose apices *are at the intersections of the weights with the curve.*

To find the greatest strain in ab, Fig. 78, we first locate the point of moments at c (Chapter III., page 27). Then the ordinate ef gives the moment at c. This moment, divided by the lever arm for ab, gives the strain in ab. In order to obtain the strain with its proper sign, plus for compression and minus for tension, observe the rule for the sign of the lever arm, Chapter III., page 27, and remember that the moments are all positive.

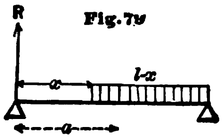
Fig. 78



Again, for the strain in the flange cd, the point of moments is at b. From *this* point, then, we drop the ordinate gh, to the polygon. The moment is given to scale by gh. Generally, we draw the ordinate *through the point of moments for the flanges in question.*

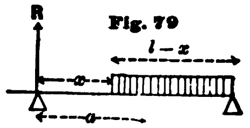
DISTRIBUTION OF UNIFORM LIVE LOAD CAUSING MAXIMUM SHEAR.—The position of a uniform live load, in order to give the greatest shear at any point, is different according as the girder is solid or framed, and in the latter case also varies according as the flanges are parallel or inclined.

Fig. 79



1*st. Solid beam without panels.*—Let the load, as before, come on from the right.

Then the left-hand reaction is, as before,



$$R = \frac{w(l-x)^2}{2l}.$$

This reaction is the shear for any and all points between the left end and the end of the load. For any point distant, a, from the left end, where a is greater than x, the shear is

$$S_a = \frac{w(l-x)^2}{2l} - w(a-x).$$

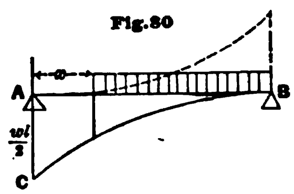
We see at once that this is less than the reaction by the amount $w(a-x)$, and that the shear will be greatest when $a = x$. That is, *the shear at any point is greatest when the load reaches from that point to the farthest support.* When the load reaches from the point to the nearest support *we have the greatest shear of opposite character.*

The same holds good for resultant shear.

Graphic Interpretation.—The equation which gives the greatest shear at any point distant x from the left end, as the load comes on from the right, is then

$$\text{Max. Shear} = \frac{w(l-x)^2}{2l}.$$

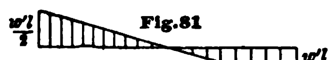
This is the equation of a parabola, Fig. 80, having its vertex at the right end, B, and the ordinate AC at the left end equal by scale to $\frac{wl}{2}$, or half the live load.



The ordinates to this parabola at any point give the maximum shear when the load comes on from the right. A similar parabola, indicated by the dotted line, gives the maximum shear for any point when the load comes on from the left.

Shear Caused by Dead Load.—If a beam or girder sustains a uniformly distributed load over its whole extent of w' per unit of length, the total load will be $w'l$, and the reaction at each end $\frac{w'l}{2}$. The shear at any point distant x from the left end, is then

$$\text{Shear} = \frac{w'l}{2} - w'x.$$



This is the equation of a straight line, as shown in Fig. 81, the end ordinates being $\frac{w'l}{2}$, and the ordinate at the centre being zero.

2d. Framed girder, uniform load, maximum shear.—The strain in any brace is, as we have seen, found by multiplying the shear, or resultant shear, by the secant of the angle which the brace makes with the vertical, regard being had to the conventions of positive and negative forces, and the direction in which θ is measured, and the definition of shear, pages 79 and 80.

In order to find the maximum strain in any brace, we must then find that position of the loading which gives the greatest shear for that brace and the corresponding shear.

This we can easily do for a uniform load. It should be noted that the position is different for parallel and inclined flanges.

Let us first take the case of parallel flanges. It is a common practice to take the load for any brace as extending beyond the brace *to the middle of the panel.* This is not strictly correct. The load reaches into the panel a variable distance, x, as will be seen from the following:

Let l = span, p = panel length, N = number of panels, m = number of panels covered by the load, w = the uniform load per lineal foot, R = the reaction at unloaded end.

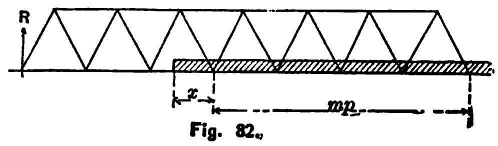


Fig. 82.

Then that portion of the load wx, which takes effect at the panel point beyond the load, is $\frac{wx^2}{2p}$. The shear at the panel point covered by the load is then

$$S = R - \frac{wx^2}{2p}.$$

But we have for the reaction

$$R = \frac{wx\left(\frac{x}{2} + mp\right)}{l} + \frac{w(mp)^2}{2l} = \frac{w}{2l}[(mp)^2 + x^2 + 2\,xmp].$$

Substituting this value of R in the expression for the shear, and placing the first differential coefficient equal to zero, we have for the condition of maximum shear

$$\frac{w}{2l}(2x + 2mp) - \frac{wx}{p} = 0, \text{ or, since } p = \frac{l}{N},$$

$$\frac{w}{2l}(2x + 2mp - 2Nx) = 0 \qquad \therefore x = \frac{mp}{N - 1}.$$

Substituting this value of x in the expression for the shear, we have, after reduction,

$$\text{Max. Shear} = \frac{wpm^2}{2(N - 1)}.$$

For the first panel point from left, $m = N - 1$ and $x = p$, or the whole span is covered, and the shear is $\frac{wp(N-1)}{2}$. These results are independent of the bracing, whether inclined, or vertical and inclined.

EXAMPLE.—Let the span $l = 140$ feet, number of panels $N = 7$, uniform load $w = 4{,}000$ lbs. per lineal foot.

Then, for the maximum shear at the first panel point on left, we have $m = 6$, $N - 1 = 6$, $x = p$, shear $= \frac{6\,wp}{2} = 3\,wp = 3$ full panel loads, or half the effective load $= 3 \times 4{,}000 \times 20 = 240{,}000$ lbs.

At the fourth panel point, $m = 3$, $x = \frac{p}{2}$, or the load reaches just to the middle of the panel, shear $= \frac{3}{4}\,wp$.

At the sixth panel point, $m = 1$, $x = \frac{1}{6}p$, shear $= \frac{1}{12}\,wp$. If we took the panel load wp as concentrated at the panel point, and disregard the portion which goes direct to the right abutment, as is the common practice, we would have shear $= \frac{1}{7}\,wp$.

In general, it will be easily seen that the shear obtained by supposing the panel load wp as concentrated at the panel point, and disregarding the half panel loads at each end, is always somewhat in excess of the strictly correct value. For this reason it is a common and allowable practice to take x as always $\frac{p}{2}$, and suppose all the load from middle to middle of panel as concentrated at the panel point.

For inclined chords the position of the load is different, as the chords themselves take a portion of the shear, and only the rest affects the braces. Let the position of the uniform load giving the maximum stress in the brace T be required, Fig. 83. Let n be the number of panels on the left of the panel point in question, c = the load which takes effect at the forward panel point, and d = the distance from the left support to the intersection of the chords cut by a section through T or P.

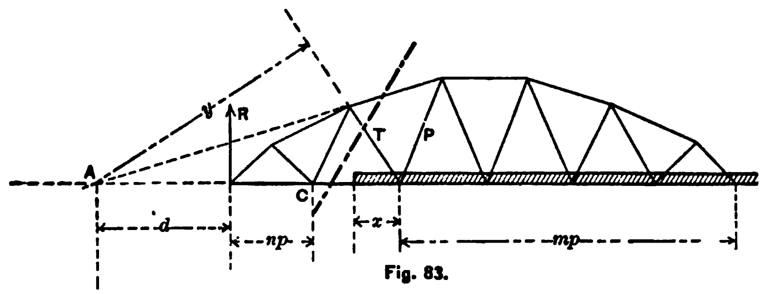


Fig. 83.

Let y = the lever arm of T about the intersection of the chords, A.
Let y' = the lever arm of P about the intersection of the chords, A.

Then we have for the reaction at left support

$$R = \frac{wx\left(\frac{x}{2} + mp\right)}{l} + \frac{w\,(mp)^2}{2l}, \text{ and } c = \frac{wx^2}{2p}.$$

Also, passing a section through T or P, completely severing the truss, and taking moments about A, we have

$$Py' = Ty = Rd - c\,(d + np) = \frac{wxd}{l}\left(\frac{x}{2} + mp\right) + \frac{w\,(mp)^2\,d}{2l} - \frac{wx^2d}{2p} - \frac{wx^2n}{2}.$$

Putting the first differential coefficient equal to zero, and $p = \frac{l}{N}$, and reducing, we have, for the condition which makes Ty, and therefore the stress in T or P, a maximum,

$$x = \frac{mpd}{d\,(N - 1) + nl}.$$

Inserting this value of x, we have

$$\text{Maximum } Ty = \frac{wm^2pd}{2N}\left[\frac{N + \frac{nl}{d}}{(N - 1) + \frac{nl}{d}}\right]$$

If the flanges are parallel, $d = \infty$, $y = \infty \cos\theta$, as on page 80, $\frac{nl}{d} = 0$, and $x = \frac{mp}{N - 1}$, and shear $= T\cos\theta = \frac{wpm^2}{2\,(N - 1)}$, as already found. The character of the bracing, whether vertical and inclined, or inclined only, makes no difference in these results.

METHOD OF CALCULATION BY CONCENTRATED LOAD SYSTEMS.*—It is the present practice to calculate all spans below 200 feet, for the system of concentrated loads actually formed by the locomotive and tenders, followed either by a uniform train load, or by a system of concentrated train loads also. Many systems are specified by engineers, and the reader should remember that we seek to illustrate the *method* of procedure, rather than to sanction any special numerical values.

The system of loads which we adopt we believe to represent good practice and to allow margin for future increase. At the same time the tendency is ever toward heavier rolling stock, and our system of loads may shortly be considered too light. Any system, however, may be handled in a precisely similar manner.

In Fig. 84, suppose a series of loads, $p_1, p_2, p_3 \ldots p_n$, to act upon the girder AB, the distances from load to load being $d_1, d_2, d_3 \ldots d_{n-1}$, and the distance of the last load from the right end being y.

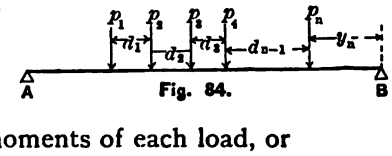


Fig. 84.

Then the total moment at B will be the sum of the moments of each load, or

$$\text{Moment at } B = p_1(d_1 + d_2 + d_3 + \ldots d_{n-1} + y_n) + p_2(d_2 + d_3 + \ldots d_{n-1} + y_n) + p_3(d_3 + \ldots d_{n-1} + y_n) + \ldots + p_n y_n.$$

Now, let us denote the total moment at the end load p_n, by M_n. We have

$$\text{Moment at } p_n = M_n = p_1(d_1+d_2+d_3+\ldots d_{n-1}) + p_2(d_2+d_3+\ldots d_{n-1}) + p_3(d_3+\ldots d_{n-1}).$$

Comparing this with the value of the moment at the right end of the span, and denoting the sum of all the wheel loads by P_n, we see at once that

$$\text{Moment at } B = M_r = M_n + (p_1 + p_2 + p_3 + \ldots p_n)\, y_n = M_n + p_n y_n.$$

This principle holds good for any other point. Thus the moment at *any point* is equal to *the moment at the preceding load on left, plus the sum of all the preceding loads multiplied by the distance from the left preceding load to the point in question.*

If a uniform train load, w per lineal foot, comes on at the right, and covers the distance y_n, then, in order to find the moment at the right end, let M_n stand for the moment *at the head of the train*, instead of the last concentrated load, and we shall have

$$M_r = M_n + P_n y_n + \frac{w y_n^2}{2},$$

which is a general expression for M_r in any case, simply taking for M_n the moment at last wheel, if there is no train load, and at the head of the train if there is; y_n in the first case being the distance from last wheel to right end, and in the second, the distance covered by the train. In both cases P_n is the sum of the wheel loads on the span.

Let us now take, for our system of concentrated loads, that given in the Table which follows. We give in column (1) the wheel loads p_1, p_2, p_3, etc., and in column (2) the distances d_1, d_2, d_3, etc., between the wheels. Any desired system can be tabulated in a similar manner. Then in column (3) we place the distances d_1, d_1+d_2, $d_1+d_2+d_3$, etc., of each wheel from the front wheel, and in column (4) the sum of the loads P_n.

By applying our principle we can find the moment M_n at any load of all preceding loads, as given in column (6).

Thus, for the moment at p_2, we have $16000 \times 8 = 128000$ ft. lbs. At p_3 we have $128000 + 41600 \times 4' \, 3'' = 304800$ ft. lbs. At p_4 we have $304800 + 67200 \times 4' \, 3'' = 590400$ ft.

* The principles and application of the method here given were worked out independently, but simultaneously, by Mr. Robert Escobar, C. E., of the Union Bridge Company, and by Theodore Cooper, C. E., *Trans. Am. Soc. C. E.*, July, 1889.

lbs., and so on. Multiplying, then, each value of P_n in column (4) by the corresponding distance in column (2), we obtain the values in column (5), and the successive additions of these give column (6). The sum of all values in (1) should check by giving the last value in (4), and the sum of all in (5) should give the last value in (6).

The Table gives locomotives and tenders, as specified by the Atlantic Coast-Line Railroad. Any desired system of loads can be treated in precisely similar manner. The loads given are the *total loads for one track.*

TABLE FOR TWO 112-TON DECAPOD ENGINES.

ATLANTIC COAST LINE.

(1)	(2)	(3)	(4)	(5)	(6)
LOADS IN POUNDS.	DISTANCES d_1, d_2, ETC., BETWEEN WHEELS.	DISTANCE FROM FIRST WHEEL.	SUMMATION OF LOADS, P_n.	PRODUCT OF P_n BY DISTANCE TO NEXT WHEEL.	MOMENT AT EACH WHEEL, M_n.
p_1 = 16000					
	8′		16000	128000	
p_2 = 25600		8′			128000
	4′ 3″		41600	176800	
p_3 = 25600		12′ 3″			304800
	4′ 3″		67200	285600	
p_4 = 25600		16′ 6″			590400
	4′ 3″		92800	394400	
p_5 = 25600		20′ 9″			984800
	4′ 3″		118400	503200	
p_6 = 25600		25′			1488000
	7′ 6″		144000	1080000	
p_7 = 20000		32′ 6″			2568000
	4′ 8″		164000	765333	
p_8 = 20000		37′ 2″			3333333
	5′ 7″		184000	1027333	
p_9 = 20000		42′ 9″			4360666
	4′ 8″		204000	952000	
p_{10} = 20000		47′ 5″			5312666
	7′ 3″		224000	1624000	
p_{11} = 16000		54′ 8″			6936666
	8′		240000	1920000	
p_{12} = 25600		62′ 8″			8856666
	4′ 3″		265600	1128800	
p_{13} = 25600		66′ 11″			9985466
	4′ 3″		291200	1237600	
p_{14} = 25600		71′ 2″			11223066
	4′ 3″		316800	1346400	
p_{15} = 25600		75′ 5″			12569466
	4′ 3″		342000	1455200	
p_{16} = 25600		79′ 8″			14024666
	7′ 6″		368000	2760000	
p_{17} = 20000		87′ 2″			16784666
	4′ 8″		388000	1810666	
p_{18} = 20000		91′ 10″			18595333
	5′ 7″		408000	2278000	
p_{19} = 20000		97′ 5″			20873333
	4′ 8″		428000	1997333	
p_{20} = 20000		102′ 1″			22870666
			448000		
				22870666	

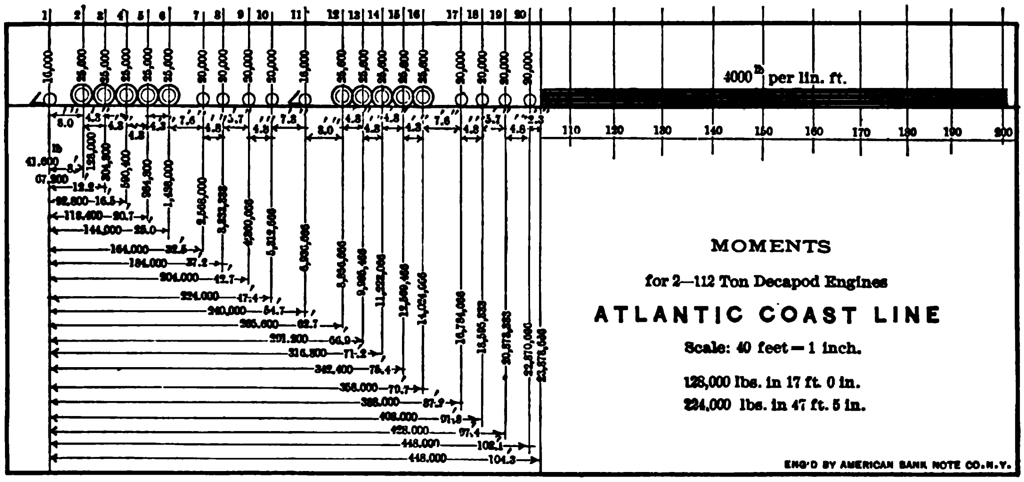
We suppose these two locomotives and tenders to be followed by a train load of 4000 lbs. per lineal foot,* and the distance from the last wheel load, p_{20}, to the uniform load to be 2′ 3″. Then the moment at the beginning of the uniform train load is

* Since the locomotive and tender concentrates 224000 lbs. on a 54 ft. wheel base, the locomotive excess for this case is 224000 − 54 × 4000 = 8000 lbs. The distance between locomotives is then *about* 50 feet.

22870666 + 448000 × $2\frac{1}{4}$ = 23878666, and this moment is to be taken for M_n in finding M_r = moment at right end, in case there is any train load on the span.

The results of our Table can now be embodied in a diagram arranged for ready use. The form of diagram here given is that used by the Phœnix Bridge Company. The diagram here given is drawn to a scale of 40 feet to an inch, merely in order to come within the limits of our page. The student should draw it anew to a scale of 20 feet to an inch, for use, and have it constantly before him while reading the following pages.*

It shows at a glance the weights and distances apart of the wheels. Below each wheel, on the horizontal line, is shown the weight of that wheel *plus* that of the preceding ones on left, together with the distance from the front wheel. On the vertical line at each



wheel is given the moment of all preceding wheels, with reference to that wheel. Thus, for the second driver, or p_8, we have 67200 lbs. for the weight of it and the preceding wheels, 12.2 feet for its distance from the front wheel, and 304800 ft. lbs. for the moment of the preceding loads with reference to p_8. At the beginning of the uniform load, we have 448000 lbs. for the weight of all preceding loads, 104.3 feet for the distance of this point from the front wheel, and 23878666 ft. lbs. for the moment of all preceding loads. The diagram extends to about 304 feet, and ten-foot intervals are marked off upon the uniform train load. The numbers of the wheels are given at top.

This diagram is used for finding reactions, shears, and moments, in any case, and it will be found convenient to draw a skeleton outline of the truss to the same scale, to be placed directly above it in proper position for the maximum stress in each member. Or a triangular boxwood scale, divided to the scale of the diagram, may be used, with two pointers, or markers. One of these markers is set on the end of the span, the other at the panel point required. The ordinary "holders" sold with the scale answer well for such markers, especially if split and ground to a point.

ILLUSTRATION OF USE OF DIAGRAM—CRITERION FOR MAXIMUM SHEAR.—Let Fig. 85 represent the skeleton of a Pratt girder with a system of concentrated loads on it. The distance of the last wheel p_n from the right end is y_n. The total load is P_n, and c is the distance of its centre of gravity from the last load. The total load in advance of the mth panel point is P_x, and a is the distance of its centre of gravity from the last load, and x from the panel point m, at which the maximum moment and shear is required. Let l be the length of span, p the panel length, N the number of panels, and R the left reaction.

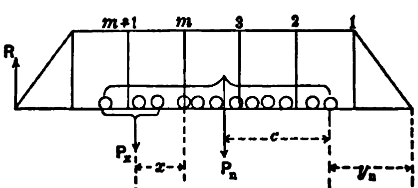


Fig. 85.

* Such a large scale diagram will be found at page 215.

Then, taking moments about the right end of span, $Rl = P_n(c + y_n)$, or $R = \frac{P_n c + P_n y_n}{l}$. The shear at m, if we disregard the panels, is then $R - P_x$. But if we regard the panel, that portion of P_x which takes effect at the $m + 1$th panel is $\frac{P_x x}{p}$. Hence the shear at the mth panel point is

$$S = \frac{P_n c + P_n y_n}{l} - \frac{P_x x}{p}.$$

But from the Fig. $mp + x = a + y_n$, or $x = a + y - mp$, and, inserting this value of x, we have

$$S = \frac{P_n c + P_n y_n}{l} - \frac{P_x}{p}(a + y_n - mp).$$

Differentiating with respect to y, and placing the first differential coefficient equal to zero, we have for the condition which makes the shear a maximum, $\frac{P_n}{l} - \frac{P_x}{p} = 0$, or, since $Np = l$,

$$\frac{P_n}{N} = P_x.$$

If in any case this condition cannot be exactly fulfilled, we should take that position of the system which makes $\frac{P_n}{N}$ just greater than P_x, so that if the wheel in question moves just past the point, $\frac{P_n}{N}$ will be less than P_x.

In general, then, for any number of panels N, the shear at any panel point is a maximum, when *one of the loads is at the point, and when the load system is so disposed that $\frac{1}{N}$th of the entire load on the span is equal to or just greater than the load P_x in front of the panel point.*

The value of P_x does *not* include the load at the point. By trial with the diagram this position can easily be determined and the maximum shear found. If any of the uniform train load, w per lineal foot, is on the span, and covers a distance y_n, it must be included in the total load, so that we have in general:

$$\frac{P_n + wy_n}{N} \geqq P_x \quad \text{. . . . . (1)}$$

When the position of the system giving maximum shear at any point is thus found, the corresponding maximum shear itself is easily calculated. Thus, if we divide the moment M_r at the right end of span by l, we have the reaction at left end of span. If from the reaction we subtract $\frac{M_x}{p}$, where M_x is the moment at the point, taken directly from the diagram, and p is the panel length, we have

$$\textbf{Shear} = \frac{M_r}{l} - \frac{M_x}{p} = \frac{M_n + P_n y_n + \frac{w}{2} y_n^2}{l} - \frac{M_x}{p} = \frac{M_n + P_n y_n + \frac{w}{2} y_n^2 - NM_x}{l} \quad . \text{ (2)}$$

In finding the moment at the right end, the moment of the train load $\frac{w}{2}y_n^2$ must be included if the train comes on, and M_n is then the moment at the head of train.

EXAMPLE.—Suppose the span $l = 140$ feet, the number of panels $N = 7$, and the maximum shear is required at 20 feet from the left end.

Set the markers on the scale at 140 feet and 120 feet from right end of scale, or draw a skeleton truss to scale, and apply it to our diagram, so that the first wheel, p_1, is at the point, 20 feet from left end of span. In this position of the system P_s is zero, the train load covers the distance $y_n = 120 - 104.3 = 15.7$ feet, $P_n = 448000$, and the total load $P_n + wy_n = 448000 + 4000 \times 15.7 = 510800$ lbs. We have, therefore, $\frac{P_n + wy_n}{7}$ greater than P_s, and if p_1 is moved a little to the left of the point, P_s will become 16000, but the total load is essentially the same, and $\frac{1}{7}$th of this is greater than 16000 also.

We therefore try for p_2 at the point. We have now $P_s = 16000, y_n = 120 + 8 - 104.3 = 23.7$ feet, $P_n = 448000$, total load $= 448000 + 4000 \times 23.7 = 542800$ lbs. Again, $\frac{1}{7}$th of this is greater than $P_s = 16000$, and if the second wheel is moved a very little to left of the point, P_s will be 41600, the total load is practically unchanged, and $\frac{1}{7}$th of it is greater than 41600 also.

We therefore try for p_3 at the point. For this position of the system $P_s = 41600, y_n = 120 + 12.2 - 104.3 = 27.9$ feet, $P_n = 448000$, the total load is $448000 + 4000 \times 27.9 = 559600$ lbs., and $\frac{1}{7}$th of this is greater than P_s. If the third wheel is moved a very little to left, P_s becomes 67200, the total load is unchanged, and $\frac{1}{7}$th of it is greater than 67200 also.

We therefore try for p_4 at the point. For this position $P_s = 67200, y_n = 120 + 16.5 - 104.3 = 32.2$ feet, $P_n = 448000$, total load $= 448000 + 4000 \times 32.2 = 576800$ lbs., and $\frac{1}{7}$th of this is greater than P_s. But if the fourth wheel is moved a little to left, P_s becomes 92800, the total load is unchanged, and $\frac{1}{7}$th of it is *less than* 92800.

The *fourth wheel at the point* gives, therefore, a maximum shear, since this is the one for which $\frac{P_n}{7}$ is greater than $P_s = 67200$, and less than 92800; that is, it is the position for which $\frac{P_n + wy_n}{N}$ is *just* greater than P_s.

Assuming this position, we have at once $M_s = 590400$, and moment at right end of span $M_r = 23878666 + 448000 \times 32.2 + \frac{4000 \times (32.2)^2}{2} = 40377946$. We have, therefore, from (2), ***maximum shear*** $= \frac{40377946 - 7 \times 590400}{140} = 258893$ lbs.

Whenever we thus determine the position for maximum shear, if when we place the next load on the point the front wheel goes off the span, we should see whether there is not another maximum which is greater than that already found.

Thus in the present case, for the fifth wheel at the point, p_1 passes off. Hence $P_s = 92800 - 16000 = 76800$ lbs. The train load covers $y_n = 120 + 20.7 - 104.3 = 36.4$ feet, and total load $= 448000 - 16000 + 4000 \times 36.4 = 577600$ lbs., and $\frac{P_n}{7}$ is greater than P_s. But for p_5 a little to left of the point P_s becomes 102400, the total load remains the same, and $\frac{P_n}{7}$ is *less* than P_s. Hence p_5 at the point also gives a maximum.

For this position we have $M_s = 984800 - 16000 \times 20.7 = 653600$, and moment at right end of span $M_r = 23878666 + 448000 \times 36.4 + \frac{4000 \times (36.4)^2}{2} - 16000 \times 140.7 = 40584586$. Hence ***maximum shear*** $= \frac{40584586 - 7 \times 653600}{140} = 257210$ lbs. As this is less than for p_4 at the point, that position gives the true maximum.

Finally, we should test and see whether the uniform load alone does not give a greater shear. In the present case the shear due to uniform load is, as found on page 85, 240000 lbs.

We may find in similar manner the maximum shear at any other panel point. The shear thus found is correct for double track and two trusses for each truss. We should take ***one half*** of it for each truss, for single track and two trusses.

CRITERION FOR MAXIMUM MOMENT.—For the moment at the panel point m, Fig. 85, we have

$$M = R(l - mp) - P_x x.$$

But

$$R = \frac{P_n c + P_n y_n}{l},$$

and inserting this value of R, we have

$$M = \left(\frac{P_n c + P_n y_n}{l}\right)(l - mp) - P_x x.$$

Substituting the value of $x = a + y_n - mp$, we have

$$M = \left(\frac{P_n c + P_n y_n}{l}\right)(l - mp) - P_x(a + y_n - mp).$$

Putting the first differential equal to zero, we have, for the condition which makes M a maximum,

$$\frac{P_n(l - mp)}{l} - P_x = 0.$$

If we denote the distance $l - mp$ of the point in question from the *left* end by z, and suppose the uniform train load to cover the distance y_n, we have for the criterion for maximum moment,

$$\frac{(P_n + wy_n)z}{l} \overset{=}{>} P_x. \qquad (3)$$

If the equality cannot be exactly satisfied, we must, as in the case of shear, take that position which gives a result *just* greater than P_x. The maximum moment, of course, will be found when some wheel is directly over the point in question.

By trial with our diagram the position of the system which satisfies this criterion can be found, and when found, the corresponding moment is obtained by dividing the moment M_r at the right end of span by l, thus obtaining the reaction at left end, multiplying this reaction by z, the distance from left end to the point, and subtracting M_x, or

$$M = \frac{M_r z}{l} - M_x = \frac{\left(M_n + P_n y_n + \frac{w}{2} y_n^2\right) z}{l} - M_x. \qquad (4)$$

It should be borne in mind that the method of application of our diagram here given holds good in all cases for *shear*, but for *moments* it holds good for *vertical and inclined* bracing only, or for Pratt Truss. For Warren, Lattice, Post, or, in general, *all* inclined bracing, it only holds good for the *un*loaded chord. For the loaded chord a modification is necessary, which will be noticed in due place.

EXAMPLE.—*Let $l = 140$ feet, $N = 7$, and let the maximum moment at 40 feet from left end be required.*

Here $z = 40, \frac{z}{l} = \frac{2}{7}$, and placing one marker on our scale at 140 feet, and the other at 100 feet, or drawing a skeleton truss and applying to our diagram, we proceed as for shear, except that we use criterion (3) and equation (4).

Thus, let us place p_6 at the point. The uniform load covers the distance $y_n = 100 + 25 - 104.3 \doteq 20.7$ feet, total load $= 448000 + 4000 \times 20.7 = 530800$, and $\frac{2}{7}$ths of this $= 151657$. This, we see, is greater than 118400 preceding, and also greater than 144000. There is no maximum for p_6.

We next place p_7 at the point. For this position $y_n = 100 + 32.5 - 104.3 = 28.2$ feet, total load $= 448000 + 4000 \times 28.2 = 560800$, and $\frac{2}{7}$ths of this $= 160228$. This is greater than $P_s = 144000$ and *less* than 164000. There is a maximum for p_7 at the point.

For this maximum we have $M_s = 2568000$, and

$$M_r = 23878666 + 448000 \times 28.2 + \frac{4000\,(28.2)^2}{2} = 38102746.$$

Hence, for p_7,

$$M = \frac{2M_r}{7} - M_s = 8318500 \text{ ft. lbs.}$$

It by no means follows, however, that this is the only maximum.

Thus, if we place p_8 at the point, $y_n = 100 + 37.2 - 104.3 = 32.9$ feet; total load $= 448000 + 4000 \times 32.9 = 579600$, and $\frac{2}{7}$ths of this $= 165600$. This is greater than $P_s = 164000$, and *less* than 184000. There is, therefore, a maximum for p_8 at the point.

For this maximum we have $M_s = 3333333$, and

$$M_r = 23878666 + 448000 \times 32.9 + \frac{4000\,(32.9)^2}{2} = 40782686.$$

Hence, for p_8,

$$M = \frac{2M_r}{7} - M_s = 8318863 \text{ ft. lbs.}$$

This maximum is, therefore, greater than for p_7.

If we continue to test we shall find no maximum until p_{11} is placed at the point.

For this position $y_n = 100 + 54.7 - 104.3 = 50.4$. But since $p_1 - p_3$ have passed off the span, total load $= 448000 + 4000 \times 50.4 - 67200 = 582400$ lbs., and $\frac{2}{7}$ths of this $= 166400$. Also, for P_s we have $224000 - 67200 = 156800$, and for next value of P_s, $240000 - 67200 = 172800$. We see that 166400 is greater than the first and *less* than the second. There is, therefore, a maximum for p_{11}.

For this maximum we have

$$M_s = 6936666 - 304800 - 67200 \times 42.5 = 3775866, \text{ and}$$

$$M_r = 23878666 + 448000 \times 50.4 + \frac{4000\,(50.4)^2}{2} - 304800 - 67200 \times 142.5 = 41657386.$$

Hence for p_{11}

$$M = \frac{2M_r}{7} - M_s = 8126244.$$

This is less than for p_8.

If we continue to test we find no maximum until we come to p_{14}.

For this position, we have $y_n = 100 + 71.2 - 104.3 = 66.9$ feet; $p_1 - p_6$ have passed off; total load $= 448000 + 4000 \times 66.9 - 144000 = 571600$.

$P_s = 291200 - 144000 = 147200$, and the next value is $316800 - 144000 = 172800$. Since $\frac{2}{7}$ths of total load $= 163314$ is greater than the first and less than the second, p_{14} gives a maximum.

For this maximum, we have

$$M_s = 11223066 - 1488000 - 144000 \times 46.2 = 3082266, \text{ and}$$

$$M_r = 23878666 + 448000 \times 66.9 + \frac{4000\,(66.9)^2}{2} - 1488000 - 144000 \times 146.2 = 40260286.$$

Hence, for p_{14}

$$M = \frac{2M_r}{7} - M_s = 8420673.$$

This is greater than for p_8.

For p_{15} at the point $y_8 = 100 + 75.4 - 104.3 = 71.1$ feet; $p_1 - p_7$ are off; total load $= 448000 + 4000 \times 71.1 - 164000 = 568400$, and $\frac{2}{7}$ths of this $= 162400$ lbs.

$P_8 = 316800 - 164000 = 152800$, and the next value is $342400 - 164000 = 178400$. Since $\frac{2}{7}$ths of total load is greater than the first and less than the second, we have a maximum for p_{15}.

For this maximum $M_8 = 12569466 - 2568000 - 164000 \times 42.9 = 2965866$;

$$M_7 = 23878666 + 448000 \times 71.1 + \frac{4000\,(71.1)^2}{2} - 2568000 - 164000 \times 142.9 = 39838286.$$

Hence for p_{15}

$$M = \frac{2M_7}{7} - M_8 = 8416500 \text{ ft. lbs.}$$

For p_{16} at the point, we find in similar manner, $M = 8407196$ ft. lbs.

We see, then, that the greatest maximum is for p_{14} at the point, and it is 8420670 ft. lbs.

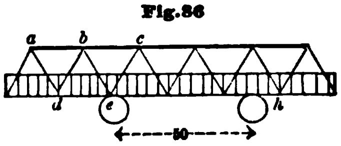
For uniform train load over the whole span, the moment is 8000000 ft. lbs. The maximum required is therefore 8420670 ft. lbs.

Method of Calculation by Locomotive Excess.—This method is at present little used except for long spans of 200 feet and over. It is, however, very simple as compared to the preceding, and, for the sake of avoiding detail, we shall first treat it quite fully in what follows, giving applications of it to all the different styles of trusses, and reserving the solution of these trusses by concentrated load system for the Appendix, page 215. By this plan we hope to bring out fundamental principles, without unnecessary detail.

To find the maximum strain in any panel by this method, we proceed as follows:

Conceive the whole span covered with the train load reduced to an equivalent uniform load per foot. Then suppose the *locomotive excess* to act, for any panel in the unloaded chord, at the centre of moments for that panel, for any panel in the loaded chord at that end of the panel next to the most distant support. If there is another locomotive, let its excess act in the longest segment of the span, at a distance from the first equal to the distance between two locomotives, or say 50 feet. If this distance does not fall exactly at an apex, it is allowable to take it as acting at the first apex beyond.

Fig. 86



Thus, suppose the train load to be one ton, or 2000 lbs. per foot. We suppose this load to cover the span, Fig. 86. Let there be in addition two locomotives. Let each locomotive and tender concentrate say 175000 lbs. upon a wheel base of 54.5 feet. The locomotive excess over 2000 lbs. per foot is, then,

$$175000 - 54.5 \times 2000 = 66000 \text{ lbs., or } 33 \text{ tons.}$$

We shall assume this value in all our illustrations of this method.

If now we wish the maximum strain in *bc*, for instance, we suppose the excess to act at *e*, the point of moments for *bc*. If there is another locomotive, then 50 feet from the first excess we let the second act. If this does not fall at an apex, then the apex *h*, further on, is the one which is loaded. So for any other upper panel, or panel in the *unloaded* chord generally. For any lower panel, as *de*, or panel in the *loaded* chord generally, whether upper or lower, the first locomotive excess acts at *e*, or that end nearest to the most distant support. In general, for any panel in the left half of the truss, place the locomotive excess on the nearest apex to the left abutment which it can occupy, *without getting between that abutment and the centre of moments* for the panel in question.

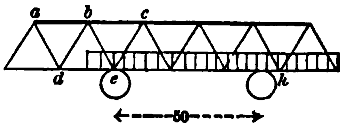
This rule for loading gives the greatest moment for any panel.

In order to find the maximum strain in any brace, take the uniform train load as extending, for positive shear, from the right, and for negative shear, from the left, *up to*

the middle of the corresponding panel in the loaded chord. As we have seen, page 85, this is not strictly correct, but for our present purpose the error may be neglected. The first locomotive excess for positive shear is at the right end of that panel, and for negative shear at the left end. The second locomotive excess follows at an interval of 50 feet on the right or left, or at the first apex just beyond this distance.

Thus, Fig. 87, the greatest positive shear for *be* or for *bd* occurs when the uniform train load reaches half a panel beyond *e*, from the right end, and the first locomotive excess acts at the apex *e* and the second at *h*. The greatest negative shear for *bd* or *be* would occur when the train load reaches from the left end to the middle of the panel *de*, and the first locomotive excess acts at *d*. The second should act 50 feet to the left of *e*, or at first apex beyond, if at all.

Fig. 87



Of course, the strains found must be divided among the trusses composing the bridge. Thus, if there are two trusses, the strains upon each will be one-half of those found. Otherwise we should take only one-half of the above loads in finding the strains.

In all cases which we shall investigate, we shall find the strains in each form of truss as though it alone supported the entire load. These strains can then be divided among the number of trusses which actually support the bridge.

The method of loading shown in Fig. 86 supposes that cars may precede as well as follow engines 50 feet apart. This may happen, but rarely, however. The maximum chord strains, therefore, are of less frequent occurrence than the maximum strains in the braces, which occur for every passage of the train.

We give in the following chapters illustrations of this method of calculation by locomotive excess, and reserve for the Appendix, page 215, the application of the method by concentrated loads, already explained.

The student will note that although all our examples are for short spans, the method *should not be applied* to short spans, or to any span under 200 feet. We take short spans simply for convenience of illustration.

Our dimensions, such as depth and panel length, also, are not to be considered as examples of practice. They are chosen to facilitate calculation only. So also as to the dead weight assumed.

Also, as specified locomotives and train vary considerably, our results should not be considered as representing usual practice.

Whatever the numerical results or values adopted, our *methods* remain unchanged, and our object is now to present these as simply as possible.

Questions for Examination.

Define what is meant by the term *shearing force* or "shear." Define shear. Illustrate by a beam supporting several weights. State the conventions which determine the proper signs for the different terms in the algebraic sum of the outer forces. Give rule for finding the strain in any brace when the flanges are horizontal. Deduce this rule. Illustrate how the angle θ is measured—when the cos θ is plus and when minus. When is the sin θ plus and when minus? Through which end of a brace is the vertical from which θ is measured—drawn?

What do you mean by resultant shear? Show how, by means of it, the strain in a brace may be found when the flanges are not horizontal. How do you determine when and how much a brace must be counterbraced when the live load acts as well as the dead? What distribution of live load causes the greatest flange strains? Deduce the equation for the maximum moment at any point. What is the graphic interpretation of this equation? Apply the diagram to a framed structure. Through what points are the ordinates which give the maximum moments drawn? How can you find the flange strain when the moment is given? What is the rule for the sign of the lever arm?

What distribution of live load causes the greatest positive and negative shear at any point? Write

down the equation for maximum shear. What is the graphic interpretation of this equation? Apply the diagram to a framed structure. From what point must we always drop the ordinates which give the shear for any brace? Show how to tell whether the brace strain is tension or compression. Write down the equation for the shear at any point due to dead load. What is the graphic interpretation of this equation?

Illustrate (Fig. 83) how to find the maximum strains in braces due to combined action of dead and live loads, when flanges are horizontal. Point out between what points on the diagram we must counterbrace, and why.

When a system of concentrated loads moves over a beam or girder, when will the greatest shear at any given point occur? Show how to find the position of the system which gives the greatest shear at any given point, and what that shear is.

When will the greatest moment at any point occur? Show how to find it.

What is the method of procedure for medium spans in order to find the greatest shears, taking into account the locomotive? What is the method for the maximum moment?

CHAPTER III.

BRIDGE GIRDERS WITH PARALLEL FLANGES—TRIANGULAR GIRDER.

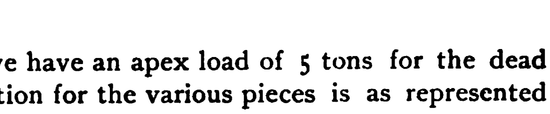
DIFFERENT METHODS OF SOLUTION.—The triangular girder is the simplest form of girder, and we choose it, therefore, as our first example of the application of preceding principles. These principles have given rise to various methods of solution for girders with horizontal flanges, some of which are advantageous in some forms of girders, and some in others. We shall give in the present chapter *all* these methods as applied to the same example, and shall then in future chapters, which discuss other forms of girder, choose in each case that method alone which seems best adapted to the case in hand.

We may distinguish four different methods, based upon the principles of the four Chapters of Section I., Part I., *viz.:* the method by graphic resolution of forces, by algebraic resolution of forces, by algebraic method of moments, and by graphic method of moments. The special form which the last two take in the case of parallel flanges, has been noticed in the preceding Chapter. The application of the first two will be apparent as we proceed.

EXAMPLE FOR SOLUTION.—We shall choose, for convenience merely, a short girder, which will serve to illustrate the methods quite as well as if it were longer.

Let the girder, Fig. 88, be 10 feet high and 80 feet long, having 8 equal bays in the lower flange and 7 in the upper. The live load passes over the lower flange, and the bridge is, therefore, a "through bridge." The bracing consists of isosceles triangles, and hence the angle made by each brace with the vertical is 26° 34′. Let the dead load be supposed to be known and equal to one half a ton per running foot, and let the live load be taken at one ton per foot.* Our data, then, are as follows:

Fig. 88



$$l = 80, \qquad d = 10, \qquad \theta = 26^\circ\ 34', \qquad p = 0.5 \text{ ton}, \qquad m = 1.0 \text{ ton};$$

where $d =$ depth of girder.

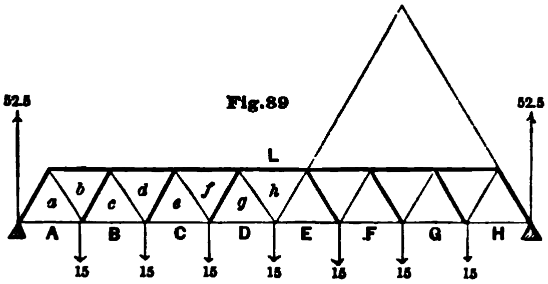
Since the length of each bay is 10 feet, we have an apex load of 5 tons for the dead load and 10 tons for the live load. The notation for the various pieces is as represented in Fig. 88.

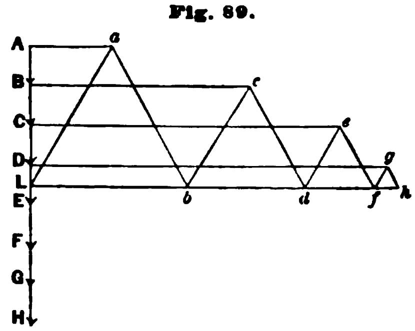
* We do not, therefore, at present take account of the action of concentrated loads. We shall do that hereafter, page 220. The above loads, it will be noted, are for the entire structure. If there are two trusses the strains will be one-half of those found. In this and all following examples the dead load and dimensions are assumed for convenience of calculation and illustration only, and are *not* to be considered as examples of practical cases. We shall see how to estimate dead load and choose best dimensions hereafter. For spans *less than* 200 *feet the method of this chapter should not be used.*

FIRST METHOD—BY GRAPHIC RESOLUTION OF FORCES.

MAXIMUM STRAINS IN THE FLANGES.—According to the principles of the preceding Chapter the flange strains will be greatest when the girder is fully loaded with both dead and live loads. When this is the case we have at each lower apex a load of $5 + 10 = 15$ tons. The reaction at each end is then 52.5 tons.

We lay off the weights, AB, BC, CD, etc., Fig. 89, then the reactions HL and LA,

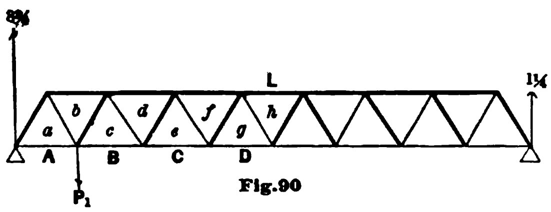




and then form the strain diagram according to the principles of Chapter I., Section I. The strains in the flanges thus obtained are the greatest which can ever occur. Making the construction we find

	Lb	Ld	Lf	Lh	Aa	Bc	Ce	Dg
strain	+ 52.5	+ 90	+ 112.5	+ 120	− 26.25	− 71.25	− 101.25	− 116.25.

It only remains to notice that since the braces are very short they will not give direction very accurately in the strain diagrams. Hence it is well to lay off carefully the directions of the diagonals to a much larger scale, as shown by the dotted lines in Fig. 89, and use these directions in forming the strain diagram.



MAXIMUM STRAINS IN THE BRACES.—In order to find the strains in the braces we may find the strains caused by each live load apex weight separately. Tabulating these strains, we can easily find the dead load strains and finally the maximum strains in each brace. Thus, Fig. 90, suppose only the first apex live load of 10 tons to act. The reaction at the left end is then $\frac{7}{8}\ 10 = 8\frac{3}{4}$, and at the right end $\frac{1}{8}\ 10 = 1\frac{1}{4}$. Lay off then AB equal to the weight $P = 10$, and BL and LA equal to the reactions and form the strain diagram. Scaling off the strains in the braces, we can enter them in a table as follows:

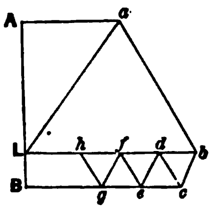


TABLE FOR STRAINS IN THE BRACES.

	La	*ab*	*bc*	*cd*	*de*	*ef*	*fg*	*gh*
P_1	+ 9.8	− 9.8	− 1.4	+ 1.4	− 1.4	+ 1.4	− 1.4	+ 1.4
P_2	+ 8.4	− 8.4	+ 8.4	− 8.4	− 2.8	+ 2.8	− 2.8	+ 2.8
P_3	+ 7.0	− 7.0	+ 7.0	− 7.0	+ 7.0	− 7.0	− 4.2	+ 4.2
P_4	+ 5.6	− 5.6	+ 5.6	− 5.6	+ 5.6	− 5.6	+ 5.6	− 5.6
P_5	+ 4.2	− 4.2	+ 4.2	− 4.2	+ 4.2	− 4.2	+ 4.2	− 4.2
P_6	+ 2.8	− 2.8	+ 2.8	− 2.8	+ 2.8	− 2.8	+ 2.8	− 2.8
P_7	+ 1.4	− 1.4	+ 1.4	− 1.4	+ 1.4	− 1.4	+ 1.4	− 1.4
Live load: Comp. +	+ 39.2	....	+ 29.4	+ 1.4	+ 21.0	+ 4.2	+ 14.0	+ 8.4
Live load: Tens. −	....	− 39.2	− 1.4	− 29.4	− 4.2	− 21.0	− 8.4	− 14.0
Dead load.	+ 19.6	− 19.6	+ 14.0	− 14.0	+ 8.4	− 8.4	+ 2.8	− 2.8
Max. com. +	+ 58.8	....	+ 43.4	....	+ 29.4	....	+ 16.8	+ 5 6
Max. tens. −	....	− 58.8	....	− 43.4	....	− 29.4	− 5.6	− 16.8

Thus the first line in the Table gives the strains in all the braces due to the first apex load P_1.

In a similar way we may find and tabulate the strains due to each of the other apex loads acting separately. This, however, need not involve a separate diagram for each apex load. We can fill up the table directly. Thus, suppose the second weight P_2 to act. It will cause at the right end a reaction twice as great as P_1 caused at that end. The strains, then, in all the braces to the right of P_2 will be twice as great as they were for P_1. As to their signs, we have only to remember that the two pieces which meet at the loaded apex *BC*, viz.: *cd* and *de*, are both tension (if the load were on the top flange both compression), and the strains alternate in sign both ways. Thus *de* would be tension and equal to $2 \times 1.4 = -2.8$, *ef* would be $+2.8$, $fg = -2.8$, gh $+2.8$, etc. In similar manner, the left hand reaction for P_2 would be $\frac{3}{4}$ $10 = 7\frac{1}{2}$, instead of $\frac{7}{8}$ 10 or $8\frac{3}{4}$. The strains in all the braces to the left of P_2, therefore, are $\frac{6}{7}$ of the strains caused by P_1 in the braces to the left of it. As to the signs, the same rule is to be observed. Thus the strain in *cd* due to P_2 is tension and equal to $\frac{6}{7} \times 9.8 = -8.4$, for *bc* we have then $+8.4$, etc. We can, therefore, fill out the table for P_2.

In similar manner for P_3 we have *fg* tension and equal to $3 \times 1.4 = -4.2$. Also *ef* tension and equal to $\frac{5}{7} \times 9.8 = -7$.

For P_4 we have *gh* tension and equal to $\frac{4}{7}$ths of the strain caused by P_1 in the left hand braces, or $\frac{4}{7} \times 9.8 = -5.6$. We can, therefore, fill out the line for P_4 in the Table.

For P_5 we have $\frac{3}{7} \times 9\,8 = 4.2$, and by reference to Fig. 90 we see that starting from the weight and remembering that the braces are alternately tension and compression, *gh* is in tension. We thus fill out the line for P_5 in the Table.

In similar manner we fill out the lines for P_6 and P_7.

Our Table now contains the strains in the braces caused by each apex live load, acting separately.

The next two lines give the compression and tension in each piece due to the live load. Thus we see at once that all the live loads cause compression in *La*. The live load compression is then 39.2 tons. In the same way we see that the greatest tension on *fg* occurs when only P_1, P_2 and P_3 act, and the greatest compression, when P_4, P_5, P_6 and P_7 act. This agrees with our principle in the preceding Chapter, that the strain in any brace is greatest when the live load reaches from the end of girder to half a bay past the brace.

Having now filled out the lines for live load compression and tension, we can easily find the dead load strains. The dead load acts at every apex simultaneously, and since it is in the present case half the live load, we have only to take the algebraic sum of all the live load strains and divide by 2 to obtain the dead load strains.* We thus fill out the line for dead load strains at once.

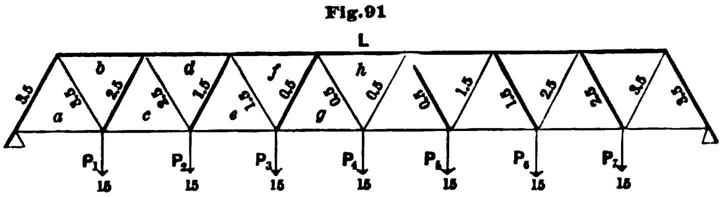
Finally we can find the maximum strains. Thus for *La*, the dead load causes + 19.6 and the live load + 39.2 tons. The maximum is therefore, + 58.8 tons. In same way $ab = -58.8$. For *bc* the greatest compression is $+ 29.4 + 14 = 43.4$. The live load tends to cause a tension of -1.4 in *bc*, but as this is less than the constant dead load strain of compression it produces no effect. The same holds good for *cd*, *de* and *ef*. In *fg* the dead load causes compression of 2.8. This, together with the live load compression of 14, gives + 16.8. But the live load may also cause a tension of -8.4. As this is greater than + 2.8 due to dead load, we must counterbrace *fg* for the difference. Hence the effective tension in *fg* is $-8.4 + 2.8 = -5.6$ tons. We find thus that *fg* and *gh* are the only pieces which need to be counterbraced, because they are the only braces in which the strain due to dead load is exceeded by the strain of opposite kind due to live load.

We have thus found the maximum strains in every piece of the girder.

SECOND METHOD—BY ALGEBRAIC RESOLUTION OF FORCES.

MAXIMUM STRAINS IN THE FLANGES.—The loading which gives the maximum strains in the flanges, is when both dead and live loads cover the whole span, that is when we have 15 tons at each apex. When this is the case, the weight P_4, Fig. 91, being at the centre, we know that it causes a reaction of $\frac{1}{2} P_4$ at each end. That is, the shear, or portion which goes each way through the braces, is 0.5 P_4. This shear multiplied by the secant θ, gives the strain in the braces due to P_4 alone, tension for *gh* and alternating toward the left.

Fig.91



If P_3 acts alone it would cause at the left end a reaction of $\frac{5}{8} P_3$ and at the right end a reaction of $\frac{3}{8} P_3$. But if P_5 acts at the same time, it will cause at the left as much as P_3 causes at the right. Hence when P_3 and P_5 act simultaneously, we can consider that the whole of P_3 goes toward the left end through the braces, and the whole of P_5 toward the right end.

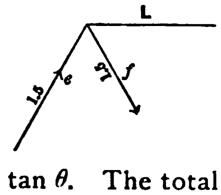
While, then, the strain in *gh* and *gf* would be 0.5 $P_4 \sec \theta$, the strains in *de* and *ef* would be given by 1.5 $P_3 \sec \theta$. In the same way for *bc* and *cd* we have 2.5 $P_2 \sec \theta$, and for *La* and *ab*, 3.5 $P_1 \sec \theta$.

We have accordingly placed upon each brace, in Fig. 91, the coefficients of *P*, which,

* This is on the supposition that the dead load takes effect only at the loaded apices.

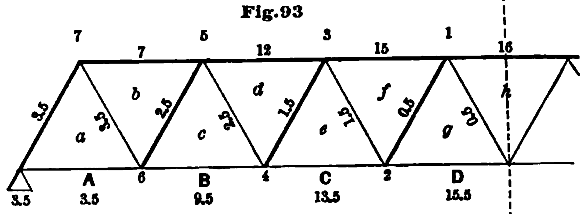
multiplied by P or 15, gives the shear for full load. This shear, multiplied by the sec θ, gives the strain in a brace, multiplied by the tangent θ it gives the strain in the flange. Thus, Fig. 92, we have the strain in ef tension and equal to $1.5\ P \sec \theta$. The horizontal component of this strain causes strain of compression in the flange Lf. This horizontal component is $1.5\ P \sec \theta \times \sin \theta = 1.5\ P \tan \theta$. But if ef is tension, de is compression and equal to ef. Hence, de causes also compression in Lf equal to $1.5\ P \tan \theta$. The total compression in Lf then is $1.5\ P \tan \theta + 1.5\ P \tan \theta = 3\ P \tan \theta$.

Fig. 92



In general, if we add together the coefficients in Fig. 91, for any two braces which meet at an apex, we shall have the coefficients which multiplied by $P \tan \theta$ will give the strain which these braces cause in the flange to the right of them. Thus, Fig. 93, we obtain at the upper apices the coefficients 1, 3, 5, 7, and at the lower apices, 2, 4, 6 and 3, 5.

Fig. 93



The strain, then, in Lb is $7\ P \tan \theta = 7 \times 15 \times 0.5 = +52.5$. In Ld, we have $5\ P \tan \theta$ due to the braces bc and cd. But the strain in Lb also acts upon Ld. The total strain in Ld is then $7\ P \tan \theta + 5\ P \tan \theta = 12\ P \tan \theta = 12 \times 15 \times 0.5 = +90$.

If, therefore, commencing at the end, *we add the apex coefficients, and place the results over each flange, the coefficients thus determined give the strains in the flanges.*

Thus

$$
\begin{aligned}
Lf &= 15\ \ P \tan \theta = 15 \times 15 \times 0.5 = +112.5 \\
Lh &= 16\ \ P \tan \theta = 16 \times 15 \times 0.5 = +120. \\
Aa &= 3.5\ P \tan \theta = 3.5 \times 15 \times 0.5 = -\ 26.25 \\
Bc &= 9.5\ P \tan \theta = 9.5 \times 15 \times 0.5 = -\ 71.25 \\
Ce &= 13.5\ P \tan \theta = 13.5 \times 15 \times 0.5 = -101.25 \\
Dg &= 15.5\ P \tan \theta = 15.5 \times 15 \times 0.5 = -116.25
\end{aligned}
$$

These are precisely the same results as those obtained by the preceding method of diagram. The method in the present case is very simple, and involves but little work.

MAXIMUM STRAINS IN THE BRACES.—In order to find the maximum strains in the braces, we might take each apex live load separately and find the shear which it sends toward each abutment. These shears multiplied by sec θ would give the strains in braces right and left of the load. We could thus easily form a Table precisely similar to the one on page 99, two simple multiplications only being necessary in order to fill out each line.

Thus let P_1 act alone, Fig. 90. The portion which goes toward the left is $\frac{7}{8} P_1$ and toward the right $\frac{1}{8} P_1$. We have then tension in both ab and bc. For ab we have $-\frac{7}{8} P_1 \sec \theta = -\frac{7}{8}\ 10 \times 1.117 = -9.8$. For bc we have $\frac{1}{8} P_1 \sec \theta = \frac{1}{8}\ 10 \times 1.117 = -1.4$. This is enough to fill out the first line in our Table, page 99, for P_1. Other lines can be filled out in similar manner. We have only to remember that the braces which meet at the weight have both the same sign, minus when the weight is below, and plus when it is at the upper apex, and that the signs alternate both ways from the loaded apex.

The Table, page 99, was rendered necessary in order to avoid making a separate diagram for each weight.

In the present case, however, it is unnecessary to draw up a Table at all. We can find the maximum strains in each diagonal directly.

a. Dead Load Strains.—Thus let us first find the dead load strains. The apex load, is 5 tons. We have, then, from our coefficients, Fig. 93.

$$La = 3.5\ P \sec \theta = 3.5 \times 5 \times 1.117 = +\ 19.54.$$

We have La compression because the reaction at its foot is upward. For ab then we have $-$ 19.54. As the signs are alternately plus and minus,

$$bc = +\ 2.5\ P \sec \theta = +\ 2.5 \times 5 \times 1.117 = +\ 13.96,$$

and $cd = -$ 13.96.

For de, we have,

$$de = +\ 1.5\ P \sec \theta = +\ 1.5 \times 5 \times 1.117 = +\ 8.38,$$

and $ef = -$ 8.38.

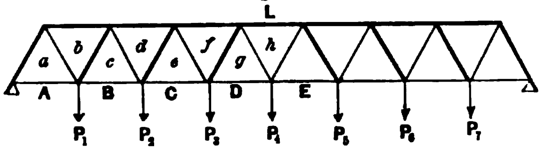
For fg,

$$fg = +\ 0.5\ P \sec \theta = +\ 0.5 \times 5 \times 1.117 = +\ 2.792,$$

and $gh = -$ 2.792.

These strains are very closely what we have found for the dead load strains in our Table, page 99. A slight discrepancy is to be expected, because in Fig. 90 we suppose $\frac{7}{8}$ of P_1 to go toward the left, and $\frac{1}{8}$ toward the right, while in Fig. 93 we suppose the whole of P_1 to go toward the left, because all the loads are supposed to act simultaneously.

Fig. 94



b. Live Load Strains.——The apex live load is 10 tons.

The greatest positive shear for the brace gh will occur when P_4, P_5, P_6, and P_7 act, the other apices being unloaded, Fig. 94. The greatest negative shear for gh will occur when only P_1, P_2, and P_3 act. We have, then, for the positive shear for gh, $(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8})\ 10 = +\ 12\frac{1}{2}$, and for the negative shear $-\ (\frac{1}{8} + \frac{2}{8} + \frac{3}{8})\ 10 = -\ 7\frac{1}{2}$.

We have, then,

$$gh \cos \theta + 12\tfrac{1}{2} = 0 \text{ or } gh = -\ 12\tfrac{1}{2} \sec \theta = -\ 13.96$$

$$gh \cos \theta - 7\tfrac{1}{2} = 0 \text{ or } gh = 7\tfrac{1}{2} \sec.\ \theta = +\ 8.38.$$

These are the greatest strains of each kind the live load can cause in gh.

For fg we have the same strains only of opposite character, hence $fg = +$ 13.96, and $-$ 8.38.

For the brace ef the greatest positive shear is when P_3, P_4, P_5, P_6 and P_7 act, and the greatest negative shear when P_1, P_2 act. We thus find the shears $+\ 18\frac{3}{4}$ and $-\ 3\frac{3}{4}$.

Hence,

$$ef \cos \theta + 18.75 = 0, \text{ or } ef = -\ 20.94,$$

$$ef \cos \theta - 3.75 = 0, \text{ or } ef = +\ 4.19.$$

for de, then, we have $de = +$ 20.94 or $-$ 4.19.

For the brace cd the greatest positive shear will be when all the weights except P_1 act. We have then for the shears, $+\ 26\frac{1}{4}$ and $-\ 1\frac{1}{4}$.

Hence,

$$cd \cos \theta + 26\tfrac{1}{4} = 0, \text{ or } cd = -\ 29.32.$$

$$cd \cos \theta - 1\tfrac{1}{4} = 0, \text{ or } cd = +\ 1.4.$$

For bc we have $bc = +29.32$ and -1.4.

For the brace ab the greatest shear is positive, and occurs when all the loads act. There is no negative shear. When all the loads act the shear is $+35$.

Hence,

$$ab \cos\theta + 35 = 0, \text{ or } ab = -39.1.$$

We have, then, $La = +39.1$.

Collecting the above results together, we have the following Table:

TABLE OF STRAINS IN THE BRACES.

	La	ab	bc	cd	de	ef	fg	gh
Dead load.	+ 19.54	− 19.54	+ 13.96	− 13.96	+ 8.38	− 8.38	+ 2.8	− 2.8
Live load. Comp. +.	+ 39.1	....	+ 29.32	+ 1.4	+ 20.94	+ 4.19	+ 13.96	+ 8.38
Live load. Tens. −	....	− 39.1	− 1.4	− 29.32	− 4.19	− 20.94	− 8.38	− 13.96
Max. comp.	+ 58.64	....	+ 43.28	....	+ 29.32	....	+ 16.76	+ 5.58
Max. ten.	....	− 58.64	....	− 43.28	....	− 29.32	− 5.58	− 16.76

The values in this Table agree well with the Table on page 99. The first three lines give the dead load strains and the live load compression and tension. From these three lines, the last two, which give the maximum strains, are easily found, just as before.

THIRD METHOD—BY ALGEBRAIC METHOD OF MOMENTS.

MAXIMUM STRAINS IN THE FLANGES.—The point of moments for any flange is at the opposite apex. We take, as before, a full load, or 15 tons per apex. This gives $52\frac{1}{2}$ tons for each reaction. Then, since the depth of truss is 10 feet and the length of panel 10 feet, we can write down the following equations (See Fig. 94):

For the upper flanges,

$$Lb \times -10 + 52.5 \times 10 = 0, \text{ or } Lb \times 10 = 52.5 \times 10,$$

hence $Lb = +52.5$.

In similar manner,

$$Ld \times 10 = 52.5 \times 20 - 15 \times 10, \text{ or } Ld = +90,$$

$$Lf \times 10 = 52.5 \times 30 - 15 \times 20 - 15 \times 10, \text{ or } Lf = +112.5,$$

$$Lh \times 10 = 52.5 \times 40 - 15 \times 30 - 15 \times 20 - 15 \times 10, \text{ or } Lh = +120.$$

For the lower flanges,

$$Aa \times 10 + 52.5 \times 5 = 0, \text{ or } Aa \times 10 = -52.5 \times 5, \text{ hence } Aa = -26.25.$$

In similar manner,

$$Bc \times 10 = -52.5 \times 15 + 15 \times 5, \text{ or } Bc = -71.25,$$

$$Ce \times 10 = -52.5 \times 25 + 15 \times 15 + 15 \times 5, \text{ or } Ce = -101.25,$$

$$Dg \times 10 = -52.5 \times 35 + 15 \times 25 + 15 \times 15 + 15 \times 5, \text{ or } Dg = -116.25.$$

These values agree perfectly with those already found.

MAXIMUM STRAINS IN THE BRACES.—We have already seen, Fig. 73, page 80, that the application of the method of moments to the braces, gives us for the strain in any brace, as bc Fig. 73,

$$bc = \text{Shear} \times \sec\theta.$$

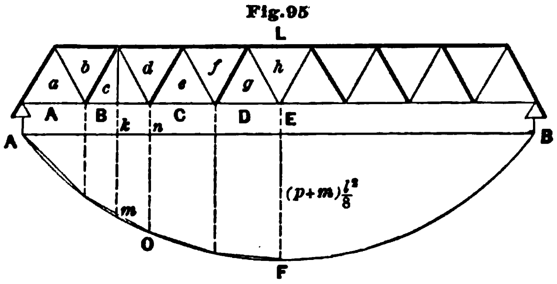
The method for the braces, then, is identical with that of the preceding method, when the flanges are horizontal.

FOURTH METHOD—GRAPHIC METHOD OF MOMENTS.

MAXIMUM STRAINS IN THE FLANGES.—The principles of this method are given on pages 83 and 84.

The dead load per unit of length is $p = 0.5$ and the live load $m = 1$. The middle ordinate of the parabola, Fig. 95, is therefore $(p + m)\dfrac{l^2}{8} = 1.5\dfrac{80^2}{8} = 1200$. We lay off then, Fig. 95, to any convenient scale $EF = 1200$ and draw the parabola AFB. Drop verticals from the *loaded apices*, and where they intersect the curve, we shall have the apices of the moment polygon. Then to find the moment for any flange, as Bc, drop a vertical *from the point of moments for that flange*. Thus, in the Fig. km, the ordinate by scale from AB to the polygon (*not* to the curve), gives the moment for the flange Bc. In like manner the ordinate nO gives the moment for Ld.

Fig. 95

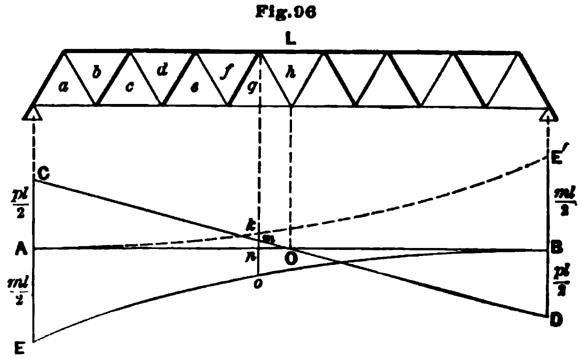


These moments divided by the depth of truss give the strains. The division can be at once effected by properly changing the scale of moments. Thus if we lay off $EF = 1200$ to a scale of 600 to an inch, and if we are to divide all the moments by 10, then the ordinate measured to a scale of 60 *tons to an inch* will give the strains directly.

If the student will make the construction carefully, he will find precisely the same values for the flanges as those already obtained by the preceding methods.

MAXIMUM STRAINS IN THE BRACES.—According to the principles of the preceding Chapter, page 84, we draw the shear diagram for dead load, Fig. 96, by laying off $\dfrac{pl}{2} = \dfrac{0.5 \times 80}{2} = 20$ at each end of the span and drawing the straight line COD. For the maximum live load shear, we lay off $AE = \dfrac{ml}{2} = \dfrac{1 \times 80}{2} = 40$ and draw the parabola EB. This parabola gives the positive shear for load coming on from the right. For load coming on from the left we have the dotted parabola AE', which gives the negative shear for any brace in the left half of truss.

Fig. 96



We have now only to remember that the shear for any brace is given by the ordinate let fall from the *middle* of the bay belonging to that brace. Thus, for gh the greatest positive shear is equal by scale to mo, where mn is the shear due to dead load, and no that due to live load. By our rule

$$gh \cos \theta + \text{shear} = o \text{ or } gh = - \text{Shear} \sec \theta.$$

For gh, as the sec θ is positive and since the shear is positive, mo sec. θ will give tension in gh. The sec θ in this case is 1.117. If, then, we divide the scale to which $\frac{pl}{2}$ and $\frac{ml}{2}$ are laid off by 1.117, mo to this new scale will give at once the strain in gh.

In the same way the greatest negative shear due to the live load is kn. But the positive shear due to dead load is mn. The difference, or km, is the effective shear which causes compression in gh. We see, therefore, that only fg and gh, and the corresponding braces on the other side of the centre, require counterbracing. For all the others the dead load positive shear exceeds the maximum negative shear due to live load.

Thus, Fig. 96, we obtain by scale $mn = 2.5$ and $no = 12.65$ or $mo = 15.15$. This multiplied by sec $\theta = 1.117$ gives 16.9 tension in gh, which agrees with the value already found by the preceding methods. In the same way we find $kn = 7.65$ and $mn = 2.5$, hence $km = 5.15$. This multiplied by sec $\theta = 1.117$ gives 5.7 tons for compression in gh, which agrees well with the values already found.

STRAINS DUE TO LOCOMOTIVE EXCESS.—In all that precedes we have supposed a uniformly distributed live load of 1 ton per foot. But as we have seen, page 94, we must for spans less than 250 feet and greater than 200 feet, also take into account the strains due to the *locomotive excess*. Whatever method, therefore, we adopt, of those just given, the solution is not complete until we have found and properly added the locomotive excess strains to those already found for uniform live load.

These strains we now proceed to find.

UPPER FLANGES.—For flange Lb, Fig. 95, we should have a concentrated load equal to the locomotive excess over 1 ton per foot, or 33 tons (page 94) acting at the 1st lower apex, Fig. 95, and another equal load acting at the 6th lower apex or 50 feet from the 1st (page 94). These two loads being conceived to act at these places, we find the left hand reaction easily from

$$R \times 80 = 33 \times 70 + 33 \times 20 \text{ or } R = 37.125.$$

Hence for the strain in Lb, we have

$$Lb \times 10 = 37.125 \times 10 \text{ or } Lb = + 37.125.$$

In the same way we have for Ld,

$$R \times 80 = 33 \times 60 + 33 \times 10, \text{ or } R = 28.875,$$

and

$$Ld \times 10 = 28.875 \times 20, \text{ hence } Ld = + 57.75.$$

$$R \times 80 = 33 \times 50, \text{ or } R = 20.625,$$

and

$$Lf \times 10 = 20.625 \times 30, \text{ hence } Lf = + 61.875 \text{ tons},$$

$$Lh \times 10 = 16.5 \times 40, \text{ hence } Lh = + 66 \text{ tons}.$$

LOWER FLANGES.—For flange Aa, Fig. 95, we have 33 tons at the first apex and at the 6th. Hence,

$$Aa \times 10 = - 37.125 \times 5, \text{ or } Aa = - 18.56.$$

For Bc we have,

$$Bc \times 10 = -28.75 \times 15, \text{ or } Bc = -43.12,$$
$$Ce \times 10 = -20.625 \times 25, \text{ or } Ce = -51.56,$$
$$Dg \times 10 = -16.5 \times 35, \text{ or } Dg = -57.75.$$

These strains must be *added* to those already found for the uniform live load of 2,000 lbs. per foot. We see at once how much the flange strains may be increased owing to the very heavy concentrated loads of the locomotive.

BRACES.—For the braces La and ab we have, according to page 101, the strains

$$La = 37.125 \times \text{sec. } \theta = 37.125 \times 1{,}117 = +41.47,$$

and

$$ab = -41.47.$$

In similar manner we have,

$$bc = 28.875 \times 1.117 = +31.5 \qquad cd = -31.5,$$

and

$$bc = -4.125 \times 1.117 = -4.5 \qquad cd = +4.5.$$

$$de = 20.625 \times 1.117 = +23.04 \qquad ef = -23.04,$$

and

$$de = -8.25 \times 1.117 = -9.21 \qquad ef = +9.21.$$

$$fg = 16.5 \times 1.117 = +18.4 \qquad gh = -18.4,$$

and

$$fg = -12.375 \times 1.117 = -13.82 \qquad gh = +13.82.$$

These values must be added to the corresponding values found for uniform live load. The actual maximum strains then are given by the following Table, where the dead load strains are as before, page 102.

TABLE FOR MAXIMUM STRAINS IN THE BRACES.

	La	ab	bc	cd	de	ef	fg	gh
Live load. Comp. +	39.1 41.47	....	29.32 31.5	1.4 4.5	20.94 23.04	4.19 9.07	13.96 18.4	8.38 13.82
Live load. Tension −	....	39.1 41.47	1.4 4.5	29.32 31.5	4.19 9.07	20.94 23.04	8.38 13.82	13.96 18.4
Dead load.	+ 19.54	− 19.54	+ 13.96	− 13.96	+ 8.38	− 8.38	+ 2.8	− 2.8
Max. comp.	100.11	....	74.78	....	52.36	4.88	35.16	19.4
Max. tens.	....	100.11	....	74.78	4.88	52.36	19.4	35.16

Comparing these strains with those found and tabulated on page 103, we see how great an influence the locomotive excess has. We see that de and ef must now also be counterbraced as well as fg and gh. All the strains are very much increased.

If there are two trusses in the bridge, the strains in each will be one half of those just found. In general we find the strains upon a truss as if it alone supported the entire load, and then divide these strains among as many trusses as may compose the bridge.

TABLES UNNECESSARY.—Reviewing all the methods, we see that in the present case the method of calculation by moments (page 103) is decidedly the simplest and best. The Table, page 99, was rendered necessary in order to avoid the necessity of making a separate diagram for each brace. With this exception, the other Tables are unnecessary, and are only given in order to show the relative influence of the dead and train loads and locomotive excess. In practice, we can and should find the maximum strain of either kind upon any piece directly by a single equation of moments. We close this Chapter, therefore, by calculating the case in hand in the manner which we recommend for all such cases.

Thus, referring to Fig. 94, let us find once more the maximum strains. Let the dead load of 5 tons at each apex be x, the train load 10 tons $= y$, and the locomotive excess 33 tons $= z$.

(*a*) STRAINS IN THE FLANGES.

The strains due to dead load alone are easily found if they are required, as on page 103, for full live load.

For the maximum strains, we proceed as follows:

At every apex of the lower flange, Fig. 94, let the dead load x and train load y act. We have, then, $x + y = 5 + 10 = 15$ tons at each apex, and the reaction at each end is $\frac{7(x+y)}{2} = 52.5$ tons.

For the bay Aa, Fig. 94, we have in addition the locomotive excess of $z = 33$ tons at P_1 and at P_6, 50 feet from P_1. These loads cause a reaction at the left end of $\frac{7}{8}z + \frac{2}{8}z = \frac{9}{8}z = 37.125$ tons. For the flange Aa then, the left reaction is $\frac{7(x+y)}{2} + \frac{9}{8}z = 89.625$ tons, and hence by moments,

$$Aa \times 10 = -\left[\frac{7(x+y)}{2} + \frac{9}{8}z\right] \times 5, \text{ or } Aa = -44.81.$$

For Bc we have z tons at P_2 and at P_7. The left reaction is, therefore, $\left[\frac{7(x+y)}{2} + \frac{7}{8}z\right] = 81.375$ tons. Hence

$$Bc \times 10 = -\left[\frac{7(x+y)}{2} + \frac{7}{8}z\right] \times 15 + (x+y)\,5 \qquad Bc = -114.56.$$

In similar manner,

$$Ce \times 10 = -\left[\frac{7(x+y)}{2} + \frac{5}{8}z\right] \times 25 + (x+y)(15+5) \qquad Ce = -152.81.$$

$$Dg \times 10 = -\left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 35 + (x+y)(25+15+5) \quad Dg = -174.$$

$$Lb \times 10 = +\left[\frac{7(x+y)}{2} + \frac{9}{8}z\right] \times 10 \qquad Lb = +89.625.$$

$$Ld \times 10 = +\left[\frac{7(x+y)}{2} + \frac{7}{8}z\right] \times 20 - (x+y)\,10 \qquad Ld = 147.75.$$

$$Lf \times 10 = + \left[\frac{7\,(x+y)}{2} + \frac{5}{8}z\right] \times 30 - (x+y)\,(20+10) \qquad Lf = +\,174.375.$$

$$Lh \times 10 = + \left[\frac{7\,(x+y)}{2} + \frac{4}{8}z\right] \times 40 - (x+y)\,(30+20+10) \quad Lh = +\,186.$$

These are the maximum strains which can ever occur in the flanges.

(*b*) STRAINS IN THE BRACES.

For the greatest tension in *gh*, Fig. 94, we have at every lower apex 5 tons $= x$, due to the dead load; also 10 tons $= y$ at all the right hand apices due to train load, and finally, 33 tons $= z$ at P_4 due to the locomotive excess. The left reaction is then $\frac{7x}{2} + \left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8}\right)y + \frac{z}{2} = \frac{7}{2}x + \frac{10}{8}y + \frac{4z}{8} = 17.5 + 12.5 + 16.5 = 46.5$ tons. The shear for *gh* is, then, the reaction minus the three dead loads at P_1, P_2, and P_3, or $\frac{7}{2}x + \frac{10}{8}y + \frac{z}{2} - 3x = 46.5 - 15 = +\,31.5$ tons. Therefore,

$$gh = -\left(\frac{7}{2}x + \frac{10}{8}y + \frac{4z}{8} - 3x\right)\sec\theta = -\,31.5 \times 1.117 = -\,35.18 \text{ tons.}$$

The greatest compression in *gh* will be when, in addition to the dead load at every lower apex, we have 10 tons $= y$ at P_1, P_2 and P_3 and 33 tons $= z$ at P_3. The reaction at the *right* end is then

$$\tfrac{7}{2}x + \left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8}\right)y + \tfrac{3}{8}z = \tfrac{7}{2}x + \tfrac{6}{8}y + \tfrac{3}{8}z = 17.5 + 7.5 + 12.375 = 37.375.$$

The negative shear for *gh* is then $-\,37.375 + 4x = -\,17.375$, and

$$gh = \left(\tfrac{7}{2}x + \tfrac{6}{8}y + \tfrac{3}{8}z - 4x\right)\sec\theta = 17.375 \times 1.117 = +\,19.4.$$

For the greatest tension in *ef*, we have, in addition to the dead load of 5 tons $= x$ at every lower apex, 10 tons $= y$ at every right hand lower apex and 33 tons $= z$ at P_3. The *left* reaction is, therefore, $\frac{7}{2}x + \frac{15}{8}y + \frac{5}{8}z = 17.5 + 18.75 + 20.625 = 56.875$. The positive shear is, therefore, $56.875 - 2x = 46.875$.

For the greatest negative shear we have 10 tons $= y$ at P_1 and P_2 and 33 tons $= z$ at P_2. Hence the *right* hand reaction is $\frac{7}{2}x + \frac{3}{8}y + \frac{2}{8}z = 29.50$. The negative shear is therefore $-\,29.50 + 5x = -\,4.50$.

We have, therefore,

$$ef = -\,46.875 \times 1.117 = -\,52.6 \text{ and } ef = +\,4.5 \times 1.117 = +\,5.02$$
$$de = +\,52.36 \qquad\qquad de = -\,5.02.$$

For tension in *cd* we have, in addition to the dead load, 10 tons at every right hand apex and 33 tons $= z$ at P_2 and at P_7 also. The left hand reaction is then $\frac{7}{2}x + \frac{21}{8} + \frac{7}{8}z = 72.625$ tons, and the positive shear is $72.625 - x = 67.625$. Hence

$$cd = -\,67.625 \times 1.117 = -\,75.53 \text{ and } bc = +\,75.53.$$

For the greatest compression, if any, in *cd*, we have 10 tons at P_1 and also 33 tons at P_1. The reaction at right is then $\frac{7}{2}x + \frac{1}{8}y + \frac{1}{8}z = 22.875$. The negative shear is, therefore $-\,22.875 + 6x = +\,7.125$. As the shear in this case comes out positive, it shows that *cd* is in tension for this loading also. In other words *cd* does not need to be counterbraced. The same holds true for all the remaining braces.

For *ab* we have finally 15 tons at every lower apex and 33 tons at P_1 and at P_6. The reaction at left end is then $52.5 + 37.125 = 89.625$. This is equal to the shear. Therefore,

$$ab = -89.625 \times 1.117 = -100.11 \text{ and } La = +100.11.$$

These values agree well with those given in the Table, page 106. The slight discrepancies are due, as explained on page 102, to the manner in which the dead load strains were formed, *viz.*, from the algebraic sum of the train strains divided by two.

We see, then, that the maximum strains may be found directly by this method from a single equation for each piece, and no Table is required. Whether a brace is to be counterbraced or not and the strain in the counter, are easily determined.

The above comprises the application of our four methods to a bridge girder sustaining a live load as well as a dead load. In the following Chapters we shall make use of one or the other of these methods, whichever may seem best adapted to the case in hand.

For the method by concentrated wheel loads see page 220. This, as has been repeatedly said, is the best method, and the student should pay special attention to it.

Questions for Examination.

What four methods of solution are there? State briefly the principles of these methods. Illustrate by an example the method by graphic resolution of forces as applied to the flanges. For what state of loading are the flange strains the greatest? Show how to find the brace strains due to a single apex load. Explain how the brace strains due to any other apex load may be at once found from this. Show how a table may be drawn up which gives the strains in the braces for each apex load separately. How can the dead load strains be found from these? How can you find finally the maximum strains?

Explain the method by algebraic resolution of forces. Show how to find the flange coefficients. By what must these coefficients be multiplied in order to find the flange strains? How are the maximum strains in the braces found? What position of live load gives the greatest strains in the braces? How can you tell whether a brace is in tension or compression?

Explain the third method, by algebraic method of moments. Apply the method to the flanges. Show that for the braces the method is the same as the preceding.

Explain the fourth method. What is the middle ordinate of the moment parabola? How can this parabola be drawn? From what point ought we to drop the ordinates for the flanges? Show how the strains can be obtained from these ordinates. Explain how to change the scale so that the ordinates may give the strains directly.

Illustrate the method for the braces. Show how to determine when a brace ought to be counterbraced. From what point ought the ordinate to be dropped for any brace? Are the tables of strains in the braces necessary? How can you find the maximum strains in any brace by a single operation? Illustrate.

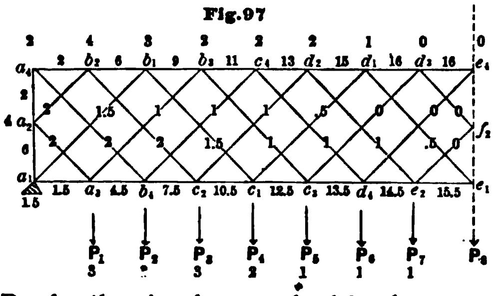
CHAPTER IV.

BRIDGE GIRDERS WITH PARALLEL FLANGES—CONTINUED.

LATTICE GIRDER—EXAMPLE FOR SOLUTION.—As the length of span becomes greater it may be advantageous to have more than one system of bracing, thus reducing the panel length. Such systems, owing to the indeterminate character of the strains, are usually avoided in practice. Lattice girders may be regarded to-day as antiquated. No more are or will be built.

In Fig. 97 we have represented the half span of a girder with four systems of bracing, load on lower flange. We have the length $l =$ 160 feet, divided into 16 bays of 10 feet each, height of girder = 20 feet, and the braces making an angle of 45° with the vertical. Therefore, $\theta =$ 45°, $\tan\theta = 1$, and $\sec\theta = 1.414$. Let the dead load $p = 0.5$ ton per foot, and the live load $m = 1$ ton per foot. Then the apex dead load is 5 tons, and the apex live load is 10 tons.*

Fig. 97



(*a*.) MAXIMUM STRAINS IN THE FLANGES.—By far the simplest method in the present case is the method by coefficients, explained in the preceding Chapter. Thus, Fig. 97, we write down the coefficients upon the diagonals, which multiplied by $P = 15$, give the shear for full load. Adding the coefficients of the two diagonals which meet at an apex, we obtain the apex coefficients as given in the Figure. Then beginning at the end and proceeding toward the centre, we find by successive addition the flange coefficients, which, multiplied by $P\tan\theta$, give the flange strains. Since $\tan\theta = 1$ and $P = 15$, $P\tan\theta = 15$.

For the upper flanges, all of which are in compression, we have, then, at once,

$$a_4 b_2 = 2 \times 15 = +30, \quad b_2 b_1 = 6 \times 15 = +90,$$

$$b_1 b_3 = 9 \times 15 = +135, \quad b_3 c_4 = 11 \times 15 = +165, \quad c_4 d_2 = 13 \times 15 = +195,$$

$$d_2 d_1 = 15 \times 15 = +225, \quad d_1 d_3 = d_3 e_4 = 16 \times 15 = +240 \text{ tons.}$$

For the lower flanges, all of which are in tension, we have,

$$a_1 a_3 = 1.5 \times 15 = -22.5, \quad a_3 b_4 = 4.5 \times 15 = -67.5, \quad b_4 c_2 = 7.5 \times 15 = -112.5,$$

$$c_2 c_1 = 10.5 \times 15 = -157.5, \quad c_1 c_3 = 12.5 \times 15 = -187.5,$$

$$c_3 d_4 = 13.5 \times 15 = -202.5, \quad d_4 e_2 = 14.5 \times 15 = -217.5, \quad e_2 e_1 = 15.5 \times 15 = -232.5.$$

For the end post,

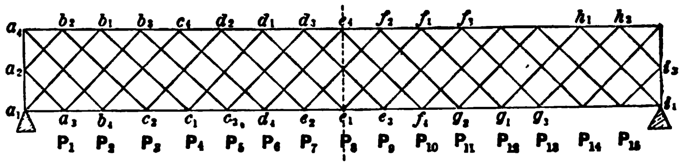
$$a_4 a_2 = 2 \times 15 = +30, \quad a_2 a_1 = 6 \times 15 = +90.$$

* In all our examples dead load and dimensions are assumed for convenience of illustration only, and are not to be considered as practical cases. We shall see how to estimate dead load and best dimensions hereafter. For spans less than 200 feet the method of this Chapter should not be used. For method by concentrated loads see Appendix, page 215.

(*b*.) Maximum Strains in the Braces.—The apex live load is 10 tons.

We find the maximum strains in the braces due to it by the method of page 101.

Fig. 98



Thus, the greatest positive shear for $d_1\,e_1$, Fig. 98, will be when P_8 and P_{12} only act, because these are the only apex weights which act on the system to which $d_1\,e_1$ belongs, on the right of $d_1\,e_1$.

This shear is $(\frac{4}{16}+\frac{8}{16})\,10=+\,7.5$. Hence

$$d_1\,e_1\cos\theta+7.5=0, \quad \text{or} \quad d_1\,e_1=-\,7.5\times 1.414=-\,10.6.$$

We therefore have $d_1\,c_1=+\,10.6$.

The greatest negative shear for $d_1\,e_1$ will be when P_4 only acts. This shear is $-\,\frac{4}{16}\,10=-\,2.5$. It causes, therefore, compression in $d_1\,e_1$ equal to $2.5\times 1.414=+\,3.53$.

In $d_1\,c_1$ we have then $-\,3.53$.

For the strain in $d_3\,e_3$, we have, from Fig. 98, the positive shear caused by P_9 and P_{13}, or equal to $(\frac{7}{16}+\frac{3}{16})\,10=+\,6.25$.

The negative shear is when P_5 and P_1 act. It is equal to $(\frac{5}{16}+\frac{1}{16})\,10=-\,3.75$. We have then $d_3\,c_3=+\,6.25\times 1.414=+\,8.84$, and $d_3\,c_3=-\,3.75\times 1.414=-\,5.3$, and $d_3\,e_3=-\,8.84$, and $+\,5.3$.

For $e_4\,f_4$, the positive shear is when P_{10} and P_{14} act, and the negative shear when P_6 and P_2 act. These shears are $(\frac{6}{16}+\frac{2}{16})\,10=+\,5$ and $(\frac{6}{16}+\frac{2}{16})\,10=-\,5$. The strains, then, in $e_4\,f_4$ are $5\times 1.414=+\,7.07$ and $-\,7.07$.

For $d_2\,e_2$ we have the positive shear when P_7, P_{11} and P_{15} act, and the negative shear when P_3 alone acts. These shears are $(\frac{9}{16}+\frac{5}{16}+\frac{1}{16})\,10=+\,9.375$ and $-\,\frac{3}{16}\,10=-\,1.875$.

We have, then,

$$d_2\,e_2=-\,9.375\times 1.414=-\,13.26,$$

and

$$d_2\,c_2=+\,1.875\times 1.414=+\,2.65.$$

The strains in $d_2\,c_2$, then, are $+\,13.26$ and $-\,2.65$.

For $c_4\,d_4$ we have in like manner P_6, P_{10} and P_{14}, causing positive shear, and P_2 causing negative shear. The positive shear is then $(\frac{10}{16}+\frac{6}{16}+\frac{2}{16})\,10=+\,11.25$ and the negative is $-\,\frac{2}{16}\,10=-\,1.25$. Therefore,

$$c_4\,d_4=-\,11.25\times 1.414=-\,15.91,$$

$$c_4\,d_4=+\,1.25\times 1.414=+\,1.77.$$

The strains in $c_4\,b_4$ are $+\,15.91$ and $-\,1.77$.

For $b_3\,c_3$ the positive shear is $(\frac{11}{16}+\frac{7}{16}+\frac{3}{16})\,10=+\,13.125$, and the negative shear is $-\,\frac{1}{16}\,10=-\,0.625$. Therefore,

$$b_3\,c_3=-\,13.125\times 1.414=-\,18.56,$$

$$b_3\,c_3=+\,0.625\times 1.414=+\,0.88,$$

and

$$b_3\,a_3=+\,18.56, \text{ and } -\,0.88.$$

For $b_1 c_1$ the positive shear is caused by P_4, P_8, P_{12}, and is $(\frac{12}{16} + \frac{8}{16} + \frac{4}{16})\ 10 = +15$. The negative shear is zero. Hence,

$$b_1 c_1 = -15 \times 1.414 = -21.21, \text{ and } b_1 a_1 = +21.21.$$

For $b_2 c_2$ the positive shear is caused by P_3, P_7, P_{11} and P_{15}, and is $(\frac{13}{16} + \frac{9}{16} + \frac{5}{16} + \frac{1}{16})\ 10 = +17.5$. The negative shear is zero. Hence,

$$b_2 c_2 = -17.5 \times 1.414 = -24.74, \text{ and } b_2 a_2 = +24.74.$$

For $a_4 b_4$ the positive shear is $(\frac{14}{16} + \frac{10}{16} + \frac{6}{16} + \frac{2}{16})\ 10 = +20$. Hence,

$$a_4 b_4 = -20 \times 1.414 = -28.28.$$

For $a_2 a_3$ the positive shear is when the loads P_1, P_5, P_9, P_{13}, act. The shear then is $(\frac{15}{16} + \frac{11}{16} + \frac{7}{16} + \frac{3}{16})\ 10 = +22.5$. Hence,

$$a_2 a_3 = -22.5 \times 1.414 = -31.81.$$

We can now collect these results in a Table, as follows:

TABLE OF STRAINS IN THE BRACES.*

	e_4d_4	d_3e_3	d_3c_3	d_1e_1	d_1c_1	d_2e_2	d_2c_2	c_4d_4	c_4b_4	b_3c_3	b_3a_3	b_1c_1	b_1a_1	b_2c_2	b_2a_2
Live load. Comp. +	+7.07	+5.3	+8.84	+3.53	+10.6	+2.65	+13.26	+1.77	+15.91	+0.88	+18.56	....	+21.21	....	+24.74
Live load. Tens. −	−7.07	−8.84	−5.3	−10.6	−3.53	−13.26	−2.65	−15.91	−1.77	−18.56	−0.88	−21.21	....	−24.74	....
Dead load.	0	−1.77	+1.77	−3.5	+3.5	−5.3	+5.3	−7.07	+7.07	−8.84	+8.84	−10.6	+10 6	−12.37	+12.37
Max. comp. +	+7.07	+3.53	+10.61	..	+14.1	....	+18.56	....	+22.98	....	+27.40	....	+31.81	....	+37.11
Max. tens. −	−7.07	−10.61	−3.53	−14.1	....	−18.56	....	−22.98	....	−27.40	....	−31.81	....	−37.11	....

The live load strains just found give us the first two lines. Since the dead load is one half of the live, the algebraic sum of the first two lines divided by 2, gives the dead load strains. The dead load strains can also be found from the coefficients, Fig. 97, upon the braces, by multiplying by $P \sec \theta = 5 \times 1.414 = 7.07$. There is a slight discrepancy in the two methods, already pointed out on page 102.

The line for dead load being thus filled out, we can find the maximum strains. We see from the Table that $f_2 e_2$, $e_4 d_4$, $d_3 c_3$ and $d_3 c_3$ are the only diagonals which require counterbracing on the left of the centre. Of course, the strains are the same in all the corresponding pieces of the right half of the girder.

In a precisely similar manner we may manage any number of systems.

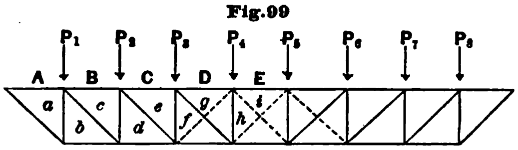
A double or triple system is generally used when the length of bay for a single system, owing to the increase of height due to great length, renders it advisable to support the flanges at more frequent intervals.

A multiple system, then, such as Fig. 98, when used for a long span, may be calculated as in the preceding pages, *disregarding locomotive excess* and considering the live load as uniformly distributed. It is not, therefore, in general necessary to take account of locomotive excess. When, however, it is necessary so to do, the method of calculation is explained further on, when treating of the Pratt truss, double system.

* Again we call attention to the fact that a Table is unnecessary, see page 103.

PRATT TRUSS.—DECK BRIDGE.—Let Fig. 99 represent a Pratt truss 90 feet long, load on the upper chord. The bridge is, therefore, a "deck" bridge. Let the depth of truss be 10 feet, and let there be 9 panels of 10 feet each in the upper chord, and 7 in the lower chord.

Fig. 99



We have then $\theta = 45°$, $\sec\theta = 1.414$. Let the train load be 1 ton per foot preceded by two standard locomotives, and the dead load 0.5 ton per foot. Then we have $P = 10$ tons per live panel load, and $P = 5$ tons for uniform panel dead load. Locomotive excess 33 tons. Trains preceded by two locomotives.

In this style of truss, the verticals are to take compression only and the inclined braces tension only. Whenever the live load would tend to cause compression in any inclined brace, that piece must be counterbraced by inserting a brace uniting the other corners of the panel. Those inclined braces which are extended by the action of the dead load, or by a full load live and dead *extending over the whole truss*, are call *ties*. They are represented in Fig. 99 by full lines. Tne dotted lines denote *counter-ties*, which are only called into play by the live load.

When the truss is fully loaded, the centre of the girder is deflected most, and on each side of the centre the curve is the same. We can always, therefore, tell which are the ties in any case, by considering the deformed panel under full load, and remembering that the tie is the longest diagonal of the deformed panel. In Fig. 99, since we have an odd number of panels, the centre panel is not deformed, but remains a rectangle. Hence the diagonals in it are both counterbraces, and are not strained by full load, at all, but only by partial or live loads not extending over the whole truss.

(*a*.) MAXIMUM STRAINS IN THE FLANGES.—Suppose at every upper apex the dead load of $x = 5$ tons, and the train load of $y = 10$ tons always acting, or 15 tons $= x + y$ at each upper apex, Fig. 99. We have, then, only to suppose in addition to this, the locomotive excess to act at the proper apices for each flange, page 94, and we can find the maximum strains at once.

Thus, for Aa, Fig. 99, we should have the locomotive excess of $z = 33$ tons at P_1 and at P_6. The reaction at the left end due to dead and live loads is, then, $\frac{8(x+y)}{2} = 60$ tons, and due to locomotive excess $\frac{8}{9}z + \frac{3}{9}z = \frac{11}{9}z = 40.33$ tons, or altogether $\frac{8(x+y)}{2} + \frac{11}{9}z = 100.33$ tons. We have, therefore,

$$Aa \times 10 = [4(x+y) + \tfrac{11}{9}z] \times 10 \qquad \text{or } Aa = +100.33$$

$$Bc \times 10 = [4(x+y) + \tfrac{9}{9}z] \times 20 - (x+y) \times 10 \qquad Bc = +171$$

$$Ce \times 10 = [4(x+y) + \tfrac{7}{9}z] \times 30 - (x+y)(20+10) \qquad Ce = +211.98$$

$$Dg \times 10 = [4(x+y) + \tfrac{5}{9}z] \times 40 - (x+y)(30+20+10) \qquad Dg = +223.33$$

$$Eh \times 10 = [4(x+y) + \tfrac{4}{9}z] \times 50 - (x+y)(40+30+20+10) \qquad Eh = +223.33$$

It makes no difference which lower apex we take as the centre of moments for Eh, the one on the right or the one on the left. Thus

$$Eh \times 10 = [4(x+y) + \tfrac{5}{9}z] \times 40 - (x+y)(30+20+10) \qquad \text{or } Eh = +223.33$$

as before. For the lower flanges we have, for Lb the locomotive excess at P_1 and P_4. Therefore

$$Lb \times 10 = -\left[4(x+y) + \tfrac{11}{9}z\right] \times 10 \qquad Lb = -100.33$$

$$Ld \times 10 = -\left[4(x+y) + \tfrac{9}{9}z\right] \times 20 + (x+y) \times 10 \qquad Ld = -171$$

$$Lf \times 10 = -\left[4(x+y) + \tfrac{7}{9}z\right] \times 30 + (x+y)(20+10) \qquad Lf = -211.98$$

$$Lh \times 10 = -\left[4(x+y) + \tfrac{5}{9}z\right] \times 40 + (x+y)(30+20+10) \qquad Lh = -223.33.$$

These are the maximum strains which can ever occur in the flanges.

(*b.*) MAXIMUM STRAINS IN THE BRACES.—We suppose each upper apex loaded with the dead load $x = 5$ tons. We take the train load $y = 10$ tons, and the locomotive excess $z = 33$ tons, at the proper apices to give the maximum strains for each brace (page 94).

Thus for the counterbrace hi, Fig. 99, we suppose $y = 10$ tons at P_5, P_6, P_7 and P_8, and $z = 33$ tons at P_5. The left reaction is then $\frac{8x}{2} = 20$ tons for dead load, $(\frac{1}{9} + \frac{2}{9} + \frac{3}{9} + \frac{4}{9})y = 11.11$ tons for train load, and $\frac{4}{9}z = 14.66$ tons for locomotive excess, or altogether $4x + \frac{10}{9}y + \frac{4}{9}z = 45.77$ tons. The positive shear for hi is then the left reaction minus all the weights between the left end and P_5, or $4x + \frac{10}{9}y + \frac{4}{9}z - 4x = \frac{10}{9}y + \frac{4}{9}z = 25.77$ tons. We have, therefore,

$$hi \cos\theta_{hi} + 25.77 = 0, \quad \text{or} \quad hi = -25.77 \times 1.414 = -36.44 \text{ tons.}$$

If there were no other diagonal in the centre panel, the greatest compression on hi would be found by supposing $y = 10$ tons at P_4, P_3, P_2 and P_1, and $z = 33$ tons at P_4. This would cause a compression of 36.44 tons, the same as the tension in the first case. As hi cannot take compression, this strain comes as tension in the other diagonal. The two centre counterbraces are, therefore, subjected to an equal maximum strain of -36.44 tons for each; under the action of the dead load alone they are not strained at all. This is in accordance with the principle that for uniform load over the entire span, the shear at the centre is zero (page 84).

The posts are always in compression. The greatest compression on gh will be when the train load extends over the *longer* segment, or when we have $y = 10$ tons, at P_4, P_5, P_6, P_7 and P_8, and $z = 33$ tons at P_4, as well as $x = 5$ tons at every upper apex. The shear for this loading will be the greatest compression on gh, and this shear multiplied by 1.414 will be the greatest tension in fg. The left reaction is then $4x + \frac{15}{9}y + \frac{5}{9}z = 55$ tons. The shear is $4x + \frac{15}{9}y + \frac{5}{9}z - 3x = 40$ tons. Hence

$$gh = x + \tfrac{15}{9}y + \tfrac{5}{9}z = +40 \text{ tons.}$$

The same loading gives the greatest tension in fg. Hence

$$fg = -40 \times 1.414 = -56.56 \text{ tons.}$$

For the greatest compression on fg, if any, or in other words the tension in the counterbrace for fg, if any counter is needed, we must have P_1, P_2 and P_3 loaded with 10 tons, and 33 tons at P_3. The left reaction is then $20 + 23.33 + 22 = +65.33$. The shear then is $65.33 - 15 - 15 - 48 = -12.66$ tons. As the shear comes out minus it will cause compression in fg or tension in the counter. Hence

$$fg = 12.66 \times 1.414 = +17.90 \text{ tons.}$$

If the shear in the second case had also come out plus, it would have denoted that

no counter was necessary. In such case both loadings would cause tension in *fg*, and the greatest would be as above, $-$ 56.56 tons.

In the same way for *ef*, we have for left reaction $20 + 23.33 + 25.66 = +69$. The maximum positive shear then is $+69 - 10 = 59$. Hence

$$ef = +59 \text{ tons.}$$

The greatest tension in *de* is, therefore,

$$de = -59 \times 1.414 = -83.43 \text{ tons.}$$

For the load coming on from the left, we have left reaction $= 20 + 16.66 + 25.66 = +62.33$. The shear is then $62.33 - 15 - 48 = -0.67$. As the shear thus comes out negative in this case, *de* requires to be counterbraced, and we have $de = 0.67 \times 1.414 = +0.95$ tons.

For *cd*, we have in similar manner, left reaction $= 20 + 31.11 + 33 = +84.11$. The greatest positive shear is, therefore, $84.11 - 5 = +79.11$. Hence

$$cd = +79.11.$$

The greatest tension in *bc* is, therefore,

$$bc = -79.11 \times 1.414 = -111.86 \text{ tons.}$$

For the load coming on from the left, the shear is positive, and there is no counterbrace needed for *bc*.

For *ab*, we have 15 tons at every upper apex and 33 tons at P_1 and P_6. Hence the reaction at left is $60 + 40.33 = +100.33$. As there are no weights between the left end and P_1, this is also the shear. Therefore,

$$ab = +100.33 \text{ tons.}$$

Finally the end tie *La* is

$$La = -100.33 \times 1.414 = -141.86 \text{ tons.}$$

These are the maximum strains in the braces.

If the girder in Fig. 99 is turned over, as shown in Fig. 100 (*a*), the load being still on the top flange, the last vertical *cd* is a simple rod diagonal to support only the centre of the last end bay, which otherwise would have to be of double length. It takes no compression. The continuation of the roadway, shown by *bc*, is not a part of the truss, neither is the end pillar *ba*, which, if needed at all, takes only a compression of one full panel load, or $15 + 33 = 48$ tons.

Fig. 100 (*a*)

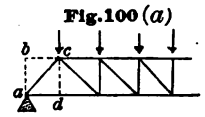


Fig. 100 (*b*)

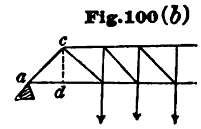
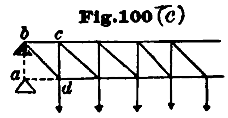


Fig. 100 (*c*)



If the load is on the bottom flange, Fig. 100(*b*), the last vertical *cd* takes tension only, if there is a cross girder at *d*, to the amount of $5 + 10 + 33 = 48$ tons. If there is no cross girder at *d*, it merely supports, as in the first case, the centre of the long double bay. If the girder is as in Fig. 99, but with the load on the lower flange, as shown in Fig. 100(*c*), the continuation of the roadway *ad* is not a part of the truss. The end pillar, *ba*, supports half the total weight of truss and train and locomotive at *d*. The support may be either directly under *b* or at *a*.

In any of these cases there can be no difficulty experienced in calculating the strains.

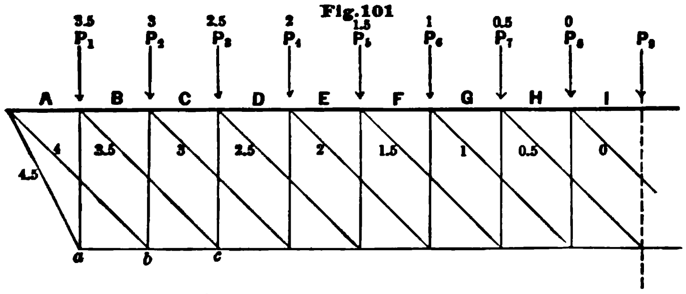
General Method for Vertical and Diagonal Bracing.—In general, then, whatever method of solution we adopt, we consider at first but one system of braces, *viz.*,

that strained by the dead load alone. Then if we find for any diagonal a strain of opposite kind to that which it is intended to resist, a counter must be inserted to take that strain.

When a piece is thus intended to take but one kind of strain, it must be so arranged that it cannot take any other. This is easily attained in practice. Thus, if the posts merely abut on the flanges and are not directly united with them, they cannot take tension under any circumstances. Or even if the posts are rigidly connected with the flanges, if the ties are rods which run through the flanges and are held by nuts on the outer side, they can never take compression. Or again, if posts and ties are connected with the flanges and each other, still if the ties are long pieces of small sectional area, they will not in practice take any great amount of compression, but will bend or buckle and thus bring strain on the counters.

PRATT TRUSS.—DOUBLE SYSTEM.*—Let Fig. 101 represent the half span. Let the height of truss be 20 feet, and panel length 10 feet. Length of span 180 feet, divided into 18 panels. Then $\theta = 45°$ and $\tan\theta = 1$ for all the diagonals except the ends, where $\theta = 26° \ 34'$ and $\tan\theta = 0.5$.

Fig. 101



Let the train load be 1 ton per foot, or 10 tons at each upper apex, and dead load be 0.5 ton per foot, or 5 tons at each upper apex. The locomotive excess is 33 tons (page 94). Train preceded by two locomotives.

(*a.*) MAXIMUM STRAINS IN THE FLANGES.—We form a diagram of coefficients as shown in Fig. 101, precisely as directed on page 101, Fig. 93. The only difference in this case is, that as the posts are vertical, the component of their strains in the direction of the flanges will be zero. Hence the coefficient for every post is omitted. In other respects the method is similar.

Thus the strain in A is compression and equal to

$$4.5\,P\tan\theta + 4\,P\tan\theta', \text{ where } P = 15 \text{ tons and } \tan\theta' = 1, \tan\theta = 0.5.$$

Hence

$$A = 4.5 \times 7.5 + 4 \times 15 = +\ 93.75.$$

For B we have

$$B = 93.75 + 3.5 \times 15 = +\ 146.25.$$

In like manner

$$C = 146.25 + 3 \times 15 = +\ 191.25,$$

$$D = 191.25 + 2.5 \times 15 = +\ 228.75.$$

and so on.

The strains due to locomotive excess must now be found separately and added. In doing this we must take each system by itself. Thus, for A, we have 33 tons at P_1 and at P_6. But P_1 acts on one system and P_6 on the other.

The left reaction for 33 tons at P_1 is $\frac{17}{18} \times 33 = 31.16$, and the centre of moments is at a. For P_6 the reaction is $\frac{12}{18} \times 33 = 22$ and the centre of moments is at b. If the second 33 tons were at P_7 instead of P_6 it would cause less strain at A, because the reaction would be less and its lever arm less. Hence

$$A \times 20 = 31.16 \times 10 + 22 \times 20 \text{ or } A = +\ 37.58.$$

* All double systems, owing to indeterminate strains, are avoided by the best practice. This system may be regarded as practically antiquated. No more will probably be built in America. When it is desirable to reduce the panel length the Baltimore Truss is preferable. For method by concentrated loads, see page 215.

In the same way for B, we have 33 tons at P_3 and at P_7. The reaction of P_3, is $\frac{16}{18}$ 33 = 29.33, and of P_7, $\frac{11}{18}$ 33 = 20.16. The centre of moments in the first case is at b and in the second at c. Hence

$$B \times 20 = 29.33 \times 20 + 20.16 \times 30, \text{ or } B = +\ 59.57.$$

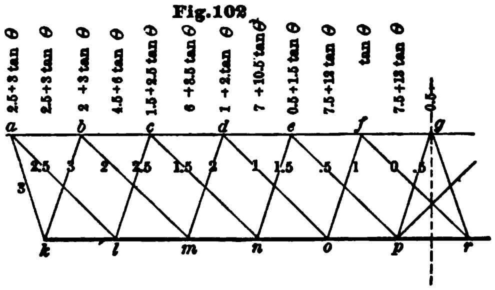
In the same way we can find the strains in the other flanges due to locomotive excess. These must be added to those already formed for dead and train loads, in order to obtain the maximum strains.

(*b.*) Maximum Strains in the Braces.—We proceed *for each system* precisely as illustrated in the preceding case, Fig. 99, and in the case of Fig. 98, page 111.

In finding the locomotive excess strains, we must take *both the loads on the same system*. Thus for the post at P_2, Fig. 101, we have a load of 33 tons at P_2 and another at P_8, and not at P_7, because P_7 belongs to the other system. The student need find no difficulty in solving the case for himself.

Post Girder.—Let Fig. 102 represent a Post truss, the span being 120 feet, divided into 12 panels in the upper chord. Depth of truss, then, will be 15 feet. The angle of the ties with the vertical is 45°, and of the inclined posts 18° 26′.

Fig. 102



We have, then, tan $\theta = 1$ for the ties and tan $\theta = 0.333$ for the posts, sec $\theta = 1.414$ for the ties and sec $\theta = 1.054$ for the posts. Let the load be on the top flange and equal 1 ton per foot for live load, and 0.5 ton per foot for dead load. The apex live load is then 10 tons and the apex dead load 5 tons. Locomotive excess as always, 33 tons (page 94). Train preceded by two locomotives.

(*a.*) Maximum Strains in the Flanges.—Suppose 15 tons at each upper apex. Then write down the coefficients for each brace as always. But we cannot now add these coefficients in order to find the apex coefficients, because the post and tie do not make equal angles with the vertical.

Thus, Fig. 102, the horizontal component of ak is $3\,P \tan\theta = 3 \times 15 \times 0.333$, and that of al is $2.5\,P \tan\theta' = 2.5 \times 15 \times 1$. If, then, since tan θ for the ties is 1, we denote by θ the angle 18° 25′ of the posts, we have at the apex a the coefficient $2.5 + 3 \tan\theta$, at b, $2 + 3 \tan\theta$, etc., where each of these coefficients is to be multiplied by 15.

The strain, then, in ab is $(2.5 + 3 \tan\theta)\,15 = (2.5 + 3 \times 0.33)\,15 = +\ 52.5$. In similar manner we have

$$bc = (4.5 + 6 \times 0.33)\,15 = +\ 97.5, \qquad cd = (6 + 8.5 \times 0.33)\,15 = +\ 132.5,$$

$$de = (7 + 10.5 \times 0.33)\,15 = +\ 157.5, \qquad ef = (7.5 + 12 \times 0.33)\,15 = +\ 172.5,$$

$$fg = (7.5 + 13 \times 0.33)\,15 = +\ 177.5.$$

In the same way we can find the strains on the lower flanges, thus:

$$kl = -\,(6 \times 0.33)\,15 = -\ 29.70 \qquad lm = -\,(2.5 + 8.5 \times 0.33)\,15 = -\ 79.57.$$

$$mn = -\,(4.5 + 10.5 \times 0.33)\,15 = -\ 119.47, \; no = -\,(6 + 12 \times 0.33)\,15 = -\ 149.40.$$

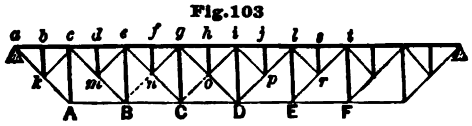
$$op = -\,(7 + 13 \times 0.33)\,15 = -\ 169.35, \; pr = -\,(7.5 + 13.5 \times 0.33)\,15 = -\ 179.32.$$

To these must be added the strains due to locomotive excess, found precisely as in the preceding case.

(*b*.) MAXIMUM STRAINS IN THE BRACES.—In order to find the maximum strains in the braces, we proceed precisely as in the preceding case, page 113, only remembering to multiply the shear by 1.414 for the ties, and by 1.054 for the struts. The student can easily solve the example for himself. This type is usually used only as a "through" girder. In either case its calculation is simple.

BALTIMORE BRIDGE COMPANY'S TRUSS.*—Fig. 103 represents this truss. Let the load be on the upper flange. The length of each bay is 10 feet and there are 16 bays in the upper flange. The depth is 20 feet. All the verticals are posts, and all inclined pieces ties. The train load is 10 tons for each upper apex and dead load 5 tons. Locomotive excess 33 tons. Train preceded by two locomotives. The angle for the ties is 45°.

Fig. 103



(*a*.) MAXIMUM STRAINS IN THE FLANGES.—Supposing 15 tons to act at every upper apex, and taking the locomotive excess at the proper apices as required for each flange, we can easily find the maximum strains. Thus for AB, Fig. 103, we have 33 tons at c and at h. The centre of moments is at c. Hence,

$$AB \times 20 = -159.94 \times 20 + 15 \times 10 \qquad AB = -152.44.$$

In similar manner,

$$BC \times 20 = -151.69 \times 40 + 15(30 + 20 + 10) \qquad BC = -258.38,$$

$$CD \times 20 = -143.44 \times 60 + 15(50 + 40 + 30 + 20 + 10) \qquad CD = -317.82.$$

The strains in ab and bc, cd and de, ef and gh, etc., must always be the same, because the posts bk, dm, fn, etc., being perpendicular to the flanges can cause no strain in them.

For the upper flanges ab or bc, then, the centre of moments is at k. We have, therefore, 33 tons at b and g. Hence,

$$ab \times 10 = +164.06 \times 10 \qquad ab = bc = +164.06 \text{ tons.}$$

For cd and de, 33 tons at d and i will evidently give the greatest strains. Taking, then, the centre of moments at B, the intersection of cm and AB, we have

$$cd \times 20 = 155.81 \times 40 - 15(30 + 20) \qquad cd = de = +274.12.$$

For ef and fg, we have the locomotive excess at f and l.

Taking the centre of moments at C, we have

$$ef \times 20 = 147.56 \times 60 - 15(50 + 40 + 30 + 20) \qquad ef = fg = +337.68.$$

For gh and hi, we have 33 tons at h and at t, 50 feet to the right of h. Taking the centre of moments at D, we have

$$gh \times 20 = +139.31 \times 80 - 15(70 + 60 + 50 + 40 + 30 + 20).$$

Hence, $gh = hi = +354.74$ tons.

These are the greatest strains which can ever occur in the flanges.

* This truss, or some modification of it, is now usually adopted when it is desired to reduce panel length, instead of the double system Pratt Truss, Post, or lattice.

(*b*.) Maximum Strains in the Braces.—It is evident at once from Fig. 103, that the greatest compression in the intermediate posts *bk*, *dm*, *fn*, etc., is equal to a full panel load, or $5 + 10 + 33 = 48$ tons.

Since *akc*, *cme*, *eng*, etc., are secondary trusses, one half the load on *bk*, *dm*, etc., is carried to *c*, *e*, *g*, etc. Thus, of any load at *d* for instance, one half goes to the right through *me*, and one half to the left through *mc*. Of the first portion, at *e*, $\frac{12}{16}$ of $\frac{1}{2}$, or $\frac{6}{16}$ of the load at *d*, causes pressure on the left abutment. Of the second portion, at *c*, $\frac{14}{16}$ of $\frac{1}{2}$, or $\frac{7}{16}$ of the load at *d*, causes pressure at the left abutment also. The total pressure at the left abutment due to a load at *d*, is then, $\frac{6}{16} + \frac{7}{16} = \frac{13}{16}$ of that load, just as should be the case, by the principle of the lever. The same holds good for any load at *b*, *f*, *h*, etc.

The maximum tension then, in all the secondary ties, *kc*, *me*, *ng*, etc. is

$$\frac{5 + 10 + 33}{2} \sec \theta = 24 \times 1.414 = 34 \text{ tons.}$$

It remains to find the maximum strains in the remaining web members.

For *ak* we have 48 tons at *b* and also at *g*, 50 feet back of *b*, and 15 tons at all the other upper apices.

The reaction at left end for this loading is easily found to be + 164.06 tons. Hence

$$ak \cos \theta + 164.06 = 0, \quad \text{or } ak = - 164.06 \times 1.414 = - 232 \text{ tons.}$$

For *kA* we have 5 tons at *b*, 48 tons at *c* and *h*, and 15 tons at all the other upper apices.

The left reaction for this loading is + 150.56 tons.

The *shear* just to the right of *k* is then $+ 150.56 - 5 = + 145.56$ tons. But a section to the right of *k* cuts *kc* also, as well as *kA*, and since, as we have seen, one half of any load at *b* is transmitted through *kc*, the upward shear in this case due to the strain in *kc* is 2.5 tons.

The *resultant* shear which acts in *kA* then, is $145.56 + 2.5 = + 148.06$ tons. We have, therefore, taking a section through *bc*, *kc* and *kA*,

$$kA \cos \theta - kc \cos \theta + 145.56 = 0,$$

or, since *kc* is already known to be in tension, and therefore minus, and $kc \cos \theta = 2.5$,

$$kA \cos \theta = - 148.06, \text{ or } kA = - 148.06 \times 1.414 = - 209.36 \text{ tons.}$$

For *cA* we have the same loading as for *kA*, and the greatest compression in *cA* is equal to the shear just found for *kA*, viz.:

$$cA = + 148.06 \text{ tons.}$$

For *cm*, in like manner, we have 5 tons at *b* and *c*, 48 tons at *d* and *i*, and 15 tons at the other apices. The left reaction is + 137.69 tons. The shear to the right of *c* is $137.69 - 10 = 127.69$, and the greatest tension in *cm* is

$$cm = - 127.69 \sec \theta = - 127.69 \times 1.414 = - 180.55 \text{ tons.}$$

For *mB*, we have 5 tons at *b*, *c* and *d*, 48 tons at *e* and *j*, and 15 tons at the other apices. The left reaction is then 125.44. The shear for *mB* is $125.44 - 5 - 5 - 5 + 2.5 = + 112.94$. The greatest tension in *mB* is then

$$- 112.94 \sec \theta = - 159.7 \text{ tons.}$$

The greatest compression in eB is $+ 112.94$ tons. In the same way we can find the strains in the other braces.

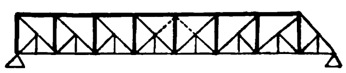
In order to find whether any diagonal, as gD, should be counterbraced, we suppose 48 tons at g and b, 15 tons at c, d, e and f, and 5 tons at the other apices. The left reaction is then $+ 136$. The shear to the right of o, is $+ 136 - 48 - 15 - 15 - 15 - 15 - 48 - 5 + 2.5 = - 22.5$ tons.

Since the resultant shear for oD thus comes out negative, it shows that a counter oC is needed. The tension in this counter is $oC = - 22.5 \times 1.414 = - 31.8$ tons.

If the resultant shear had come out positive, no counter would be required. The same method is to be observed for counter Bn. Working out the numerical results, it will be found that no counter for Bn is required.

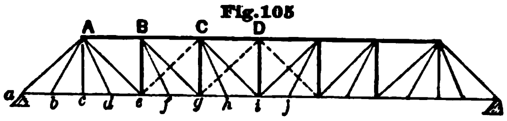
This type of truss is also usually built as a through girder, as shown in Fig. 104, and the ends may be either square or inclined. In either case the calculation offers no special difficulties in view of what has preceded.

Fig. 104



THE KELLOGG TRUSS.—We have represented this truss in Fig. 105. The verticals, except Ac, are posts, and all inclined pieces are ties, except the two ends, which are struts. The truss is 160 feet long, lower chord divided into 16 equal bays of 10 feet each. Height of truss 20 feet, and the angle for the main ties Ci, Bg and Ae is, therefore, 45°. The angle for the secondary ties Ab, Ad, Bf and Ch, is 26° 34'. Hence sec 45° = 1.414 and sec 26° 34' = 1.118. Let the load be on the bottom flange, 1 ton per foot train load and 0.5 ton per foot dead load. Locomotive excess 33 tons. Hence the apex weights are 10 tons for train and 5 tons for dead load. The train is preceded by two locomotives.

Fig. 105



(*a.*) MAXIMUM STRAINS IN THE FLANGES.—Let all the lower apices be loaded with 15 tons. Then for AB, Fig. 105, we have locomotive excess at e and at j. Hence,

$$AB \times 20 = + 151.69 \times 40 - 15\,(30 + 20 + 10) \qquad AB = + 258.38,$$

$$BC \times 20 = + 143.44 \times 60 - 15\,(50 + 40 + 30 + 20 + 10) \qquad BC = 317.82,$$

$$CD \times 20 = + 135.18 \times 80 - 15\,(70 + 60 + 50 + 40 + 30 + 20 + 10 \qquad CD = + 330.72.$$

For the lower flanges ab, bc, cd and de, the centre of moments is at A. For ef and fg at B, for gh and hi at C. For hi, then, we have a locomotive excess at g, and another 50 feet to the right of g. Hence,

$$hi \times 20 = - 143.44 \times 60 + 15\,(50 + 40 + 30 + 20), \text{ or } hi = - 325.32.$$

Observe here particularly, that the moment of the weight at f balances that at h, and the moment of the weight at g is zero.

For gh we have,

$$gh \times 20 = - 143.44 \times 60 + 15\,(50 + 40 + 30 + 20 + 10) \qquad gh = - 317.82.$$

In similar manner,

$$fg \times 20 = - 151.69 \times 40 + 15\,(30 + 20) \qquad fg = - 265.88,$$

$$ef \times 20 = - 151.69 \times 40 + 15\,(30 + 20 + 10) \qquad cf = - 258.38,$$

$de \times 20 = -159.93 \times 20 \qquad de = -159.93,$

$cd \times 20 = -159.93 \times 20 + 15 \times 10 \qquad bc = cd = -152.43,$

$bc \times 20 = -159.93 \times 20 + 15 \times 10$

$ab \times 20 = -159.93 \times 20 \qquad ab = de = -159.93.$

These are the maximum strains which can ever occur in the flanges under the action of the assumed loads.

(*b*.) MAXIMUM STRAINS IN THE BRACES.—The secondary ties, *Ch*, *Bf*, *Ad* and *Ab*, Fig. 105, have simply to support a full panel load, or $15 + 33 = 48$ tons. They are all in tension then, and the greatest strain which can ever occur in each of them is

$$-48 \sec \theta = -48 \times 1.118 = -53.66 \text{ tons.}$$

Ac is also a tie, and the greatest strain is a full panel load, or 48 tons.

For *Ci* we have the shears $+47.67$ and -33.56, the first when the train is on the right hand half and the locomotive excess is at *i* and 50 feet to the right of *i*, the dead load acting at every lower apex. The second when the load reaches from the left up to and including *h*, the locomotive excess being at *h* and *c*. Hence,

$$Ci = -47.67 \times 1.414 = -67.4, \text{ and } Ci = +33.56 \times 1.414 = +47.45.$$

Ci must, therefore, be counterbraced, and the strain in the counter *gD* is -47.45. This gives the compression in $Di = +33.56$.

For *Cg* the greatest compression is when the train advances to *h*. We have, therefore,

$$Cg = +62.43 \text{ tons.}$$

In similar manner for *Bg* we have the shears $+77.81$ and -7.18. Hence,

$$Bg = -77.81 \times 1.414 = -110.02 \qquad Bg = +7.18 \times 1.414 = +10.15.$$

Therefore, *Bg* must be counterbraced, and the strain in the counter *Ce* is -10.15. For *Be*, the greatest compression is when the train advances to *f*. Hence,

$$Be = +93.81.$$

For *Ae* we have the shear $+110.44$, there is no negative shear. Hence,

$$Ae = -110.44 \times 1.414 = -156.16.$$

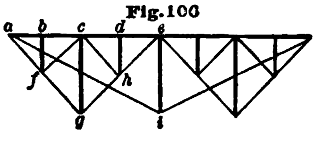
For *Ac* the strain as already remarked is -48 tons.

For *Aa* we have the shear $+164.06$. Hence,

$$Aa = +164.06 \times 1.414 = +232 \text{ tons.}$$

FINK TRUSS.—We have represented this truss in Fig. 106. The span is 80 feet, divided into 8 equal bays of 10 feet each. The vertical pieces, *ei*, *dh*, *cg* and *bf*, are all posts, and will take only compression. All the inclined pieces are ties. We take $ei = cg = 20$ feet. Then the angle θ which *ag* nd *eg* make with the vertical is 45°, and the angle which *ai* makes with the vertical is 63° 26′. Hence $\sec 45° = 1.414$, $\cos 45° = 0.70711$, $\sec 63° 26' = 1.236$, $\cos 63° 26' = 0.44724$, $\sin 63° 26' = 0.89441$. The lengths of *dh* and *bf* are each 10 feet.

Fig. 106



We take 1 ton per foot train load and 0.5 ton per foot dead load, or 10 and 5 tons per apex respectively. Locomotive excess 33 tons.

We see at once from the Figure that the greatest strain which can come on the short posts, bf and dh, is a full panel load. Hence $bf = dh = 15 + 33 = 48$ tons.

We see also that every apex load causes strain in ei and ai. The greatest strains in these pieces will then be for 15 tons at each upper apex, and 33 tons locomotive excess at e. We can easily find the strain in ai for this loading by moments. Thus, for a section cutting de, he and ai, the centre of moments for ai is at e. The lever arm for ai is

$$ei \times \sin 63^\circ\ 26' = 20 \times 0.89441 = 17.8882.$$

For train and dead load, then, since the reaction is 52.5 at the left end, we have,

$$ai \times 17.8882 = -52.5 \times 40 + 15\,(30 + 20 + 10), \text{ or}$$

$$ai = -\frac{1200}{17.8882} = -67.07 \text{ tons.}$$

For the locomotive excess at e, we have

$$ai \times 17.8882 = -16.5 \times 40, \quad \text{or, } ai = -36.9 \text{ tons.}$$

For the strain in ei, we have for train and dead loads,

$$ei = 2\,ai \cos 63^\circ\ 26' = 2 \times 67.07 \times 0.44724 = +60.$$

Therefore, the load upon ei is equal to four apex loads. This is also evident from Fig. 106. For since the point e is supported by means of ei and ai, we can consider the secondary truss age as an independent truss supported at a and e. Therefore, a load at b of 15 tons causes at e a pressure of $\frac{1}{4}$th of 15, at c $\frac{1}{2}$, and at d $\frac{3}{4}$ths of 15 tons. Hence we have at e $(\frac{1}{4} + \frac{1}{2} + \frac{3}{4})\,15$. The secondary truss on the right causes an equal pressure. Finally, we have 15 tons at e. Therefore, $2\,(\frac{1}{4} + \frac{1}{2} + \frac{3}{4})\,15 + 15 = 3 \times 15 + 15 = 4 \times 15 = 60$, is the pressure upon ei.

The locomotive excess at e causes in ei a compression of 33 tons.

If ai is known we can easily find the strain in af. Thus for train and dead load the reaction at left end is 52.5 tons. But of this the tie ai furnishes $ai \cos 63^\circ\ 26' = 30$ tons, leaving only $52.5 - 30 = 22.5$ to be supplied by af. We have, then,

$$af \cos 45^\circ = -22.5, \quad \text{or } af = -22.5 \times 1.414 = -31.815 \text{ tons.}$$

For locomotive excess the strain in af will be greatest for 33 tons at b. We have, then,

$$af = -\tfrac{3}{4}\,33 \times 1.414 = 35 \text{ tons.}$$

The strain in ab is equal to the sum of the horizontal components of ai and af. Hence for train and dead loads

$$ab = 67.07 \sin 63^\circ\ 26' + 31.815 \sin 45^\circ = +82.5.$$

The strain in bc is evidently the same as in ab.

For locomotive excess, the strain in ab will be greatest for 33 tons at e. Hence,

$$bc = ab = 36.9 \sin 63^\circ\ 26' = +33 \text{ tons.}$$

We easily find fc by resolving bf into af and fc. Thus for train and dead loads,

$$fc = -15 \cos 45^\circ = -15 \times 0.70711 = -10.60.$$

For locomotive excess,

$$fc = -33 \cos 45^\circ = -23.33.$$

The shear at f which causes strain in fg is the algebraic sum of the vertical components of the strains in af, fc, and the strain in bf. We have this shear for train and dead loads equal to

$$-15 + af \cos \theta_{af} + fc \cos \theta_{fc}.$$

Substituting numerical values,

$$-15 - 31.815 \times -0.70711 - 1060 \times -0.70711 = -15 + 22.5 + 7.5 = +15.$$

The strain in fg then is,

$$fg = -15 \times 1.414 = -21.21.$$

For locomotive excess at b, we have the shear equal to

$$-33 - 35 \times -0.70711 - 23.33 \times 0.70711 = +8.25.$$

Hence,

$$fg = -8.25 \times 1.414 = -11.66.$$

For cg we have for train and live loads,

$$cg = 2\,fg \cos 45^\circ = 2 \times 21.21 \times 0.70711 = +30,$$

or cg sustains 2 apex weights. For locomotive excess $cg = +33$ tons. By reason of the symmetry of the Figure we have $gh = fg$, $ch = cf$, and $he = af$.

For de we can take moments about i. The strain in he is -31.81 for strain and dead loads, and its lever arm is 14.1422. Hence,

$$de \times 20 = +52.5 \times 40 - 15\,(30 + 20 + 10) + 31.81 \times 14.1422,$$

or

$$de = +82.5.$$

This is precisely the same as the strain already found for ab.

In this form of truss, then, *the strain in the upper flange is uniform from end to end, and the strains in all the braces are greatest for train load over the entire span.*

To recapitulate, we have,

$bf = dh = +15 + 33 = +48$ tons, $\quad cg = +30 + 33 = +63$ tons,

$ei = +60 + 33 = +93$ tons, $\quad ai = -67.07 - 36.9 = -103.97$ tons.

$af = he = -31.815 - 35 = -66.815$ tons. $\quad fc = ch = -10.6 - 23.33 = -33.93.$

$fg = gh = -21.21 - 11.66 = -32.87.$ $\quad ab = bc = cd = de = +82.5 + 33 = +115.5$ tons.

CONCLUDING REMARKS.—The preceding comprise all those bridge trusses with horizontal flanges in common use. Our principles, if comprehended, will render easy the solution of any other form which ingenuity may suggest. The strains found in all our examples are in excess of general practice. This is due to the assumption of an engine weighing 90,000 lbs. on a 12-foot base. The methods and principles remain the same, whatever assumption be made in this respect. The bill reported by the Joint Committee of the Ohio Legislature, appointed to investigate the Ashtabula accident, recommends the adop-

tion of a standard locomotive weighing 91,200 lbs. on a 12½-foot wheel base. As locomotives exceeding this in weight are in use on some roads, and the tendency is to greater loads, we do not consider our assumed load as excessive. Taking, as we do, the length of locomotive and tender at 50 feet, and 2,000 lbs. per foot over the 38 feet not covered by the drivers, we have for weight of locomotive and tender $90,000 + 38 \times 2,000 = 166,000$ lbs. This is not an excessive estimate of our large engines of to-day.

As all pieces expand or contract under the influence of heat and cold in direct proportion to their length, it is not customary to consider temperature as having any influence upon the strains. It will be sufficient to rest one end of the truss upon friction rollers, so as to allow of change of length. As no deformation is caused, no strains are caused. If it be required to find the strains for a moving system of concentrated loads, our methods remain the same, regard being had to the principles of pages 87 and 215.

We have chosen in each case that method which seems best adapted to give the required results. But the student is by no means limited to the methods of procedure laid down. Thus it is unnecessary to form Tables as on pages 99, 103, 106, etc., giving the dead and live load strains and locomotive excess strains separately. The maximum strains in any piece may be found by the method of moments by a single equation for each piece, the dead load, live load and locomotive excess being taken as all acting together at the proper apices to give the maximum strain. We have, as we have seen, Chapter III. page 97, four methods, either of which may be employed.

We attempt no comparison of the different types of trusses. Practical details of construction and extra material required for stiffening long struts affect the cost and quantity of materials to such an extent, that a comparison based upon a strain sheet alone is often misleading. The bill of materials is the best means of comparison, and this the student is not yet prepared to draw up. Even then a comparison for a given length only would be imperfect, as a girder which compares unfavorably for one length may often give a better result for another. Comparison of well-executed designs and the results of practice are the only reliable tests. Estimated by this standard, the single intersection Pratt Truss is, in all respects, the best and most common. For spans up to about 65 feet the best practice gives the preference to the plate girder. This length requires two ordinary flat cars 33 feet long for transport. The span is rarely increased to a maximum of 90, which requires three cars. Riveted Warren Girders, when used at all now, are also limited to short spans, intermediate between the longest plate girders and the shortest pin-connected spans, say between 60 and 120 feet. They possess the advantage of superior rigidity for short spans over pin-connected trusses, but less security and rigidity than plate girders, as faulty rivets make a greater reduction of strength. Plate girders are also cheaper up to 65 feet, cost less for maintenance, and possess fewer corners and recesses for the accumulation of dirt and moisture, and are therefore cleaner and less exposed to oxidation. As to pin-connected trusses, the old forms of Bollman, Fink, Kellogg, and Post have become wholly obsolete. The double intersection Pratt or Whipple is disappearing also. The best practice avoids, as much as possible, all double systems of bracing, owing to the indeterminate character of the strains. As already stated, the single intersection Pratt, or, for long spans, some modification of the Baltimore Truss, are the standard forms.

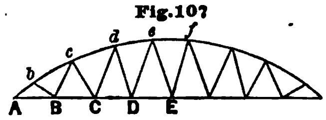
For the proper computation of the cases of this Chapter, for concentrated load system, see Appendix, page 220. The student who wishes the most recent practice ***should not proceed further*** till he has checked the results there given. The method by locomotive excess, here given, is seldom used for spans less than 200 feet.

CHAPTER V.

BRIDGE GIRDERS WITH INCLINED FLANGES.

BOWSTRING GIRDER.—In Fig. 107 we have represented a bowstring girder with isosceles bracing. The span is 120 feet, divided into 8 equal bays of 15 feet each. The bow is a polygon whose apices *Abcdef* lie upon a circle whose depth at the centre is 20 feet. As the upper flanges are of course straight, the centre depth of the inscribed polygon is 19.74 feet instead of 20 feet. We take the train load at 1 ton per foot, or 15 tons per lower apex, and the dead load at 0.5 ton per foot, or 7.5 tons per lower apex. The train is preceded by two locomotives. Since the bracing is isosceles, the apices *e*, *d*, *c*, etc., are vertically over the centre of each lower flange, and the horizontal projection of each upper flange is constant and equal to 15 feet, except the two end upper flanges, whose horizontal projection is 7.5 feet.*

Fig. 107



(*a*.) *The Flanges.*

METHOD OF CALCULATION.—The maximum strains in the flanges occur for a full load or 22.5 tons at each lower apex, together with the locomotive excess at the proper apices for each flange. Perhaps the simplest and readiest method of solution for all such cases of curved flanges, is to diagram the strains according to the method of Section I., Chapter I., page 8, as illustrated by Fig. 61, page 63. The readiest method of calculation is by moments, according to the principles of Section I., Chapter III.

(*b*.) *The Braces.*

For the braces we can diagram the strains caused by a single live load weight at *B*, and then form a Table as explained on page 99. The best method of calculation is by the principles of page 28.

We shall calculate the strains in the pieces and leave the checking of them by diagram to the student.

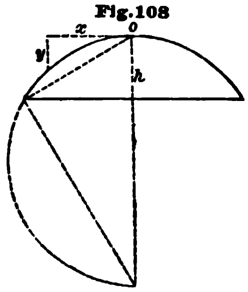
LEVER ARMS AND ANGLES OF INCLINATION.—Before proceeding to calculate, it is necessary to know the lever arms for the flanges and the angles of inclination of the various pieces with the vertical. This, the dimensions being given, is a simple trigonometrical operation. Much time may often be saved, however, by carefully drawing the frame in Fig. 107 to scale. The lever arms can then be measured directly from the drawing with all requisite accuracy and without the possibility of error.

We shall, however, calculate all the necessary data in the present case, as an example for all.

* In all our examples dead load and dimensions are assumed for convenience of illustration only, and are not to be regarded as practical cases. We shall see how to estimate dead load and choose best dimensions hereafter. For spans less than 200 feet the method of this Chapter should not be used. For method by concentrated loads see Appendix, page 225.

If the curve of the bow is a circle, as shown in Fig. 108, where S = the length of span and h = the height of arc at centre of span, we can easily determine the radius r of the arc from the proportion

Fig. 108



$$2r - h : \frac{S}{2} :: \frac{S}{2} : h,$$

or

$$r = \frac{h}{2} + \frac{S^2}{8h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Taking the crown o as an origin, we have, from the well known equation of the circle, $x^2 = 2ry - y^2$,

$$y = r - \sqrt{r^2 - x^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the curve, Fig. 108, is a parabola, we have, from the well known equation of the parabola, $x^2 = 2py$, hence

$$y = \frac{x^2}{2p}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

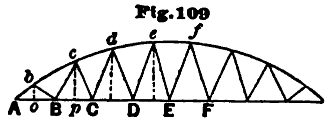
where we have

$$p = \frac{S^2}{8h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From these equations we can always find y for any point on the curve, that is for any apex in Fig. 107. Subtracting then y thus found from h, *we shall have the lever arms for the lower flanges.*

We find thus in the present case, Fig. 109, since $S = 120$, $h = 20$, the radius of the circle

Fig. 109



$$r = 10 + \frac{14{,}400}{160} = 100 \text{ feet.}$$

Making then $x = 7.5$, 22.5, 37.5, 52.5 in equation (2) above, and subtracting the values of y thus found from h, we have the verticals let fall from e, d, c and b, for the lever arms for the lower flanges. Thus

lever arm for DE = 19.74 feet,

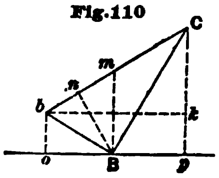
lever arm for CD = 17.43 feet,

lever arm for BC = 12.7 feet,

lever arm for AB = 5.11 feet.

For the upper flanges, take, for instance, the flange bc, Fig. 110. We have just found $bo = 5.11$ and $cp = 12.7$. We have, then, $mB = \dfrac{5.11 + 12.7}{2} = 8.905$. The lever arm nB, then, is equal to $mB \cos mBn = 8.905 \cos mBn$. But the angle mBn is equal to the angle Cbk.

Fig. 110



The $\tan Cbk = \dfrac{Ck}{bk} = \dfrac{12.7 - 5.11}{15} = \dfrac{7.59}{15} = 0.506$. Hence the

angle $Cbk = mBn = 26° 50'$. We have, therefore, the lever arm of $bc = 8.905 \times 0.89232 = 7.95$ feet.

In similar manner we find

$$\text{lever arm of } Ab = 8.43 \text{ feet,}$$

$$\text{lever arm of } cd = 14.36 \text{ feet,}$$

$$\text{lever arm of } de = 18.37 \text{ feet,}$$

$$\text{lever arm of } ef = 19.74 \text{ feet.}$$

Denoting by θ the angle made by any pieces *with the vertical*, we find easily

$$\theta_{de} = 81° 15', \theta_{cd} = 72° 27', \theta_{bc} = 63° 10', \theta_{Ab} = 55° 47',$$

$$\theta_{Bb} = 55° 46', \theta_{Bc} = \theta_{Cc} = 30° 34', \theta_{Cd} = \theta_{Dd} = 23° 17, \theta_{De} = \theta_{Ee} = 20° 48'.$$

Collecting these results, we have for the data necessary for calculation, length of span = 120 feet; panel length = 15 feet; apex live load = 15 tons; apex dead load = 7.5 tons.

For the lever arms of the flanges we have

	DE	CD	BC	AB	Ab	bc	cd	de	ef
lever arm =	19.74 ft.	17.43	12.7	5.11	8.43	7.95	14.36	18.37	19.74.

For the angle θ of pieces with the vertical

	de	cd	bc	$Ab = Bb$	$Bc = Cc$	$Cd = Dd$	$De = Ee$
$\theta =$	81° 15′	72° 27′	63° 10′	55° 47′	30° 34′	23° 17′	20° 48′
$\cos\theta =$	0.15212	0.30154	0.45140	0.56232	0.86104	0.91856	0.93483.

We are now ready for the calculation.

(*a.*) *Maximum Strains in the Flanges.*

CALCULATION OF STRAINS IN THE PIECES.—For the flanges consider a full load of $y + x = 15 + 7.5 = 22.5$ tons at each lower apex. Then, having reference to the rule for proper sign of lever arm, page 27, we have, since the reaction at each end is 78.75 tons, the following equations.

For the lower flanges, Fig. 109, we have for the flange AB, in addition to the above load, the locomotive excess $z = 33$ tons at the apex B and 50 feet to the right of B. Fifty feet to the right of B gives a point between E and F. We take the second locomotive excess therefore at F, the first apex beyond. We have then

$$AB \times 5.11 = -\left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 7.5, \qquad AB = -176.12 \text{ tons,}$$

$$BC \times 12.7 = -\left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 22.5 + (x+y) \times 7.5, \qquad BC = -184.7 \text{ tons,}$$

$$CD \times 17.43 = -\left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 37.5 + (x+y)(22.5 + 7.5), \qquad CD = -183.95 \text{ tons,}$$

$$DE \times 19.74 = -\left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 52.5 + (x+y)(37.5 + 22.5 + 7.5), DE = -176.38 \text{ tons}$$

In similar manner for the upper flanges we have

$$Ab \times 8.43 = \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15 \qquad Ab = +213.52$$

$$bc \times 7.95 = \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15 \qquad bc = +226.41$$

$$cd \times 14.36 = \left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 30 - (x+y) \times 15 \qquad cd = +210$$

$$de \times 18.37 = \left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 45 - (x+y)(30+15) \qquad de = +198.42$$

$$ef \times 19.74 = \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 60 - (x+y)(45+30+15) \qquad ef = +186.9$$

We see that the maximum strains in the flanges, especially in the upper chord, are very nearly uniform.

(b.) Maximum Strains in the Braces.

We find the strains in the braces according to the method of page 81. Thus the greatest tension in Ee, Fig. 109, will be when all the lower apices on the right are loaded with $x + y = 22.5$ tons, those on the left with $x = 7.5$ tons, and the locomotive excess $z = 33$ tons is at E. When we have this loading, since ef and DE are horizontal, the vertical components of their strains are zero, and the strain in Ee will be the shear, or the left reaction minus $3x$, multiplied by the sec θ_{Ee}. The left reaction is

$$\frac{7x}{2} + \left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8}\right)y + \frac{4}{8}z = \frac{7}{2}x + \frac{10}{8}y + \frac{4}{8}z = 61.5 \text{ tons.}$$

The shear is, therefore, $61.5 - 3x = 39$ tons. Hence

$$Ee = -39 \sec\theta_{Ee} = -39 \times 1.0697 = -41.72 \text{ tons.}$$

The greatest compression on Ee will be when the left apices are loaded with $x + y = 22.5$ tons, the right with $x = 7.5$ and the locomotive excess $z = 33$ tons is at D. For this loading the left reaction is $\frac{7}{2}x + (\frac{7}{8} + \frac{6}{8} + \frac{5}{8})y + \frac{5}{8}z = 80.625$. The shear is $80.625 - 3(x + y) - z = -19.875$. Hence

$$Ee = +19.875 \times 1.0697 = +21.26.$$

In order to find De, consider a section cutting de, De and DE. Then, according to the principles of page 80, the algebraic sum of the vertical components of the strains in the cut pieces must be in equilibrium with the shear. The plus shear for De is the same as for Ee just found, viz.: $+39$ tons, and the minus shear is -19.875 tons. We have, then, since DE is horizontal and the vertical component of its strain zero,

$$De \cos \theta_{De} + de \cos \theta_{de} + 39 = 0. \quad \ldots \ldots \ldots \quad (a)$$

$$De \cos \theta_{De} + de \cos \theta_{de} - 19.875 = 0. \quad \ldots \ldots \ldots \quad (b)$$

These equations give the strain in De when the train extends from the right to E, or from the left to D, provided we know the strains in the flange de for these loadings. These we can easily find by moments. Thus in the first case,

$$de \times 18.37 = 61.5 \times 45 - 7.5\,(30 + 15) \quad \text{or } de = +\,132.28,$$

and in the second case,

$$de \times 18.37 = 80.625 \times 45 - 22.5\,(30 + 15) \qquad de = +\,142.38.$$

These values inserted in equations (a) and (b) will enable us to find the strains in De. We must remember to measure θ according to our rule, page 80, from the vertical ***through the left hand ends*** of the cut pieces. We have, then, from (a) and (b),

$$De \times -0.93483 + 132.28 \times -0.15242 + 39 = 0, \quad \text{or } De = +\,20.15,$$

$$De \times -0.93483 + 142.38 \times -0.15242 - 19.875 = 0, \quad \text{or } De = -\,44.47.$$

For Dd the reactions at the left end for the two methods of loading which give maximum strains are, when the train reaches from right end to D, $\frac{7}{8}x + \frac{15}{8}y + \frac{6}{8}z = 79.125$, and when the train reaches from left end to C, $\frac{7}{8}x + \frac{13}{8}y + \frac{6}{8}z = 75.375$ tons. The shear, then, for Dd and Cd is $79.125 - 2x = +\,64.125$ in the first case, and $75.375 - 2\,(x + y) - 33 = -\,2.625$.

The corresponding values of de are given by

$$de \times 18.37 = 79.125 \times 45 - 7.5\,(30 + 15), \quad \text{or } de = +\,175.15,$$

$$de \times 18.37 = 75.375 \times 45 - 22.5\,(30 + 15) - 33 \times 15, \quad \text{or } de = +\,102.57.$$

Observe that in the second of these equations we must introduce the moment of the locomotive excess at C, Fig. 109. Hence,

$$Dd \times 0.91856 - 175.15 \times 0.15242 + 64.125 = 0, \quad \text{or } Dd = -\,42.9,$$

$$Dd \times 0.91856 - 102.57 \times 0.15242 - 2.625 = 0, \quad \text{or } Dd = +\,19.8.$$

For Cd we have the same reactions and shears as for Dd. We find first cd for each case of loading. Thus,

$$cd \times 14.36 = 79.125 \times 30 - 7.5 \times 15, \quad \text{or } cd = +\,157.46,$$

$$cd \times 14.36 = 75.375 \times 30 - 22.5 \times 15, \qquad cd = +\,141.8.$$

Observe that since the point of moments is now at C, Fig. 109, the moment of the locomotive excess does *not* enter the second equation. Hence,

$$Cd \times -0.91856 + 157.46 \times -0.30154 + 64.125 = 0, \qquad Cd = +\,18.12,$$

$$Cd \times -0.91856 + 141.8 \times -0.30154 - 2.625 = 0, \qquad Cd = -\,49.4.$$

For Cc we have the left hand reaction $\frac{7}{8}x + \frac{21}{8}y + \frac{8}{8}z = 98.625$, and

$\frac{7}{8}x + \frac{7}{8}y + \frac{7}{8}z = 68.25$, and the shears $98.625 - x = +91.125$, and $68.25 - (x + y) - z = +12.75$.

We first find cd. Thus,

$$cd \times 14.36 = 98.625 \times 30 - 7.5 \times 15, \qquad cd = +198.2 \text{ tons.}$$
$$cd \times 14.36 = 68.25 \times 30 - 55.5 \times 15, \qquad cd = +84.61 \text{ "}$$

Hence,

$$Cc \times 0.86104 - 198.2 \times 0.30154 + 91.125 = 0, \qquad Cc = -36.4 \text{ tons.}$$
$$Cc \times 0.86104 - 84.61 \times 0.30154 + 12.75 = 0, \qquad Cc = +14.82 \text{ "}$$

For Bc we have the same reactions and shears as for Cc. Therefore,

$$bc \times 7.95 = 98.625 \times 15, \text{ or } bc = +186.08,$$
$$bc \times 7.95 = 68.25 \times 15, \quad bc = +128.77.$$
$$Bc \times -0.86104 + 186.08 \times -0.45140 + 91.125 = 0, \qquad Bc = +8.28 \text{ tons.}$$
$$Bc \times -0.86104 + 128.77 \times -0.45140 + 12.75 = 0, \qquad Bc = -52.70 \text{ "}$$

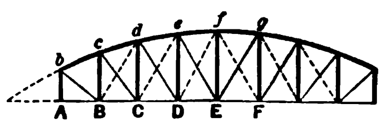
For Bb we have the left reaction $= 119.75$,

$$bc \times 7.95 = 119.75 \times 15, \text{ or } bc = 226 \text{ tons,}$$
$$Bb \times 0.56256 - 226 \times 0.45140 + 119.75 = 0, \text{ or } Bb = -32.05.$$

BOWSTRING SUITED FOR LONG SPANS.—If we were to find the strains due to dead load alone, we should find that all the braces are in tension. As the span increases, therefore, the dead load strains will increase while the live load remains always the same. It is evident that for a very long span the dead load tension may be greater than the compression in any brace due to live load. In such case the braces will always be in tension. Triangular bracing, such as is shown in Fig. 109, is then the best, as we thus have no long struts and can save the extra material required for stiffening. For a short span, such as the present, vertical posts and inclined ties are preferable, as then each piece has to resist only one kind of strain.

TRUNCATED BOWSTRING.—In Fig. 111 we have represented a form of truss which, for lack of a better name, we shall call the "truncated bowstring," because it resembles a bowstring with the ends cut off.

Fig. 111



Let the span be 120 feet, divided into 8 panels of 15 feet each, and the bracing be vertical and diagonal, as shown in the Figure. The vertical braces take compression only, and the inclined braces tension. The load is on the lower flange, and equal to 1 ton per foot for live load and 0.5 ton per foot for dead load, or 15 tons per apex for live load, and 7.5 tons per apex for dead load. The locomotive excess is 33 tons. The train is preceded by two locomotives.

The upper flange has its apices in a parabola, the height of truss at centre being 20 feet, and at ends 10 feet. The rise of the parabola at centre, therefore, is 10 feet, and the equation of the curve, page 126, is

$$y = \frac{4hx^2}{S^2},$$

where s is the span, h the rise at centre, and x the distance of any point right or left of the highest point f. In the present case this becomes

$$y = \frac{x^2}{360}$$

LEVER ARMS AND ANGLES OF INCLINATION.—If we make a section cutting ef, eE and ED, Fig. 111, the centre of moments for ED is at e, the intersection of the other two strained pieces cut. This section may really cut four pieces, viz.: the counter Df, as well as the others named. We consider, however, only that system of bracing which would be called into play by the action of the dead load only, shown in the Figure by the full lines. If, then, we find any diagonal of this system in compression, the amount of compression is the tension in its counterbrace. If any post is found to be in tension, that is the compression upon it caused by the counter.

The point of moments, therefore, for CD is at d, for BC at c, etc.

We obtain, then, the lever arms for the lower flanges by substituting $x = 15$, 30, 45, and 60, in the equation

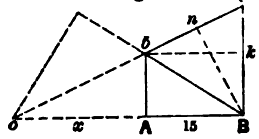
$$y = 20 - \frac{x^2}{360}$$

We thus find for the lever arms of the lower flanges,

Lower flanges,	AB,	BC,	CD,	DE,
Lever arm =	10	14.375	17.5	19.375 feet.

The point of moments for the upper flange bc is at B, for cd at C, etc. The lever arm nB, then, for any upper flange, as bc, Fig. 112, is $cB \cos cBn$. But cB is already found. The angle cBn is equal to the angle cbk. The tan of this angle is $\frac{ck}{bk}$. The difference between cB and bA gives ck, and bk is known to be 15 feet. We thus find for bc, $ck = 4.375$, $cbk = cBn = 16° \ 14'$, hence $nB = 14.375 \times \cos 16° \ 14' = 14.375 \times 0.96013 = 13\ 8$ feet.

Fig. 112



In like manner we find

$$\text{lever arm of } cd = 17.5 \times \cos 11° \ 46' = 17.5 \times 0.979 = 17.13,$$

$$\text{lever arm of } de = 19.375 \cos 7° \ 8' = 19.375 \times 0.9923 = 19.22,$$

$$\text{lever arm of } ef = 20 \cos 2° \ 23' = 20 \times 0.99913 = 19.98.$$

The centre of moments for the vertical bA, Fig. 112, will be at o, where bc meets AB produced. The distance $oA = x$ may be easily found from the proportion,

$$x : Ab :: x + 15 : Bc, \quad \text{or } x : 10 :: x + 15 : 14.375.$$

Hence,

$$x = \frac{150}{4.375} = 34.285.$$

We have, then, from Fig. 111, for the lever arm for cB, $34.285 + 15 = 49.285$.

In the same way we have for the lever arm of dC,

$$x : 17.5 :: x - 15 : 14.375, \quad \text{or } x = 84 \text{ feet.}$$

Lever arm of eD,

$$x : 19.375 :: x - 15 : 17.5, \quad \text{or } x = 155 \text{ feet.}$$

Lever arm of fE,

$$x : 20 :: x - 15 : 19.375, \quad \text{or } x = 480 \text{ feet.}$$

In order to find the lever arms for the inclined braces, we see from Fig. 112 that the lever arm for $bB = (x + 15) \sin bBA$. The angle bBA is easily found to be $33° 41'$, hence for lever arm of bB,

$$(34.285 + 15) \sin 33° 41' = 49.285 \times 0.5546 = 27.33 \text{ feet.}$$

Lever arm of cC,

$$84 \sin 43° 59' = 84 \times 0.69445 = 58.33 \text{ feet.}$$

Lever arm of dD,

$$155 \sin 49° 24' = 155 \times 0.75927 = 117.69 \text{ feet.}$$

Lever arm of eE,

$$480 \sin 52° 15' = 480 \times 0.79069 = 379.53 \text{ feet.}$$

We have, then, the following lever arms, Fig. 111:

Lower flanges,	AB	BC	CD	DE	
Lever arms,	10	14.375	17.5	19.375.	
Upper flanges,	bc	cd	de	ef	
Lever arms,	13.8	17.13	19.22	19.98.	
Vertical braces,	bA	cB	dC	eD	fE
Lever arms,	34.285	49.285	84	155	480.
Inclined braces,	bB	cC	dD	eE	
Lever arms,	27.33	58.33	117.69	379.53.	

We are now ready for the calculation.

(a.) *Maximum Strains in the Flanges.*

CALCULATION OF STRAINS IN THE PIECES.—Suppose at each lower apex, Fig. 111, $x + y = 22.5$ tons, and take the locomotive excess at the proper apices for each flange. Thus for the flange BC, we have $z = 33$ tons at B and at F, Fig. 111. Therefore,

$$AB \times 10 = -\left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 0, \qquad AB = 0.$$

$$BC \times 14.375 = -\left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 15, \qquad BC = -116.6.$$

$$CD \times 17.5 = -\left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 30 + (x+y) \times 15, \qquad CD = -158.15.$$

$$DE \times 19.375 = -\left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 45 + (x+y)(30+15), \qquad DE = -168.9.$$

$$bc \times 13.8 = \left[\frac{7(x+y)}{2} + \frac{10}{8}z\right] \times 15, \qquad bc = +130.43.$$

$$cd \times 17.13 = \left[\frac{7(x+y)}{2} + \frac{8}{8}z\right] \times 30 - (x+y) \times 15, \qquad cd = +176.$$

$$de \times 19.22 = \left[\frac{7(x+y)}{2} + \frac{6}{8}z\right] \times 45 - (x+y)(30+15), \qquad de = +189.6.$$

$$ef \times 19.98 = \left[\frac{7(x+y)}{2} + \frac{4}{8}z\right] \times 60 - (x+y)(45+30+15), \qquad ef = +185.$$

(*b*.) MAXIMUM STRAINS IN THE BRACES.—We shall find the strains in the braces by moments also in this case. Thus, Fig. 111, the centre of moments for Bb is at the intersection of bc and AB, of cB at the same point, of cC at the intersection of cd and BC, and so on. Remembering, then, the rule for the sign of the lever arm (page 27), and for positive and negative rotation, we can write down an equation of moments which shall give directly the strain in any brace for that loading which causes the maximum strain. Thus for bA we have $x + y = 22.5$ tons at every lower apex, and $z = 33$ tons at B and F. The reaction at left end is, therefore, 120 tons. Hence

$$bA \times 34.285 - 120 \times 34.285 = 0, \qquad bA = +120$$

$$-bB \times 27.33 - 120 \times 34.285 = 0, \qquad bB = -150.5.$$

For cB and cC we have the reaction at the left when the train reaches from the right end up to C, 26.25 tons for dead load, 39.375 tons for train load, and 33 tons due to locomotive excess, or 98.625 tons. Hence,

$$cB \times 49.285 - 98.625 \times 34.285 + 7.5 \times 49.285 = 0, \qquad cB = +61.13.$$

$$cC \times -58.33 - 98.625 \times 54 + 7.5 \times 69 = 0, \qquad cC = -82.46.$$

In the same way,

$$dC \times 84 - 79.125 \times 54 + 7.5\,(69 + 84) = 0, \qquad dC = +37.20.$$

$$dD \times -117.69 - 79.125 \times 110 + 7.5\,(125 + 140) = 0, \quad dD = -57.06.$$

We have also for the train coming on from the other end

$$dD \times -117.69 - 74.125 \times 110 + 22.5 \times 125 + 55.5 \times 140 = 0, \qquad dD = +20.6.$$

So also for cC we have

$$cC \times -58.33 - 68.5 \times 54 + 55.5 \times 69 = 0, \qquad cC = +2.23.$$

Again, for eD and eE, we have

$$eD \times 155 - 61.50 \times 110 + 7.5\,(125 + 140 + 155) = 0, \qquad eD = +23.32.$$

$$eE \times -379.53 - 61.50 \times 420 + 7.5\,(435 + 450 + 465) = 0, \quad eE = -41.3.$$

For train coming on from left,

$$eE \times -379.53 - 80.625 \times 420 + 22.5(435 + 450) + 55.5 \times 465 = 0, \quad eE = +31.2.$$

Finally, for fE the greatest compression will be when the train comes on either from right or left as far as F or D. In either case,

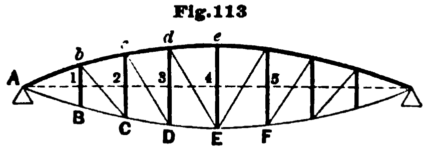
$$fE \times 480 - 49.875 \times 420 + 7.5\,(435 + 450 + 465 + 480) = 0, \qquad fE = +\ 15.$$

These are the maximum strains which can ever come upon the braces. We see that cC, dD and eE must be counterbraced, or the diagonals dB, eC and fD, Fig. 111, must be inserted, the strains in these counters being the compression which would otherwise occur in the ties.

For any case of vertical and diagonal bracing, we simply find, as above, the strains in the system strained by the dead load alone, and then if any tie of this system is found to have compression in it due to live load, this compression is the strain in the counter. The posts will then always be in compression, and the greatest compression will be when the train covers the longest segment of the span.

BOWSTRING SUSPENSION.—We have represented in Fig. 113 this form of truss, sometimes called the *double bow*, or *lenticular girder*.

Fig. 113



We choose the same span as in the preceding cases, viz.: 120 feet, divided into 8 panels of 15 feet each. Vertical and diagonal bracing. Train load 1 ton per foot, or 15 tons per panel, dead load 0.5 ton per foot, or 7.5 tons per panel. Locomotive excess 33 tons. Train preceded by two locomotives. Each bow is a polygon inscribed in a parabola, the centre rise of which is 10 feet.

The equation (page 126),

$$h - y = 20 - \frac{x^2}{180}$$

gives, therefore, the length of the verticals eE, dD, etc., for corresponding values of x, measured horizontally from the crown e.

The load is supposed to pass along the centre line. The verticals take compression only, and the diagonals tension only.

LEVER ARMS AND ANGLES OF INCLINATION.—We find easily, by inserting $x = 0$, 15, 30, 45 in the equation

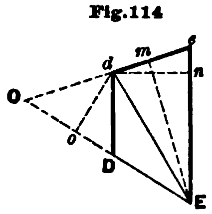
$$h - y = 20 - \frac{x^2}{180}$$

the lengths of the verticals, *viz.*,

eE	dD	cC	bB
20	18.75	15	8.75.

The centre of moments for each flange is at the opposite upper or lower apex.

Fig. 114



For the flange de, Fig. 114, the lever arm $Em = Ee \cos eEm$. The angle $eEm = edn$, and this latter angle can be easily found, since its tangent $= \frac{en}{dn} = \frac{eE - Dd}{2 \times 15} = \frac{1.25}{30} = 0.041666$. Hence, $Em = 20 \cos 2°\ 23'$ $= 20 \times 0.99913 = 19.98$. We have, similarly, the lever arm of DE or $do = Dd \cos 2°\ 23' = 18.75 \times 0.99913 = 18.73$.

In like manner we find for

	cd	CD	bc	BC	Ab	AB
lever arm =	18.6	14.88	14.68	8.57	8.4	8.4.

The distance of the point of intersection of de and DE from the vertical Ee is easily found from the proportion

$$x : 20 :: x - 15 : 18.75, \quad \text{or } x = 240 \text{ feet.}$$

In the same way we find the intersection of cd and CD distant from dD, 75 feet; of bc and BC, from cC, 36 feet; and of Ab and AB from bB, 15 feet.

The angle which dE makes with the vertical is easily found. Thus, Fig. 114, $\tan \theta_{dE} = \frac{dn}{nE} = \frac{15}{19.375} = 0.77420$, hence $\theta_{dE} = 37° 45'$. In the same way we find

$$\theta_{cD} = 41° 38', \qquad \theta_{bC} = 51° 38'.$$

We can now find the lever arms of the diagonals. Thus, for dE we have $232.5 \cos 37° 45' = 232.5 \times 0.79068 = 183.83$ feet. In the same way we find for cD, $67.5 \times \cos 41°38' = 50.45$, and for bC, $28.5 \cos 51° 38' = 17.62$.

The point of moments for the vertical bB is at the intersection of Ab and BC, because these are the two flanges cut by a section through Ab, bB and BC. The lever arm for bB, therefore, is 17.5 feet.

The point of moments for cC is at the intersection of bc and CD.* Hence the lever arm for cC is 45 feet.

The point of moments for dD is at the intersection of cd and DE. Hence the lever arm for dD is 112.5 feet.

The point of moments for eE is at the intersection of de and EF. Since these are parallel, the lever arm for eE is infinitely great.

To recapitulate, then, we have the following lever arms:

	de	DE	cd	CD	bc	BC	Ab	AB
lever arms	19.98	18.73	18.6	14.88	14.68	8.57	8.4	8.4

	bB	bC	cC	cD	dD	dE	eE
lever arms,	17.5	17.62	45	50.45	112.5	183.83	∞

Intersection of de and DE 180 feet to the left of A.
" " cd and CD 30 " " " " " "
" " bc and BC 6 " " " " " "

We are now ready for the calculation.

(a.) Strains in the Flanges.

CALCULATION OF STRAINS IN THE PIECES.—The student should draw a Figure similar to Fig. 113 and mark upon it plainly the above lever arms and intersections With this before him, he can check easily the following equations.

For the flanges we suppose 22.5 tons at each cross-girder, 1, 2, 3, and 4, etc., and let the locomotive excess act at the proper points for each flange. Thus for de we have 33 tons at 4, Fig. 113. Hence,

$$de \times 19.98 = 95.25 \times 60 - 22.5\,(45 + 30 + 15), \qquad de = +\,184.6 \text{ tons.}$$

$$cd \times 18.6 = 103.5 \times 45 - 22.5\,(30 + 15), \qquad cd = +\,194.7 \;\text{"}$$

$$bc \times 14.68 = 111.75 \times 30 - 22.5 \times 15, \qquad bc = +\,205.3 \;\text{"}$$

* The distance x of the point of intersection of any two bays, as bc and CD, on opposite sides of the vertical cC, is given by $x = \frac{2p\overline{c2}}{D3 - b1}$, where p is the panel length.

$Ab \times 8.4 = 120 \times 15,$ $Ab = + 214.3$ tons.

$DE \times 18.73 = - 103.5 \times 45 + 22.5 (30 + 15),$ $DE = - 194.6$ "

$CD \times 14.88 = - 111.75 \times 30 + 22.5 \times 15,$ $CD = - 202.6$ "

$BC \times 8.57 = - 120 \times 15,$ $BC = - 210$ "

$AB \times 8.4 = - 120 \times 15,$ $AB = - 214.3$ "

(b.) Strains in the Braces.

If the load occupies the axis as shown in Fig. 113, each vertical is divided into two parts, the strains in each of which will be different.

Thus the vertical eE, Fig. 113, is composed of two parts, $e4$ and $4E$. The train load which gives the greatest compression for $e4$ reaches from the right end to 5, or from the left end to 3. For $4E$ the compression is equal to that found for $e4$, increased by a full panel load, or 55.5 tons.

Generally for any post, the compression in the bottom half is greater by 55.5 tons than that in the upper.

We have then,

$$e4 \times \infty - 49.875 \times \infty + 7.5 \times 4 \infty = 0, \qquad e4 = + 19.875.$$

$$4E = 19.875 + 55.5 = + 75.37 \text{ tons.}$$

In similar manner,

$$d3 \times 112.5 - 61.50 \times 67.5 + 7.5 (82.5 + 97.5 + 112.5) = 0, \quad d3 = + 17.4, \quad 3D = + 72.$$

$$c2 \times 45 - 79.125 \times 15 + 7.5 (30 + 45) = 0, \qquad c2 = + 13.87, \quad 2C = + 69.37.$$

$$b1 \times 17.5 - 98.625 \times 2.5 + 7.5 \times 17.5 = 0, \qquad b1 = + 6.58, \quad 1B = + 62.$$

The inclined braces are also easily found by moments. Thus,

$$dE \times - 183.83 - 61.50 \times 180 + 7.5 (195 + 210 + 225) = 0, \qquad dE = - 34.51$$

$$dE \times - 183.83 - 80.625 \times 180 + 22.5 (195 + 210) + 55.5 \times 225 = 0, \quad dE = + 38.55$$

$$cD \times - 50.45 - 79.125 \times 30 + 7.5 (45 + 60) = 0, \qquad cD = - 31.44$$

$$cD \times - 50.45 - 75.375 \times 30 + 22.5 \times 45 + 55.5 \times 60 = 0, \qquad cD = + 42$$

$$bC \times - 17.62 - 98.625 \times 6 + 7.5 \times 21 = 0, \qquad bC = - 24.64$$

$$bC \times - 17.62 - 68.5 \times 6 + 55.5 \times 21 = 0, \qquad bC = + 42.22$$

METHOD BY DIAGRAM.—It will be seen from the preceding that the calculation of girders with curved flanges, though sufficiently simple in principle, is tedious in computa-

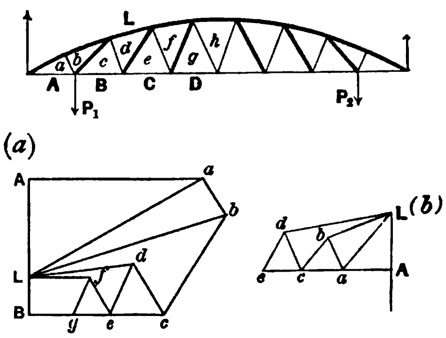
tion. There is also considerable liability to error through carelessness in writing down the equations. The student would do well to make it a rule always to check the computation by diagram.

The diagram is best applied by taking a single apex weight and finding the strains it causes.

Thus, let Fig. 115 represent a bowstring girder; span 80 feet, divided into 8 equal bays. Bow circular, the versine being 10 feet, hence the central depth of inscribed polygon is 9.85 feet. The load is supposed to traverse the lower flange and to be equal to 1 ton per foot. Dead load 0.5 ton per foot.

First suppose only the train load P_1 of 10 tons to act, and diagram its strains as in Fig. 115 (*a*).

Fig. 115*



Then suppose the load $P_7 = 10$ tons to act, and diagram its strains. We can now easily form a Table giving the strains in every brace due to each separate apex live weight.

TABLE OF STRAINS IN THE BRACES.

	Pieces.	*ab*	*bc*	*cd*	*de*	*ef*	*fg*	*gh*
	P_1	**−2.7**	**−11.4**	+4.8	−4 3	+2.4	−2.3	+1.4
	P_2	−2.3	+1.4	**−3.4**	**−8.6**	+4.7	−4.6	+2.8
	P_3	−2.0	+1.1	−2.8	+2.6	**−4.5**	**−6.9**	+4 2
	P_4	−1.6	+0.9	−2.2	+2.0	−3.6	+3.5	−5.6
	P_5	−1.2	+0.7	−1.7	+1.5	−2.7	+2.6	−4.2
	P_6	−0.8	+0.5	−1.1	+1.0	−1.8	+1.8	−2.8
	P_7	−0.39	+0.23	−0.56	+0.51	−0.90	+0.88	−1.4
Compression +	Live load.	....	+4.8	+4.8	+7.6	+7.1	+8.8	+8.4
	Locomotive excess.	....		+15.84		**+15.51**		
Tension −	Live load.	−11.0	−11.4	−11.8	−12.9	−13.5	−13.8	−14.0
	Locomotive excess.	−11.55		−13.07		**−14.85**		
	Dead load.	−5.5	−3.3	−3.5	−2.6	−3.2	−2.5	−2.8
	Max. compression.	....		17.14		20		
	Max. tension.	28.05		28.37		31		

Thus we set down in the Table the strains in all the braces, caused by P_7, as found from diagram (*b*). Then the strains due to P_6 will be twice those caused by P_7. Those due to P_5 and P_4, three and four times those caused by P_7, respectively. This is evident from Fig. 115, where P_6 is twice as far from the right end as P_7. Its left reaction is, there-

fore, twice as great, and causes in all the braces to the left a double strain. We can thus fill the lines for P_7, P_6, P_5 and P_4. For P_3 we see at once that *cd* and *de* will both be tension. The signs alternate both ways. The strains in these two pieces for P_3 are in different type in the Table. For all braces on the right of P_3 the strains will be twice what they were for P_1, and for all on the left six times what they were for P_7.

In the same way for P_2 the strains on *ef* and *fg* (given in Table in black type), are both minus and signs alternate right and left from these. For all pieces on the right the strains are three times what they were for P_1, and for all on the left five times what they were for P_7. Generally, then, the strains are all multiples of either P_1 or P_7, and we can easily fill up the Table.

We can now fill out the lines for live load compression and tension. Then adding these algebraically and dividing by the ratio of live to dead load, we find the dead load strains.

It remains to take account of the locomotive excess. This is easily done. Thus for *ef* the greatest tension occurs when we have 33 tons at the third apex. This weight will cause in *ef*, therefore, $\frac{33}{10} = 3.3$ times as much tension as P_2 caused, or $3.3 \times 4.5 = 14.85$. The total tension in *cf*, then, taking account of locomotive excess, is $-13.5 - 14.85 - 2.7 = -31$ tons.

In the same way the compression on *cf* given by the Table, or 4.4, is to be increased by the compression in this piece due to locomotive excess. This compression is 3.3 times the compression in *ef* due to P_2, or $4.7 \times 3.3 = 15.51$. Therefore, the greatest compression on *ef* is $7.1 + 15.51 - 2.7 = 20$ tons.

In similar manner we can find and add the locomotive excess strains for the other pieces, and thus find the maximum strains. The student is left to fill up these lines in the Table for himself.

The flanges are found by a similar Table, the locomotive excess strains being determined in an analogous manner.

General Remarks.—The foregoing is sufficient to show the application of our principles to any bridge girder with curved or inclined flanges.

In finding the lever arms the student should check the computation of each one by measuring it to scale from a properly drawn frame. In this way errors may be avoided.

Instead of finding the dead load strains from the computed live load strains, as is done in our Table, the dead load strains may be easily diagramed or computed separately, if it is thought desirable. No comparison of the girders in this Chapter has been attempted, but the double bow is easily found, so far as strains are concerned, to be the best. This might be expected, as both flanges being curved, each acts to sustain the load, while in the bowstring, Fig. 115, the lower chord simply resists the spread of the upper chord. The bowstring ranks next, and the "truncated bowstring" last of all.

The best bracing in all cases for long span is the triangular, as in such case all the braces may always be in tension, and the material required for stiffening long struts is avoided.

The *Pauli* truss (page 60) resembles the double bow, but the flanges are so curved that the strain in them is constant. Such a truss with triangular bracing is, therefore, somewhat superior to the double bow for long spans.

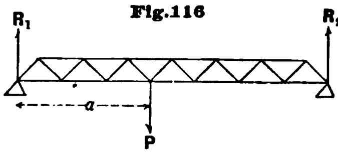
The student who has checked the examples in the Appendix, page 215, will have no difficulty in solving the examples of this Chapter for concentrated loads.

CHAPTER VI.

THE CONTINUOUS GIRDER.

Definition of Shear, Reaction.—A continuous girder is one which rests upon more than two supports. When a girder rests upon two supports only, a weight placed anywhere upon it causes pressures or reactions at the two supports, which may be at once determined from the law of the lever. Thus in Fig. 116, a weight P, placed at a distance, a, from the left end, causes the reactions $R_1 = \frac{P\,(l-a)}{l}$ and $R_2 = \frac{Pa}{l}$. These reactions being thus known, the strains in every piece can be readily calculated by moments, or otherwise.

Fig. 116

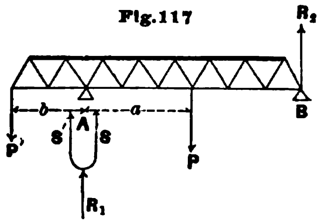


But suppose one end of this girder to overhang the support, as in Fig. 117, and to have a weight P' at the end, as well as the weight P, as before. The reaction R_2 at the right end will then be found by moments, as follows:

$$- R_2 \times l + Pa - P'b = 0, \quad \text{or } R_2 = \frac{Pa}{l} - \frac{P'b}{l}.$$

The reaction R_2 is, therefore, no longer the same as before, but is diminished by $\frac{P'b}{l}$.

Fig. 117



The reaction at A is also no longer the same as before, but is composed of two parts, viz.: the shear S at A due to P, and the shear S' at A due to P'. The shear due to P, or the portion of P which goes toward the left, is equal to $S = P - R_2$, or $S = \frac{P\,(l-a)}{l} + \frac{P'b}{l}$. The shear at A due to P' is $S' = P'$. Hence the entire reaction at A is $R_1 = S' + S = P' + \frac{P\,(l-a)}{l} + \frac{P'b}{l}$ The same result can also be found by moments. Thus,

$$R_1\, l - P\,(l-a) - P'\,(b + l) = 0, \quad \text{or } R_1 = P' + \frac{P\,(l-a)}{l} + \frac{P'b}{l}.$$

We see, then, and the above is simply intended to illustrate this point, that the reaction at a support, when the girder extends past this support, is composed of two parts, viz., the shear due to loads on the right, and the shear due to loads on the left. *Shear* and *reaction*, then, must now be distinguished from each other and never be confounded. In the case of the simple girder upon two supports only, the shears and reactions at the supports are the same, but in a continuous girder they are not.

Now in Fig. 117, the weight P' and the shear $S' = P'$, form a couple, the moment of which is,

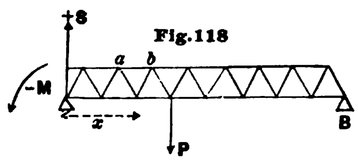


Fig. 118

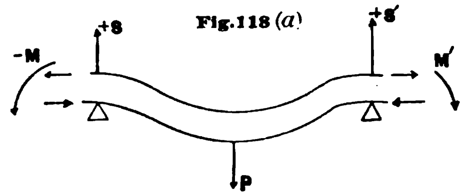


Fig. 118 (*a*)

therefore, constant and equal to $P'b$ for all points of the truss to the right of A (see page 26). If, then, Fig. 118, we suppose acting at A the shear S due to P, viz., $\frac{P(l-a)}{l} + \frac{P'b}{l}$, and *in addition* the moment $-M = -P'b$, we can find the strains in every piece just as for the simple girder, the only difference being that we have the moment $-M$ at the support, whereas in the simple girder we have the shear or reaction at the support only.

Thus let ab, Fig. 118, be any flange, the point of moments for which is distant x from A, and let d be the depth of girder.

Then for the simple girder the strain in ab would be $ab \times d = Sx$, or $ab = \frac{Sx}{d}$, where S would be, as in Fig. 116, equal to $R_1 = \frac{P(l-a)}{l}$.

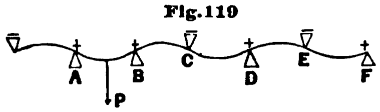
But for the overhanging girder, we should have $ab \times d = Sx - M$, or $ab = \frac{Sx}{d} - \frac{M}{d}$, where now, $S = \frac{P(l-a)}{l} + \frac{P'b}{l}$ and $M = P'b$. If, therefore, S and M can be found for any loading, the calculation of the strains offers no difficulty.

Continuous Girder—Exterior and Interior Loading.—Now Fig. 118 (*a*) represents precisely the state of a span of a continuous girder. A load placed anywhere upon the span causes at each end *positive* shears and *negative* moments. One portion of the problem, therefore, which we must solve, is to find for any position of the load what these shears and moments are. Any system of loading in the span itself we call *interior loading.*

But in the case of the continuous girder, not only do loads in the span itself cause strains in all the pieces of that span, but also loads in other spans. We have, therefore, to find the moment and shear at the ends of any span caused by loads in any of the others.

In Fig. 119 let there be a weight in the span AB. As we have seen, this causes positive shears at A and B. But as the other spans are unloaded the curve of the girder must be as shown in the figure. That is, *the shears at both supports of any loaded span are positive, and are alternately minus and plus either way from that span.*

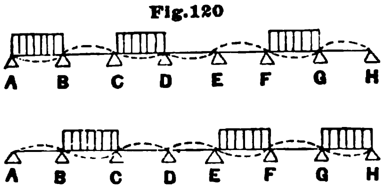
Fig. 119



In the same way we see that the *moments at the ends of a loaded span are both negative, that is, cause tension in the upper flange, and are alternately plus and minus either way from that span.*

For any span, then, as DE, Fig. 120, the greatest positive shear and negative moment at the end D, due to exterior loading, will be caused when the spans AB, CD, FG, etc., are fully loaded and the others are empty. The greatest negative shear and positive moment at D will be when BC, EF, GH, etc., are loaded.

Fig. 120

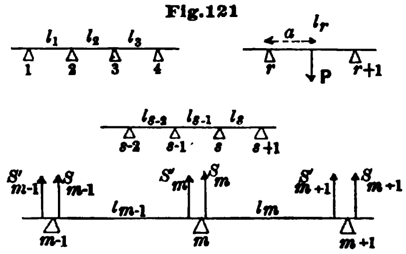


The second part of our problem is, then, to determine for any span the shear and moment at the end of that span caused by a full load over any other span. We can thus find the strains due to "exterior" loading.

The calculation, then, of the strains in any span of a continuous girder offers no especial diffi-

culty, provided we can find, 1st, the shear and moment at the end of that span due to a concentrated load placed anywhere within it, and 2d, the shear and moment at the same end for a full load over any other span.

Fig. 121



GENERAL FORMULÆ.*—We give here, therefore, the formulæ which will enable us to determine the shears and moments. The development of these formulæ is given in the Appendix to Part I. page 278.

NOTATION.—The notation we adopt is as follows, Fig. 121:

Whole numbers of spans are indicated by s.

Hence, whole number of supports is $s + 1$, numbered from left to right. Number of any support in general, always from left, is m.

The supports *adjacent to the loaded span*, left and right, are indicated by r and $r + 1$.

The length of span is denoted by l. The subscript denotes which span is referred to. Thus l_2 is the second span, l_3 the third from left, and so on. l_r is the length of the loaded span, l_m any span in general. The subscript is thus always the number of the left hand support.

A concentrated load is denoted by P.

Its distance from the left hand support is a.

The ratio of a to length of loaded span l_r is $k = \dfrac{a}{l_r}$.

The moment at any support in general is M_m, where m may be 1, 2, 3, r, $r + 1$, s, etc., indicating in every case the moment at corresponding support from left.

In same way the shear just to the right of any support is denoted by S_m. Thus S_r is the shear just to the right of the left end of the loaded span. The shear just to the left of any support is denoted by S'_m.

The uniform live load is w per unit of length.

These comprise all the symbols we shall have occasion to use. By reference to Fig. 121, the reader can familiarize himself with their signification, and will then find no difficulty in understanding and using the following formulæ.

FORMULÆ FOR MOMENTS AND SHEARS.—ALL SUPPORTS ON LEVEL.—For the moment at any support to the left of the loaded span, or 1st when $m < r + 1$,

$$M_m = - c_m \frac{A_r d_{s-r+2} + B_r d_{s-r+1}}{l_2 d_{s-1} + 2\,(l_1 + l_2)\, d_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I.)}$$

For the moments at any support on the right of the loaded span, or 2d when $m > r$,

$$M_m = - d_{s-m+2} \frac{A_r c_r + B_r c_{r+1}}{l_{s-1} c_{s-1} + 2\,(l_s + l_{s-1})\, c_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

For the shear at the left support of loaded span,

$$S_r = \frac{M_{r+1} - M_r}{l_r} + q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.}a\text{)}$$

at the right support of loaded span,

$$S'_{r+1} = \frac{M_r - M_{r+1}}{l_r} + q' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III.}b\text{)}$$

For *un*loaded spans,

$$S_m = \frac{M_{m+1} - M_m}{l_m}, \quad S'_m = \frac{M_{m-1} - M_m}{l_{m-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IV.)}$$

* These formulæ were first given by Prof. Merriman, *London Phil. Magazine*, Sept. 1875.

In these formulæ we have for concentrated loading,

$$q = P(1 - k), \qquad q' = Pk, \qquad k = \frac{a}{l_r},$$

$$A_r = Pl_r^2(2k - 3k^2 + k^3), \qquad B_r = Pl_r^2(k - k^3).$$

For uniform load entirely covering one span, we have $q = q' = \frac{1}{2} w_r l_r$ and $A = B = \frac{1}{4} w_r l_r^3$.
The numbers c and d have the following values:

$$c_1 = 0, \qquad c_2 = 1, \qquad c_3 = -\frac{2(l_1 + l_2)}{l_2}, \qquad c_4 = -2c_3\frac{l_2 + l_3}{l_3} - c_2\frac{l_2}{l_3},$$

$$c_5 = -2c_4\frac{l_3 + l_4}{l_4} - c_3\frac{l_3}{l_4}, \qquad c_6 = -2c_5\frac{l_4 + l_5}{l_5} - c_4\frac{l_4}{l_5},$$

and so on.

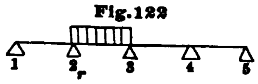
$$d_1 = 0, \qquad d_2 = 1, \qquad d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}}, \qquad d_4 = -2d_3\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\frac{l_{s-1}}{l_{s-2}},$$

$$d_5 = -2d_4\frac{l_{s-2} + l_{s-3}}{l_{s-3}} - d_3\frac{l_{s-2}}{l_{s-3}}, \qquad d_6 = -2d_5\frac{l_{s-3} + l_{s-4}}{l_{s-4}} - d_4\frac{l_{s-3}}{l_{s-4}}.$$

and so on. The numbers can be written out by simple inspection to any extent desired.

These formulæ hold good for any number of spans of different lengths, *provided all the supports are on the same level, or at a constant elevation.* They are also based upon the supposition of a constant coefficient of elasticity and constant moment of inertia of cross-section.

Fig. 122



A few examples will make the use and application of the preceding formulæ clear.

EXAMPLE 1. *A continuous girder of four equal spans has the second span from the left covered with the live load. What are the moments and shears at the supports?*

We have in this case, Fig. 122,

$$s = 4, \quad r = 2, \quad c_1 = 0, \quad c_2 = 1, \quad c_3 = -4, \quad c_4 = +15, \text{ and } d_1 = 0,$$
$$d_2 = 1, \quad d_3 = -4, \quad d_4 = +15.$$

For supports 1 and 2, we have $m < r + 1$, hence from equation (I.), page 141,

$$M_1 = 0, \quad M_2 = -c_2\frac{Ad_4 + Bd_3}{ld_3 + 4ld_4} = \frac{-15A + 4B}{56l} = -\frac{11}{224}wl^2.$$

For supports 3, 4, and 5, we have $m > r$, hence from equation (II.), page 141,

$$M_3 = -d_3\frac{Ac_2 + Bc_3}{lc_3 + 4lc_4} = 4\frac{A - 4B}{-4l + 60l} = -\frac{12}{224}wl^2,$$

$$M_4 = -d_2\frac{Ac_2 + Bc_3}{lc_3 + 4lc_4} = -\frac{A - 4B}{56l} = +\frac{3}{224}wl^2, \quad M_5 = 0.$$

For the shear (or reaction) at the first support, we have from equation (IV.),

$$S_1 = \frac{M_2 - M_1}{l} = -\frac{11}{224}wl.$$

For the second support, the shear on the *left* is,

$$S'_2 = \frac{M_1 - M_2}{l} = +\frac{11}{224}wl.$$

The shear on the *right* of the second support, is from equation (III.*a*),

$$S_2 = \frac{M_3 - M_2}{l} + \tfrac{1}{2}\,wl = -\frac{1}{224}\,wl + \tfrac{1}{2}\,wl = +\frac{111}{224}\,wl.$$

The shear on the *left* of the third support, is from equation (III.*b*),

$$S'_3 = \frac{M_2 - M_3}{l} + \tfrac{1}{2}\,wl = +\frac{1}{224}\,wl + \tfrac{1}{2}\,wl = +\frac{113}{224}\,wl.$$

The shear on the right of the third support is from equation (IV.),

$$S_3 = \frac{M_4 - M_3}{l} = +\frac{15}{224}\,wl.$$

In the same way,

$$S'_4 = \frac{M_3 - M_4}{l} = -\frac{15}{224}\,wl,$$

$$S_4 = \frac{M_5 - M_4}{l} = -\frac{3}{224}\,wl,$$

$$S'_5 = \frac{M_4 - M_5}{l} = +\frac{3}{224}\,wl.$$

A positive shear acts upward, a negative shear downward.

A positive moment causes compression in the upper flanges, a negative moment tension in the upper flange above the support.

EXAMPLE 2. *In the preceding case, what is the moment and shear at the second support for a concentrated load P, placed anywhere on the span?*

Answer

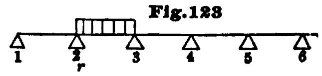
$$M_2 = -\frac{1}{56}\,(26\,k - 45\,k^2 + 19\,k^3)\,Pl,$$

$$S_2 = \frac{P}{56}\,(56 - 38\,k - 57\,k^2 + 39\,k^3).$$

EXAMPLE 3. *A continuous girder of five spans, the centre and adjacent spans being* 100 *feet and the end spans each* 75 *feet long, has a uniform load extending over the second span. What are the moments at the supports? What is the shear on the right of the* 4*th support?*

In this case, Fig. 123, we have

Fig. 123



$$s = 5, \qquad l_1 = l_5 = 75 = \tfrac{3}{4}\,l_2, \qquad l_2 = l_3 = l_4 = 100, r = 2,$$

also

$$c_1 = 0, \qquad c_2 = 1, \qquad c_3 = -\tfrac{7}{2}, \qquad c_4 = +13, \qquad c_5 = -48.5, \qquad \text{and } d_1 = 0,$$
$$d_2 = 1, \qquad d_3 = -\tfrac{7}{2}, \qquad d_4 = +13, \qquad d_5 = -48.5.$$

Since then $A = B = \frac{1}{4}\,wl^3_2$ for uniform load, we have from equation (I.),

$$M_1 = 0, \qquad M_2 = -\frac{35.5}{627}\,wl_2^2,$$

and from equation (II.),

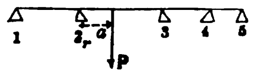
$$M_3 = -\frac{65}{1254}\,wl_2^2, \qquad M_4 = +\frac{35}{2508}\,wl_2^2, \qquad M_5 = -\frac{5}{1254}\,wl_2^2, \qquad M_6 = 0.$$

For the shear on the right of the 4th support, we have from equation (IV.),

$$S_4 = \frac{M_5 - M_4}{l_4} = - \frac{45}{2508} wl_2.$$

EXAMPLE 4. *A continuous girder of four spans, $l_1 = 80$, $l_2 = 100$, $l_3 = 50$, and $l_4 = 40$ feet, has a load of 10 tons in the second span, at a distance of 40 feet from the second support. What are the moments at the supports? What is the shear just on the right of the second support?*

Fig.124



In this case, Fig. 124, we have

$$s = 4, \quad r = 2, \quad a = 40, \quad k = \frac{a}{l_2} = \frac{40}{100} = 0.4, \quad c_1 = 0,$$

$$c_2 = 1, \quad c_3 = -3.6, \quad c_4 = +19.6, \quad c_5 = -83.7, \quad d_1 = 0, \quad d_2 = 1,$$
$$d_3 = -3.6, \quad d_4 = +10.3, \quad d_5 = -41.85.$$

We have then from equations (I.) and (II.),

for $m < 3$

$$M_m = - \frac{c_m}{3348}(Ad_4 + Bd_3),$$

for $m > 2$

$$M_m = - \frac{d_{6-m}}{3348}(Ac_2 + Bc_3).$$

Hence,

$$M_1 = 0, \quad M_2 = - \frac{10.3 A - 3.6 B}{3348} = - \frac{Pl_2^2}{3348}(17k - 30.9k^2 + 13.9k^3) = -82.01,$$

$$M_3 = +3.6 \frac{A - 3.6 B}{3348} = - \frac{3.6 Pl_2^2}{3348}(1.6k + 3k^2 - 4.6k^3) = -88.77,$$

$$M_4 = - \frac{A - 3.6 B}{3348} = \frac{Pl_2^2}{3348}(1.6k + 3k^2 - 4.6k^3) = +24.65, \quad M_5 = 0.$$

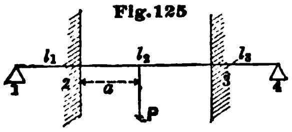
For the shear S_2, we have from equation (III.*a*)

$$S_2 = \frac{M_3 - M_2}{l_2} + P(1 - k) = +5.9324 \text{ tons.}$$

CONTINUOUS GIRDER WITH FIXED ENDS.—It is worthy of remark that if we make l_1 or $l_s = 0$, our formulæ still hold good for a girder with either or both ends fastened or walled in horizontally. We must, however, remember that when we thus make l_1 or l_s or both equal to zero, the value of s must still remain unchanged, *and the supports must be numbered as they were before the end spans were taken away.*

EXAMPLE 1. *A beam of one span is fixed horizontally at the ends. What are the end moments and shears for a concentrated load distant $a = kl$ from left end?*

Fig.125



In this case, Fig. 125, the two outer spans l_1 and l_3 are zero. But we have still $s = 3$ and $r = 2$, just the same as if the outer spans still existed.

We have then,

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -2, \quad d_1 = 0, \quad d_2 = 1, \quad d_3 = -2,$$

Hence for $m = 2$ we have

$$M_2 = - c_2 \frac{Ad_3 + Bd_2}{ld_2 + 2ld_3},$$

and for $m = 3$ we have

$$M_3 = - d_2 \frac{Ac_2 + Bc_3}{lc_2 + 2lc_3}.$$

Inserting the values of c and d, we have

$$M_2 = - Pl\,(k - 2\,k^2 + k^3) \quad \text{and } M_3 = - Pl\,(k^2 - k^3).$$

For the shear at the left end, we have

$$S_2 = \frac{M_3 - M_2}{l} + P\,(1 - k), \quad \text{or } S_2 = P\,(1 - 3k^2 + 2\,k^3).$$

For a load anywhere, we have simply to give proper values to k, and we have at once the moment and shear (which in this case is the same as the reaction) at the end.

Thus for a load in the centre, $k = \frac{1}{2}$ and

$$M_2 = M_3 = \tfrac{1}{8}\,Pl, \qquad S_2 = S_3 = \tfrac{1}{2}\,P \text{ as should be.}$$

EXAMPLE 2. *For a uniform load over the same beam, what are the end moments and shears?*

We have simply to introduce the proper values of A and B for this case, and we have at once

$$M_2 = - \tfrac{1}{12}\,wl^2 = M_3 \quad \text{and } S_2 = S_3 = \tfrac{1}{2}\,wl.$$

EXAMPLE 3. *A girder of three equal spans is "walled in" at the ends, and has a concentrated load in the first span. What are the moments and shears at the ends and intermediate supports?*

In this case we have $s = 5$, $r = 2$, and hence

$$M_2 = - \frac{Pl}{45}\,(45\,k - 78\,k^2 + 33\,k^3), \qquad M_3 = - \frac{21}{45}\,Pl\,(k^2 - k^3),$$

$$M_4 = + \frac{2Pl}{45}\,(3\,k^2 - 3\,k^3) \qquad M_5 = - \frac{Pl}{45}\,(3\,k^2 - 3\,k^3).$$

For the shears we have,

$$S_2 = \frac{P}{45}\,(45\,k - 99\,k^2 + 54\,k^3),$$

$$S'_3 = \frac{P}{45}\,(99\,k^2 - 54\,k^3), \qquad S'_4 = - \frac{P}{45}\,(27\,k^2 - 27\,k^3),$$

$$S_3 = \frac{P}{45}\,(27\,k^2 - 27\,k^3), \qquad S_4 = - \frac{P}{45}\,(9\,k^2 - 9\,k^3),$$

$$S'_5 = \frac{P}{45}\,(9\,k^2 - 9\,k^3).$$

The *reactions* are equal to the sum of the shears at each support. Thus the reaction at the second support is

$$R_3 = S'_3 + S_3 = \frac{P}{45}\,(126\,k^2 - 81\,k^3).$$

Observe that the moments are negative at each end of the loaded span, and alternate in sign each way. A negative moment always denotes tension in the upper flange.

The shears are positive at the ends of the loaded span, and alternate in sign each way. A pos-

itive shear acts upward, and requires the support to be *below* the girder. Disregarding, then, the weight of the girder itself, it would have to be *held down* at the first pier from the right end.

Since the sum of all the reactions should equal the weight, this fact affords a ready check upon the accuracy of our results.

EXAMPLE 4.—*A beam of one span is fixed horizontally at the right end, what are the shears and moments for concentrated load?*

We have here,

$$s = 2, \quad r = 1, \quad l_2 = 0, \quad c_1 = 0, \quad c_2 = 1, \quad d_1 = 0, \quad d_2 = 1, \quad d_3 = -2.$$

Hence,

$$M_1 = 0, \quad M_2 = -d_2 \frac{Ac_1 + Bc_2}{lc_1 + 2lc_2} = -\frac{Bc_2}{2lc_2} = -\frac{B}{2l} = -\frac{Pl}{2}(k - k^3),$$

$$S_1 = \frac{M_2}{l} + P\ (1 - k) = \frac{P}{2}(2 - 3k + k^3),$$

$$S_2' = \frac{P}{2}(3k - k^3).$$

If the beam is uniformly loaded, we have

$$M_1 = 0, \quad M_2 = -\tfrac{1}{8} wl^2, \quad S_1 = \tfrac{3}{8} wl, \quad S_2' = \tfrac{5}{8} wl.$$

EXAMPLE 5.—*A beam of three spans of* 25, 50 *and* 40 *feet respectively, is fixed horizontally at the right end, and has a concentrated load of* 10 *tons at* 12 *feet from the third support from the left. What are the moments at the supports?*

Here,

$$l_1 = 25, \quad l_2 = 50, \quad l_3 = 40, \quad l_4 = 0, \quad P = 10, \quad kl_3 = 12, \quad k = 0.3, \quad s = 4, \quad \text{and } r = 3.$$

Also,

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -3, \quad c_4 = 12.25, \quad d_1 = 0, \quad d_2 = 1, \quad d_3 = -2, \quad d_4 = 6.4, \quad d_5 = -34.4.$$

When, then, $m < 4$,

$$M_m = -\frac{c_m}{860}(-2A + B) = +\frac{c_m Pl_3^2}{860}(3k - 6k^2 + 3k^3).$$

Inserting $k = 0.3$, and the values of c, we have,

$$\text{for } m = 1, \quad M_1 = 0\,; \quad m = 2, \quad M_2 = +8.20\,; \quad m = 3, \quad M_3 = -24.62.$$

When $m = 4$,

$$M_4 = \frac{-d_5}{860}(-3A + 12.25\,B) = -\frac{Pl_3^2}{860}(6.25\,k + 9\,k^2 - 15.25\,k^3).$$

Or

$$M_4 = -42.29 \text{ foot tons.}$$

Find the shears. Also moments and shears for uniform load over the third span.

UNIFORM LOAD OVER ENTIRE LENGTH OF GIRDER.—Our formulæ, page 141, enable us to find the moment and shear at any support, for a uniform load over any single span. If, then, we suppose each span in turn uniformly loaded, the algebraic sum of the moments and shears thus obtained at each support, will give the moments and shears for any uniform load over the entire length of girder.

This is, however, unnecessarily tedious. We can find the moment at any support in this case, directly by the following formulæ.*

$$M_m = \frac{u}{4}\left[b_m - \frac{c_m \left[(l^3_{s-1} + l^3_s)\, d_2 + (l^3_{s-2} + l^3_{s-1})\, d_3 + \ldots \ldots (l_1^3 + l_2^3)\, d_s \right]}{d_{s-1}\, l_2 + 2d_s\, (l_2 + l_1)} \right]$$

where u is the uniform load per unit of length, and the numbers c and d are the same as in the general formulæ, page 142. The numbers b are as follows:

$$b_1 = 0, \quad b_2 = 0, \quad b_3 = -\frac{l_1^3 + l_2^3}{l_2}, \qquad b_4 = -\frac{l_2^3 + l_3^3}{l_3} - 2b_3\,\frac{l_2 + l_3}{l_3},$$

$$b_5 = -\frac{l_3^3 + l_4^3}{l_4} - 2b_4\,\frac{l_3 + l_4}{l_4} - b_3\,\frac{l_3}{l_4},$$

$$b_6 = -\frac{l_4^3 + l_5^3}{l_5} - 2b_5\,\frac{l_4 + l_5}{l_5} - b_4\,\frac{l_4}{l_5}, \text{ etc.}$$

Here again we may fix one or both ends horizontally by making l_1 or l_s or both equal to zero, and proceeding as directed on page 144.

If the spans *are all equal*, the preceding formula becomes much simpler. Thus *for equal spans:*

$$M_m = \frac{A}{3\,c_{s+1}}\left[c_m\,(1 - c_{s+2}) - c_{s+1}\,(1 - c_{m+1})\right].$$

where $A = \dfrac{ul^2}{4}$, and the numbers indicated by c are

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -4, \quad c_4 = +15, \quad c_5 = -56, \quad c_6 = +209, \text{ etc.},$$

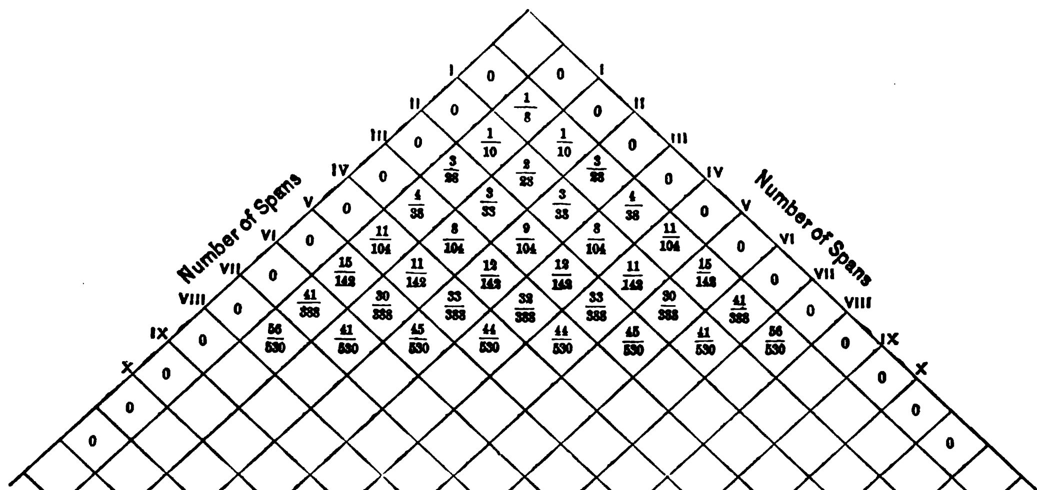
alternating in sign, and each one being numerically equal to four times the preceding, *minus* the next preceding.

The shears at any support are given in any case, whether the spans are equal or not, by the general formulæ (III.a), and (III.b), of page 141.

In the case of equal spans, the moment at any support can be easily found without formulæ or calculation. Thus the following Table gives the coefficients of $-ul^2$ for any number of spans.

The Roman numerals at the sides indicate the number of spans, and the horizontal line to which they belong give the moments.

MOMENTS AT SUPPORTS—TOTAL UNIFORM LOAD—ALL SPANS EQUAL—COEFFICIENTS OF $-ul^2$ GIVEN IN TABLE.



* These formulæ and the following for equal spans are here given for the first time.

The Table may be easily continued to any number of spans desired. Thus for any *even* number of spans, as VIII. for example, the coefficients are obtained by multiplying the fraction preceding in the same diagonal row, both numerator and denominator, by 2, and adding the numerator and denominator of the fraction preceding that. Thus

$$\frac{15 \times 2 + 11}{142 \times 2 + 104} = \frac{41}{388}, \qquad \frac{11 \times 2 + 8}{142 \times 2 + 104} = \frac{30}{388}.$$

In like manner,

$$\frac{12 \times 2 + 9}{142 \times 2 + 104} = \frac{33}{388} \text{ or } \frac{11 \times 2 + 11}{142 \times 2 + 104} = \frac{33}{388}$$

in the other diagonal row.

For any *odd* number of spans, as IX. for instance, we have simply to add, numerator to numerator and denominator to denominator, the two preceding fractions in the same diagonal row.

Thus, $$\frac{41 + 15}{388 + 142} = \frac{56}{530}, \qquad \frac{33 + 12}{388 + 142} \text{ or } \frac{30 + 15}{388 + 142} = \frac{45}{530}$$

and so on. We can thus, independently of the formula, produce the table to any required number of spans.

For seven equal spans, then, we have at once from the table,

$$M_1 = M_8 = 0, \qquad M_2 = M_7 = \frac{-15}{142} ul^2, \qquad M_3 = M_6 = \frac{-11}{142} ul^2, \qquad M_4 = M_5 = \frac{-12}{142} ul^2.$$

The moments are all negative, showing that the upper flange is in tension over every support.

Similar tables * may easily be drawn up for shears and reactions. It is unnecessary to give them here.

The moments being known, the shears can easily be found by the formulæ of page 141.

General Method of Calculation Indicated.—Thus we see that the simple formulæ of page 141 are all that we need for the complete solution of any case of level supports—whether the spans be all equal, or the end ones only different, or all different; whether the girder merely rests on the end supports or is fastened horizontally at one or both ends. We have only to remember that a positive moment causes compression and a negative moment tension in the *upper* flange. Also that a positive shear acts upward and a negative shear downward. Also, that the moment and shear are respectively negative and positive at the supports of the loaded span, and alternate in sign both ways. This is all we need in order to form properly the equation of moments for any apex, and determine the quality of the strains in flanges and diagonals. We can thus solve any practical case of framed continuous girder, with the same ease as the simple girder.

Thus, for any span, as DE, Fig. 120, we have only to find by the formulæ of page 141 the moments at D and E due to every position of the apex load P in the span DE, and the corresponding shears at D. These once known, we can find and tabulate the strains in every piece due to each apex weight. An addition of these strains gives then the maxima of each kind due to interior loading.

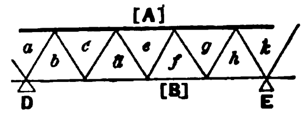
We have then to find, in like manner, the strains due to the two cases of *exterior loading*, as rep-

* Many such tables will be found in "*Elements of Graphical Statics*," Du Bois: Wiley & Sons, 1879.

resented in Fig. 120. An addition of these strains gives the maxima of each kind due to exterior loading. We can then deduce the dead load strains, and finally the total maximum strains of each kind for every piece.

EXAMPLE.—Let us take as an illustration of the preceding, a continuous girder of seven equal spans, and seek the maximum strains which can ever occur in the middle span. Let Fig. 126 represent the centre span DE. Length 80 feet, divided into 4 panels, and let the live load per panel be, for instance 40 tons or $w = 2$ tons, the uniform or dead load being *half* as much, or 20 tons per panel. Height of truss 10 feet. Load on top chord.

Fig. 126



We have then, by the application of our formulæ, page 141.

For 1st span loaded (Fig. 120) . . . . . moment at $D = -61.56$ ft. tons
shear at $D = +0.97$ tons.

For 2d span loaded . . . . moment at $D = +184.68$ ft. tons
shear at $D = -2.93$ tons.

For 3d span loaded . . . . moment at $D = -677.15$ ft. tons
shear at $D = +10.73$ tons.

For 5th span loaded . . . . moment at $D = +181.38$ ft. tons
shear at $D = -10.73$ tons.

For 6th span loaded . . . . moment at $D = -49.47$ ft. tons
shear at $D = +2.93$ tons.

For 7th span loaded . . . . moment at $D = +16.49$ ft. tons
shear at $D = -0.97$ tons.

Also for the loads in the span DE itself,

For the 1st load P_1 . . . . moment at $D = -158.97$ ft. tons
shear at $D = +36.2$ tons.

For the 2d load P_2 . . . . moment at $D = -271.97$ ft. tons
shear at $D = +25.86$ tons.

For the 3d load P_3 . . . . moment at $D = -203.2$ ft. tons
shear at $D = +14.14$ tons.

For the 4th load P_4 . . . . moment at $D = -62.89$ ft. tons
shear at $D = +3.8$ tons.

These quantities are easily found from the formulæ of page 141. The student, if he has followed our explanation of the use of the formulæ, will have no difficulty in checking the above results. Once known, the complete calculation of the continuous girder offers no special difficulty.

Thus for live load over the 1st, 3d, and 6th spans, the others being unloaded, we have the moment at $D = -788.18$ and the shear $+14.63$. We have, therefore, for the strains in the upper flanges of the 4th span, due to this loading,

$$Aa \times 10 = -788.18 \qquad \text{or, } Aa = -78.82 \text{ tons,}$$
$$Ac \times 10 = -788.18 + 14.63 \times 20 \qquad \text{or, } Ac = -49.56 \text{ tons,}$$
$$Ae \times 10 = -788.18 + 14.63 \times 40 \qquad \text{or, } Ae = -20.30 \text{ tons,}$$
$$Ag \times 10 = -788.18 + 14.63 \times 60 \qquad \text{or, } Ag = +\ 8.96 \text{ tons,}$$
$$Ak \times 10 = -788.18 + 14.63 \times 80 \qquad \text{or, } Ak = +38.22 \text{ tons.}$$

In similar manner, for the lower flanges,

$$Bb \times 10 = +788.18 - 14.63 \times 10 \qquad \text{or, } Bb = +64.19 \text{ tons,}$$
$$Bd \times 10 = +788.18 - 14.63 \times 30 \qquad \text{or, } Bd = +34.93 \text{ tons,}$$
$$Bf \times 10 = +788.18 - 14.63 \times 50 \qquad \text{or, } Bf = +\ 5.67 \text{ tons,}$$
$$Bh \times 10 = +788.18 - 14.63 \times 70 \qquad \text{or, } Bh = -23.59 \text{ tons.}$$

For the diagonals, since the angle made by these with the vertical is $\theta = 45°$, we have sec $\theta = 1.414$, and hence, $ab = 14.63 \times 1.414 = +20.68$, $bc = -20.68$, etc.

In this way we can fill up the column for L_1, in the table which follows. This column gives the strains in every piece in the span, due to the first, third, and sixth spans loaded. In the same way we can easily calculate the strains in every piece of the span DE, due to the live load extending over the second, fifth, and seventh spans. We thus find the columns L_1 and L_2 of the table.

We can now find the strains in every piece due to each apex load in the span DE. Thus, for P_1 we have moment at $D = -158.92$, and shear at $D = +36.17$. We have, then,

$$Aa \times 10 = -158.97 \qquad \text{or, } Aa = -15.9 \text{ tons,}$$
$$Ac \times 10 = -158.97 + 36.2 \times 20 - 40 \times 10 \qquad \text{or, } Ac = +16.5 \text{ tons,}$$
$$Ae \times 10 = -158.97 + 36.2 \times 40 - 40 \times 30 \qquad \text{or, } Ae = +\ 8.9 \text{ tons,}$$

and so on.

So also for the lower flanges,

$$Bb \times 10 = +158.97 - 36.2 \times 10 \qquad Bb = -20.3,$$
$$Bd \times 10 = +158.97 - 36.2 \times 30 + 40 \times 20 \qquad Bd = -12.7,$$

and so on.

Also, for the diagonals, we have

$$ab = 36.2 \times 1.414 = +51.19, \qquad bc = (40 - 36.2)\ 1.414 = +5.37, \qquad cd = -5.37, \text{ etc.}$$

We can thus fill out the column for P_1, and in similar manner the columns for P_2, P_3, etc.

The shear at any point is equal to the shear just to the right of the left support, minus all the weights between the support and the point. Thus, for P_1 we have for diagonal bc, the shear $36.2 - 40$, or a downward force of 3.8, since the weight 40 tons acts down. This downward shear causes compression in bc, since it acts at the upper end. For ab we have 36.2 acting up at the foot of ab, and, therefore, also causing compression. The diagonals which meet at the weight are always either both tension or both compression, according as the weight acts at the bottom or top. Right and left from the weight the diagonals alternate in sign.

We have thus the following Table for the strains in the various pieces of the span DE, Fig. 126. The strains due to a locomotive excess at any panel points can be easily inserted. Thus, for an excess of 33 tons at P_1, all the strains in the column for P_1 will be increased by $\frac{33}{40}$ths.

TABLE OF STRAINS IN THE PIECES.

PIECES.	LIVE LOADS IN FOURTH SPAN. P_1	P_2	P_3	P_4	EXTERIOR LOADING. L_1	L_2	LIVE LOAD STRAINS. COMP. +	TENS. −	DEAD LOAD = ½ LIVE.	TOTAL MAXIMUM STRAINS. COMP. +	TENS. −
Aa	− 15.90	− 27.20	− 20.32	− 6.30	− 78.82	+ 38.22	38.22	148.54	− 55.16	......	203.70
Ac	+ 16.50	+ 24.52	+ 7.96	+ 1.30	− 49.56	+ 8.96	59.24	49.56	+ 4.84	64.08	44.72
Ae	+ 8.90	+ 36.24	+ 36.24	+ 8.90	− 20.30	− 20.30	90.28	40.60	+ 24.84	115.12	15.76
Ag	+ 1.30	+ 7.96	+ 24.52	+ 16.50	+ 8.96	− 49.56	59.24	49.56	+ 4.84	64.08	44.72
Ah	− 6.30	− 20.32	− 27.20	− 15.90	+ 38.22	− 78.82	38.22	148.54	− 55.16	......	203.70
Bb	− 20.30	+ 1.34	+ 6.18	+ 2.50	+ 64.19	− 23.59	74.21	43.89	+ 15.16	89.37	28.73
Bd	− 12.70	− 50.38	− 22.10	− 5.10	+ 34.93	+ 5.67	40.60	90.28	− 24.84	15.76	115.12
Bf	− 5.10	− 22.10	− 50.38	− 12.70	+ 5.67	+ 34.93	40.60	90.28	− 24.84	15.76	115.12
Bh	+ 2.50	+ 6.18	+ 1.34	− 20.30	− 23.59	+ 64.19	74.21	43.89	+ 15.16	89.37	28.73
ab	+ 51.19	+ 36.57	+ 20.00	+ 5.37	+ 20.68	− 20.68	133.81	20.68	+ 56.56	190.37	......
bc	+ 5.37	− 36.57	− 20.00	− 5.37	− 20.68	+ 20.68	26.05	82.62	− 28.28	......	110.90
cd	− 5.37	+ 36.57	+ 20.00	+ 5.37	+ 20.68	− 20.68	82.62	26.05	+ 28.28	110.90	......
de	+ 5.37	+ 20.00	− 20.00	− 5.37	− 20.68	+ 20.68	46.05	46.05	0	46.05	46.05
ef	− 5.37	− 20.00	+ 20.00	+ 5.37	+ 20.68	− 20.68	46.05	46.05	0	46.05	46.05
fg	+ 5.37	+ 20.00	+ 36.57	− 5.37	− 20.68	+ 20.68	82.62	26.05	+ 28.28	110.90	......
gh	− 5.37	− 20.00	− 36.57	+ 5.37	+ 20.68	− 20.68	26.05	82.62	− 28.28	......	110.90
hk	+ 5.37	+ 20.00	+ 36.57	+ 51.19	− 20.68	+ 20.68	133.81	20.68	+ 56.56	190.37	......

For the dead load strains we simply have to add algebraically all the other columns horizontally, and divide by 2 in this case, or by the proper number in any case, whatever that is. The Table then gives at once the maximum strains in every piece, as well as the position of the loads which cause these maximum strains. We can also tell at once whether any piece needs to be counterbraced, or is subject to strains of two kinds. Thus the dead load acts always and causes in *Aa*, for instance, a tension of 55.16 tons. All the interior loads, P_1, P_2, P_3, etc., also cause tension in *Aa*, as do also the live loads of the 1st, 3d, and 6th spans. The maximum tension, since all these loads *may* act together, is, therefore, the sum, or 203.70 tons tension. On the other hand, the only loads which can cause compression in *Aa* are those in the 2d, 5th, and 7th spans. If these three spans are all loaded simultaneously, the united compression in *Aa* is less than the tension due to the dead load. This piece, then, does not need to be counterbraced. It is always in tension, and the greatest tension upon it is 203.79 tons.

Again, for *Ac* we have a dead load compression of 4.84 tons, which may be increased by all the interior loads, and by the live load in the 2d, 3d, and 7th spans to 64.08 tons. The live load in 1st, 3d, and 6th spans causes tension in *Ac* of 49.56 tons. The sum of all these tensions is given in L_1, and subtracting the dead load compression, we have 44.72 tons tension remaining. The piece *Ac*, then, is subjected to 44.72 tons tension and 64.08 tons compression, and must be made to resist both. So for each and every piece, the two columns for total maximum strains are easily made out. We also see at once from the Table what weights, and where placed, give these two strains.

CONTINUOUS GIRDER—SUPPORTS NOT ON A LEVEL.—We have, then, on page 141, all the formulæ required for the solution of the continuous girder for supports on a level, or all on line, when the deviation from level is small, whatever may be the number or relative length of the spans. If for a continuous girder of *constant cross-section* the supports are properly lowered, a considerable saving, of 20 per cent. or more over the same girder with supports on a level, may be obtained. If, however, the cross-section varies according to the strain—in other words, if the girder is of constant strength—no advantage is thus gained from lowering intermediate supports. Such disposition of the supports may even act injuriously.

The formulæ for shear and moments, which we have given, are, indeed, based upon the hypoth-

esis of constant cross-section; but if the strains in every piece being found for the shears and moments thus obtained, each piece is proportioned to its strain, the actual girder erected is not of constant cross-section, but more nearly one of uniform strength. *Formulæ for the case of supports out of level, as well as determinations of the best differences of level, are, hence, of little practical importance.* If, however, it is desired to find the effect due to a change of level of any one pier, we may make use of the following formulæ:

Let the nth support be out of level by the distance h_n. Then the moments at all the supports are changed. The moments at n, and at each alternate support from n, are diminished, and at the others increased.

For the sake of convenience, let

$$H = \frac{36\, h_n EI}{l^2}$$

where E is the coefficient of elasticity, and I the moment of inertia of the cross-section. When the support is lowered h_n is minus; when raised, h_n is plus. For the moment due to the lowering of the supports alone, we have, then, *when all the spans are equal,*

for $m < n$

$$M_m = \frac{c_m\, c_{s-n+2}}{c_{s+1}} H,$$

for $m = n$

$$M_n = \frac{H}{6} + \frac{c_n\, c_{s-n+2}}{c_{s+1}} H,$$

for $m > n$

$$M_m = \frac{c_{s-m+2}\, c_n}{c_{s+1}} H,$$

where n is the number of the lowered support from the left, and

$$c_1 = 0, \quad c_2 = +1, \quad c_3 = -4, \quad c_4 = +15, \quad c_5 = -56, \quad c_6 = +209, \text{ etc.},$$

the numbers alternating in sign, and each one being equal to four times the preceding, minus the one preceding that.

From the moments at the supports the shears can be determined by the formulæ III. and IV. of page 141.

2d. When the spans are equal:

for $m < n$,

$$M_m = -\frac{c_m \left(\dfrac{d_{s-n+1} - d_{s-n+2}}{l_n} + \dfrac{d_{s-n+3} - d_{s-n+2}}{l_{n-1}}\right) 6\, h_n EI}{d_{s+1}\, l_1}$$

for $m = n$,

$$M_n = \frac{6\, EI\, h_n}{l^2_{n-1}} - \frac{6\, EI\, c_n\, h_n}{d_{s+1}\, l_1}\left[\frac{d_{s-n+1} - d_{s-n+2}}{l_n} + \frac{d_{s-n+3} - d_{s-n+2}}{l_{n-1}}\right]$$

for $m > n$,

$$M_m = -\frac{d_{s-m+2}\left(\dfrac{c_{n-1} - c_n}{l_{n-1}} + \dfrac{c_{n+1} - c_n}{l_n}\right) 6\, h_n EI}{c_{s+1}\, l_s}$$

where

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -2\frac{l_1 + l_2}{l_2}, \quad c_4 = -2\, c_3 \frac{l_2 + l_3}{l_3} - c_2 \frac{l_2}{l_3}, \quad c_5 = -2\, c_4 \frac{l_3 + l_4}{l_4} - c_3 \frac{l_3}{l_4}, \text{ etc.},$$

and

$$d_1 = 0, \quad d_2 = 1, \quad d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}}, \quad d_4 = -2\,d_3\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\frac{l_{s-1}}{l_{s-2}}$$

$$d_5 = -2\,d_4\frac{l_{s-2} + l_{s-3}}{l_{s-3}} - d_3\frac{l_{s-2}}{l_{s-3}}, \text{ etc.}$$

The reader who has learned the use of the formulæ of page 141, will have no difficulty in applying the above to any particular case. In the same way as explained on page 144, by making l_1 or l_n or both zero, we may fix the girder horizontally at one or both ends. The formulæ for shear at any support are the same as on page 141.

EXAMPLE 1.—*Let a beam of two equal spans be uniformly loaded throughout its whole length, and let the centre support be lowered by an amount* $h_2 = \dfrac{wl^4}{48\,EI}$. *What are the moments, shears and reactions?*

The moments due to the full load alone, before the support is lowered, are $M_1 = 0$, $M_2 = -\dfrac{wl^2}{8}$ $M_3 = 0$. For the moment due to the lowering of the support alone, we have from the formulæ just given, since $H = \dfrac{3\,wl^2}{4}$, $s = 2$, $m = n = 2$,

$$M_1 = 0, \quad M_2 = +\frac{wl^2}{16}, \quad M_3 = 0.$$

Hence the total moment is

$$M_2 = -\frac{wl^2}{8} + \frac{wl^2}{16} = -\frac{wl^2}{16},$$

or only one half as much as before the support was lowered. For the shears we have,

$$S_1 = \frac{M_2}{l} + \frac{wl}{2} = \frac{7}{16}wl, \qquad S_2' = \frac{9}{16}wl.$$

$$S_2 = \frac{9}{16}wl, \qquad S_3' = \frac{7}{16}wl.$$

Hence,

$$R_1 = \frac{7}{16}wl, \qquad R_2 = \frac{18}{16}wl, \qquad R_3 = \frac{7}{16}wl.$$

EXAMPLE 2.—*How much must we lower the second support in the preceding example, in order that the reaction at the centre support may be just zero?*

In this case we have,

$$M_2 = \frac{H}{12} - \frac{wl^2}{8} = \frac{3\,h_2\,EI}{l^2} - \frac{wl^2}{8}$$

$$R_2 = -\frac{2\,M_2}{l} + wl = +\frac{wl}{8} - \frac{3\,h_2\,EI}{l^3} + \frac{wl}{2} = \frac{5}{8}wl - \frac{3\,h_2\,EI}{l^3}.$$

If this is to be zero, we have,

$$h_2 = \frac{5\,wl^4}{24\,EI},$$

and hence

$$M_2 = +\tfrac{1}{2}\,wl^2, \text{ and } R_1 = wl, \quad R_2 = 0, \quad R_3 = wl,$$

or precisely as for a beam of single span and length $2l$.

EXAMPLE 3.—*A beam of four equal spans is unloaded, and the third support is lowered by an amount* $h_3 = \dfrac{wl^4}{24\,EI}$. *What are the reactions?*

Answer:

$$R_1 = -\frac{12}{112}\,wl, \quad R_2 = +\frac{44}{112}\,wl, \quad R_3 = -\frac{64}{112}\,wl, \quad R_4 = R_2, \quad R_5 = R_1.$$

EXAMPLE 4.—*A beam of five equal spans rests as a continuous girder over six supports. Having given the dimensions of the beam, length of span, and coefficient of elasticity, to determine the reactions due to a sinking of the third support one eighth of an inch.*

Let the beam be of wood, 1 foot wide and 1.5 deep.

$l = 20$ feet, $s = 5$, $n = 3$, $E = 288{,}000{,}000$ lbs. per sq. foot, $h_3 = \frac{1}{8}$ in. $= 0.010417$ feet,

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = -4, \quad c_4 = 15, \quad c_5 = -56, \quad c_6 = 209.$$

$$M_2 = -\frac{540\,EI\,h_3}{209\,l^2}, \quad M_3 = +\frac{906\,EI\,h_3}{209\,l^2}, \quad M_4 = -\frac{576\,EI\,h_3}{209\,l^2}, \quad M_5 = +\frac{144\,EI\,h_3}{209\,l^2},$$

$$M_1 = M_6 = 0.$$

Or inserting the constants above, and $I = \frac{1}{12}\,bd^3 = \dfrac{3.375}{12}$,

$$M_1 = M_6 = 0, \quad M_2 = -5{,}448, \quad M_3 = +9{,}142, \quad M_4 = -5{,}812, \quad M_5 = +1{,}453 \text{ ft. lbs.}$$

when all the spans are unloaded. For the reactions necessary to bend the beam down and keep it to its supports,

$$R_1 = -272 \text{ lbs}, \quad R_2 = +1{,}002, \quad R_3 = -1{,}477, \quad R_4 = 1{,}111, \quad R_5 = -436, \quad R_6 = +73.$$

If unloaded, then, the beam must be fastened down at the first, third and fifth supports.

If the beam weighs 75 lbs. per foot, what deflection of the third support will raise the left end off the abutment?

Ans.

$$R_1 = \frac{15}{30}\,wl = \frac{540\,EI\,h_3}{209\,l^3} \quad \text{or } h_3 = 0.0287 \text{ feet} = 0.3712 \text{ inches.}$$

It will be observed that a small difference of level in the supports has a very considerable effect.

EXAMPLE 5.—*Two equal spans are uniformly loaded. How high must the centre be raised in order that the ends may just touch the supports?*

This is the case of the pivot span when the centre support is properly raised.

The reactions at the end are zero. At the centre $R_2 = 2\,wl$, hence $M_2 = -\frac{1}{2}\,wl^2$. But the moment when the supports are on level is $M_2 = -\frac{1}{8}\,wl^2$, hence $-\frac{3}{8}\,wl^2$ must be due to the elevation of the support. From our formulæ

$$-\tfrac{3}{8}\,wl^2 = \frac{3\,EIh_2}{l^2} \qquad \text{or } h_2 = -\frac{wl^4}{8\,EI}.$$

This is precisely the same as the deflection of a horizontal beam, fastened at one end and free at the other. (See page 253.)

ECONOMY OF THE CONTINUOUS GIRDER.—Although a comparison of strains alone is not sufficient to demonstrate economy in all cases, owing to increased cost of construction, etc., yet when the strain sheet shows a great saving, it may point the way to improvement.

Upon page 151 we have given the strains for a continuous girder, the centre span of seven. For the girder discontinuous, we find for the same load,

	Aa	*Ac*	*Ae*	*Ag*	*Ak*	*Bb*	*Bd*	*Bf*	*Bh*
Continuous		+ 64.08	+ 115.12	+ 64.08		+ 89.37	+ 15.76	+ 15.76	+ 89.37
	− 203.7	− 44.72	− 15.76	− 44.72	− 203.7	− 28.73	− 115.12	− 115.12	− 28.73
Simple	0	+ 180	+ 180	0	− 90	− 210	− 210	− 210	− 90

	ab	*bc*	*cd*	*de*	*ef*	*fg*	*gh*	*hk*
Continuous	+ 190.37	− 110.9	+ 110.9	± 46.05	± 46.05	+ 110.9	− 110.9	+ 190.37
Simple	+ 127.3	− 127.3	+ 56.5	+ 56.5	− 56.5	+ 56.5	− 127.3	+ 127.3

It will be seen at once that there is a saving in the flanges—about 11 per cent. in all—but the bracing is heavier, giving little or no saving. The span is too short to bring out the relative economy of the continuous girder.

For a girder of 200 feet, height 20 feet, 10 panels, double system of triangulation, live load 20 tons per panel, dead load 10 tons, we have the following results:

	ONE SPAN.	TWO SPANS.	FIVE SPANS CENTRE.
Bracing,	1,398.6	1,428.2	1,596.2
Lower Chord,	2,400.	1,793.2	1,395.7
Upper Chord,	2,550.	1,981.6	1,622.6
Total	6,348.6	5,203.	4,614.5
Per cent. saving		18 per cent.	27 per cent.

We see, then, that the saving increases rapidly with length of span. Although undoubtedly the full theoretical saving cannot be obtained in practice, the above is sufficient to point out that for very long girders the system may have its legitimate place amongst bridge constructions.

Disadvantages of the Continuous Girder.—In order, then, that we may properly estimate this system and be able to make use of it in such circumstances as render its use desirable, it will be well to consider the objections to it as contrasted with the simple girder.

1st. The chords at certain points undergo strains of opposite character. Strains of alternating kind have a more injurious effect than strains of one kind only, and require a greater area of cross section for the same safety. This tends to reduce our theoretical saving. It must, however, be borne in mind, that these alternating strains in the chords occur at those points where the strain is least, and where the cross section in the simple girder is generally considerably larger than the strain sheet demands. This tends to balance the above objection.

2d. Extra work and cost of chords and chord connections necessary to secure flanges against both compressive and tensile strain. For long spans this objection decreases in force.

3d. The changes of strains, unforeseen and often considerable, which a small settling of the piers or change of level of supports may occasion.

This is a strong objection. As we have seen a very slight change of level causes great changes in the strains. It is, therefore, indispensable that the supports of the continuous girder should be invariable. All cases where the piers are iron columns are, then, unsuitable for the employment of this system, as a slight change of temperature would affect the system. Even for masonry supports the girder cannot be erected until after the first season, when the piers have settled. Any subsequent change due to insecure foundations would be disastrous. We recognize, therefore, another of the necessary conditions which must be complied with in all cases where the continuous girder is used. The span must not only be long, *but the supports must be practically immovable.*

4th. Changes of temperature. The greater the number and length of spans the greater is the elongation due to rise of temperature. It would seem advisable, therefore, to limit the use of the continuous girder to three or four long spans.

5th. Difficulty of calculation. The method given in the preceding pages involves no special difficulty. Its application to any special case involves considerable labor, but in a construction costing many thousands of dollars, the labor involved in calculation is not a legitimate question. The

only points of interest are whether the method of calculation is clear, simple, easily systematized and reliable in its results and not liable to errors of computation. The method here given is believed to answer satisfactorily all these requirements.

6th. Accuracy of formulæ. Our formulæ are based upon the assumption of constant moment of inertia of cross-section and constant coefficient of elasticity of the material. Neither of these assumptions are strictly correct. The theory of flexure as applied to girders of variable cross-section shows that the results obtained by assuming a constant cross-section err upon the safe side, in giving somewhat greater strains than the actual ones. The slight gain in accuracy is more than counterbalanced by the increased complexity of the resulting formulæ. As to the second objection, while it is true that the coefficient of elasticity varies in the case of iron between tolerably wide limits, these limits become consideraby less in iron manufactured for a special purpose, as nearly as possible of uniform quality, by the same establishment, from the same ore. An assumption of a constant mean value is allowable in such case, and leads to no errors of practical importance.

ADVANTAGES OF THE CONTINUOUS GIRDER.—The principal advantages of the continuous girder are :

1st. Saving in width of piers as compared with width required for separate successive spans. The girder may, indeed, theoretically be set upon knife edges at the piers. In fact such a construction would be preferable, as better insuring the calculated strains. Width of piers is undesirable.

2d. Ease of erection, where false works are difficult or expensive. The girder may be put together on shore and pushed out over the piers.

3d. Saving of material, which for long spans would appear to be considerable.

SUMMARY.—It will be seen, then, that of the objections or disadvantages enumerated, 3 and 4 have considerable weight. The use of the continuous girder must, therefore, be confined to the comparatively rare cases of a number of successive very long spans. Even in such cases the question of economy is of less importance than that of ease of erection. The remaining objections have less weight. The proper employment of the continuous girder may then be stated as confined to a few occasional situations. When the situation justifies its use, it offers special advantages well worth consideration.

BEST RATIO OF SPANS.—The length of the various spans has some influence upon the economy of construction. The best ratio of spans for minimum material is given by Winkler* in the following Table :

LENGTH OF SPAN.	FOR 3 SPANS.	FOR 4 SPANS.
150 feet.	1 : 1.111 : 1	1 : 1.129 : 1.129 : 1
300 feet.	1 : 1.125 : 1	1 : 1.136 : 1.136 : 1
450 feet.	1 : 1.148 : 1	1 : 1.168 : 1.168 : 1
Or about	7 : 8 : 7	7 : 8 : 8 : 7

Deviations from the above ratios have, however, but slight effect. Thus, if for 3 spans we choose the ratios 1 : 1 : 1, 1 : 1.1 : 1, 1 : 1.2 : 1, 1 : 1.3 : 1, we have for 150 feet span, respectively 1.6, 0.1, 0.9, 1.6 per cent., for 300 feet span, 1.6, 0.1, 0.5, 1.1 per cent., and for 450 feet span, 1.4, 0.1, 0.2, 0.7 per cent. more material than for the ratios given above. These ratios, can, therefore, be deviated from, when circumstances render it advisable, without much loss.

HINGED CONTINUOUS GIRDERS.—The hinged continuous girder of Gerber, page 59, is free from all the objections which apply to the continuous girder proper, and has its principal advantage. That is, it may be built on shore and pushed out over the piers, and the flanges afterward cut or

* Vorträge über Brückenbau.—Wien, 1875.

hinged. A calculation of the strains in such a girder shows considerable saving when the spans are long, over the simple girder, and the system is worth more attention than it has yet received. The most remarkable girder of this kind in this country is the Kentucky River Bridge, designed by C. Shaler Smith, consisting of three spans of 375 feet each. It was erected without scaffolding, the girder being pushed out from each end and united at the centre. The flanges were afterwards cut at a distance of 75 feet on the land side of each of the central piers, thus making the middle portion a continuous girder 525 feet long, with two discontinuous spans, each 300 feet in length, at the ends of the projecting cantilevers, extending 75 feet from each pier.

Literature Upon the Continuous Girder.

We give for the benefit of students and those interested a short list of the more important works which treat the continuous girder. The literature is very extended, and no attempt is made at completeness; only a few of the more important works are cited. For a much fuller list we refer to the author's, "Elements of Graphical Statics."

CLAPEYRON.—"*Calcul d'une poutre élastique reposant librement sur des appuis inégalement espacés.*" Comptes Rendus, 1857. [Giving the well known Clapeyronian method and "Theorem of three moments."]

MOHR.—"*Beitrag zur Theorie der Holz-und Eisen constructionen.*" Zeitschr. des Hannöv. Arch u., Eng. Ver., 1860. [Theory of continuous girder with reference to relative height of supports. Application to girders of two or three spans. Best sinking of supports for constant cross-section. Disadvantage of accidental changes of height of supports. Influence of breadth of piers.]

WINKLER.—"*Beiträge zur Theorie der continuirlichen Brücken-träger.*" Civil Ingénieur, 1862. [General Theory. Determination of methods of loading, causing maximum strains.]

WINKLER.—"*Die Lehre von der Elasticitaet und Festigkeit,*" 1867. [Complete treatment of continuous girder for all spans equal and unequal, uniform and concentrated loading.]

WINKLER.—"*Vorträge über Brückenbau,*" 1875. [Complete graphic and analytic treatment. Also discussions of girder of varying cross-section.]

CULMANN.—"*Die graphische Statik,*" 1866. [Graphical treatment of simple and continuous girder of constant and variable cross-section.]

WEYRAUCH.—"*Allgemeine Theorie und Berechnung der continuirlichen und einfachen Träger,*" 1873. [Gives the general theory for constant and variable cross-section for any number of spans and all kinds of loading. Difference of level of supports; most unfavorable position of load; examples illustrating use of formulæ.]

GREENE, CHAS. E.—"*Graphical method for the analysis of Bridge Trusses.*" Van Nostrand, 1875. [Application of equilibrium polygon by balancing of moment areas.]

DU BOIS.—"*Elements of Graphical Statics.*" Wiley, 1877. [Graphic and analytic treatment.]

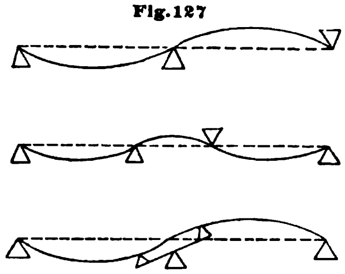
MERRIMAN.—"*On the Theory and Calculation of Continuous Bridges.*" Van Nostrand, 1876. [Analytic treatment, with illustrations of method of using formulæ.]

CHAPTER VII.

THE PIVOT OR SWING BRIDGE.

THREE KINDS OF PIVOT OR DRAW SPANS.—The pivot or draw span may be considered as a girder continuous over three, or it may be in some cases, over four supports. The small intermediate span, in this case, is the width of the turn-table. In this latter case, again, it may happen that both the centre supports are pivoted upon a third, so that they can *tip* under the action of a weight in the end span, one up and the other down. In Fig. 127 we have represented these three cases. The first two are only particular cases of our general formulæ, page 141. The third case is discussed in the Appendix, page 287.

Fig. 127



The reaction at any support is in general the *sum of the shears on each side.* For an end support, however, the reaction is the same as the shear. Our formulæ give shears, and these must not be confounded with reactions.

In the case of the pivot span, it is evident that if the shear at the end, due to a weight placed anywhere, is known, then, since at this support there is no moment, we have all we need in order to find the strains. The middle span, if any, when it rests directly upon the turn-table, is not affected by loads placed in it. Otherwise it can be calculated precisely as for a span of a continuous girder, as already illustrated.

RAISING OF CENTRE SUPPORT.—The centre support should be raised above the level of the ends by precisely the amount that the draw deflects, or else when once opened it would be difficult to shut it again.

The dead load strains, then, found for the draw when open, *exist in the draw even when it is shut.* The apex live loads can then be considered, each by itself, for draw shut, and the fact that the centre support is thus raised *will not affect the shears as given by our formulæ.* Moreover, it is not necessary to enter into elaborate computations as to the precise amount by which the centre support must be raised. It is only necessary in practice to raise the centre support, or lower the end supports, till the ends just bear when the bridge is empty. Thus, even when shut, there is no pressure upon the end supports except when the live load comes on.

It may seem strange at first sight that under these circumstances the live load presures are just what they would be for the level supports. If the beam, originally straight, were *held* down at the ends, then the reactions would have to be investigated for supports out of level. These reactions would be negative, and a live load coming on would diminish them, or, if great enough, convert the reactions from negative into positive. But such is not the state of things. The end reactions are zero *in the beginning*, and any load gives, therefore, at the end a positive reaction, just as for level supports.

An analytical discussion would be out of place here, but assuming the expression to which such a discussion would lead us, we may show that this is so.

Thus, for a beam over three supports, A, B, and C, *not* on a level, c_1 being the distance of A below B, and c_2 the distance of C below B, the coefficient of elasticity being E and the moment of inertia I, we have for the moment at the centre support * due to any number of weights in both spans,

$$4\,M_B\,l = \left(\frac{c_1 + c_2}{l}\right) 6\,EI + \frac{1}{l}\,\Sigma\,P\,a\,(l-a)\,(l+a)$$

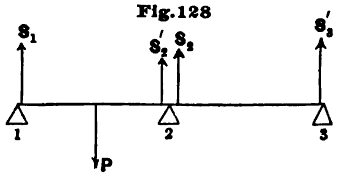
$$+ \frac{1}{l}\,\Sigma\,P\,a\,(l-a)\,(2l-a),$$

a being always measured from the left support.

In this expression the last two terms are precisely the same as for supports on a level. The influence of the different levels is contained in the first term on the right only. Now by supposition, c_1 and c_2 must be taken equal to the deflection due to the dead load, and the value of this term will, therefore, be entirely independent of the live load.

METHOD OF CALCULATION.—We have, then, to make two calculations, one for draw open and strains due to dead load, the other for draw shut and strains due to live load. The union of the two will give us the maximum strains. For the draw open, the girder becomes a cantilever, and the strains are easily found. For the draw shut, we have simply to find the shear at the end supports for each and every apex load in all the spans, omitting the centre span when it rests upon the turn-table. It therefore only remains to give the formulæ which give these shears.

Fig. 128



BEAM OF TWO EQUAL SPANS.—This is only a particular case of our general formulæ, page 141. Thus if we make $s = 2$, $r = 1$, we have for a load P in the first span, at a distance a from the left end (Fig. 128) for the moments,

$$M_1 = 0, \quad M_2 = -\frac{Pl}{4}(k - k^3), \quad M_3 = 0, \quad \text{where } k = \frac{a}{l}.$$

For the shears, then, we have,

$$\left.\begin{aligned} S_1 &= \frac{P}{4}(4 + k^3 - 5k) \\ S'_3 &= \frac{P}{4}(k^3 - k), \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Equations (1) are all we need for the complete calculation of a draw of two equal spans. The shears at the centre support are not needed. They are,

$$S_2' = \frac{P}{4}(5k - k^3),$$

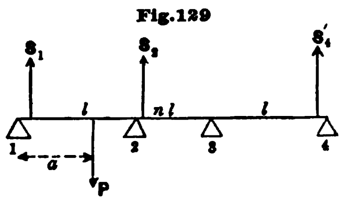
$$S_2 = \frac{P}{4}(k - k^3).$$

The sum of all the shears is equal to the weight P, as should be the case. The reac-

* See Appendix to Part I., page 289.

tion at the centre support is $R_2 = S'_2 + S_2 = \frac{P}{4}(6k - 2k^3)$. A negative moment denotes tension in the upper flange. A positive shear acts upward. If, then, S_3' comes out negative, it shows that the support should be above, or that the girder should be held down at the right end.

BEAM OVER FOUR SUPPORTS.—Let the end spans be equal, and denote them by l, and let the centre span be nl, where n is the ratio of the length of the centre span to the end spans, Fig. 129.



For purposes of calculation we only need to know the shears S_1, S_2, and S'_4 for a load anywhere in the first span (Fig. 129). Loads in the second span come directly on the turn-table, and hence cause no strains. Loads in the third span have the same effect upon the first as loads in the first have upon the third.

We have then in our general formulæ, page 141,

$$r = 1, \quad s = 3, \quad c_1 = d_1 = 0, \quad c_2 = d_2 = 1, \quad c_3 = d_3 = -2\frac{1+n}{n}.$$

Hence

$$M_1 = 0, \quad M_2 = -\frac{2P(k - k^3)(l + nl)}{H}, \quad M_3 = \frac{P(k - k^3)nl}{H}, \quad M_4 = 0,$$

where

$$H = 4 + 8n + 3n^2, \text{ and } k = \frac{a}{l}.$$

For the shears we have

$$\left.\begin{aligned} S_1 &= \frac{P}{H}[H - Hk - (k - k^3)(2 + 2n)] \\ S_2 &= \frac{P}{H}\left[(k - k^3)\left(3 + \frac{2}{n}\right)\right] \\ S'_4 &= \frac{P}{H}(k - k^3)n \end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

Equations (2) are all we need for the complete calculation of a draw with end spans equal to l and centre span nl. The shears at the other supports are not needed.

They are

$$S'_2 = \frac{P}{H}[Hk + (k - k^3)(2 + 2n)]$$

$$S'_3 = -\frac{P}{H}\left[(k - k^3)\left(3 + \frac{2}{n}\right)\right]$$

$$S_3 = -\frac{P}{H}(k - k^3)n.$$

The sum of all these shears is equal to the weight P, as should be the case. The reaction at the second support is

$$R_2 = S'_2 + S_2 = \frac{P}{H}\left[Hk + (k - k^3)\left(5 + 2n + \frac{2}{n}\right)\right].$$

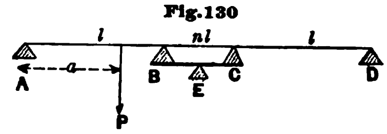
The reaction at the third support is

$$R_3 = S'_3 + S_3 = -\frac{P}{H}\left[(k - k^3)\left(3 + n + \frac{2}{n}\right)\right].$$

A negative moment denotes tension in the upper flange. A positive shear acts upward.

BEAM OVER FOUR SUPPORTS—TIPPER.*—When the two centre supports are themselves supported upon a fixed point E, Fig. 130, the case is essentially different from that of four fixed supports. Whether in any case the girder is to be treated as having three or four supports, or as supported as in Fig. 130, depends upon the construction of the turn-table. If under the action of a load in the span AB the table is free to tip, the following formulæ may be used. We have ventured to call such a construction of the pivot span, the "*Tipper*," or "tip" span. The formulæ for this case are developed in the Appendix. Taking, then, as shown in Fig. 130, the centre span equal to nl, and the outer ones each equal to l, and also $k = \frac{a}{l}$ and $H = 4 + 8n + 3n^2$, we have for a weight anywhere in the left end span,

Fig. 130



$$M_1 = 0, \quad M_2 = -\frac{Pl}{2H}[2Hk - (10 + 15n + 3n^2)k + (2 + n)k^3].$$

$$M_3 = -\frac{Pl}{2H}[(2 + 3n + 3n^2)k - (2 + n)k^3], \quad M_4 = 0.$$

For the shears we have

$$\left.\begin{aligned} S_1 &= \frac{P}{2H}[2H - (10 + 15n + 3n^2)k + (2 + n)k^3] \\ S_2 &= -\frac{P}{2H}(4 + 6n)k \\ S'_4 &= -\frac{P}{2H}[(2 + 3n + 3n^2)k - (2 + n)k^3] \end{aligned}\right\} \quad . \; . \; . \; . \quad (3)$$

Equations (3) are all that are really needed for the calculation. The remaining shears are,

$$S'_2 = \frac{P}{2H}[(10 + 15n + 3n^2)k - (2 + n)k^3],$$

* The formulæ for this case were first given by Clemens Herschel, C. E., "Continuous Revolving Draw Bridges," Little, Brown & Co., 1875.

$$S'_3 = \frac{P}{2H}(4 + 6n)k,$$

$$S_3 = \frac{P}{2H}[(2 + 3n + 3n^2)k - (2 + n)k^3].$$

The sum of all these shears is equal to the weight P, as should be the case. The reaction at the second support is,

$$R_2 = S'_2 + S_2 = \frac{P}{2H}[(6 + 9n + 3n^2)k - (2 + n)k^3].$$

The reaction at the third support is

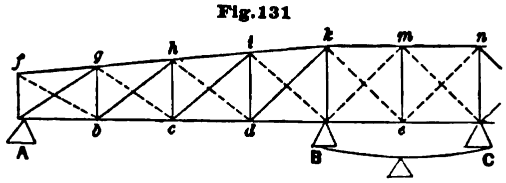
$$R_3 = S'_3 + S_3 = \frac{P}{2H}[(6 + 9n + 3n^2)k - (2 + n)k^3],$$

or just the same as at the second, as should be the case, since the supports tip.

A negative moment denotes tension in the upper flange. A positive shear acts upward.

EXAMPLE OF PIVOT SPAN—DIMENSIONS.—In Fig. 131, we have represented a Tipper draw, the two outer spans being each 80 feet long and the centre span being 20 feet long.

Fig. 131



The outer spans are divided into 4 panels of 20 feet each, and the centre spans into 2 panels of 10 feet each. The bracing is vertical and diagonal. The verticals are to take compression only and the diagonals tension only. The end height of truss is 6 feet and the centre height 10 feet. The live load is taken at 2 tons per foot, or 40 tons at each lower apex. The dead load we assume for the sake of illustration at one half of the live load, or 20 tons per apex. Our proportions are, of course, taken for the sake of illustration merely, and not as an example of actual practice. All the points necessary to be brought out are, however, illustrated as well as they would be for a much longer span and more usual proportions. We have drawn both systems of diagonals, but of course, only one diagonal can act at a time in any panel. Thus when the draw is open, we have Ag, bh, ci, dk strained by the dead load and the other diagonals not at all. If the supports are properly arranged (page 158) these strains still exist *even when the draw is shut.* A live apex load at d then will only cause tension in dh, when the compression it would otherwise cause in ci is greater than the dead load tension already existing in ci. The *ties* then, or the diagonals strained by the dead load, are the pieces Ag, bh, ci, and dk. For the dead load the diagonals in the centre span are not strained. The full lines, therefore, represent the diagonals strained by the dead load, and the dotted diagonals are only called into play, if at all, by the live load.

If by reason of the construction the diagonals cannot take compression, there can be no ambiguity as to the pieces strained by any weight. Thus a weight at b can cause no strain in bg, gc or gA, but must go through bf and bh, hc, ci, etc., to the left and right.

SHEARS.—Let us consider the case of the "Tipper." The method for four fixed supports is precisely similar. We have first to put our formulæ into proper shape for use in the particular case under consideration, Thus, $l = 80$, $nl = 20$, hence $n = \frac{1}{4}$, and $H = 4 +$

$8n + 3n^2 = 4 + 2 + \frac{3}{16} = \frac{99}{16}$, $k = \frac{a}{80}$; where a has for each apex load the successive values, 20, 40, 60, etc., or k is successively $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$. Equations (3) page 161, become then after reduction,

$$S_1 = \frac{20}{99}\left[198 - 223\,k + 36\,k^3\right],$$

$$S_2 = -\frac{1760}{99}\,k, \quad M_2 = -\frac{1600}{99}\left[25\,k - 36\,k^3\right],$$

$$S'_4 = -\frac{20}{99}\left(47\,k - 36\,k^3\right).$$

Now, in the present case, the denominator of k is always 4, of k^3 always 64, the numerator only changing according to the position of the weight. These equations can then be written,

$$S_1 = \frac{5}{396}\left[3168 - 892\,a + 9\,a^3\right],$$

$$S_2 = -\frac{440}{99}\,a, \quad M_2 = -\frac{25}{99}\left[400\,a - 36\,a^3\right],$$

$$S'_4 = -\frac{5}{396}\left[188\,a - 9\,a^3\right].$$

These are then the practical formulæ for the present case, and from them we can easily find the reactions for the apex load of 40 tons.

Thus for P_1 we have $a = 1$, and hence

$$S_1 = +\,28.85 \text{ tons}, \quad S_2 = -\,4.44 \text{ tons}, \quad S'_4 = -\,2.26 \text{ tons},$$
$$M_2 = -\,91.91 \text{ ft. tons}.$$

For P_2 we make $a = 2$ and hence

$$S_1 = +\,18.38 \text{ tons}, \quad S_2 = -\,8.88 \text{ tons}, \quad S'_4 = -\,3.83 \text{ tons},$$
$$M_2 = -\,129.3 \text{ ft. tons}.$$

For P_3 we make $a = 3$, and hence

$$S_1 = +\,9.28 \text{ tons}, \quad S_2 = -\,13.33 \text{ tons}, \quad S'_4 = -\,4.05 \text{ tons},$$
$$M_2 = -\,57.57 \text{ ft. tons}.$$

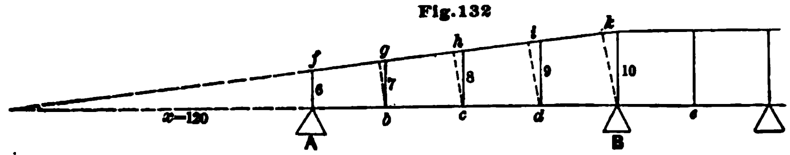
The weight P_4 acts at the support B, Fig. 131, but since this support tips, we have a reaction at A also. Hence for P_4 we make $a = 4$, and have

$$S_1 = +\,2.22 \text{ tons}, \quad S_2 = -\,17.77 \text{ tons}, \quad S'_4 = -\,2.22 \text{ tons},$$
$$M_2 = +\,177.77 \text{ ft. tons}.$$

A load at e, Fig. 131, rests directly upon the turn-table. For loads in the right-hand span the reaction at A is the same as the reaction at the right end, S_4', for corresponding loads in the left-hand span.

We thus know the shears at A and B, due to every apex load in either end span, and can now proceed to find the strains. The formulæ for shears, it will be observed, are very simple for any particular case.

Fig. 132



Lever Arms.—The lengths of the verticals are 6, 7, 8, 9, and 10 feet (Fig. 132). These are the lever arms for the lower flanges. The upper flange meets the lower flange at a distance x to the left of A, which is easily found from the proportion,

$$x : 6 :: x + 80 : 10, \quad \text{or } x = 120 \text{ feet.}$$

The angle of inclination of the upper flange with the horizontal is $2° 52'$. Hence the perpendicular from B upon the flange ik is $200 \sin 2° 52' = 10$ feet. For hi the perpendicular from d is $180 \sin 2° 52' = 9$ feet. For gh the perpendicular from c is $160 \sin 2° 52' = 8$ feet. For fg the perpendicular from b is $140 \sin 2° 52' = 7$ feet. The perpendicular from A upon fg is $120 \sin 2° 52' = 6$ feet.

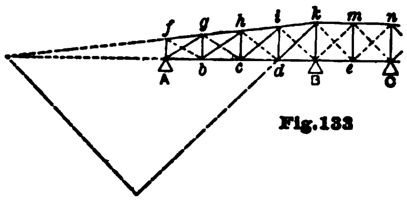


Fig. 133

For the angles which the diagonals make with the vertical, we have, Fig. 133,

$$\theta_{Ag} = \theta_{gc} = 70° 43', \quad \theta_{fb} = 73° 18', \quad \theta_{bh} = \theta_{hd} = 68° 12', \quad \theta_{Bi} = \theta_{ci} = 65° 47',$$
$$\theta_{dk} = 63° 27', \quad \theta_{Bm} = \theta_{ke} = 45°.$$

The centre of moments for any diagonal is at the intersection of the flanges, or 120 feet to the left of A. Hence we have lever arm of $Ag = 120 \cos \theta_{Ag} = 39.63$ feet. In the same way we find

	fb	bh	gc	ci	dh	dk	Bi	Bm	ke
Lever arm =	40.23	52	52.84	65.63	66.85	80.45	82	∞	∞

For the verticals, we have,

	fA	gb	hc	id	kB
Lever arm =	120	140	160	180	200

We are now ready to proceed to the calculation.

Calculation of Strains.—We have only to apply the method of moments, and the work is so simple that an example or two will suffice.

Thus suppose the draw closed and P_1 to act. The shear or reaction at the left end is 28.85 tons. The weight P_1 causes strains in the diagonals bf and bh, Fig. 133. For the flange Ab, then, the point of moments is at f; for bc, cd and dB, at h, i and k. For the flange fg the point of moments is at b; for gh, hi and ik, it is at b, c and d. We have, then, for the weight P_1 acting at b,

$$Ab \times 6 = 0, \qquad bc \times 8 = -28.85 \times 40 + 40 \times 20, \quad \text{or } bc = -44.25 \text{ tons.}$$

$$cd \times 9 = -28.85 \times 60 + 40 \times 40, \quad \text{or } cd = -14.6 \text{ tons.}$$

$$dB \times 10 = -28.85 \times 80 + 40 \times 60, \quad \text{or } dB = +9.2 \text{ tons.}$$

For the upper flanges, we have,

$$fg \times 7 = 28.85 \times 20, \quad \text{or } fg = +82.42 \text{ tons.}$$

$$gh \times 7 = 28.85 \times 20, \quad \text{or } gh = +82.42 \text{ tons.}$$

$$hi \times 8 = 28.85 \times 40 - 40 \times 20, \quad \text{or } hi = +44.25 \text{ tons.}$$

$$ik \times 9 = 28.85 \times 60 - 40 \times 40, \quad \text{or } ik = +14.55 \text{ tons.}$$

For the central span, we have at B, when P_1 acts, the shear $S_2 = -4.44$ tons, and $M_2 = -91.91$ ft. tons.

The shear being minus, acts down, and the moment being minus, causes tension in the upper flange. Therefore,

$$km \times 10 = -91.91 \qquad km = -9.2 \text{ tons.}$$

$$mn \times 10 = -91.91 - 4.44 \times 10 \qquad mn = -13.6 \text{ tons.}$$

$$Be \times 10 = +91.91 + 4.44 \times 10 \qquad Be = +13.6 \text{ tons.}$$

$$eC \times 10 = +91.91 + 4.44 \times 20 \qquad eC = +18.1 \text{ tons.}$$

In precisely similar manner we may find the strains in the flanges due to the other apex weights.

We can thus form a Table giving the strains in every flange due to each apex weight. The strains due to locomotive excess at any point can easily be inserted. Thus, for an excess of 33 tons at P_1 the strains in each member due to P_1 should be increased by $\frac{33}{40}$ths.

STRAINS IN THE FLANGES—DRAW CLOSED.

	fg	*gh*	*hi*	*ik*	*km*	*mn*	*Ab*	*bc*	*cd*	*dB*	*Be*	*eC*
P_1	+ 82.42	+ 82.42	+ 44.25	+ 14.55	− 9.2	− 13.6	0.00	− 44.25	− 14.6	+ 9.2	+ 13.6	+ 18.1
P_2	+ 52.51	+ 91.9	+ 91.9	+ 33.64	− 12.96	− 21.81	0.00	− 52.51	− 33.64	+ 12.96	+ 21.81	+ 30.7
P_3	+ 26.52	+ 46.4	+ 61.87	+ 61.87	− 5.76	− 19.1	0.00	− 26.5	− 46.4	+ 5.76	+ 19.1	+ 32.42
P_4	+ 6.34	+ 11.1	+ 14.8	+ 17.76	+ 17.77	0.00	0.00	− 6.34	− 11.1	− 14.8	0.00	+ 17.77
P_5	− 6.34	− 11.1	− 14.8	− 17.77	0.00	+ 17.77	0.00	+ 6.34	+ 11.1	+ 14.8	+ 17.77	0.00
P_6	− 11.57	− 20.25	− 27.0	− 32.4	− 19.1	− 5.76	0.00	+ 11.57	+ 20.25	+ 27.0	+ 32.42	+ 19.1
P_7	− 10.94	− 19.15	− 25.53	− 30.64	− 21.81	− 12.93	0.00	+ 10.94	+ 19.15	+ 25.53	+ 30.7	+ 21.81
P_8	− 6.46	− 11.3	− 15.1	− 18.1	− 13.6	− 9.2	0.00	+ 6.46	+ 11.3	+ 15.1	+ 18.1	+ 13.6
Total Live Load Strains.	+ 167.79	+ 231.82	+ 212.82	+ 127.82	+ 17.77	+ 17.77	0.00	+ 35.31	+ 61.8	+ 110.35	+ 153.50	+ 153.50
	− 35.31	− 61.80	− 82.43	− 98.91	− 82.4	− 82.4	0.00	− 129.6	− 105.74	− 14.8	....	....

The dead load strains occur when the draw is open. In this case we have 10 tons at A and 20 tons at b, c and d. The same holds good for the other end. Loads in the centre span cause no strains. We have, then,

DEAD LOAD STRAINS—DRAW OPEN.

fg	*gh*	*hi*	*ik*	*km*	*mn*	*Ab*	*bc*	*cd*	*dB*	*Be*	*eC*
0	−28.6	−100	−200	−320	−320	+28.6	+100	+200	+320	+320	+320

These strains exist even when the draw is shut, if the end supports are properly lowered. We can, therefore, draw up the following Table for strains in the flanges.

MAXIMUM STRAINS IN THE FLANGES.

	fg	*gh*	*hi*	*ik*	*km*	*mn*	*Ab*	*bc*	*cd*	*dB*	*Be*	*eC*
Live Load strains	+ 151.2	+ 202.7	+ 173.4	+ 89.1	+ 17.77	+ 17.77	0.00	+ 35.3	+ 61.8	+ 156.7	+ 236.2	+ 236.2
	− 35.3	− 61.8	− 82.4	− 98.9	− 82.4	− 82.4	0.00	− 113.0	− 77.4	− 14.8	....	....
Dead load strains.	0.0	− 28.6	− 100	− 200	− 320	− 320	+ 28.6	+ 100	+ 200	+ 320	+ 320	+ 320
Maximum strains.	+ 151.2	+ 174.1	+ 73.4	....	....	....	+ 28.6	+ 135.3	+ 261.8	+ 476.7	+ 556.2	+ 556.2
	− 35.3	− 90.4	− 182.4	− 298.9	− 402.4	− 402.4	....	− 13.0	....	....	....	....

The strains in the diagonals can be found in a precisely similar manner. We first find the strains due to each apex load, and then form a Table giving the live load strains. Combining these strains with those due to dead load, we find the maximum strains.

It is also easy to deduce directly from our Tables the strains due to any other loads, such as snow, etc., by simply taking proportional parts of the apex load strains.

It will be seen that every load in the right-hand span causes a negative shear or reaction at A, Fig. 133. If, therefore, the draw when shut exerts little or no pressure upon the end supports, it should be *latched down* at the ends.

The complete calculation of a draw offers, then, no special difficulty, whether we treat it as three spans, as a *tipper*, or as two spans. Our method by tabulation of apex weights is open to the charge of tediousness in the case of a long span. On the other hand, it is simple and easily systematized. The work, consisting of repetitions of similar operations, is easily checked, and the results showing, as they do, the action of each load upon all the pieces, are more satisfactory than those obtained by other methods.

The ordinary method of calculating a pivot span, is to find the strains when shut, precisely as for a single discontinuous span, and when open to consider it loaded not only with the dead load, but for $\frac{1}{3}$ of the length from centre on each side, with the live load also. This method, it is hardly necessary to state, while perhaps *safe*, does not properly distribute the material in accordance with the actual strains, and is hardly more convenient or simple than the more accurate method of this Chapter.

Having calculated the strains as directed in this Chapter, the closed draw should also be calculated precisely as for a single span also, and the members made to take the resulting greatest strains, and counterbraced, if they have strains of opposite kinds. This is rendered necessary from the fact that practically the span is not perfectly continuous nor perfectly discontinuous, but in an intermediate condition.

For the application of the method by concentrated wheel loads, see page 232.

CHAPTER VIII.

THE BRACED ARCH.

THREE KINDS OF BRACED ARCH.—We may distinguish three kinds of braced arch, viz.: 1st hinged at crown and at ends; 2d., hinged at ends only; 3d., without hinges.

The strains in the various pieces may be easily found either by diagram or calculation, for any loading, if only all the outer forces acting upon the arch for that loading are known; that is, so soon as in addition to the load we know the horizontal thrust and vertical reactions at the ends, and the moments, if any, which exist at the ends.

ARCH HINGED AT CROWN AND ENDS.—This form of construction, Fig. 134, owing to the hinges at crown and ends, is an arch only in form, but in principle is simply two curved and braced rafters, the thrust of which is taken by the abutments instead of by a tie rod.

The calculation of strains present, then, no special difficulty. They may be found by diagram or by the method of moments, provided not more than two pieces, the strains in which are unknown, meet at any apex.

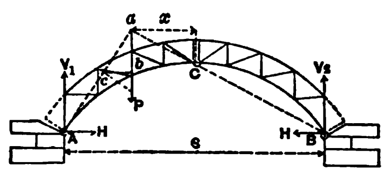


Fig. 134

HORIZONTAL THRUST AND REACTIONS.—The resultant at the crown for the unloaded half, Fig. 134, must pass through C and B. Its direction is, therefore, constant for every weight P upon the other half. The resultant for the other half must then pass through a and A.

We have, then, simply to draw the line CB and prolong it to intersection a with P, and then draw aA. Aa and Ba are the directions of the resultant pressures at A and B. By resolving P along these lines we can find the magnitude of these pressures, viz., ac and Pc. Resolving these pressures vertically and horizontally, we have the vertical reactions $V_1 = ab$, $V_2 = Pb$, and the horizontal thrust $H = cb$.

We can also easily compute V and H. Thus let the span be s, the rise of the arch r, and the distance of the weight P from the crown be x, negative to the left. Then taking moments about B, we have,

$$V_1 \times s = P\left(\frac{s}{2} - x\right) \text{ or } V_1 = \frac{P\left(\frac{s}{2} - x\right)}{s}.$$

Also taking moments about the crown,

$$V_1 \times \frac{s}{2} - Hr = -Px, \text{ or } H = \frac{Px + V_1\frac{s}{2}}{r} = \frac{2\,Px + Ps}{4r}.$$

In these equations x is minus when on the left of the crown, and positive to the right of the crown. The rise r is always measured from the chord AB *to the hinge* at the crown.

These values of V_1 and H are independent of the shape of the arch. Having found them, we

can calculate the strains in the pieces by moments, or by diagram, for each apex weight. A tabulation of the strains for each weight will then give us the strains for uniform load as well as for live load.

UNNECESSARY PIECES.—There should be only two pieces meeting at the abutments. Thus the pieces in Fig. 134, represented by broken lines, can only serve to support a superstructure or to transmit load to the arch. They are not pieces of the arch proper, and have no influence upon the strains. In the same way at the crown, the upper flanges are not connected.

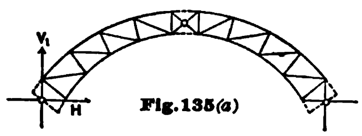


Fig. 135(a)

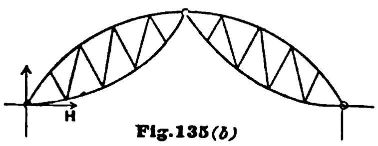


Fig. 135(b)

The hinges may be in either flange at the crown and at the abutments, provided we have regard to the above. The depth of the arch may vary at will above and below the centre line. This will affect the lever arm of the pieces, but V_1 and H remain unchanged. Thus in Fig. 135, whatever may be the shape of arch and character of bracing, V_1 and H can always be found from our formulæ, and these two forces being known for any apex weight, the strains caused by that weight are easily found.

BEST FORM AND BEST DEPTH OF ARCH.—If the arch is hinged in the lower flange, as shown in Fig. 134, the resultant for weights on the right half passes through A and C. Weights on the right-hand half will then cause tension in the upper flange. It is, therefore, well *to pivot the arch either at the upper flange or at the centre line*, as shown in Fig. 135(a). In such case if the depth is made large enough the flanges will always be in compression.

Fig. 136

As to the proper depth, let ACB, Fig. 136, be the centre line. If we draw the line AC, we can easily find the greatest vertical ordinate between this line and the arch. Thus for a *circular arch*, this ordinate mn is equal to

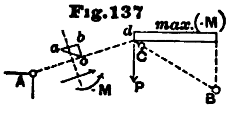
$$\frac{R}{s}\sqrt{s^2+4r^2}-R=\frac{2R}{s}\sqrt{\frac{s^2}{4}+r^2}-R,$$

where R is the radius of the arch. The vertical depth of the arch should, therefore, not be less than twice this ordinate, if the depth is constant. If the depth is variable, it should at any point be greater than the ordinate to AC at that point from the centre line. In other words, the resultant AC should lie *inside of the flanges*.

LOADING GIVING MAXIMUM STRAINS.—As the tabulation of the strains due to each apex load involves considerable calculation, the labor of computation may be much lessened by considering for each piece all those loads acting at once whose effect upon the piece is of the same kind. We can thus find the maximum strains at once.

FLANGES.—For any flange as ab, Fig. 137, the centre of moments is at o, the intersection of the other pieces cut by a section through the arch at ab.

Fig. 137



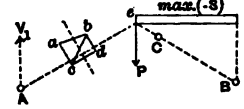
Now it is evident that if we draw a line through A and o, and produce it to intersection d with BC, that a weight at d causes no strain in ab, because the resultant for that weight passes through the point of moments o for ab. Any weight to the right of P, in Fig. 137, has a resultant which lies to the right of Ad, and causes, then, a *negative* moment, that is, *tension* in the upper flange ab. The maximum negative moment for ab will occur, then, when all the weights from the right end up to d act at once. When all the weights from the left end up to d act, we have the greatest compression in ab.

So for any flange, whether upper or lower, we have simply to draw a line through A and the point of moments for the flange and produce to intersection with BC. When the load reaches from the right up to this point we have the greatest negative moment, and when it reaches from the left up to this point we have the greatest positive moment. If d falls on the right of the hinge C at the crown, then every weight causes compression in ab, and there is no tension under any loading. A negative moment denotes tension in an upper flange, and compression in a lower flange. The moment divided by the lever arm for the flange, or the depth of arch, gives at once the maximum strain.

BRACES.—For the braces we must first find that position of load which causes the greatest shear. Next, we must find the strains in the two flanges cut by a transverse section through the arch and brace, for the given loading. Then since the algebraic sum of the vertical components of the strains in the cut pieces and the shear must equal zero, we can find the strain in the brace.

Let bc, Fig. 138, be any brace, and ab and cd the two flanges cut by a transverse section through the arch and brace. Draw Ae parallel to that flange which passes through the *left end* of the brace in question. Thus for the brace bc we draw Ae parallel to cd. For the brace ac, draw Ae parallel to ab. Produce Ae to intersection e with BC, the line through the other two hinges. A weight P at e has a resultant Ae parallel to cd. This weight then, causes no shear at c. For all weights right of e the shear at c is downward or negative. For all weights left of c the shear at c is also downward or negative. For all weights between e and d the shear is positive. *

Fig. 138



When the load extends from the right end to e, and also from A to c, we have the greatest negative shear. If e falls *on the right* of the hinge at the crown, then every weight will cause a positive shear at c.

Let φ be the angle which cd makes with the horizontal. Then max. $(-S) = V_1 - H \tan \varphi - \Sigma_0^c P$, and max. $(+S) = V_1 - H \tan \varphi$, where $\Sigma_0^c P$ denotes the sum of all the weights from A to c.

In writing down the algebraic sum of the vertical components of the strains in the flanges measure their angles always from a vertical through their *left* ends, and follow the rule for signs of page 16.

We have then $$ab \cos \theta_{ab} + bc \cos \theta_{bc} + cd \cos \theta_{cd} + S = 0.$$

If S is determined as above and ab and cd found by moments for that loading which gives S, we can easily find bc.

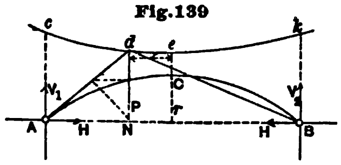
It is unnecessary to give an example. In view of the preceding principles the student will find no difficulty in calculating any given case, both for dead and live loads. The flanges are in general best found by moments, while the bracing is best found by the method of diagram of Chapter I. Section I.

Changes of temperature occasion no strains in this arch. Each half is free to turn about the hinges, and thus accommodate itself to any change of shape. For very long spans, therefore, such a construction as Fig. 135 (b) would seem well adapted.

PARABOLIC ARCH HINGED AT ABUTMENTS—CONTINUOUS AT CROWN.—We shall confine ourselves in what follows to the consideration of very long spans, to which alone the braced arch is applicable. In such cases the parabola is decidedly the best curve, as giving the best shape and the simplest formulæ. When the rise is small compared with the span, or $\frac{r}{s}$ is not more than $\frac{1}{10}$, no great error will be committed by the application of our method and formulæ to circular arches also.

If we suppose the hinge at the crown removed, those at the ends being retained, then for any position of a weight P, Fig. 139, the resultant pressures must, for equilibrium, pass, as in the preceding case, through the end hinges. This case differs from the preceding only in that now the intersection of the weight and resultant pressures has a different position. Thus, in the case of three hinges, the intersection d, Fig. 139, of Ad and P, was always in the line BC. Now it is no longer in this line, but is situated in a curve or *locus* $cdek$.

Fig. 139



If we can find this locus or curve, in which the point d must always lie, we can easily find, as before, the reactions, by simply prolonging the line of direction of the weight P, till it meets this locus, and then drawing from the point of intersection lines to A and P, and resolving P in these directions. V_1, V_2 and H can thus be easily found.

* This construction only holds good in case the chords ab and cd *are parallel.* If they are not parallel, draw Ae to *the point of intersection*, for the brace bc, the intersection of ab and cd, for brace ac, the intersection of ab and the bay on left of cd. Produce this direction of Ae to intersection e with Bc.

The equations of this locus for a parabolic arc or a flat circular arc, where $\frac{r}{s} < \frac{1}{10}$, is

$$y = \frac{32 s^2 r}{25 s^2 - 20 x^2},$$

where s is the span, r the rise, x the distance of the weight from the crown, and y the ordinate Nd of the locus, measured from the chord AB.

For a given arc, then, that is when s and r are given, we have only to substitute different values for x, as $x = 0$, $0.1\,s$, $0.2\,s$, etc., and we can easily find the corresponding ordinates y, and can thus construct the locus $cdek$. It is then easy to find the reactions of A and B for any position of P. By resolving these reactions horizontally and vertically we find H_1, V_1 and V_2.

The vertical reactions at the abutments may also be easily calculated. Thus, as in the case of three hinges, taking moments about B, we have

$$V_1 \times s = P\left(\frac{s}{2} - x\right) \text{ or } V_1 = \frac{P\left(\frac{s}{2} - x\right)}{s},$$

where x is minus when measured to the left of the crown. We have also *

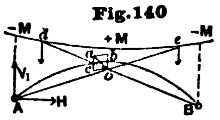
$$H = \frac{5}{128} P \frac{(5 s^2 - 4 x^2)(s^2 - 4 x^2)}{r s^3}.$$

From these equations we can find the values of V_1 and H without making the construction of Fig. 139. We measure r from the chord to the lower or upper chord, or to the centre line, according as the arch is hinged at the extremities of the lower chord, upper chord, or centre line.

Loading giving Maximum Strains.—By means of the preceding formulæ for H and V_1 we can find by moments or by diagram, the strains in every piece due to each apex load. Tabulating these strains we can find the dead load and maximum strains. This method is, however, unnecessarily tedious. We can readily determine for each piece that loading which causes the greatest strains in it, and then find the maximum strains at once.

Flanges.—In Fig. 140, let o be the point of moments for any flange as ab. Through o draw Ao and Bo and produce to intersections e and d with the *locus*. We see at once that all loads between d and e cause a positive moment at o, or compression in the upper flange and tension in the lower. The greatest tension in ab will be when all loads to the right of e and left of d act. Finding V_1 and H for these loadings, by the construction of Fig. 139 for each apex weight, we can easily find by moments or diagram the maximum strains in any flange.

Fig. 140



Braces.—If we draw, Fig. 141, a line Ae parallel to ab, all loads to the right of e cause a negative shear at a. Since the vertical reaction V_1 must be less than the weights which cause it, the shear at a due to the loads on the left of a must also be negative. Fig. 141 gives, then, the loading which causes negative shear at a. The greatest positive shear will be when all loads between e and b act. If e falls to the right of B, then all loads from B up to b cause positive shear, and all loads between a and A cause negative shear.

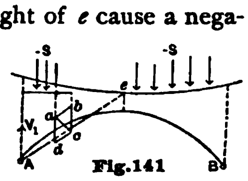


Fig. 141

As before, we have,

$$\max(-S) = (V_1 - \Sigma_0^a P) - H \tan \varphi,$$

where φ is the angle made by ab with the horizontal, and $\Sigma_0^a P$ the sum of all the weights between a and A,

$$\max(+S) = V_1 - H \tan \varphi.$$

---

* For the demonstrations of the analytical results made use of in this Chapter we refer to "*Die Lehre von der Elasticitaet und Festigkeit*," by E. Winkler, Prague, 1867.

The shear being known we find the strains in ab and dc by moments. Then

$$ab \cos \theta_{ab} + ac \cos \theta_{ac} + dc \cos \theta_{dc} + S = 0,$$

from whence we can find ac. Measure the angle θ always from the vertical through the *left end* of piece, and observe the rule for signs of page 16. Or we can find V_1 and H for the required loading by successive applications of Fig. 139, and then diagram the strains due to this loading up to ac.

TEMPERATURE STRAINS.—While in the arch with three hinges there are no temperature strains, in the arch with only two hinges the strains caused by change of temperature may be considerable. Disregarding the weight of the arch, the effect of a rise of temperature above the mean temperature for which there is no strain, is to cause a horizontal thrust at the ends. If there is a rise of temperature, this thrust acts, as in Fig. 142, causing a negative moment at every point, or tension in the outer flanges. If there is a fall of temperature, the arch contracts and H acts in the opposite direction, causing a positive moment at every point, or compression in the outer flanges. If this thrust is known, the strains caused by it can be easily found. This thrust is given by the formula,

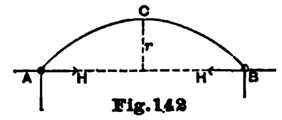


Fig. 142

$$H = \frac{15}{8} \frac{EIF \varepsilon t}{Fr^2 + 15 I};$$

where E is the coefficient of elasticity, I the moment of inertia of the cross-section, F the area of cross-section, ε the coefficient of expansion, t the rise or fall of temperature above or below the mean temperature for which there is no strain. A positive H acts as in Fig. 142.

For cast iron $\varepsilon = 0.00000617$ for one degree Fahrenheit.

For wrought iron $\varepsilon = 0.00000686$. For steel (untempered) 0.00000599.

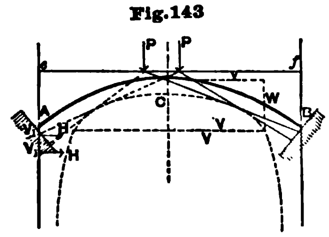
Tables giving the values of ε and E are to be found in any work upon the strength of materials. F is the total cross-section of the flanges and $I = \frac{F d^2}{4}$, where d is the distance between the centres of gravity of the cross sections. We take for t the greatest anticipated range in degrees above and below the temperature of erection, for which no temperature strains exist.

The temperature strains thus found for greatest rise and fall of temperature must be taken in connection with the maximum strains already found for the live and dead loads, in order to find the total maximum strains. Thus the compression in any piece must be increased by the compression, if any, due to temperature.

PARABOLIC ARCH WITHOUT HINGES.—In this case, as in the preceding, the intersection of the weight and of the reactions lies in a curve or *locus*, the equation of which may be found and the curve plotted for any given case.

But the present case differs from the preceding in that the reactions *no longer pass through the ends of the arch A* and B, Fig. 143, but pass through points above or below the ends. This is the same thing as saying that we have not only at each end a horizontal thrust and vertical reaction, but also a moment which varies with the position of the weight, and is always of such a magnitude as to keep the tangent at the end of the arch constant in direction.

Fig. 143



For a parabolic arch we have for the locus ef, *a straight line at* $\frac{1}{5}$*th the rise of the arch above the crown.* The reactions pass through the intersection of the weight P with this locus. The reactions, also, are tangent to a curve, whose equation is

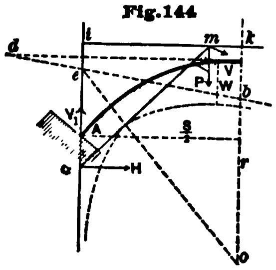
$$w = \frac{5 s^2 - 10 sv + 8 v^2}{15 s (s - 2v)} r;$$

where v is the abscissa on either side of the vertical through the crown, and w is the ordinate measured from the horizontal through the crown.

The curve is, therefore, an hyperbola on each side of the vertical through the crown. The hyperbola on each side has the vertical through the end for one asymptote. The other asymptote is a line which cuts the axis of symmetry at a distance of $\frac{1}{8}r$ below the crown, the tangent at the crown at a distance of $\frac{3}{4}s$ from the crown, and the chord at a distance of $3\,s$ from the centre. The centre of each hyperbola is at a distance of $\frac{1}{18}r$ below the crown.

Fig. 144 shows one of these hyperbolas. The line eA is one asymptote. The line eb is the other. The centre is at e, and eo is the axis. The hyperbola can, therefore, be drawn, as also that on the other side. Then for a weight P placed anywhere, we prolong P till it meets ik in m. From m draw tangents to the hyperbolas on each side. These tangents are the directions of the reactions. P resolved in these directions gives the reaction at each end. Each reaction resolved vertically and horizontally gives the horizontal and vertical reactions at each end. These horizontal and vertical reactions must be considered as acting not at the ends of the arch, but at the points of intersection of the tangent reactions with the verticals through the ends.

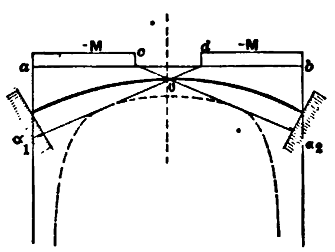
Fig.144



The reactions V_1 and H being thus constructed for any portion of any apex weight, and the point of application α being known of these reactions, we can easily calculate the strains as in the preceding cases.

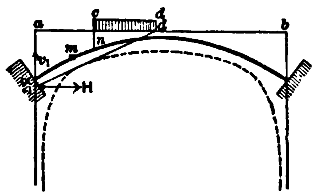
LOADING GIVING MAXIMUM STRAINS.—It is, however, unnecessary to find thus the strains due to every apex weight. We can easily find for each piece the loading which gives the maximum strains in that piece and thus find the strains directly. Thus, if o, Fig. 145, is the point of moments for any flange, we draw through o the resultants $o\alpha_1$ and $o\alpha_2$, which meet the locus ab at the points c and d. Then every load to the right of d or left of c causes a negative moment at o. Loads at d and c cause no moment about o. Loads between c and d give a positive moment about o.

Fig.145



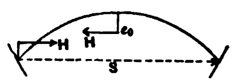
In similar manner, for the greatest shear at any point as n, Fig. 146, let mn be the flange and draw $\alpha_1\,d$ parallel to it. Then a weight at d causes no shear at n. Any load on the right of d or left of c causes negative shear at n, and any load between c and d causes positive shear at n.

Fig.146



TEMPERATURE STRAINS.—In the present case, when there are no hinges, the effect of a change of temperature is to cause a thrust, H, at a distance e_0 from the crown, or a thrust, H, at a distance $(r - e_0)$ from the ends. For rise of temperature this thrust acts as shown in Fig. 147, at a distance $r - e_0$ above the ends, and causing at the ends, therefore, a positive moment. For equal fall of temperature we have an equal negative moment. The thrust is given by the formula

Fig.147



$$H = \frac{45\,EIF\varepsilon T}{4\,Fr^2 + 45\,I},$$

and e_0 by

$$e_0 = \frac{\left(\frac{Fs^2}{4} + 6\,I\right)r}{3\,F\frac{s^2}{4}} = \frac{r\,(Fs^2 + 24\,I)}{3\,Fs^2};$$

where E is the coefficient of elasticity, I the moment of inertia of the cross-section, ε the coefficient of expansion, t the rise or fall of temperature above or below the mean temperature. A positive H acts as shown in Fig. 147.

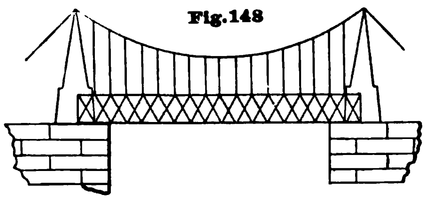
CHAPTER IX.

COMPOSITE STRUCTURES. SUSPENSION SYSTEM WITH STIFFENING TRUSS.*

EACH of the structures of the preceding Chapter may be inverted, and constitutes in such case an inverted arch, or suspension system. The method of calculation is then precisely similar, the only difference being that the horizontal thrust at the end of the arch becomes a horizontal pull at the ends of the cable, and, therefore, pieces which were in compression are now in tension and *vice versa.*

SUSPENSION SYSTEM.—A very common construction for long spans, however, is that shown in Fig. 148. Such a structure we may call "composite," that is, it consists of two different systems which act together. Fig. 148 represents the most important of these, known as the "suspension system." It consists of a flexible chain or cable, which is stiffened under the action of partial loads, by a truss. The truss is slung on to the cable by suspenders, and may be of any design, either double or single intersection, Post, Pratt, etc. The cable carries the entire dead weight of the truss, that is, the suspenders are screwed up until the ends of the truss just bear on the abutments. The office of the truss is then mainly to stiffen the cable and prevent change of shape and oscillation, due to partial and moving loads. It also acts to support a share of the moving load. There are usually side spans at each end. In any case the cable is not attached to the towers, but passes over rollers at the top and is carried on beyond and firmly fastened to large anchorages of masonry.

Fig.148



DEFECTS OF THE SYSTEM.—The principal defect of this system is its lack of rigidity. The cable possesses no inherent rigidity except such as is due to its weight and inertia; and any stiffness which it may have, therefore, is due almost entirely to the truss.

A second disadvantage is that a rise of temperature, by increasing the deflection of the cable, throws considerable load upon the truss. To obviate this objection, the truss is often hinged at the centre and placed on rollers at the ends.

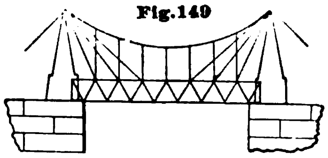
ADVANTAGES OF THE SYSTEM.—It is evident from the preceding, that the system is most advantageous for very long spans. The cable then carries the dead weight in the most advantageous manner, and by reason of its own very considerable weight in such case, resists in some degree the deforming action of partial loads. The truss is then very light compared to what it would have to be if there were no cable.

STAYS UNNECESSARY.—The system is accordingly in practice applied only to very long spans. But owing to a lack of rigidity even in such cases, additional stiffness is sought to be obtained by the introduction of *stays*, reaching from the top of the tower to various points of the truss, as shown in Fig. 149. The use of these is not to be recommended, for two reasons. They render the correct

* The student before reading this Chapter should either first read the Appendix or be familiar with the Theory of Flexure.

determination of the strains indeterminate. A load at any point may be carried entirely by the suspender and stay at that point, or by the suspender and truss, or by the stay and truss. It is impossible to tell exactly the duty performed by each ; and even if it were, it would be impossible to so adjust the several systems that they shall take their proper share. If such adjustment were possible, it would not last. Variations of strain, set and elongation of the metal, shocks and vibrations, and rise and fall of temperature, would constantly disturb the adjustment.

Fig. 149



In the second place the stays are unnecessary. The truss is a rigid construction in and of itself, and ought to render rigid any system of which it is a member. If properly proportioned, then, the truss is sufficient. Stays are superfluous additions, and their use is unscientific. Their employment is a confession of improper design in the truss. We shall, therefore, suppose in what follows that no stays are employed.

TRUSS SHOULD BE FIXED HORIZONTALLY AT THE ENDS.—As the main object of the truss is to secure rigidity, that construction of truss should be adopted which possesses the greatest stiffness. The truss should, therefore, be securely and firmly bolted down at several successive points at the piers and abutments, so that under all circumstances it is *fixed horizontally at the ends*. It should also be *continuous and without hinge at the centre*.

It may be objected to this, that in such case there are strains due to change of temperature. This is quite true. But such strains can be accurately found, and the truss proportioned to withstand them.

The common practice of hinging the truss in the centre and placing it on rollers at the ends, in order to avoid temperature strains, is at the expense of rigidity. Moreover, nothing is gained in economy when the endeavor is made to make good this loss of rigidity by the employment of stays. The material saved in the truss and cable is balanced by the material in the stays, and the result is a shaky and unscientific combination. It is far better to put the material of the stays into the truss, where it is needed to resist temperature strains. We thus obtain the greatest stiffness the system admits of, and have a combination, the strains in which can be accurately determined, easy of adjustment, and which once adjusted will remain so.

Instead of rollers at the ends, a *sliding joint* at the centre of the truss may be permitted, since such a joint, if properly constructed, does not break the continuity of the flanges, and hence does not impair the rigidity, while it does reduce the strains due to temperature. Rollers may also be used, provided that they do not affect the condition that the truss shall be *horizontally fixed at the ends*.

BEST FORM OF SUSPENSION SYSTEM.—The best form of the suspension system, then, and the one which we shall investigate in the following pages, consists simply of truss, cable and suspenders, as shown in Fig. 148. The truss is assumed to be fixed horizontally at the ends, and to have, if desirable, rollers or a sliding joint at the centre or ends, which does not interfere, however, with the continuity of the flanges at that point.

The truss may be of any of the usual patterns. The flanges are usually horizontal, and the bracing either single or double intersection with vertical posts, or triangular. This, however, is by no means necessary. Any form of truss may be employed, whether the flanges are horizontal, or parallel, or not.

It is necessary to point out that such a system as the above does not at present exist. Of the large suspension bridges in existence, none are fixed horizontally at the ends, and most employ stays and are hinged at the centre. Our discussion and formulæ, therefore, *do not apply to such*, and will not enable one to find the strains in them. Indeed, from the preceding, we see that it is not possible to find the strains in them with any degree of certainty, and hence it is useless to devote space here to their discussion. It is not, therefore, surprising that the system has been considered, and is still considered, unsatisfactory in railroad practice, except under certain regulations as to allowable speed and load, and, as regards its calculation, still more unsatisfactory in theory.

We claim that the system proposed secures the greatest rigidity possible to the combination, and

admits of satisfactory calculation, and is, therefore, not only the best, but the only scientific combination. Finally, that the bugbear of temperature strains is one only in appearance, since what is saved in the truss and cable by the ordinary system, is lost in the stays.

THEORY OF COMPOSITE STRUCTURES GENERALLY.—Before proceeding to investigate the suspension system, we shall illustrate the principles which must govern the discussion of any composite system.

Let λ_1 be the alteration in length, per unit of length, due to the strains in one system, and λ_2 that due to the strains in the other. Then in order that there may be the same unit strain at all points in both members, we must have

$$\lambda_1 = \lambda_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I.)$$

But a common unit strain in both systems implies that both act together and are made of the same material. Equation (I.), then, expresses the condition that both systems reach the elastic limit simultaneously, *provided they are both made of the same material.*

In any special case, as we shall see hereafter, λ_1 and λ_2 can easily be found in terms of the depth, span, and deflection. Since the span and deflection are the same for both systems, they will cancel in equation (I.), and we shall then have from it the relation that should exist between the depths, for the least amount of material. Equation (I.), then, under the limitation of a common material, enables us to determine the dimensions of each system, in order that the elastic limit may be simultaneously reached by both, that is, in order that the two may act together as one system.

But whether both systems are made of the same material or not, or whether the elastic limit is reached simultaneously or not, still, if both systems are rigidly connected, they must have a common deflection at each point of connection. One system cannot deflect without causing the other to deflect an equal amount. If, then, δ_1 is the deflection of one system, and δ_2 that of the other, we have

$$\delta_1 = \delta_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II.)$$

Since the deflection varies directly as the load, the ratio $\frac{\delta_1}{\delta_2}$ found from equation (II.) in any special case, will give the ratio of the loads. If, then, we know the total load, we can find the load carried by each system.

The two principles expressed by equations (I.) and (II.) lie at the foundation of the discussion of all composite structures.

CHANGE OF LENGTH.—The coefficient of elasticity E is that theoretical unit force which would stretch a piece an amount equal to its original length, if the law of proportionality of elongation to stretching force were to hold good throughout such extreme elongation.

Thus, if S is the stretching force, and F the cross section, $\frac{S}{F}$ is the unit force. If the elongation produced by this unit force is λ per unit of length, and the length of the piece is L, the elongation due to the unit force $\frac{S}{F}$ is λL. It will, then, according to the assumed law, require as many times $\frac{S}{F}$ to produce the elongation L, as λL is contained in L. Hence,

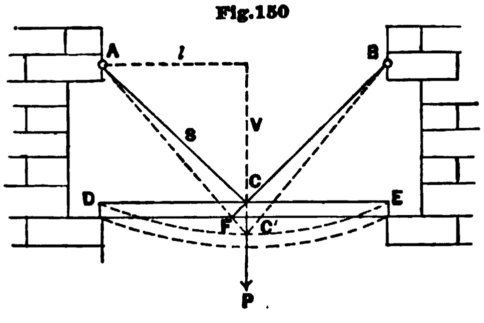
$$E = \frac{S}{F} \times \frac{L}{\lambda L} = \frac{S}{F\lambda}.$$

From this we have for the elongation per unit of length,

$$\lambda = \frac{S}{FE} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (III.)$$

From this equation we can always find the elongation per unit of length λ, when the cross section of the piece F, and the applied force S are known.

Fig. 150



APPLICATION OF PRECEDING PRINCIPLES—ILLUSTRATION.—Let us take as an illustration of these principles and of the use of our fundamental equations (I.) and (II.), a single composite structure, represented in Fig. 150.

It consists of a beam DE and two tie-rods or stays AC and BC, united to the beam at its centre C. A single weight P is placed at the centre.

Let Δ_1 be the deflection of the stays, CC', and Δ_2 the deflection of the beam. Let λ_1 be the coefficient of elongation for the stays, and λ_2 for the beam. Let the length of each stay be s, the half span be l, and the vertical projection of each stay be v.

1. *Deflection of Stays.*

The increase of length of each stay is $\lambda_1 s$. Draw CF at right angles to AC'. Since $\lambda_1 s$ is very small, $AF = AC$ approximately, and hence $FC' = \lambda_1 s$. We have, then, by similar triangles,

$$\frac{CC'}{FC'} = \frac{s}{v} \text{ or } \frac{\Delta_1}{\lambda_1 s} = \frac{s}{v}.$$

From this we obtain,

$$\Delta_1 = \lambda_1 \frac{s^2}{v} \quad \ldots\ldots\ldots \quad (1)$$

Let nP be the portion of the weight carried by the stays. Then $(1 - n)P$ is the portion carried by the beam.

Each stay then carries $\frac{nP}{2}$. The strain in each stay is then

$$\frac{nP}{2} \sec\theta = \frac{nP}{2} \cdot \frac{s}{v}.$$

From equation (III.) the elongation caused by this strain is

$$\lambda_1 = \frac{S}{F_1 E_1} = \frac{nPs}{2\,F_1 E_1\,v}.$$

Inserting this value in (1), we have,

$$\Delta_1 = \frac{nPs^3}{2\,F_1 E_1\,v^2} \quad \ldots\ldots\ldots \quad (2)$$

where $\frac{nP}{2}$ is the weight carried by the stay, and E_1 F_1 are the coefficients of elasticity and the area of cross section of stays.

2. *Deflection of Beam.*

The deflection of a beam supported at the ends, with a weight $(1 - n)P$ in the middle, is, from the Theory of Flexure (see Appendix):

$$\Delta_2 = \frac{(1 - n)\,Pl^3}{6\,E_2\,I_2};$$

where I_2 is the moment of the inertia of the cross section. If the beam is of rectangular constant cross section, $I_2 = \frac{1}{12}bh^3$, where b is the breadth and h the height of cross section. The deflection at the centre is, therefore,

$$\Delta_2 = \frac{2(1-n)Pl^3}{E_2 F_2 h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where E_2 is the coefficient of elasticity, F_2 the area of cross section. Let ρ, Fig. 151, be the radius of curvature of the neutral axis of the beam. Considering the curve of deflection as an arc of a circle, which is approximately true when the unit strain is constant, or when the deflection is small, as is the case in practice, we have

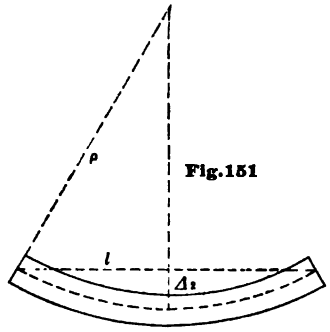


Fig.151

$$\Delta_2 : l :: l : 2\rho - \Delta_2;$$

or neglecting Δ_2 as very small in comparison with 2ρ,

$$\Delta_2 = \frac{l^2}{2\rho}.$$

But when Δ_2 is very small, the length of arc is the same as the length of chord, approximately. The length of the neutral axis, then, is $2l$, and of the lower edge $2l + \lambda_2 2l = 2l(1 + \lambda_2)$. Since the lengths are proportional to the radii,

$$\frac{\rho}{\rho + \frac{h}{2}} = \frac{2l}{2l(1+\lambda_2)} \quad \text{or } \rho = \frac{h}{2\lambda_2}.$$

Substituting this value of ρ in the value for Δ_2, above, we have,

$$\Delta_2 = \frac{\lambda_2 l^2}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We have thus found in equations (1) and (4) the deflection in terms of the elongation, and in equations (2) and (3) the deflection in terms of the load.

We are now ready to apply our principles.

1. *To Find the Best Ratio of* $\frac{v}{h}$.

Since the deflections at C, Fig. 150, must be equal, we can equate equations (1) and (4). We thus have,

$$\Delta_1 = \Delta_2, \text{ or } \lambda_1 \frac{s^2}{v} = \lambda_2 \frac{l^2}{h}, \text{ or } \frac{v}{h} = \frac{\lambda_1 s^2}{\lambda_2 l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equation (5) gives, then, the best ratio of $\frac{v}{h}$, whether the material is the same in both systems or not, if we put for λ_1 and λ_2 their values at the limit of elasticity for each material.

If each system is made of the same material, $\lambda_1 = \lambda_2$, and

$$\frac{v}{h} = \frac{s^2}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

2 *To Find the Load Carried by Each System.*

If we equate (2) and (3), instead of (1) and (4), we have,

$$\frac{nPs^3}{2F_1E_1v^2} = \frac{2(1-n)Pl^3}{F_2E_2h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From this equation we can easily find n, whether the material in the systems is the same or not. If, however, the material is the same, we have from (4) and (3)

$$\lambda_2 = \frac{2(1-n)Pl}{E_2 F_2 h};$$

and if we put this equal to the value for λ_1, we have, since now $E_2 = E_1$,

$$\frac{2(1-n)Pl}{F_2 h} = \frac{nPs}{2F_1 v};$$

or if we introduce the best ratio of $\frac{v}{h}$ as given by (6),

$$\frac{2(1-n)Pl}{F_2 h} = \frac{nPl^2}{2F_1 s h}, \text{ or } n = \frac{4F_1 s}{F_2 l + 4F_1 s} \quad \ldots \ldots \quad (8)$$

The preceding is sufficient to illustrate the use of our equations (I.), (II.), (III.), and the method which must be adopted for any composite structure. We shall now proceed to discuss in a similar manner the structure represented by Fig. 148.

Notation.—We group together here the principal notation which we shall employ for convenience of reference.

E_1 = coefficient of elasticity of the cable.
E_2 = " " " " truss.
F_1 = cross-section due to strains at centre of cable.
F_2 = " " " " truss.
F_0 = " " " ends of cable.
λ_1 = elongation per unit of length of cable at centre.
λ_2 = " " " truss " "
λ = " " " cable at any point.
δ_1 = deflection of cable, δ_2 = deflection of truss at any point.
Δ_1 = " " " Δ_2 = " " " centre.
I_1 = moment of inertia of cable, I_2 of truss, both at centre.
f_1 = greatest allowable unit strain for cable, f_2 for truss.
s = length of cable supporting dead load only.
t = number of degrees rise or fall above or below mean temperature.
e = elongation per unit of length due to a rise of temperature of one degree.
q = unit load due to rise or fall of temperature.
h = depth of truss, $2l$ = span, v = versine of cable.
r = height of towers, p = permanent or dead unit load.
m = moving or live unit load.
M_x = moment at any point, S_x = shear at any point.
M_1 = moment at left end of truss, M_2 at right end.
H = horizontal pull of cable, V_1 = vertical reaction of cable at left.

Equilibrium Curve—Horizontal Force Constant.—The curve or polygon which a perfectly flexible string, hung from two fixed points, assumes when acted upon by a given distributed load, or by given concentrated loads, is called the "equilibrium curve" or polygon.

In Fig. 152, let A and B be the ends of the string. In order that these ends may be fixed, we

must have at A and B the upward forces V_1 and V_2. The sum of these upward forces must equal the sum of the downward forces. Also if A and B are on the same level, the moment of V_1 with reference to B must be equal and opposite to the sum of the moments of the downward forces with reference to B. So also for V_2 with reference to A. If the forces P_1 P_2, etc., are known, and their points of application given, we can then easily find V_1 and V_2. But in order that A may be fixed, we must have not only the upward force V_1, which can be found as above, but also a horizontal force H, equal and opposite to the horizontal pull of the string at that point.

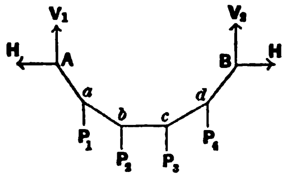


Fig.152

The portion of the string Aa must then lie in the direction of the resultant of H and V_1, and the tension in Aa must be equal to that resultant.

The portion ab must then lie in the direction of the resultant of Aa and P_1, and the tension in it is equal to that resultant. But the resultant of P_1 and Aa is the same as the resultant of H, V_1 and P_1. In like manner the tension in any portion, as bc, is the resultant of all the forces to the left or right of that portion.

But all these forces are parallel and vertical, except H, which is horizontal. The tension of any portion, as bc, then, is the resultant of $V_1 - P_1 - P_2$ and H. But $V_1 - P_1 - P_2$ is the shear just to the right of b.

Therefore, at any point of an equilibrium curve or polygon, *the vertical load is the shear at that point, and the horizontal force at every point is constant* and equal to the horizontal pull at A or B.

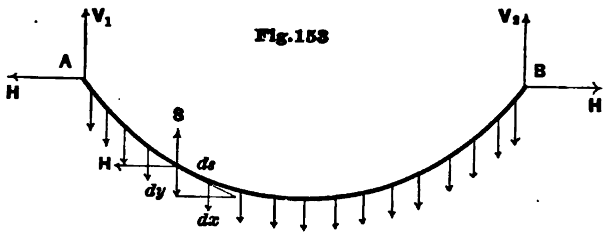


Fig.153

As the loads are more numerous, the polygon approaches a curve. When the load is distributed, we have an equilibrium curve, Fig. 153. The tension in any element of this curve at any point, as ds, is, then, the resultant of the shear S at this point and H, and its direction is the direction of this resultant. We have, then, from similar triangles, completing the parallelogram on H and S,

$$\frac{dx}{dy} = \frac{H}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

That is, the tangent of the angle which the tangent to the curve at any point makes with the vertical is equal to H divided by the shear at that point.

EQUILIBRIUM CURVE FOR UNIFORM LOAD A PARABOLA.—Let the distributed load be uniform and equal to p per unit of length. Let the span be $2\,l$ and the versine at the centre be v. Then, since the load is uniform, it is evident that v_1 must equal v_2, and whatever the form of the curve, each half must be symmetrical. That is, the lowest point of the curve is at the centre and at the point $\frac{dy}{dx} = 0$.

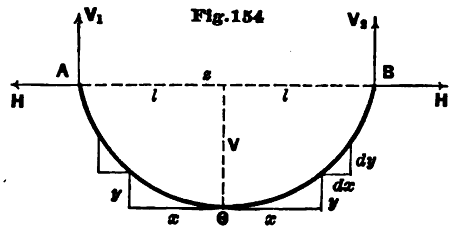


Fig.154

Take C as an origin. Then from (9) we have for the vertical force S at any point,

$$S = H\frac{dy}{dx}.$$

The elementary load at any point is then,

$$\frac{dS}{dx} = H\frac{d^2y}{dx_2} = p.$$

Integrating this, we have

$$H\frac{dy}{dx} = px + C.$$

Since for $x = 0$,

$$\frac{dy}{dx} = 0,$$

the constant $C = 0$ and

$$H\frac{dy}{dx} = px.$$

Integrating again we have

$$Hy = \frac{px^2}{2} + C.$$

Since for $x = l$, $y = v$, we have $C = Hv - \frac{pl^2}{2}$,

and

$$Hy = \frac{px^2}{2} + Hv - \frac{pl^2}{2}.$$

But taking moments about B, we have for the equilibrium of the half BC, since the vertical force at C is zero,

$$Hv = pl \times \frac{l}{2} = \frac{pl^2}{2}.$$

Substituting, we have

$$y = v\frac{x^2}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which is the equation of a parabola.

Hence, *the curve of a flexible string uniformly loaded along the horizontal, is a parabola; and inversely, if a flexible string has the form of a parabola, it must be uniformly loaded.*

Deflection of Cable.—Uniform Live Load over entire Span.—Now in the system shown in Fig. 148, the cable carries the entire dead load of the truss, as the suspenders are supposed to be so adjusted during erection that the unloaded truss just bears at the ends upon the abutments. The weight of truss, flooring, suspenders, etc., may be taken as very nearly uniform, as the variation of weight due to variation of cross section of flanges and braces can be neglected as very small, compared to the uniform dead weight of flooring, wind bracing, etc. Moreover, this uniform load is very great compared to weight of cables. The curve of the cables under the action of the dead load alone may be then considered as very closely a parabola, and therefore given by equation (10).

Differentiating, we have

$$\frac{dy}{dx} = \frac{2\,vx}{l^2}.$$

If the length of arc is s, we have

$$ds = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

Inserting the value for $\frac{dy}{dx}$,

$$ds = dx\sqrt{1 + \frac{4v^2x^2}{l^4}} = dx\left(1 + \frac{2v^2x^2}{l^4} - \frac{2v^4x^4}{l^8} + \ldots\right)$$

If the ratio $\frac{v}{l}$ is small, as is the case for long spans, we can neglect $\frac{v^4}{l^8}$, and have approximately

$$ds = dx\left(1 + \frac{2v^2x^2}{l^4}\right) \quad \ldots \ldots \ldots \quad (11)$$

Integrating between the limits $-l$ and $+l$, we have for the length of the cable, when only the dead load acts,

$$s = 2l\left(1 + \frac{2v^2}{3l^2}\right) \quad \ldots \ldots \ldots \quad (12)$$

Equation (12) gives the original length of cable, when the span and versine are given, and the ratio of $\frac{v}{l}$ is small. That is, for long spans.

Now let the entire span be covered from end to end with the uniformly distributed live load. It is required to find the deflection of the cable.

The new curve will evidently also be a parabola of the same span, whose new length s_1 will be

$$s_1 = 2l\left(1 + \frac{2v_1^2}{3l^2}\right);$$

where v_1 is the new versine.

Let λ be the elongation per unit of length at any point caused by the live load acting over the whole span. Now the cable may be composed of links and pins, or of wire. The last is more common. In the first case, the cross section may be varied according to the strain. The unit strain will then be constant and λ will be constant. In the second case, the cross section of the cable is constant and equal at any point to the cross section required by the greatest strain, that is to the cross section at the ends. The unit strain will then vary as the secant of the angle of inclination, and λ will vary as the unit strain.

Let, then, λ_1 be the elongation per unit of length at the centre C, Fig. 154. If the cable is composed of links and pins, λ_1 will be constant at every point. If the cable is of wire, and of constant cross section, the elongation per unit of length will be

$$\lambda_1 \frac{ds}{dx}.$$

From (11) we have, then, the elongation at any point

$$\lambda = \lambda_1\left(1 + \frac{2v^2x^2}{l^4}\right) \quad \ldots \ldots \ldots \quad (13)$$

We shall find the deflection of the cable for these two cases separately.

1*st. When* λ_1 *is constant,* or the cross section of cable varies so that the unit strain is constant.

In this case, the new length of the chain cable will be

$$s + \lambda_1 s = 2l\left(1 + \frac{2v_1^2}{3l^2}\right); \text{ or from (12)}$$

$$2l\left(1 + \frac{2v^2}{3l^2}\right)(1 + \lambda_1) = 2l\left(1 + \frac{2v_1^2}{3l^2}\right).$$

From this equation we can find the new versine v_1. Thus

$$v_1 = v\sqrt{1 + \lambda_1\left(1 + \frac{3\,l^2}{2\,v^2}\right)} = v\left[1 + \frac{\lambda_1}{2}\left(1 + \frac{3\,l^2}{2\,v^2}\right) - \frac{\lambda_1^2}{8}\left(1 + \frac{3\,l^2}{2\,v^2}\right)^2 + \ldots\right]$$

Since λ_1 is a small fraction, this becomes approximately

$$v_1 = v\left[1 + \frac{\lambda_1}{2}\left(1 + \frac{3\,l^2}{2\,v^2}\right)\right] \quad \ldots \ldots \ldots \quad (14)$$

The distance of any point of the cable below the horizontal *AB*, Fig. 154, is

$$v - y = v - \frac{vx^2}{l^2};$$

where x is the distance of the point right or left of the centre. The distance of the same point of the deflected cable below the horizontal is $v_1 - \frac{v_1\,x^2}{l^2}$.

The difference of these two distances is the deflection at any point, or

$$\delta_1 = v_1 - v - \frac{x^2}{l^2}(v_1 - v).$$

But from (14)

$$v_1 - v = \frac{\lambda_1 v}{2}\left(1 + \frac{3\,l^2}{2\,v^2}\right) = \tfrac{3}{4}\,\lambda_1\frac{l^2}{v}\left(1 + \frac{2\,v^2}{3\,l^2}\right);$$

or, since $\frac{v}{l}$ is small for long spans

$$v_1 - v = \tfrac{3}{4}\,\lambda_1\frac{l^2}{v}.$$

Hence

$$\delta_1 = \frac{3\,\lambda_1}{4\,v}(l^2 - x^2) \quad \ldots \ldots \ldots \ldots \quad (15)$$

where x is the distance of any point right or left of the centre.

For the deflection at the centre, $x = 0$, and

$$\Delta_1 = \frac{3\,\lambda_1\,l^2}{4\,v} \quad \ldots \ldots \ldots \ldots \quad (16)$$

where λ_1 is constant and is the elongation per unit of length due to the strain at the centre, caused by the uniform live load when it covers the whole span.

Let the uniform live load be m per unit of length. Then the vertical reactions at the ends, Fig. 154, are $V_1 = V_2 = ml$. Taking moments about the centre, we have

$$Hv_1 = ml \times \frac{l}{2}, \text{ or } H = \frac{ml^2}{2\,v_1},$$

where v_1 is the new versine of the elongated cable. The value of this new versine can be found from (14).

H is the tension at the centre of the cable. If we denote the constant unit strain by f_1 we have from (III.)

$$\lambda_1 = \frac{f_1}{E_1} = \frac{H}{F_1 E_1} = \frac{ml^2}{2\, E_1 F_1 v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

Equation (15) then becomes

$$\delta_1 = \frac{3\; ml^2}{8\; E_1 F_1 v_1 v}\,(l^2 - x^2)\, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

and hence the deflection at the centre of the cable is

$$\varDelta_1 = \frac{3\; ml^4}{8\; E_1 F_1 v_1 v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

where F_1 is the cross section at the centre of the cable required by the strain at that point, and v_1 is given by the equation

$$v_1 = v\left[\,1 + \frac{f_1}{2\, E_1}\left(1 + \frac{3\, l^2}{2\, v^2}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (20)$$

2d. When the cross section of cable is constant.—In this case the elongation at any point is from (13)

$$\lambda = \lambda_1\left(1 + \frac{2\, v^2 x^2}{l^4}\right).$$

The elongation of any element ds of the curve is λds, and the new length of the element is $ds + \lambda ds = ds_1$.

From (11), then, we have,

$$ds\,(1 + \lambda) = ds_1 = dx\left(1 + \frac{2\, v^2 x^2}{l^4} + \lambda_1 + \frac{4\, v^2 \lambda_1 x^2}{l^4} + \frac{4\, v^4 \lambda_1 x^4}{l^8}\right).$$

Integrating between $-\,l$ and $+\,l$,

$$s_1 = 2l\,(1 + \lambda_1)\left(1 + \frac{2v^2}{3\,l^2} + \frac{2\, v^2\, \lambda_1}{3\, l^2\,(1 + \lambda_1)}\right).$$

Since $\frac{v}{l}$ is a small fraction, and $\frac{\lambda_1}{1 + \lambda_1} = \frac{1}{1 + \frac{1}{\lambda_1}}$ is also a very small fraction, we have approximately

$$s_1 = 2\, l\,(1 + \lambda_1)\left(1 + \frac{2\, v^2}{3\, l^2}\right).$$

This is apparently the same length as in the first case. But it must be remembered that λ_1 or the elongation due to the strain at the centre is now less than before, because the cross section there is greater and equal to the cross section at the ends. Let the cross section at the ends be F_0. Then, since the cross section is constant,

$$\lambda_1 = \frac{H}{F_0 E_1} = \frac{ml^2}{2\, E_1\, F_0\, v_1}.$$

We have, then, from (15) for the deflection at any point,

$$\delta_1 = \frac{3\; ml^2}{8\; E_1\, F_0\, v_1\, v}\,(l^2 - x^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18a)$$

$$\Delta_1 = \frac{3}{8}\frac{ml^4}{E_1 F_0 v_1 v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19a)$$

where we must put for F_0 the cross section required by the strain at the ends of the cable. Equation (20) gives, as before, the value of v_1.

In *either case*, then, the deflection at any point is

$$\delta_1 = \frac{3}{8}\frac{ml^2}{E_1 F v_1 v}(l^2 - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (IV.)$$

If we remember that in the first case F is the cross section F_1 due to the *strain at centre*, and in the second case F is the cross section F_0 due to the strains *at the ends*.

The second case is the most common, as a wire cable is more often used than a chain and is more economical, and such a cable must have a uniform cross section. We shall suppose in what follows, therefore, a wire cable of constant cross section, and, therefore, $F = F_0$. If the formulæ for the first case are required, we have only to replace F_0 by F_1.

BEST RATIO OF HEIGHT OF TRUSS TO VERSINE OF CABLE.—The deflection of the cable at the centre is from (16),

$$\Delta_1 = \frac{3}{4}\lambda_1 \frac{l^2}{v};$$

where $\lambda_1 = \dfrac{H}{E_1 F_0}$ and F_0 is the cross section due to strains at ends.

If ε is the elongation per unit of length due to a rise of temperature of 1°, and t is the number of degrees rise above the mean temperature of erection, the deflection due to temperature is

$$\frac{3}{4}\varepsilon t \frac{l^2}{v}.$$

The total deflection of cable, then, is

$$\Delta_1 = \frac{3 l^2}{4v}(\lambda_1 + \varepsilon t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The deflection at the centre of a beam fixed horizontally at both ends, when uniformly loaded with the load m per unit of length is (Appendix, page 264),

$$\Delta_2 = \frac{ml^4}{24 E_2 I_2}.$$

For a braced girder we may take the radius of gyration equal to $\dfrac{h}{2}$, where h is the depth of girder. Hence $I_2 = \dfrac{F_2 h^2}{4}$; where F_2 is the total cross section of both the flanges at the centre. We have, then,

$$\Delta_2 = \frac{ml^4}{6 E_2 F_2 h^2}.$$

If we take moments about the centre of the span, we have for the strain in either flange, since the moment at the centre is $\dfrac{ml^2}{6}$ (Appendix, page 264),

$$\text{strain} = \frac{ml^2}{6h}.$$

From (III.), then,

$$\lambda_2 = \frac{S}{\frac{F}{2} E_2} = \frac{ml^2}{3 E_1 F_2 h}.$$

We have, then,

$$\Delta_2 = \frac{1}{2} \lambda_2 \frac{l^2}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Equating (22) and (21), we obtain for the best ratio of height of truss to versine of cable,

$$\frac{h}{v} = \frac{2}{3} \frac{\lambda_2}{(\lambda_1 + \varepsilon t)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (V.)$$

Since the cable is always in tension, we can take a higher unit strain for it than for the truss, even when they are of the same material. If of wire, the unit strain may be considerably greater than the unit strain in the truss. Moreover, the tops of the towers may give somewhat, and this has the same effect as an increase of λ_1.

EXAMPLE.—Let the material in cable and truss be wrought iron. Take the coefficient of elasticity at, say 24,000,000 lbs. per. sq. inch, or 12,000 tons. If, then, we assume the safe unit strain for the truss 4 tons, and for the cable, if of wire, 7 tons per inch, we have,

$$\lambda_1 = \frac{S}{F_1 E_1} = \frac{7}{12,000} \text{ and } \lambda_2 = \frac{S}{F_2 E_2} = \frac{4}{12,000}.$$

Take $\varepsilon = 0.00000686$ and $t = 40°$. Then $\varepsilon t = \frac{1}{3,000}$ nearly.

We have, then, from (V.),

$$\frac{h}{v} = \frac{2}{3} \times \frac{4}{7 + 4} = \frac{8}{33} = 0.25 \text{ nearly.}$$

In a similar manner we can find $\frac{h}{v}$ in any special case.

LOAD DUE TO CHANGE OF TEMPERATURE.—A fall of temperature causes the cable to shorten. The effect of this is the same as a uniform upward load over the truss and an equal downward load over the cable. Let this load be equivalent to q per unit of length. We may call q the unit "cold load." This load pulls the truss up and acts as a downward load, therefore, on the cable. The actual rise at the centre of cable is, then, the rise due to change of length, minus the deflection due to cold load.

The rise due to change of length is from (16) $\frac{3}{4} \varepsilon t \frac{l^2}{v_1}$. The horizontal tension at centre of cable due to load q per unit of length is approximately, since in this case v and v_1 are nearly equal, $H = \frac{ql^2}{2 v_1}$. The elongation at centre due to this tension is for constant cross section F_0 from (III.),

$$\lambda_1 = \frac{H}{E_1 F_0} = \frac{ql^2}{2 v_1 E_1 F_0};$$

where F_0 is the cross section due to strain *at ends*. The deflection corresponding to this elongation is from (16),

$$\frac{3}{4} \frac{ql^2}{2 v_1 E_1 F_0} \frac{l^2}{v_1} = \frac{3 ql^4}{8 v_1^2 E_1 F_0}.$$

The actual rise of the cable is, then, at centre.

$$\Delta_1 = \frac{3}{4} \varepsilon t \frac{l^2}{v_1} - \frac{3 ql^4}{8 v_1^2 E_1 F_0}$$

The upward deflection of the truss at the centre is (Appendix, page 264),

$$\varDelta_2 = \frac{ql^4}{24 E_2 I_2};$$

or putting $I_2 = \frac{F_2 h^2}{4}$

$$\varDelta_2 = \frac{ql^4}{6 E_2 F_2 h^2}.$$

Equating these two deflections we obtain

$$q = \frac{2 \varepsilon t E_1 F_0 v_1}{l^2 \left(1 + \frac{4}{9} \frac{E_1 F_0 v_1^2}{E_2 F_2 h^2}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VI.)}$$

From this equation we can find the temperature load q, or that unit load which would give the same strains as are caused by change of temperature.

For a fall of temperature this load, or the "cold load," is carried by the cable and bends the girder upward. For a rise of temperature the "hot load" is carried by the truss alone.

The strains in each case must be combined with those caused by the dead and live loads, in such a way as to give the greatest strains which can ever occur.

EXAMPLE.—If we take, as before, $\varepsilon t = \frac{1}{2,000}$, and assume $E_1 = E_2$, $\frac{h}{v} = \frac{1}{3}$ and $\frac{F_0}{F_2} = \frac{1}{2}$, we have, taking $E_1 = E_2 = 12,000$ tons per sq. inch, $l = 400$ feet, $v = 50$ feet, and $F_0 = 200$ sq. inches, from (VI.),

$$q = 0.25 \text{ tons per foot,}$$

as the load on cable for fall of temperature, or the load on truss for rise of temperature.

LIVE LOAD OVER ENTIRE SPAN.—Let the live load m per unit of length cover the entire span. A portion of this load acts as a uniform load upon the cable, and a portion as a uniform load upon the truss. Let nm be the portion carried by the cable. Then $(1 - n)\, m$ is the load of the truss. We have from (18*a*) for the deflection of cable at centre,

$$\varDelta_1 = \frac{3\, mnl^4}{8 E_1 F_0 v_1 v}.$$

The deflection of the truss (Appendix, page 264) is

$$\varDelta_2 = \frac{(1 - n)\, ml^4}{24 E_2 I_2};$$

or putting $I_2 = F_2 \frac{h^2}{4}$, where h is the depth of truss,

$$\varDelta_2 = \frac{(1 - n)\, ml^4}{6 E_2 F_2 h^2}.$$

Equating these two deflections we obtain,

$$n = \frac{1}{1 + \frac{9}{4} \frac{E_2 F_2 h^2}{E_1 F_0 v_1 v}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VII.)}$$

PARTIAL UNIFORM LOAD.—Under the action of a partial load which extends only over a portion of the span, the curve of the cable is no longer a parabola, and the greatest deflection is no longer

at the centre. As the office of the truss is to prevent deformation, it acts to distribute the partial load over the cable. But as the curve of the cable is no longer parabolic, the distributed load cannot be regarded as uniform, and, in fact, is not uniform.

If we suppose, however, that the cable load *is* uniform, we can easily find that portion of the partial load which would act in such case, upon the cable as a uniform load. Although the supposition is not correct, we shall have future use for this result, and we therefore deduce it here.

Suppose a partial load m per unit of length to extend over a distance a *from each end.* These two loads being equal and symmetrically placed, will cause a uniform load over the cable, the curve of which is, therefore, still a parabola. Of the whole load $2ma$, let the amount $2km$ act as a uniform load per unit of length on the cable, km being the unit load due to *each partial load.*

Then the deflection of the cable at the centre is from (19*a*).

$$\Delta_1 = \frac{6\,kml^4}{8\,E_1 F_0 v_1 v}.$$

The deflection of the truss at centre is equal to the deflection which would be caused by the two weights acting alone, minus the upward deflection due to the reaction of cable, or the upward load on the truss.

The first deflection, putting $I_2 = F_2\,\dfrac{h^2}{4}$, is (Appendix, page 272)

$$\frac{m}{6\,E_2 F_2 h^2}\left[2\,la^3 - a^4\right].$$

The second deflection is (Appendix 272)

$$\frac{2\,kml^4}{6\,E_2 F_2 h^2}.$$

The actual deflection of the truss at the centre is, then,

$$\Delta_2 = \frac{m}{6\,E_2 F_2 h^2}\left[2\,la^3 - a^4\right] - \frac{2\,kml^4}{6\,E_2 F_2 h^2}.$$

Equating Δ_1 and Δ_2, we obtain,

$$k = \frac{2\,la^3 - a^4}{2\,l^4\left[1 + \dfrac{9\,E_2 F_2 h^2}{4\,E_1 F_0 v_1 v}\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VIII.)}$$

This equation gives the value of k for any value of a from o to l. If the load extends from the left end *beyond* the centre, we have,

$$k = \frac{2\,l^4 - a\,(2\,l - a)^3}{2\,l^4\left[1 + \dfrac{9}{4}\,\dfrac{E_2 F_2 h^2}{E_1 F_0 v_1 v}\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(VIII.}a\text{)}$$

In either of these equations, when $a = l$, the value of k becomes equal to half of n, as given by (VII.), as should be. For varying cross section of cable, put F_1 for F_0.

Approximate values of F_0, F_1 and F_2.—The use of the equations thus far deduced requires that F_0 and F_2, the cross sections at each end of cable and centre of truss, should be known. But these are the very quantities which are to be found. It is, therefore, necessary to make a preliminary, if not a very exact estimate of the cross sections, before we can use our formulæ. The strains being then found, we can determine the corresponding cross sections. If they do not agree well enough with their assumed values, a second approximation can be made.

We see from (VI.), (VII.) and (VIII.) that very considerable variations in the ratio $\frac{F_2}{F_0}$, will have but slight effect upon the resulting values of q, n and k. Our approximations need not, then, be very exact.

We may suppose the cable to carry the load $p + m$ per unit of length. This is an error in excess, because really the cable carries only a portion of the live load m. The "cold load" q, however, which for the present we neglect, tends to balance this error. We may also take $v = v_1$.

The horizontal tension at centre of cable would then be approximately

$$H = \frac{(p + m)\, l^2}{2v}.$$

The secant of the angle of inclination at the ends would be from [11],

$$\frac{ds}{dx} = 1 + \frac{2v^2}{l^2}.$$

Hence the strains at the ends is

$$S_0 = \frac{(p + m)\, l^2}{2v}\left(1 + \frac{2v^2}{l^2}\right).$$

Let the greatest allowable unit strain for the cable be f, then, approximately,

$$F_0 = \frac{S_0}{f_1} = \frac{(p + m)\, l^2}{2vf_1}\left(1 + \frac{2v^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.)}$$

From equation (IX.) we can compute approximately the cross section F_0 at the ends of cable, or the constant cross section of cable.

The cross section F_1 at the centre of the cable, if the cross section varies with the strain is also approximately,

$$\text{approx. } F_1 = \frac{H}{f_1} = \frac{(p + m)\, l^2}{2vf_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(IX.}a\text{)}$$

We may also find a preliminary value of n by putting $\frac{F_2}{F_0} = 1$, and $v_1 = v$, in equation (VII.) We have then, approximately,

$$\text{approx. } n = \frac{1}{1 + \frac{9\, E_2\, h^2}{4\, E_1\, v^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

We can now find the preliminary value of q by putting $\frac{F_0}{F_2} = 1$, in (VI.). Thus,

$$\text{approx. } q = \frac{2\, \varepsilon t\, E_1\, F_0\, v}{l^2\left(1 + \frac{4\, E_1\, v^2}{9\, E_2\, h^2}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

For varying cross section put F_1 in place of F_0.

We can now compute an approximate value for F_2. Thus we may consider the truss loaded with $(1 - n)\, m + q$ per unit of length, where q is the "hot load." The moment at centre of truss (Appendix, page 264) is, then,

$$\frac{[(1 - n)\, m + q]\, l^2}{6}.$$

The strain in each flange at the centre is, then,

$$\frac{[(1 - n)m + q]\,l^2}{6h}.$$

The total area of both flanges at the centre is, then, if f_2 is the greatest allowable unit strain for truss,

$$\text{approx. } F_2 = \frac{[(1 - n)m + q]\,l^2}{3hf_2} \quad \text{(X.)}$$

Equations (IX.), (IX.*a*), (23), (24) and (X.), give us approximations to the values of F_0, F_1 and F_2, sufficiently exact for our purpose.

GREATEST DEFLECTION AND STRAIN IN THE CABLE.—We can now find the strain at any point of the cable.

The greatest strain in the cable will occur when the dead load, live load and temperature load, all act. The uniform load of the cable, then, will be $p + nm + q$. The dead load, p per unit of length, and the live load, m per unit of length, are supposed to be known. We can find n and q from (VII.) and (VI.) with the aid of (X.) and (IX.).

The deflection at the centre, then, of cable is from (IV.),

$$\Delta_1 = \frac{3\,(p + nm + q)\,l^4}{8\,E_1 F_0 v_1 v} \quad \text{(XI.)}$$

For varying cross sections we must put F_1 in place of F_0 in all formulæ. The new versine v_1 is given by (14). It will in general be so nearly equal to v that small error will be committed by taking them equal.

The greatest horizontal tension at centre of cable is, then,

$$H = \frac{(p + nm + q)\,l^2}{2v_1} \quad \text{(XII.)}$$

The strain at any point is $H \sec i$, where i is the angle of inclination of cable at that point to horizontal. From (11) we have,

$$\sec i = \frac{ds}{dx} = 1 + \frac{2v_1^2 x^2}{l^4}.$$

Hence the strain at any point is,

$$S = \frac{(p + nm + q)\,l^2}{2v_1}\left(1 + \frac{2v_1^2 x^2}{l^4}\right) \quad \text{(XIII.)}$$

where x is the distance of point from centre. If the cross section is constant, it must be determined from the greatest strain, or making $x = l$ in (XIII.). We have, then, the constant cross section

$$F = \frac{S_{x=l}}{f_1} = \frac{(p + nm + q)\,l^2}{2v_1 f_1}\left(1 + \frac{2v_1^2}{l^2}\right) \quad \text{(XIV.)}$$

If the cross section varies, the cross section at the centre is

$$F_1 = \frac{H}{f_1} = \frac{(p + nm + q)\,l^2}{2v_1 f_1} \quad \text{(XV.)}$$

These cross sections thus found ought to agree satisfactorily with our approximate values found

from (X.) and (IX.). If they do not, another and better approximation can be made. We can thus easily find the deflection at the centre and the strain at any point, whether the cross-section is constant or varies according to the strain.

A discussion of the side spans, if any, is unnecessary so far as the cable is concerned. If the cable is of uniform cross-section it must evidently remain of the same cross-section for the side spans also. If it varies in cross-section, it will be sufficient to make the area of the cable at any point in the side span equal to its area at the corresponding point of the centre span, whatever the length of the side span.

Estimate of the Dead Weight.—In any given case n and q are given by (VII.) and (VI.). We also know m or the unit live load. But p, or the unit dead load, depends upon the weight of the truss and cable, or upon F_0, or F_1 and F_2, and these are the very quantities it is required to determine. We must, therefore, make a preliminary estimate of the unit dead load p. The dead load consists of the weight of the floor system and wind bracing, etc., the weight of the cable, and the weight of the truss. Of these, the weight of the floor system, wind bracing, etc., can be very exactly estimated for any special case and system. The weight of cable can be easily estimated from (IX.) and (IX.a) Thus if the cross-section of cable is uniform, the cubic contents of cable are,

$$F_0\, s_1 = 2\, F_0\, l\left(1 + \frac{2v_1^2}{3l^2}\right) \quad \ldots \ldots \quad (25)$$

where v_1 is given by (14).

This multiplied by the weight of a cubic unit of cable, which is known, of course, will give the weight of cable. We may take this weight as uniformly distributed and add it to weight of floor system, etc.

If cross-section of cable varies, it will be near enough to take the mean area, or $\frac{F_1 + F_0}{2}$, and multiply by the length, or

$$\frac{F_1 + F_0}{2}\, s_1 = (F_1 + F_0)\, l\left(1 + \frac{2\, v_1^2}{3\, l^2}\right) \quad \ldots \ldots \quad (26)$$

Lastly, for the weight of truss, we have for the cubic contents of the flanges $2F_2\, l$, assuming that the flanges are constant. This multiplied by the weight of a cubic unit of the material of the truss will give an estimate of weight of girder. As the flanges are not uniform in cross-section, the estimate is too large. But the error in this direction is balanced by the weight of bracing, which we neglect.

We can thus estimate p closely enough for our purposes. The strains in the cable can then be found.

Recapitulation of Formulæ Necessary for Calculation of Cable.—For convenience of reference let us now group together the formulæ thus far deduced, in the order required for calculation.

We have for the best ratio of height of truss to versine of cable, neglecting temperature,

$$\frac{h}{v} = \frac{2\lambda_2}{3\lambda_1} = \frac{2f_2}{3E_2\,\frac{f_1}{E_1}} \quad \ldots \ldots \quad (\text{V.})$$

For preliminary estimate we have next,

$$\text{approx. } n = \frac{1}{1 + \frac{9\,E_2\,h^2}{4\,E_1\,v^2}} \quad \ldots \ldots \quad (23)$$

$$\text{approx. } F_0 = \frac{(p + m)\, l^2}{2v\, f_1}\left(1 + \frac{2v^2}{l^2}\right) \quad \ldots \ldots \quad (\text{IX.})$$

$$\text{approx. } F_1 = \frac{(p+m)\,l^2}{2v\,f_1} \quad \text{(IX.}a\text{)}$$

$$\text{approx. } q = \frac{2\,\varepsilon t\,E_1\,F_0\,v}{l^2\left(1 + \frac{4}{9}\frac{E_1\,v^2}{E_2\,h^2}\right)} \quad (24)$$

$$\text{approx. } F_2 = \frac{[(1-n)\,m + q]\,l^2}{3h\,f_2} \quad \text{(X.)}$$

These approximate values being thus determined, we have,

$$v_1 = v\left[1 + \frac{\lambda_1}{2}\left(1 + \frac{3l^2}{2v^2}\right)\right] \quad (14)$$

where $\lambda_1 = \frac{f_1}{E_1}$, when the cross-section of cable varies according to the strain, and $\lambda_1 = \frac{F_1}{F_0}\frac{f_1}{E_1}$ when the cross-section of cable is constant.

We have now the accurate value

$$n = \frac{1}{1 + \frac{9\,E_2\,F_2\,h^2}{4\,E_1\,F_0\,v_1\,v}} \quad \text{(VII.}$$

Where we put for F_2 and F_0 their approximate values as determined by (X.) and (IX.). For varying cross-section of cable, we put F_1 in place of F_0. For a fall of temperature of t^0 below the mean, the uniform "cold load" of cable per unit of length is

$$q = \frac{2\,\varepsilon t\,E_1\,F_0\,v_1}{l^2\left[1 + \frac{4\,E_1\,F_0 v_1^2}{9\,E_2\,F_2\,h^2}\right]} \quad \text{(VI.)}$$

For varying cross-section of cable, put F_1 in place of F_0. For rise of temperature, q is the "hot load" of truss.

For a partial distributed load ma, covering a distance a from the left end, upon the supposition that the curve of the cable remains a parabola, with the vertex at the centre, the equivalent uniform load on the cable per unit of length is, when a is less than the half span, or when $a < l$

$$k = \frac{a^3\,(2l - a)}{2l^4\left[1 + \frac{9\,E_2\,F_2\,h^2}{4\,E_1\,F_0\,v_1\,v}\right]} \quad \text{(VIII.)}$$

when $a > l$,

$$k = \frac{2l^4 - a\,(2l - a)^3}{2l^4\left[1 + \frac{9\,E_2\,F_2\,h^2}{4\,E_1\,F_0\,v_1\,v}\right]} \quad \text{(VIII.}a\text{)}$$

For the deflection due to a uniform load u per unit of length, we have

$$\delta_1 = \frac{3ul^2}{8\,E_1\,F_0\,v_1\,v}\,(l^2 - x^2) \quad \text{(IV.)}$$

where x is the distance of the point from the centre. For varying cross-section of cable, always put F_1 in place of F_0.

For the maximum deflection at the centre of cable,

$$\Delta_1 = \frac{3\,(p + nm + q)\,l^4}{8\,E_1\,F_0\,v_1\,v} \quad \text{(XI.)}$$

For varying cross-sections put F_1 in place of F_0.

The greatest horizontal strain at centre of cable is now

$$H = \frac{(p + nm + q)\, l^2}{2v_1} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(XII.)}$$

The strain at any point of cable distant x from centre, is

$$S = \frac{(p + nm + q)\, l^2}{2v_1}\left(1 + \frac{2v_1^2 x^2}{l^4}\right) \quad \ldots\ldots\ldots \quad \text{(XIII.)}$$

The constant cross-section of cable is

$$F_0 = \frac{(p + nm + q)\, l^2}{2v_1 f_1}\left(1 + \frac{2v_1^2}{l^2}\right) \quad \ldots\ldots\ldots \quad \text{(XIV.)}$$

The cross-section at centre, if cross-section varies with the strain, is

$$F_1 = \frac{(p + nm + q)\, l^2}{2v_1 f_1} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(XV.)}$$

EXAMPLE.—Take, for instance, the Niagara Bridge, the span of which is $2l = 800$ feet, $v = 60$ feet. Take $E_1 = E_2 = 12{,}000$ tons per square inch, and let the greatest allowable strain in the cable be $f_1 = 10$ tons, and in the truss $f_2 = 4$ tons per square inch. Take $\varepsilon = 0.00000686$ for one degree, Fahr., and let $t = 70°$ above or below the mean temperature of erection. Then $\varepsilon t = \frac{1}{2000}$.

Let also $m = 1$ ton per foot, and estimated $p = 0.5$ ton per foot.

Then from (V.) we have for best height of truss,

$$h = \frac{2 \times 4 \times 60}{3 \times 12000 \times \frac{10}{12000}} = 16 \text{ feet.}$$

From (23) we obtain,

$$\text{approx. } n = \frac{1}{1 + \frac{9 \times 256}{4 \times 3600}} = 0.9 \text{ nearly.}$$

We have also from (IX.) and (IX.a),

$$\text{approx. } F_0 = \frac{1.5 \times 160000}{2 \times 60 \times 10}\left(1 + \frac{2 \times 3600}{160000}\right) = 209 \text{ sq. inches.}$$

$$\text{approx. } F_1 = \frac{1.5 \times 160000}{2 \times 60 \times 10} = 200 \text{ sq. inches.}$$

We can now estimate from (24),

$$\text{approx. } q = \frac{2 \times \frac{1}{2000} \times 12000 \times 209 \times 60}{160000\left(1 + \frac{4 \times 3600}{9 \times 256}\right)} = 0.055 \text{ tons per foot.}$$

From (X.) we have

$$\text{approx. } F_2 = \frac{(0.05 + 0.055)\ 160000}{3 \times 16 \times 4} = 133 \text{ sq. inches.}$$

We have also from (14), since λ_1 for constant cross-section is $\frac{200 \times 4}{209 \times 12000}$, or $\lambda_1 = \frac{1}{3135}$

$$v_1 = v\left[1 + \frac{1}{6270}\left(1 + \frac{3 \times 160000}{2 \times 3600}\right)\right] = 1.01\, v.$$

We may, therefore, take $v_1 = v$ without appreciable error.

We have next from (VII.),

$$n = \frac{1}{1 + \frac{9 \times 133 \times 256}{4 \times 209 \times 3600}} = 0.9 \text{ ton per foot,}$$

or about the same as our first estimated value.

From (VI.) we have, then,

$$q = \frac{2 \times \frac{1}{3000} \times 12000 \times 209 \times 60}{160000 \left(1 + \frac{4 \times 209 \times 3600}{9 \times 133 \times 258}\right)} = 0.04.$$

We have, then, for constant cross section of cable, from (XIV.),

$$F_0 = \frac{(0.5 + 0.9 + 0.04)\,160000}{2 \times 60 \times 10} \left(1 + \frac{2 \times 3600}{160000}\right) = 209 \text{ sq. inches},$$

or the same as our estimated value. For the given unit strain this would require 4 cables, of 8 inches each in diameter. From (XI.), finally, we have for the deflection of cable at the centre,

$$\Delta_1 = \frac{3 \times 1.5 \times 160000 \times 160000}{8 \times 12000 \times 209 \times 3600} = 1.5 \text{ feet}.$$

THEORY OF THE STIFFENING TRUSS.*

ORDINARY METHOD INCORRECT.—The usual method adopted for the discussion of the stiffening girder, is to assume that the truss distributes any partial load over the cable, *so as to cause it to take effect as a uniform load.* The new curve of the cable is thus assumed to be still a parabola with its vertex at the centre, whatever may be the loading. By finding, then, the deflections at centre of cable and truss and equating them, that portion of the partial load which acts on the cable as a uniformly distributed load, is easily found.

This is the same as the upward load on the truss. The moment at any point of the truss can then be determined.

This method is, however, essentially incorrect. The truss, it is true, does act to distribute any partial load over the cable, but we have no right to assume this distributed load as uniform. In point of fact it is not uniform. The new curve of the cable is not a parabola whose vertex is at the centre, but is a curve similar to that shown in Fig. 155. The distributed load on the cable is, therefore, *not* uniform, but follows some other law of distribution.

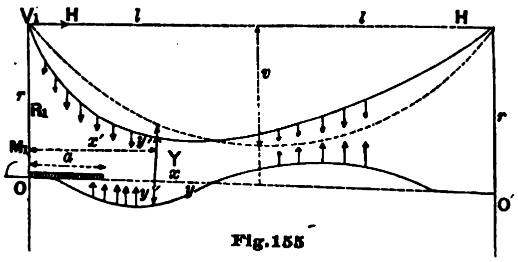


Fig. 155

This distributed load, whatever its law may be, acts through the suspenders as an equal upward load upon the truss. The elastic curve of the truss is, then, similar to that shown in Fig. 155. If we neglect the slight increase of length of the suspenders, we can assume that *the deflections of truss and cable at any point are equal.*

NOTATION.—Let the moving load per unit of length be m, and let this load extend over a distance a from the left end. Let R_1 be the reaction at the left end, *positive* when it acts upward, and M_1 be the moment at the left end. Any moment tending to cause compression in the upper flange is *positive*. Take O, Fig. 155, as the origin of co-ordinates. For any point distant x_1 from the left end, let y' be the ordinate to the *original* curve of cable x_1 is positive toward the right, and y' is positive when laid off above OO'. Let the deflection of the truss at the same point x_1 be y''; y'' is also positive when laid off *above* OO'. Let Y be the ordinate at the same point x_1 of the *new* curve of cable. Then at any point x_1 we have generally,

$$Y = y' + y'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Let x be any point of the truss on the right of x_1, and y be the corresponding deflec-

* The method of this discussion was first applied by *Charles Bender, C.E.*, Van Nostrand's *Engineering Magazine*, November, 1881. It will also be found, extended in treatment, in "Principles of Economy in the Design of Metallic Bridges" by the same author (Wiley & Sons).

tion. Let v, as always, be the versine of the original curve of cable, and r the height of tower, and l the half span.

MOMENT OF DISTRIBUTED CABLE LOAD.—Since the curve of the cable is originally a parabola, under the action of the entire dead load, which may be considered as uniform, its equation is easily found from (10) by transferring the origin from the centre to O,

$$y' = r - \frac{2vx_1}{l} + \frac{vx_1^2}{l^2}.$$

We have, then, from (27),

$$Y = r - \frac{2vx_1}{l} + \frac{vx_1^2}{l^2} + y''$$

for the ordinate to new curve of cable at any point x_1, y'' being negative when below horizontal.

The tangent of the angle of inclination with horizontal of the new curve of cable at any point x_1 is, then,

$$\frac{dY}{dx_1} = -\frac{2v}{l} + \frac{2vx_1}{l^2} + \frac{dy''}{dx_1} \quad \ldots \ldots \ldots \ldots \quad (28)$$

The new curve is an equilibrium curve. Its horizontal force at all points is, therefore, constant (page 179). Let this constant horizontal force, or pull of cable, be H.

Then $H\frac{dY}{dx_1}$ is the vertical force at any point. The differential of this will give the elementary load on cable at any point x_1,

$$H\frac{d^2Y}{dx_1^2}dx_1 = H\frac{2v}{l^2}dx_1 + H\frac{d^2y''}{dx_1^2}dx_1 \quad \ldots \ldots \ldots \quad (29)$$

The moment of this elementary load with reference to any point x on the right of x_1, is $H\frac{d^2Y}{dx_1^2}dx_1(x - x_1)$.

Therefore, $\int_{x_1=0}^{x_1=x} H\frac{d^2Y}{dx_1^2}dx_1(x - x_1)$ = moment of entire distributed load on cable between left end and any point x, with reference to x. We have, then, from (29),

$$\int_{x_1=0}^{x_1=x} H\frac{d^2Y}{dx_1^2}dx_1(x - x_1) = \int_{x_1=0}^{x_1=x} Hx\frac{d^2Y}{dx_1^2}dx_1 - \int_{x_1=0}^{x_1=x} H\frac{d^2Y}{dx_1^2}x_1 dx_1$$

$$= H\int_{x_1=0}^{x_1=x}\frac{2vx}{l^2}dx_1 + H\int_{x_1=0}^{x_1=x} x\frac{d^2y''}{dx_1^2}dx_1 - H\int_{x_1=0}^{x_1=x}\frac{2v}{l^2}x_1 dx_1 - H\int_{x_1=0}^{x_1=x}\frac{d^2y''}{dx_1^2}x_1 dx_1.$$

Performing the integrations, we have, since when $x_1 = 0$, $\frac{dy_0''}{dx_1} = 0$, if the truss is fixed horizontally at the ends,

$$\int_{x_1=0}^{x_1=x} H\frac{d^2Y}{dx_1^2}dx_1(x - x_1) = H\frac{2vx^2}{l^2} + H\,x\,\frac{dy}{dx} - H\frac{vx^2}{l^2} - H\int_{x_1=0}^{x_1=x}\frac{d^2y''}{dx_1^2}x_1 dx_1$$

$$= H \frac{vx^3}{l^2} + H x \frac{dy}{dx} - H \int_{x_1=0}^{x_1=x} \frac{d^2 y''}{dx_1^2} x_1 \, dx_1 . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Let us now consider the final integral in equation (30).

We have from the differential calculus,

$$d(xy) = x\,dy + y\,dx,$$

hence,

$$x\,dy = d(xy) - y\,dx.$$

Now let $x = x_1$ and $y = \frac{dy''}{dx_1}$. Then $dy = \frac{d^2 y''}{dx_1^2} dx_1$ and $x\,dy = \frac{d^2 y''}{dx_1^2} x_1\,dx_1$.

Hence $$\int_{x_1=0}^{x_1=x} \frac{d^2 y''}{dx_1^2} x_1\,dx_1 = H \int_{x_1=0}^{x_1=x} d\left[\frac{dy''}{dx_1} x_1\right] - H \int_{x_1=0}^{x_1=x} \frac{dy''}{dx_1} dx_1.$$

Performing the integration, and remembering that when $x_1 = 0$, $y'' = 0$, and $\frac{dy''}{dx_1} = 0$, we have

$$H \int_{x_1=0}^{x_1=x} \frac{d^2 y''}{dx_1^2} x_1\,dx = Hx\frac{dy}{dx} - Hy.$$

Substituting this in (30) we have,

$$\int_{x_1=0}^{x_1=x} H \frac{d^2 Y}{dx_1^2} dx_1 (x - x_1) = H \frac{vx^3}{l^2} + Hy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Equation (31) gives the moment of the entire distributed cable load between the left end and any point distant x from the left end, with reference to that point as a centre of moments.

Elastic Curve of the Truss.—From the theory of flexure we have, for the differential equation of the elastic curve, the moment at any point from $x = 0$ to $x = a$, or when $x < a$,

$$EI \frac{d^2 y}{dx^2} = R_1 x - \frac{mx^2}{2} + \frac{Hvx^3}{l^2} + Hy + M_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32a)$$

For any point from $x = 2l$ to $x = a$, or when $x > a$,

$$EI \frac{d^2 y_1}{dx^2} = R_1 (x - 2l) - ma(x - 2l) + \frac{Hv}{l^2}(x^3 - 4l^3) + Hy_1 + M_2 \quad . \quad . \quad (32b)$$

where $M_2 = 2R_1 l - ma\left(2l - \frac{a}{2}\right) + 4\,Hv + M_1$; where y_1 denotes the deflection at any point beyond the load.

If we differentiate equation (32a) twice, we have

$$EI\frac{d^3y}{dx^3} = R_1 - mx + \frac{2Hvx}{l^2} + H\frac{dy}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad (33a)$$

$$EI\frac{d^4y}{dx^4} = \frac{2Hv}{l^2} + H\frac{d^2y}{dx^2} - m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34a)$$

Equation (34*a*) can be put in a simpler shape. Thus let

$$\left.\begin{array}{l} H\tau^2 = EI \\ z = \dfrac{2v}{l^2} + \dfrac{d^2y}{dx^2} - \dfrac{m}{H} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35a)$$

Then

$$\frac{dz}{dx} = \frac{d^3y}{dx^3}, \quad \frac{d^2z}{dx} = \frac{d^4y}{dx^4}$$

and equation (34*a*) becomes

$$\frac{d^2z}{dx^2} = \frac{z}{\tau^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36a)$$

Integrating * between the limits $x = 0$ and x, we have

$$z = Ae^{\frac{x}{\tau}} - Be^{-\frac{x}{\tau}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37a)$$

* The integration is performed as follows:

$$\frac{d^2z}{dx^2} = \frac{z}{\tau^2}.$$

Multiply both sides by $2\,dz$, then

$$\frac{2\,dzd^2z}{dx^2} = \frac{2\,zdz}{\tau^2}.$$

Integrating,

$$\frac{dz^2}{dx^2} = \frac{z^2}{\tau^2} + c_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now in the present case, when $x = 0$, $\frac{dy}{dx} = 0$, and from (33*a*)

$$\frac{dz^2}{dx^2} = \left(\frac{d^3y}{dx^3}\right)^2 = \frac{R_1^2}{H^2\tau^4}.$$

Also, when $x = 0$, $y = 0$, and from (32*a*),

$$\frac{d^2y}{dx^2} = \frac{M_1}{H\tau^2}.$$

Hence for $x = 0$,

$$\frac{z^2}{\tau^2} = \frac{\left(\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right)^2}{\tau^2}.$$

Hence

$$c_1 = \frac{R_1^2}{H^2\tau^4} - \frac{\left(\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right)^2}{\tau^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where e is the base of the Naperian system of logarithms, or $e = 2.7182818$, and A and B are constants of integration to be determined by the special conditions of the case.

In the present case,

$$\left.\begin{array}{ll} A = \frac{1}{2}[K+N] & B = \frac{1}{2}[K-N] \\ K = \frac{R_1}{H\tau} & N = \frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2} \end{array}\right\} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (38a)$$

From $(37a)$ we have, after replacing z by its value in $(35a)$,

$$\frac{d^2y}{dx^2} = Ae^{\frac{x}{\tau}} - Be^{-\frac{x}{\tau}} - \frac{2v}{l^2} + \frac{m}{H} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (39a)$$

Differentiating $(39a)$ twice, we have,

$$\frac{d^3y}{dx^3} = \frac{A}{\tau}e^{\frac{x}{\tau}} + \frac{B}{\tau}e^{-\frac{x}{\tau}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (40a)$$

$$\frac{d^4y}{dx^4} = \frac{A}{\tau^2}e^{\frac{x}{\tau}} - \frac{B}{\tau^2}e^{-\frac{x}{\tau}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (41a)$$

From (1) we have

$$dx = \frac{dz}{\sqrt{\frac{z^2}{\tau^2} + c_1}}.$$

Integrate again and we have

$$x = \tau \log\left[\frac{z}{\tau} + \sqrt{\frac{z^2}{\tau^2} + c_1}\right] + c_2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (3)$$

In the present case, for $x = 0$, we have

$$c_2 = -\tau \log\left[\frac{\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}}{\tau} + \frac{R_1}{H\tau^2}\right] \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

From (3) we have

$$e^{\frac{x-c_2}{\tau}} - \frac{z}{\tau} = \sqrt{\frac{z^2}{\tau^2} + c_1}, \quad \text{or } z = \frac{\tau}{2}\left(e^{\frac{x-c_2}{\tau}} - c_1 e^{-\frac{x-c_2}{\tau}}\right)$$

Put $A = \frac{\tau}{2}e^{-\frac{c_2}{\tau}}$ and $B = \frac{\tau}{2}c_1 e^{\frac{c_2}{\tau}}$, and we have

$$z = Ae^{\frac{x}{\tau}} - Be^{-\frac{x}{\tau}}$$

which is the equation in the text.

From (4) we have

$$e^{-\frac{c_2}{\tau}} = \frac{R_1}{H\tau^2} + \frac{\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}}{\tau}$$

Hence

$$A = \frac{1}{2}\left[\frac{R_1}{H\tau} + \left(\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right)\right]$$

$$B = \frac{1}{2}\left[\frac{R_1}{H\tau} - \left(\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right)\right]$$

We have also, by integration, from (39*a*),

$$\frac{dy}{dx} = \tau A e^{\frac{x}{\tau}} + \tau B e^{-\frac{x}{\tau}} - \frac{2vx}{l^2} + \frac{mx}{H} + C \quad \ldots \ldots \quad (42a)$$

when $x = 0$, $\frac{dy}{dx} = 0$, and hence

$$C = -\tau (A + B) = -\frac{R_1}{H} \quad \ldots \ldots \quad (43a)$$

Also,

$$y = \tau^2 A e^{\frac{x}{\tau}} - \tau^2 B e^{-\frac{x}{\tau}} - \frac{vx^2}{l^2} + \frac{mx^2}{2H} + Cx + D \quad \ldots \ldots \quad (44a)$$

when $x = 0$, $y = 0$, and hence

$$D = -\tau^2(A - B) = -\tau_2 \left(\frac{2v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right) \quad \ldots \ldots \quad (45a)$$

The values of all the constants are thus given in terms of H, M, and R_1, which still remain to be found.

For that portion of the elastic curve beyond the load, equation (32*b*) holds good. Differentiate this twice and we have

$$EI\frac{d^3 y_1}{dx^3} = R_1 - ma + \frac{2Hvx}{l^2} + H\frac{dy_1}{dx} \quad \ldots \ldots \quad (33b)$$

$$EI\frac{d^4 y_1}{dx^4} = \frac{2Hv}{l^2} + H\frac{d^2 y_1}{dx^2} \quad \ldots \ldots \quad (34b)$$

Equation (34*b*) can be put in a simpler shape. Thus, let as before,

$$\left.\begin{aligned} H\tau^2 &= EI \\ \text{and} \qquad z_1 &= \frac{2v}{l^2} + \frac{d^2 y_1}{dx^2} \end{aligned}\right\} \quad \ldots \ldots \quad (35b)$$

Then equation (34 *b*) becomes

$$\frac{d^2 z_1}{dx^2} = \frac{z_1}{\tau^2} \quad \ldots \ldots \quad (36b)$$

Integrating between the limits $x = 2l$ and x, we have

$$z_1 = \alpha e^{\frac{x-2l}{\tau}} - \beta e^{-\frac{x-2l}{\tau}} \quad \ldots \ldots \quad (37b)$$

where, as before, e is the base of the Naperian system of logarithms, and α and β are constants of integration, to be determined by the special conditions of the case.

In the present case,

$$\left.\begin{aligned}\alpha &= \frac{\tau}{2}(\eta + \phi) \qquad \beta = \frac{\tau}{2}(\eta - \phi)\\ \eta &= \frac{R_1 - ma + \dfrac{4\,Hv}{l}}{H\tau^2}\\ \phi &= \frac{2\,v}{\tau l^2} + \frac{2\,R_1\,l - ma\left(2\,l - \dfrac{a}{2}\right) + 4\,Hv + M_1}{H\tau^3}\end{aligned}\right\} \quad . \; . \; . \; . \quad (38b)$$

From $(37b)$ we have, after replacing z by its value in $(35b)$,

$$\frac{d^2 y_1}{dx^2} = \alpha e^{\frac{x-2l}{\tau}} - \beta e^{-\frac{x-2l}{\tau}} - \frac{2\,v}{l^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (39b)$$

From $(39b)$ by differentiating twice, we have,

$$\frac{d^3 y_1}{dx^3} = \frac{\alpha}{\tau} e^{\frac{x-2l}{\tau}} + \frac{\beta}{\tau} e^{-\frac{x-2l}{\tau}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (40b)$$

$$\frac{d^4 y_1}{dx^4} = \frac{\alpha}{\tau^2} e^{\frac{x-2l}{\tau}} - \frac{\beta}{\tau^2} e^{-\frac{x-2l}{\tau}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (41b)$$

We have also from $(39b)$ by integration between the limits $2l$ and x,

$$\frac{dy_1}{dx} = \tau\alpha e^{\frac{x-2l}{\tau}} + \tau\beta e^{-\frac{x-2l}{\tau}} - \frac{2v}{l^2}(x - 2\,l) + \gamma \quad . \; . \; . \; . \; . \; . \quad (42b)$$

when $x = 2l$, $\dfrac{dy_1}{dx} = 0$, and

$$\gamma = -\tau(\alpha + \beta) = -\tau^2\eta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (43b)$$

Integrating again,

$$y_1 = \tau^2\alpha e^{\frac{x-2l}{\tau}} - \tau^2\beta e^{-\frac{x-2l}{\tau}} - \frac{v}{l^2}(x - 2\,l)^2 + \gamma(x - 2\,l) + \delta \quad . \; . \; . \; . \quad (44b)$$

when $x = 2\,l$, $y_1 = 0$, and

$$\delta = -\tau^2(\alpha - \beta) = -\tau^3\phi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (45b)$$

Equation $(44a)$ gives the elastic curve of that portion of the truss covered by the load, and $(44b)$ of all that portion beyond the load.

All the constants, A, B, C, D, and α, β, γ and δ, are given in terms of R_1, M_1 and H. It remains to determine these quantities.

VALUE OF H, R_1 and M_1.—At the point where $x = a$, the two portions of the elastic curve right and left have the same deflection and a common tangent. Also the moment and shear at this point are the same for both curves. Hence for $x = a$, equations $(44b)$ and $(44a)$, $(42b)$ and $(42a)$, $(40b)$ and $(40a)$, $(41b)$ and $(41a)$ are simultaneous. We have, then, the following equations of condition:

$$\tau^2 Ae^{\frac{a}{\tau}} - \tau^2 Be^{-\frac{a}{\tau}} - \frac{va^2}{l^2} + \frac{ma^2}{2H} + Ca + D = \tau^2 \alpha e^{\frac{a-2l}{\tau}} - \tau^2 \beta e^{-\frac{a-2l}{\tau}} - \frac{v}{l^2}(a-2l)^2$$
$$+ \gamma(a-2l) + \delta \quad . \quad . \quad . \quad . \quad (a)$$

$$\tau Ae^{\frac{a}{\tau}} + \tau Be^{-\frac{a}{\tau}} - \frac{2\,va}{l^2} + \frac{ma}{H} + C = \tau\alpha e^{\frac{a-2l}{\tau}} + \tau\beta e^{-\frac{a-2l}{\tau}} - \frac{2\,v}{l^2}(a-2\,l) + \gamma \quad . \quad . \quad (b)$$

$$Ae^{\frac{a}{\tau}} - Be^{-\frac{a}{\tau}} + \frac{m}{H} = \alpha e^{\frac{a-2l}{\tau}} - \beta e^{-\frac{a-2l}{\tau}} \quad . \quad . \quad . \quad . \quad (c)$$

$$\frac{A}{\tau}e^{\frac{a}{\tau}} + \frac{B}{\tau}e^{-\frac{a}{\tau}} = \frac{\alpha}{\tau}e^{\frac{a-2l}{\tau}} + \frac{\beta}{\tau}e^{-\frac{a-2l}{\tau}} \quad . \quad . \quad . \quad . \quad (d)$$

Combining (c) and (d) by addition and subtraction, we obtain

$$\left.\begin{aligned} 2\,Ae^{\frac{a}{\tau}} - 2\,\alpha e^{\frac{a-2l}{\tau}} &= -\frac{m}{H} \\ 2\,\beta e^{-\frac{a-2l}{\tau}} - 2\,Be^{-\frac{a}{\tau}} &= -\frac{m}{H} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (\text{XVI.})$$

Combining (a) and (b) in the same way, we have, after substituting the values of C, D, γ and δ, from (43a), (45a), (43b) and (45b), and reducing, precisely the same two equations as (XVI.). In like manner the combination of (b) and (c) gives us precisely the same two equations. So also for the combination of (a) and (d).

Our four equations of condition give us, then, in reality only two equations containing R_1, M_1, and H or τ.

It is necessary to find a third equation. This we can easily do by assuming, in accordance with the ordinary theory, that the curve of the cable remains parabolic even for partial loading. Although this assumption is not theoretically correct, still, in practice, the real curve differs so little from the parabolic that, *so far as the cable alone is concerned*, the value of H thus found is very exactly the real value of H.

We have, then,

$$H = \frac{kml^2}{2\,v} \quad . \quad . \quad . \quad . \quad . \quad (\text{XVII.})$$

where we have already found for k the values:

when $a < l$,
$$k = \frac{a^3(2\,l - a)}{2\,l^4\left[1 + \frac{9\,E_2 F_2 h^2}{4\,E_1 F_0 v^2}\right]} \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII.})$$

when $a > l$,
$$k = \frac{2\,l_4 - a\,(2\,l - a)^3}{2\,l^4\left[1 + \frac{9\,E_2 F_2 h^2}{4\,E_1 F_0 v^2}\right]} \quad . \quad . \quad . \quad . \quad . \quad (\text{VIII.}a)$$

If we put for I_2 its value, viz.: $I_2 = F_2 \frac{h^2}{4}$, we have

$$\tau = \sqrt{\frac{E_2 F_2 h^2 v}{2\, kml^2}} \quad \text{(XVIII.)}$$

These values of τ and H, inserted in equations (XVI.), will give us two equations containing only R_1 and M_1. We can, therefore, find these quantities. Thus the values of H, τ, R_1 and M_1, can be easily determined for any given value of a.

TEMPERATURE LOAD.—The effect of a rise of temperature of t^0 above the mean, is to load the truss with a uniform downward load, or "hot load" of q per unit of length. A fall of temperature of t^0 below the mean, causes an equal uniformly distributed upward load, or "cold load." In either case we have already found

$$q = \frac{2\, \varepsilon t\, E_1 F_0 v}{l^2 \left[1 + \frac{9}{4} \frac{E_1 F_0 v^2}{E_2 F_2 h^2} \right]} \quad \text{(VI.)}$$

The moment, then, at any point of the truss due to the temperature load is (Appendix, page 263),

$$M_x = qlx - \frac{qx^2}{2} - \frac{ql^2}{3} \quad \text{(XIX.)}$$

and the shear at any point is

$$S_x = ql - qx \quad \text{(XX.)}$$

The strains due to each case of temperature load must be combined with those due to live load, so as to give the greatest possible strains.

RECAPITULATION OF FORMULÆ NECESSARY FOR COMPLETE CALCULATION OF TRUSS.—For convenience of reference we group here all the formulæ thus far deduced, which are requisite for the calculation of the truss, in the order in which they must be used.

$$\text{approx. } F_0 = \frac{(p+m)\, l^2}{2 v f_1} \left(1 + \frac{2 v^2}{l^2} \right) \quad \text{(IX.)}$$

$$\text{approx. } F_1 = \frac{(p+m)\, l^2}{2 v f_1} \quad \text{(IX.}a\text{)}$$

$$\text{approx. } q = \frac{2\, \varepsilon t\, E_1 F_0 v}{l^2 \left(1 + \frac{4}{9} \frac{E_1 v^2}{E_2 h^2} \right)} \quad (24)$$

$$\text{approx. } n = \frac{1}{1 + \frac{9}{4} \frac{E_2 h^2}{E_1 v^2}} \quad (23)$$

$$\text{approx. } F_2 = \frac{[(1-n)\, m + q]\, l^2}{3\, h f_2} \quad \text{(X.)}$$

These approximate values being found, the accurate values are

$$n = \frac{1}{1 + \frac{9}{4} \frac{E_2 F_2 h^2}{E_1 F_0 v^2}} \quad \text{(VII.)}$$

$$q = \frac{2\,\varepsilon t\,E_1 F_0 v}{l^2\left[1 + \frac{4\,E_1 F_0 v^2}{9\,E_2 F_2 h^2}\right]} \quad \ldots\ldots \quad \text{(VI.)}$$

$$F_1 = \frac{(p + nm + q)\,l^2}{2\,v f_1} \quad \ldots\ldots \quad \text{(XV.)}$$

$$F_0 = \frac{(p + nm + q)\,l^2}{2\,v f_1}\left[1 + \frac{2\,v^2}{l^2}\right] \quad \ldots\ldots \quad \text{(XIV.)}$$

For varying cross-section of cable, put F_1 in place of F_0, in (VII.) and (VI.)
We can now find

$$\tau = \sqrt{\frac{E_2 F_2 h^2}{2\,kml^2}} \quad \ldots\ldots \quad \text{(XVIII.)}$$

where k is given by,
when $a < l$,

$$k = \frac{a^2 (2\,l - a)}{2\,l^4\left[1 + \frac{9\,E_2 F_2 h^2}{4\,E_1 F_0 v^2}\right]} \quad \ldots\ldots \quad \text{(VIII.)}$$

when $a > l$,

$$k = \frac{2\,l^4 - a(2\,l - a)^3}{2\,l^4\left[1 + \frac{9\,E_2 F_2 h^2}{4\,E_1 F_0 v^2}\right]} \quad \ldots\ldots \quad \text{(VIII.}a\text{)}$$

Also,

$$H = \frac{kml^2}{2\,v} \quad \ldots\ldots \quad \text{(XVII.)}$$

In these values for k, put F_1 in place of F_0, for varying cross-section of cable.
Knowing now τ and H, we can substitute their values in

$$\left.\begin{aligned} 2\,Ae^{\frac{a}{\tau}} - 2\,\alpha e^{\frac{a-2l}{\tau}} &= -\frac{m}{H} \\ 2\,\beta e^{-\frac{a-2l}{\tau}} - 2\,Be^{-\frac{a}{\tau}} &= -\frac{m}{H} \end{aligned}\right\} \quad \ldots\ldots \quad \text{(XVI.)}$$

where

$$\left.\begin{aligned} A &= \frac{1}{2}\left[\frac{R_1}{H\tau} + \left(\frac{2\,v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right)\right] \\ B &= \frac{1}{2}\left[\frac{R_1}{H\tau} - \left(\frac{2\,v}{l^2} - \frac{m}{H} + \frac{M_1}{H\tau^2}\right)\right] \end{aligned}\right\} \quad \ldots\ldots \quad (38a)$$

and

$$\left.\begin{aligned} \alpha &= \frac{\tau}{2}(\eta + \varphi) \qquad \beta = \frac{\tau}{2}(\eta - \varphi) \\ \eta &= \frac{R_1 - ma + \frac{4\,Hv}{l}}{H\tau^3} \\ \varphi &= \frac{2v}{\tau\,l^2} + \frac{2\,R_1 l - ma\left(2\,l - \frac{a}{2}\right) + 4\,Hv + M_1}{H\tau^3} \end{aligned}\right\} \quad \ldots\ldots \quad (38b)$$

R_1, M_1, H and τ being thus found, we can find the deflection at any point from the following equations: when $x < a$,

$$y = \tau^2 A e^{\frac{x}{\tau}} - \tau^2 B e^{-\frac{x}{\tau}} - \frac{vx^2}{l^2} + \frac{mx^2}{2H} + Cx + D \quad \text{. . . . .} \quad (44a)$$

where

$$C = -\frac{R_1}{H}, \qquad D = -\tau^2 (A - B)$$

when $x > a$,

$$y_1 = \tau^2 \alpha e^{\frac{x-2l}{\tau}} - \tau^2 \beta e^{-\frac{x-2l}{\tau}} - \frac{v}{l^2}(x - 2l)^2 + \gamma(x - 2l) + \delta \quad \text{. . . . . .} \quad (44b)$$

where

$$\gamma = -\tau^2 \eta \qquad \delta = -\tau^2 \varphi.$$

Having thus found y, we can find the moment at any point from the following equations: when $x < a$,

$$M_x = R_1 x - \frac{mx^2}{2} + \frac{Hvx^2}{l^2} + Hy + M_1 \quad \text{. . . . .} \quad (32a)$$

when $x > a$,

$$M_x = R_1 x - ma\left(x - \frac{a}{2}\right) + \frac{Hvx^2}{l^2} + Hy_1 + M_1 \quad \text{. . . . .} \quad (32b)$$

Finally, the shear at any point is given by the following equations: when $x < a$,

$$S_x = R_1 - mx + \frac{2Hvx}{l^2} + H\frac{dy}{dx} \quad \text{. . . . .} \quad (33a)$$

when $x > a$,

$$S_x = R_1 - ma + \frac{2Hvx}{l^2} + H\frac{dy_1}{dx} \quad \text{. . . . .} \quad (33b)$$

where

$$\frac{dy}{dx} = \tau A e^{\frac{x}{\tau}} + \tau B e^{-\frac{x}{\tau}} - \frac{2vx}{l^2} + \frac{mx}{H} + C \quad \text{. . . . .} \quad (42a)$$

$$\frac{dy_1}{dx} = \tau \alpha e^{\frac{x-2l}{\tau}} + \tau \beta e^{-\frac{x-2l}{\tau}} - \frac{2v}{l^2}(x - 2l) + \gamma \quad \text{. . . . .} \quad (42b)$$

For temperature load we have

$$q = \frac{2\varepsilon t E_1 F_0 v}{l^2\left[1 + \frac{9 E_1 F_0 v^2}{4 E_2 F_2 h^2}\right]} \quad \text{. . . . .} \quad (\text{VI.})$$

where for varying cross-section of cable we put F_1 in place of F_0.

The moment at any point is, then,

$$M_x = qlx - \frac{qx^2}{2} - \frac{ql^2}{3} \quad \text{. . . . .} \quad (\text{XIX.})$$

and the shear is

$$S_x = ql - qx \quad \text{. . . . .} \quad (\text{XX.})$$

For a single concentrated load P, at a distance a from the left end, we have only to put $\frac{m}{H} = 0$ in equations XVI. and (38 a); $ma = P$ and $ma\left(2l - \frac{a}{2}\right) = 2Pl$ in equations (38 b); $\frac{mx^2}{2H} = 0$ and $\frac{mx^2}{2} = 0$ in equations (44 a) and (32 a); $ma\left(x - \frac{a}{2}\right) = P(x - a)$ in (32 b); $mx = 0$ in (33 a); $ma = P$ in (33 b); $\frac{mx}{H} = 0$ in (42 a).

CALCULATION OF STRAINS.—These are all the formulæ necessary for finding the strains in the truss.

Thus we can first find the temperature strains, from (VI.), (XX.) and (XXI.). Thus, the moment at the centre of any flange divided by the depth of truss, gives the strain in the flange. A negative moment denotes tension in the upper flange. A positive moment compression.

The shear at any apex multiplied by the secant of the angle which the brace makes with the vertical, gives the strain in the brace. A positive shear acts upward at the left end of the brace. A negative shear acts downward at the left end of the brace.

The strains thus found for temperature load may be of either kind for each piece, according as the "cold load" or "hot load" acts, but in each case are equal in amount.

For the moving load it will be sufficient to take a for a few points of the truss; thus $a = \frac{1}{8}l, \frac{2}{8}l, \frac{3}{8}l$, etc. For each value of a find the moments at a few points, such as $x = 0$, $\frac{1}{8}l, \frac{2}{8}l$, etc. Also the shears. We can then plot these moments and shears to scale. Thus in Fig. 156 we can plot the curves of moments for $a = \frac{1}{8}l, \frac{2}{8}l$, etc. Then at any point the maximum moment will be given by the greatest ordinate at that point to any one of these curves. The curve enveloping them all will, therefore, give the maximum moments. Thus in Fig. 156, the greatest positive moment at any point in the right hand half of truss, is given by the ordinates to the upper enveloping curve on *right*. The greatest negative moments by ordinates to lower enveloping curve on *right*. The strains in each half are of course the same.

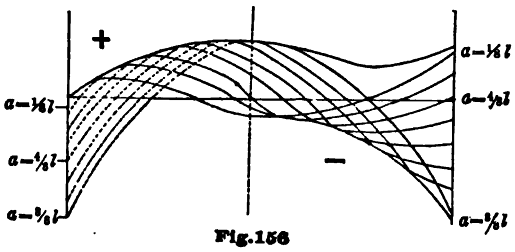


Fig. 156

We may treat the shears in similar manner. The maximum moments and shears being found, we can find the corresponding strains as detailed for temperature load.

SUPPLEMENT TO CHAPTER IX.

COMPOSITE STRUCTURES. SUSPENSION SYSTEM WITH STIFFENING TRUSS.

IN the preceding Chapter we have given what we conceive to be a more accurate theory of the suspension system than that usually accepted. We shall give here a discussion based upon the generally received theory, *viz.*, that the truss distributes any partial load *so as to make it take effect upon the cable as a uniform load.*

DEFLECTION OF STAYS.—Let s be the length of a stay, Fig. 5, page 207, v its vertical projection and z the distance from foot of stay to foot of tower. Then, as we have already seen from equation (1) page 176, of the preceding Chapter, the deflection δ_s of the stay is

$$\delta_s = \lambda_s \frac{s^2}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the stay supports a load P, the strain in the stay is $P \sec \theta = P \frac{s}{v}$. This strain produces the elongation

$$\lambda_s = \frac{Ps}{vF_sE_s} \cdot$$

Hence the deflection is

$$\delta_s = \frac{Ps^3}{F_sE_sv^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The weight P produces also a horizontal strain in the flange where it is attached equal to $P \tan \theta = \frac{Pz}{v}$.

DIFFERENT KINDS OF STIFFENED SUSPENSION.—We are now ready to proceed to our discussion of the stiffened suspension system. We may distinguish three kinds, viz: 1st, when the stiffening truss merely rests upon the piers and abutments; 2d, when the ends are simply tied or anchored down; 3d, when the truss passes continuously over the piers, or is anchored down firmly at several continuous points over each pier. In this latter case the girder may be regarded as fixed horizontally at the piers. In the first case there can be no negative reactions, and the action of a heavy partial load may be to lift the girder entirely off the supports at one end. As this ought never to occur, the truss, if necessary, should be tied down. The first case, therefore, never occurs, or ought never to occur in practice, and we have only two cases to consider, *viz.*, that in which the girder is simply tied down, and that in which it is fixed horizontally at each pier. In the latter case, the truss should not be considered or constructed as continuous over the piers, since nothing is gained by such continuity. It is necessary and sufficient that the ends be so bolted down that they may be considered as rigidly fixed in a horizontal direction. The trusses, should, therefore, be disconnected over the piers, or at least the bracing should not be continuous, and strains in one span do not pass over into an

adjacent one, as in the case of a continuous girder. The girders are thus practically *fixed* horizontally at the ends and are not continuous as to the bracing.

OFFICE OF THE STIFFENING TRUSS.—The truss acts, of course, as a supporting member, but its principal object is to prevent change of shape of the cables under the action of a partial load. Any single load, therefore, placed anywhere in the span, is distributed over the cable and acts as a uniformly distributed load upon it. The curve of the cable is thus always a parabola, whose versine varies according to the load and temperature.

A partial load on the truss, therefore, causes at all points of the cable a uniform load per unit of horizontal projection. At all points of the truss not acted upon by the load there is a uniform *upward* load, due to the reaction of the cable upon the truss.

Thus if, as in Fig. 1, we have a partial load $p\ (z_2 - z_1)$, a portion of this load acts as shown in Fig. 2, as a uniform load upon the cable. This same load acts as a uniform upward load upon the truss, as shown in Fig. 3. The deflection of the truss, is therefore equal to the deflection which would be caused by the load $p\ (z_2 - z_1)$ alone, as shown in Fig. 4, *minus* the upward deflection due to the upward load of Fig. 3. This holds good whether the girder is simply tied down at the ends, or is fixed horizontally at the ends.

EFFECT OF LOAD IN SIDE SPANS UPON THE CENTRE SPAN.—The effect of a load in each side span, the centre span being empty, is to cause a horizontal pull at the top of the towers. This pull

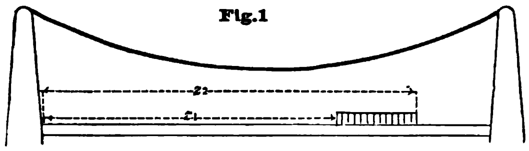


Fig. 1

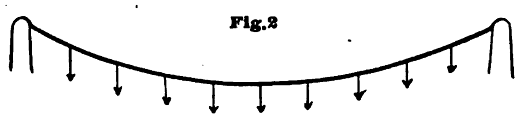


Fig. 2

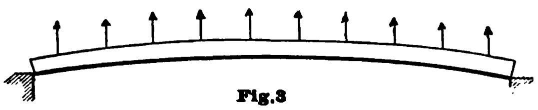


Fig. 3

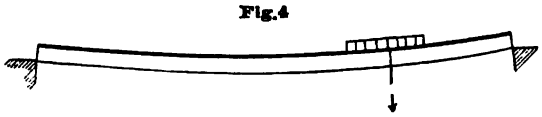


Fig. 4

is equivalent to a uniform load upon the centre cables. This load upon the cables acts also as an upward load on the centre truss. This load can be found, provided that the horizontal pull H is known. This horizontal pull is easily found. Thus taking moments about the centre,

$$Hv = nml_1 \times \frac{l_1}{2} = \frac{nml_1^2}{2}$$

or

$$H = \frac{nml_1^2}{2v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where m is the load per unit of horizontal length in the side spans, n is the portion of that load carried by the cable, and l_1 is the length of the side spans.

We have, then, the load of the centre cable, or the upward load on the centre truss,

$$pl = \frac{2\ vH}{l} = \frac{nml_1^2}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where l is the *half* span of the centre cables and p is the unit load of the centre span caused by the full live load m in the side spans.

This load of pl being found from equation (4) we can easily calculate the strains it causes in the centre span of the cable and of the truss. These strains are to be added to the strains due to loads in the centre span itself, in such a way as to give the greatest strains which can ever occur.

CABLE, STAYS AND TOWERS.—It is customary to insert stays, as shown in Fig. 5, page 207, in order to relieve the cable, stiffen the structure and give additional security to the towers.

The truss is only called into action by the live load. Its office is chiefly to stiffen by preventing change of shape of the cable. It adds nothing whatever to the supporting power of the cables, though it does add to the supporting power of the entire system, since it supports its share of the live load also.

The cable supports, then, the entire dead load and also its share of the live load. The truss supports its share of the live load only, and stiffens the structure under the action of partial loads.

The stays are called into action by local live loads. Each stay may cause compression in the flange to which it is attached. This compression should not be taken by the flanges, but by special compression members bolted to the truss and independent of the truss bracing. The flanges of the truss are thus unaffected by the stays.

The cable passes, generally, over rollers on the top of the towers, so that, neglecting friction, the resultant pressure is vertical. Even if friction is not neglected, or even if there are no rollers, the stays will furnish ample security.

In all cases, the truss should be constructed with a *sliding joint* in the top and bottom flanges at the centre, so as to allow of expansion and contraction under changes of temperature. This joint should not break the continuity of the flanges, like a hinge, or be equivalent to cutting the truss in two. Thus although the girder is free to expand and contract, the flanges still furnish resistance to bending at the centre, as though the truss were not cut.

STRAINS INDETERMINATE WHEN THE THREE SYSTEMS ARE COMBINED.—So far as the dead load is concerned, the cable supports it all, and there are no strains in the truss or stays.

With regard to the live load, there is a necessary indeterminance. Thus a partial live load causes strains in both cable, truss and stays. These strains we could easily find theoretically from the principles and by the method already indicated on page 176. But it would be practically impossible to adjust so many independent pieces so that they shall all act together as theory demands. Even if it were possible, imperfections of workmanship, the working loose of joints, the stretch and set under working loads, changes of temperature, etc., would perpetually cause disturbance.

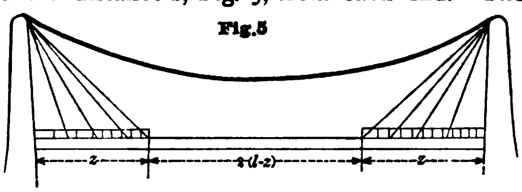
We adopt, therefore, the principle of considering each two systems as though the third did not act. We thus allow for all possible variations in the distribution of strain between the different systems.

Thus for the live-load strain in the cable we suppose the live load to cover the whole span, and consider it entirely sustained by the cable and truss alone. We thus ignore the action of the stays, which in reality help the cable. The cable, then, supports the entire dead load and also its portion of the full live load.

For the truss, we consider each apex live load by itself, and form a table for the maximum strains. We consider each apex live load as carried by the cable and truss alone. This is true for that portion of the truss where the stays do not extend. For other portions the action of the stays is ignored and the truss strains thus obtained are, therefore, in excess of the actual.

For the stays, we suppose the live load to extend the distance z, Fig. 5, from each end. The distance z is equal to the distance out to which the stays extend. We have thus the load mz at each end. We find the deflection at any point of the truss where a stay is attached, due to this loading, when only the truss and cable act. This deflection is greater than the actual deflection, because the stays in reality act also. If then we put the deflection of the stay equal to this deflection, we obtain a greater strain than the actual. This is compensated in some degree by not taking into account the deflection at the same points due to the live load in the portion $2 (l - z)$, Fig. 5, which evidently also causes strains in the stays.

Fig. 5



There is no practical difficulty in securing the distribution of strains assumed. Thus, in the process of erection, the cable is first put in position. Then the truss is built out from each end. The cable thus takes the entire dead load, if the suspenders are properly screwed up. Lastly, the

stays are attached and brought to a bearing. They will, therefore, only be called into action by the live load. The same holds true of the truss.

In accordance with these views, we shall discuss the strains in the combination.

Best Ratio of Height of Truss to Versine of Cable—We have already found in the preceding Chapter, for this ratio, when the girder is fixed horizontally at the ends,

$$\frac{h}{v} = \frac{2\lambda_2}{3(\lambda_1 + \varepsilon t)} \quad \ldots\ldots\ldots\ldots \quad (5)$$

If the girder is not fixed horizontally, but simply tied down at the ends, if necessary, we find in precisely similar manner

$$\frac{h}{v} = \frac{20\lambda_2}{18(\lambda_1 + \varepsilon t)} \quad \ldots\ldots\ldots\ldots \quad (6)$$

Strains Caused by Change of Temperature.—We have already found in the preceding Chapter, the unit "cold load" or "hot load," for girder fixed horizontally at the ends.

$$q = \frac{2\,\varepsilon t\,E_1 F_0 v}{l^2\left(1 + \frac{4}{9}\frac{E_1 F_0 v^2}{E_2 F_2 h^2}\right)} \quad \ldots\ldots\ldots\ldots \quad (7)$$

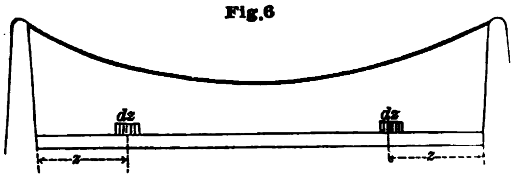
If the truss is not fixed horizontally at the ends, but is simply tied down if necessary, we find in precisely similar manner

$$q = \frac{2\,\varepsilon t\,E_1 F_0 v}{l^2\left(1 + \frac{20}{9}\frac{E_1 F_0 v^2}{E_2 F_2 h^2}\right)} \quad \ldots\ldots\ldots\ldots \quad (8)$$

In these formulæ F_0 is the cross section at ends. If the cable varies in cross section according to the strain, we must put for F_0, the value of F_1 or the cross section at the centre.

Partial Load—Cable and Truss Acting.—Suppose at any distance z from each end, Fig. 6, the elementary load mdz. These weights cause a uniform load over the cable and a uniform *upward* load over the truss.

Fig. 6



Let the uniform load per unit of horizontal projection which the weights cause on the cable be $2\,nmdz$, where nm is the unit load due to each partial load mdz. Then, since the two loads are symmetrically placed with respect to the centre, the greatest deflection of both truss and cable will be at the middle.

We have from the preceding Chapter, for the deflection of the cable at the centre,

$$\Delta_1 = \frac{6\,nmdzl^4}{8\,v^2 E_1 F_1}.$$

The deflection of the truss is equal to the deflection which would be caused by the two weights acting alone, *minus* the upward deflection due to the reaction of the cable.

For the first deflection we have, from the theory of flexure, page 272, when $I_2 = F_2\frac{h^2}{4}$,

$$\frac{mdz}{3\,E_2 F_2 h^2}\left[3\,lz^2 - 2\,z^3\right],$$

and for the second, page 272,

$$\frac{2\,nmdzl^4}{6\,E_2F_2h^2}.$$

The actual deflection of the truss at centre is then

$$\Delta_2 = \frac{mdz}{3\,E_2F_2h^2}\left[3\,lz^2 - 2\,z^3\right] - \frac{nmdzl^4}{6\,E_2F_2h^2}.$$

Equating these two deflections, we have

$$ndz = \frac{dz\,(3\,lz^2 - 2\,z^3)}{l^4\left[1 + \dfrac{9\,E_2F_2h^2}{4\,E_1F_1v^2}\right]},$$

where z is the distance from each end to the nearest load. Integrating between the limits z_2 and z_1 on one side, and z_2 and z_1 on the other, we have for the fraction of the whole load m per unit of length, carried by the cable,*

$$n_{1-2}\,(z_2 - z_1) = \frac{2\,l\,(z_2^3 - z_1^3) - (z_2^4 - z_1^4)}{2\,l^4\left(1 + \dfrac{9\,E_2\,F_2\,h^2}{4\,E_1\,F_1\,v^2}\right)} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

If the truss is simply supported at the ends, we have, in similar manner from (32) and (20),

$$n_{1-2}\,(z_2 - z_1) = \frac{6\,l^2\,(z_2^2 - z_1^2) - (z_2^4 - z_1^4)}{10\,l^4\left(1 + \dfrac{9\,E_2\,F_2\,h^2}{20\,E_1F_1\,v^2}\right)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

where F_1 and F_2 are the cross sections at centre of cable and truss. For a uniform load over the whole span we have, neglecting the reaction of the stays, from (9), by making $z_1 = 0$ and $z_2 = l$,

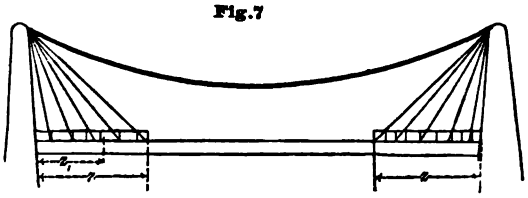
$$n_{1-2} = \frac{1}{1 + \dfrac{9\,E_2\,F_2\,h^2}{4\,E_1\,F_1v^2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

and from (10)

$$n_{1-2} = \frac{1}{1 + \dfrac{9\,E_2\,F_2\,h^2}{20\,E_1\,F_1\,v^2}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

PARTIAL LOAD—STAYS AND CABLE ACTING.—Suppose we have a uniform load m per unit of length extending over a distance of z from each end, Fig. 7. Let the distance z be equal to the farthest distance out to which the stays extend, and let the uniform load on the cable due to the two loads mz be nm per unit of length. From (IV.) of preceding Chapter, we have, for the deflection of the cable at any point,

Fig. 7



* n denotes always the portion of load carried by the cable. 1, 2 and 3 as subscripts, refer to cable, truss and stay respectively. Hence n_{1-2} is that fraction of the load carried by the cable when cable and truss act together and the stays are neglected.

$$\delta_1 = \frac{3}{8} \frac{nml^2}{E_1 F_1 v^2} (l^2 - x^2),$$

where x is the distance of any point from the centre. If z_1 is the distance of any point from the left end, and z_2 the distance of the same point from the right end, this becomes

$$\delta_1 = \frac{3}{8} \frac{nml^2 z_1 z_2}{E_1 F_1 v^2} \quad . \quad . \quad . \quad (13)$$

Now if the stays did not act, the value of n in this equation would be given by (9) or (10) according as the truss is fixed at the ends or simply supported at the ends. The deflection of a stay must always be the same as that of the truss at the point where the stay is attached. But if the stays do act, the actual deflection at any point is less than that given by (13). It can never be greater. If, then, we assume the deflection of a stay as equal to that given by (13) we shall get strains in excess of the actual.

The deflection of a stay is from (2)

$$\delta_2 = \frac{Ps^3}{F_3 E_3 r^2} \quad . \quad . \quad . \quad (14)$$

where s is the length of stay, and r is its vertical projection, and P is the load carried by the stay.

Equating (13) and (14) we obtain for the load P carried by any stay, whose point of attachment is distant z_1 from the left end, and z_2 from the right end of span,

$$P = \frac{3n_{1-2}\, m\, E_3 F_3 r^2 l^2 z_1 z_2}{8\, E_1 F_1 s^3 v^2} \quad . \quad . \quad . \quad (15)$$

where n_{1-2} is given by equations (9) or (10) by making in them $z_1 = 0$, and $z_2 = z$. Thus, for the truss fixed horizontally at the ends,

$$n_{1-2} = \frac{2lz^3 - z^4}{l^4 \left(1 + \frac{9}{4} \frac{E_2 F_2 h^2}{E_1 F_1 v^2}\right)} \quad . \quad . \quad . \quad (16)$$

and for truss supported at the ends,

$$n_{1-2} = \frac{6\, l^2 z^2 - z^4}{5l^4 \left(1 + \frac{9}{20} \frac{E_2 F_2 h^2}{E_1 F_1 v^2}\right)} \quad . \quad . \quad . \quad (17)$$

These values substituted in (15) will give the load carried by any stay.

Dead load causes no strain in the stays.

Concentrated Load—Cable and Truss Acting.—Suppose we have two concentrated loads P, P, acting each at the distance z_1 from each end. Since the loads are symmetrical with respect to the centre, the deflection of both truss and cable is greatest at the centre.

The deflection of the cable at the centre is, from theory of flexure, page 183,

$$\Delta_1 = \frac{3}{8} \frac{NPl^4}{E_1 F_1 v^2} \quad . \quad . \quad . \quad (18)$$

where $2NP$ is the portion of the total load $2P$ which takes effect upon the cable as a uniform load.

The deflection of the truss is equal to the deflection which would be caused by the two weights alone, minus the deflection due to the uniform upward load of the truss. The first deflection is from theory of flexure, page 272, putting $I_2 = F_2 \frac{h^2}{4}$,

$$\frac{P}{3\,E_2 F_2 h^2}\left[3z_1^2 l - 2z_1^3\right]$$

for a beam fixed horizontally at the ends.

The second deflection is, from theory of flexure, page 272,

$$\frac{N P l^3}{6\,E_2 F_2 h^2}$$

The deflection of the truss at the centre is, therefore,

$$\Delta_2 = \frac{P}{3\,E_2 F_2 h^2}\left[3z_1^2 l - 2z_1^3\right] - \frac{N P l^3}{6\,E_2 F_2 h^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Equating (18) and (19), we have,

$$N_{1-2} = \frac{2\left[3z_1^2 l - 2z_1^3\right]}{l^3\left[1 + \dfrac{9\,E_2 F_2 h^2}{4\,E_1 F_1 v^2}\right]}$$

Evidently each of the loads P must produce the same effect upon the cable. When only one of them acts, therefore, the portion of P which acts as a uniform load over the cable, is, per unit of length,*

$$N_{1-2} = \frac{\left[3z_1^2 l - 2z_1^3\right]}{l^4\left[1 + \dfrac{9\,E_2 F_2 h^2}{4\,E_1 F_1 v^2}\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

If the truss is simply supported at the ends, we have, in similar manner,

$$N_{1-2} = \frac{2\left[3l^2 z_1 - z_1^3\right]}{5\,l^4\left[1 + \dfrac{9\,E_2 F_2 h^2}{20\,E_1 F_1 v^2}\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

These equations will give the portion of any concentrated load carried by the cable, when cable and truss are alone supposed to act, and the action of the stays is ignored. The distance z_1 is always the distance to the *nearest* end.

APPROXIMATE VALUES OF F_0, F_1 AND F_2.—The use of our equations, thus far deduced, requires that F_0, F_1 and F_2, the cross sections at end and centre of cable and centre of truss, should be known.

We can find these quantities approximately, by the formulæ (IX.), (IX.*a*) and (X.) of the preceding Chapter.

STRAINS IN THE CABLE.—The strain in the cable at any point can now be calculated precisely as in the preceding Chapter, page 190. Indeed, so far as the cable is concerned, there is no difference in the present theory from that already deduced. The only difference is in the computation of the truss. We only call attention here to the fact that we have now given two values for $\frac{h}{v}$ and n and q, according as the truss is supposed to be fixed at the ends, or simply tied down.

STRAIN IN THE STAYS.—The stays are strained by the live apex loads only. The live load carried by any stay is given by equation (15), or

$$P = \frac{3n_{1-2}\, m\, E_2 F_2 r^2 l^2 z_1 z_2}{8\,E_1 F_1 s^3 v^2},$$

* The letter N denotes that portion of a concentrated load P which the cable sustains as a uniformly distributed load. The subscripts, 1, 2 and 3 refer to cable, truss and stay respectively. N_{1-2} denotes, then, that cable and truss act together.

where the value of n_{1-2} is given by (16) or (17), and z_1, z_2 are the distances from point of attachment of stay to left and right ends of span.

This load, multiplied by the secant of the angle which the stay makes with the vertical, or by $\frac{s}{v}$, gives the strain in the stay. Multiplied by the tangent of that angle, or by $\frac{z_1}{v}$ or $\frac{z_2}{v}$, the smallest value of z_1 or z_2 being taken, it gives the compression at the foot of the stay, in the compression member provided for that purpose. Each stay adds its own increment to this member, and the compression, therefore, increases toward the towers at each end.

The strain in any stay, then, is

$$\frac{s}{v}\left[\frac{3n_{1-2}\, m\, E_3 F_3 r^2 l^2 z_1 z_2}{8\, E_1 F_1 s^3 v^2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where s is the length of stay, v its vertical projection, F_3 its cross section, and z_1 and z_2 the distances from left and right ends to foot of stay.

The strains in the suspenders can be simply taken at one full panel load, or $(p + m)\, b$, where b is the length of a bay. This is greater than can ever come upon any suspender, as both truss and stays act also to support the load. Still, owing to imperfect local adjustment at each apex, it is impossible to say exactly how the load may be distributed between stay, truss and suspender, and it is well, therefore, to be on the safe side. Owing to shock and sudden strains, also, an excess of strength in the suspenders is desirable.

STRAINS IN THE TRUSS.—To find the strains in the truss we proceed by apex weights. Since, as already explained, we do not consider the dead load as affecting the truss, we have only to consider the live load. We neglect the action of the stays, and consider each apex live load as carried by cable and truss. Equations (20) or (21) give then the uniform cable load caused by each apex load. This cable load acts as an upward load on the truss. The moment at any point, Fig. 8, is, then, equal to the moment due to the weight P, minus the moment due to the upward load of the cable.

Both for moment and shear the truss is to be considered as acted upon by a single concentrated

Fig.8

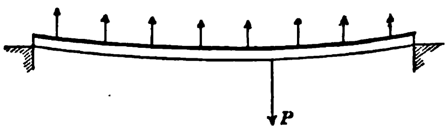
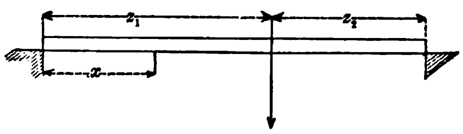


Fig.9



load and by a uniform upward load at the same time.

The moment at any point, due to the weight alone, is, from theory of flexure, page 304,

For truss supported at ends, Fig. 9,

when $x < z_1$ $$M_x = \frac{P z_2 x}{2l};$$

when $x > z_1$ $$M_x = \frac{P z_2 x}{2l} - P(x - z_1).$$

For truss fixed horizontally at ends,

when $x < z_1$, $$M_x = \frac{P z_2^2 (3z_1 + z_2)\, x}{8l^3} - \frac{P z_1 z_2^2}{4l^2};$$

when $x > z_1$, $$M_s = \frac{P z_1^2 (3z_2 + z_1)}{8l^3} - \frac{P z_2 z_1^2}{4l^2}.$$

The moment due to the upward load of the cable is, for beams supported at ends,

$$M_s = \frac{N_{1-2}\, x P}{2}(2l - x),$$

and for beam fixed horizontally at the ends,

$$M_s = N_{1-2}\frac{x(2l - x)P}{2} - \frac{N_{1-2}\, l^2 P}{3}.$$

We can thus find the moment at any point for each apex load, and can then find and tabulate the flange strains due to each load, just as for a simple truss (page 99).

So also for the shears and strains in the braces.

Side Spans.—We are not able to place loads symmetrically on each side of the centre in the side spans. Hence as the end of the side span is supported and cannot deflect like the centre of the main span, the vertex of the parabola formed by the two side cables shifts for every partial load. The equation for the deflection thus becomes very involved.

We shall always be on the safe side, however, without undue excess, if we take the strain at any point of the side cables the same as at the corresponding point of the main cables, counting each way from the tower if the side span is half the main span, or $l = l_1$. If the side span is less than half the main span, or $l_1 < l$, we may take the strain at any point of the side cables $\frac{l_1}{l}$ times that at the corresponding point of the main cables.

Corresponding stays on each side of the tower may have equal areas.

For the side truss, the depth being always the same as the main truss, the strain at any point may be taken at $\frac{l_1^2}{4l^2}$ of that at the corresponding point of the main truss. Thus if $l_1 = l$, or the side span is $\frac{1}{2}$ the main span, the strain at the centre, quarters, etc., of the side span may be taken as $\frac{1}{4}$th of the strains at the same points of the main span.

Towers.—Each stay on each side of a tower causes a vertical compression upon it. To the sum of these we must add half the entire weight, dead and live, carried by the cables at both sides and centre. The towers must sustain this load.

In addition to this, when the centre span carries both dead and live load, and the side spans the dead load only, each tower is pulled inwards by a horizontal force at the top. This horizontal force is equal to the vertical pressure of the side and main cables when thus loaded, multiplied by the coefficient of friction. When the cables pass over rollers on top of the towers, the friction is slight. In any case, the stays act to resist the inward or outward pull, and thus the tower is sufficiently braced.

Accuracy of the Preceding Method.—Several inaccuracies are implicitly committed in the preceding method. It is desirable to notice them particularly, in order to be convinced that their influence is properly disregarded.

Thus the curve of the deflected cable is a parabola. That of the truss is the elastic curve. These two curves cannot coincide. Thus the ends of the suspenders, after deflection of the cable, would lie in a parabola instead of a straight line.

This is, however, counteracted by the fact that each suspender is elongated according to its length. As the lengths vary as the ordinates to a parabola, the ends are thus brought nearly to a straight line again. The two actions balance, and the error of disregarding both is slight.

Again, the tops of the towers are not strictly fixed points, but "*give*" somewhat, thus increasing the deflection of the cable and the load carried by the truss. But since in our method we have considered any two systems as acting without the third, our method gives an excess to the truss as it is, and thus the "*give*" of the towers is already allowed for and can be neglected.

We do not consider the method here outlined as accurate as that given in the preceding Chapter. It will, however, be found simpler of application, and is based upon the theory commonly received.

TRUSS HINGED AT THE CENTRE, SUPPORTED AT ENDS.—The calculation of strains for this case is very simple. The strain in the cable is found as before, as given on page 190. The strain in any stay is given by the apex load at its foot multiplied by the secant of the angle which the stay makes with the vertical.

For the truss, the moment due to a weight P at any point, is given on page 212.

The moment due to the upward load of the cable and its left reaction, is simply

$$M_s = \frac{P z_1 x}{2 l^2} (2 l - x),$$

where z_1 is the distance from the left end to the weight P, l the half span, and x any distance from left.

For the moment at any point, therefore, we have

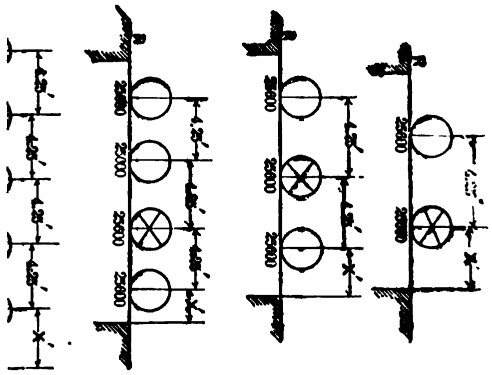
$$M_s = \frac{P x}{2l} \left[z_2 - \frac{z_1 (2 l - x)}{l} \right].$$

The reaction at the left end is $\frac{P (2 l - z_1)}{2 l}$. The uniform upward load of the cable per unit of length is $\frac{P z_1}{l^2}$.

The shear at any point between the weight and left end is, therefore,

$$\frac{P (2 l - z_1)}{2 l} - \frac{P z_1 x}{l^2}.$$

The truss is assumed as rigid, and the curve of cable a parabola.



Span in feet.	Maximum Bending Moment per track.
8	58200
9	67200
10	83200
11	102400
12	121600
13	140800
14	160000
15	179200
16	199200
17	224400
18	249600
19	281600
20	313600
21	343600
22	377600
23	409600
24	441600
25	473600
26	505600

The demonstrations of pages 85 and 86, the diagram of page 89, the tables of moments here given, and the demonstrations of pages 216 and 228 are due to

P. L. SZLAPKA, C.E.,

Asst. Engr. in charge of the Computing Dept. of the Phœnix Bridge Co.

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$$M_s = \frac{P z_1 x}{2 l^2}(2 l - x),$$

where z_1 is the distance from the left end to the weight P, l the half span, and x any distance from left.

For the moment at any point, therefore, we have

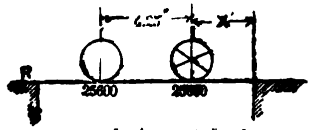
$$M_s = \frac{P x}{2l}\left[z_2 - \frac{z_1 (2 l - x)}{l}\right].$$

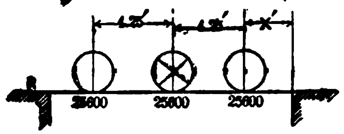
The reaction at the left end is $\frac{P(2 l - z_1)}{2 l}$. The uniform upward load of the cable per unit of length is $\frac{P z_1}{l^2}$.

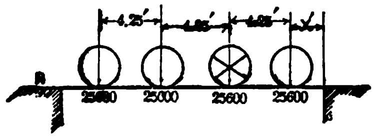
The shear at any point between the weight and left end is, therefore,

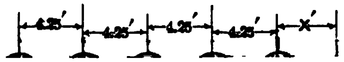
$$\frac{P(2 l - z_1)}{2 l} - \frac{P z_1 x}{l^2}.$$

The truss is assumed as rigid, and the curve of cable a parabola.

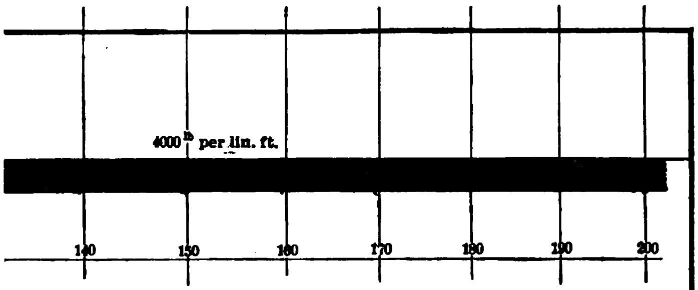








Span in feet.	Maximum Bending Moment per track.	Limits.	
8	59200	7.2′	$x = \frac{l}{2} - 1.0625$
9	67200	to	$R = 25600 + \frac{54400}{l}$
10	83200	9.4′	$M = 12800l - 54400 + \frac{57800}{l}$
11	102400		
12	121600		
13	140800	9.4′	$x = \frac{l}{2} - 4.25$
14	160000	to	$R = 38400$
15	179200	15.8′	$M = 19200l - 108800$
16	199200		
17	224400		
18	249600	15.8′	$x = \frac{l}{2} - 5.3125$
19	281600	to	$R = 51200 + \frac{108800}{l}$
20	313600	18.0′	$M = 25600l - 217600 + \frac{115600}{l}$
21	345600		
22	377600		
23	409600		
24	441600	18.0′	$x = \frac{l}{2} - 8.5'$
25	473600	to	$R = 64000$
26	50[illegible]600		



MOMENTS

ır 2-112 Ton Decapod Engines

ANTIC COAST LINE

Scale: 20 ft.=1 inch.

128,000 lbs. in 17 ft. 0 in.

224,000 lbs. in 47 ft. 5 in.

APPENDIX.

CHAPTER I.

CONCENTRATED LOAD SYSTEM.

WE have already given, page 87, the general method of dealing with a system of concentrated loads, and explained the manner of formation and method of use of our diagram.

There are certain special modifications required in special cases, which we shall notice here.

APPLICATION TO INCLINED WEB SYSTEM.*—The method as already given, page 87, is general for *shear*, whatever the character of the bracing, and also, in any case, for the moments for the *unloaded chord*. But when all braces are inclined, a modification is necessary for the *loaded chord* moments, as follows:

Let the load be on the lower chord, and suppose we wish the maximum moment at the point c, Fig (a), distant z from the left end, in order to find the strain in the panel ab.

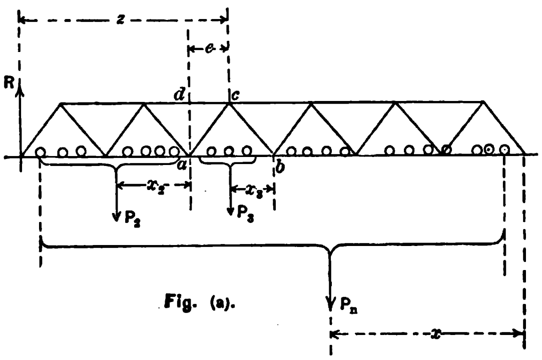


Fig. (a).

Let p = the panel length; e = the distance dc, which is a constant for any given case, and for equal inclination of braces is $\frac{p}{2}$. Let P_n = the total weight of all wheels on the span, or, if any uniform train load comes on, $P_n + wy_n$. Let x = the distance of the resultant of this total weight from the right end. Let P_2 = the total weight of all wheels in front of the panel ab, and x_2 = the distance of the resultant of this weight from a; P_3 = the weight of all wheels in panel ab, and x_3 = the distance of its resultant from b.

The portion of P_3 which takes effect at a is $\frac{P_3 x_3}{p}$. The moment at c is,

$$M = \frac{P_n x}{l} z - P_2 (x_2 + e) - \frac{P_3 x_3}{p} e. \qquad (1)$$

Differentiating and placing the first differential coefficient equal to zero, we have, for the condition which makes the moment a maximum, since $dx = dx_2 = dx_3$,

$$\frac{z}{l} = \frac{P_2 + \frac{P_3}{p} e}{P_n}.$$

* The student should prepare a diagram, like that given on page 89, to a scale of 20 feet to an inch, and have it constantly at hand before him while reading the following pages.

If the uniform train load covers the distance y_n, the total load is $P_n + wy_n$. If the braces are equally inclined $\frac{e}{p} = \frac{1}{2}$. The condition is then, in general,

$$\frac{2\,(P_n + wy_n)}{l} \underset{>}{=} P_s + \frac{P_s}{p}\,e. \qquad (2)$$

We can easily shift our diagram on the span, just as explained in the case of vertical and inclined braces, page 92, until this condition is satisfied, or as nearly satisfied as possible, and find the maximum moment for this position, from (1). We should try, as on page 93, for all maximums, and take the largest. In using (1) remember that $P_n x = M_r$, $P_s(x_s + e)$ is the moment of all the weights in front of the panel with reference to c, and $P_s x_s$ of all the weights in the panel with reference to b, Fig. (a).

An example of application of this will be given, page 220.

INCLINED CHORDS.—When the chords are inclined they take a portion of the shear, and only the resultant shear takes effect in the web. On this account the position of the load system for maximum strains in any web member is not the same as for horizontal chords.

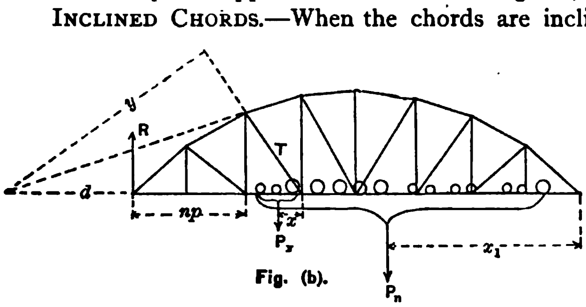


Fig. (b).

Let T, Fig. (b), be the strain in any web member; l = length of span; p = panel length; N = number of panels, so that $Np = l$; P_x = sum of all loads in advance of the point at which maximum shear is required; x = the distance of the resultant of P_x from this point; P_n = total load on span; x_1 = the distance of its resultant from right end; n = number of panels in advance of the panel in question; np = the distance from left end to this panel; d = distance from left end to intersection of the chords for T; y = lever arm for T; R = left reaction.

Then $R = \frac{P_n x_1}{l}$, and the portion of P_x which takes effect at left end of panel c, is $\frac{P_x x}{p} = \frac{NP_x x}{l}$.

We have, therefore,

$$Ty = Rd - \frac{NP_x x}{l}(d + np) = \frac{P_n d x_1}{l} - \frac{NP_x x}{l}(d + np).$$

Differentiating and putting the first differential equal to zero, we have, since diff. of x = diff. of x_1, for the condition which gives the maximum strain,

$$P_n d = NP_x\,(d + np).$$

If any uniform train load comes on the span and covers the distance y_n, the total load is $P_n + wy_n$, and the condition is, in general,

$$\frac{P_n + wy_n}{N} \underset{>}{=} P_x\,\frac{d + np}{d}.$$

This condition enables us to find, by trial, the position of the load system for maximum shear just as on page 91.

If the chords are horizontal $d = d + np = \infty$ and $\frac{P_n}{N} \underset{>}{=} P_x$ as found, page 90.

After this position is found, we have, for the maximum resultant shear required,

$$\text{Shear} = T\cos\theta = \frac{\cos\theta}{y}\left[\frac{M_n d}{l} - \frac{M_x\,(d + np)}{p}\right],$$

where θ is the angle of the brace with the vertical. Or since

$$\frac{\cos\theta}{y} = \frac{1}{d + (n+1)p},$$

$$\textbf{Shear} = \left[\frac{M_r d}{l} - \frac{M_x(d + np)}{p}\right]\frac{1}{d + (n+1)p},$$

where M_r is the moment at right end of all the loads, and M_x is the moment at the point of all the loads in front of the point. The shear thus found, divided by cos θ, or multiplied by the sec θ, will give the strain required.

For the counter stress in T we have the load coming on from left, or we may find the strain for the corresponding brace on right for load coming on from right, provided we take the right-hand intersection.

In the latter case we should put $d + l$ in place of d, and np is the distance of c *from the right end.*

Maximum Moment in a Plate Girder.—In designing plate girders and stringers, it is necessary to find the maximum moment at or near the centre, caused by the load system. The maximum moment will be at a point at or near the centre, and some wheel must be at the point.

In Fig. (c) let the point at which the maximum moment occurs be at a distance x on left of the centre, and let one of the wheels occupy this point. Let the sum of the loads in front of the point be P_x, and the distance of their centre of gravity from the point, or wheel at the point, be a. Let the total load on the girder be P_n, and the distance of its centre of gravity from the point, or wheel at the point, be b. For any given wheel at the point, a and b are constant. Let the span be l.

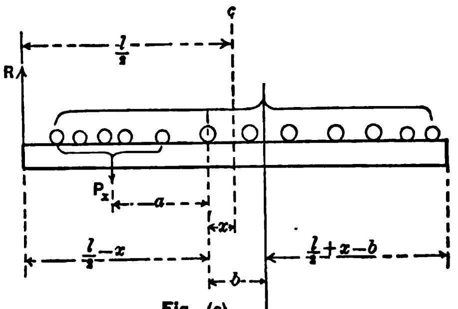


Fig. (c).

Then we have for the reaction,

$$R = \frac{P_n}{l}\left(\frac{l}{2} + x - b\right),$$

and the moment at the point is

$$M = \frac{P_n}{l}\left(\frac{l}{2} + x - b\right)\left(\frac{l}{2} - x\right) - P_x a.$$

Differentiating and putting the first differential coefficient equal to zero, we have $-2x + b = 0$, or $x = \frac{b}{2}$.

That is, the moment is a maximum *when the system is so placed that the wheel at the point, causing the maximum, is as far on one side of the centre of the span as the centre of gravity of the total load is on the other.*

If any uniform train load comes on the span, it must be included in the value of the total load P_n.

The distance of the centre of gravity, or resultant of the total load, from the centre of the span is

$$\frac{b}{2} = \frac{l}{2} - \frac{M_r}{P_n},$$

where M_r is the moment at the right end of the span, and is easily found from our diagram.

We can, in general, find a maximum for each one of a number of wheels on the left of the centre. Judgment must be exercised, therefore, in selecting the wheels to test for, in order to determine the greatest maximum moment. In general, we should select that position which *brings the greatest load on the girder*, and at the same time brings the resultant of the total load *nearest the centre of the span.*

Guided by this, we can usually select not more than three wheels, one of which will give the greatest maximum moment, and can be found by trial.

EXAMPLE.—A PLATE GIRDER IS 60 FEET LONG. REQUIRED THE MAXIMUM MOMENT FOR THE SYSTEM OF LOADS OF OUR DIAGRAM.

Set the markers on the scale at 30 and 60 feet, or mark these points on a strip of paper to the scale of the diagram, and apply to the diagram as follows :

We see, by shifting the scale so that each weight is successively at the centre of the span, that we bring the greatest load on the span, and at the same time the resultant of the total load is nearest the centre of the span, either for p_{14}, p_{16}, or p_{18} at the centre. We have, therefore, only to test for these loads.

For p_{14} at the centre we have $y_n = 30 + 71.2 - 97.4 = 3.8$ feet; loads $p_1 - p_8$ are off the span, p_9 is distant $30 + 71.2 - 37.2 = 64$ feet from the right end. There is no uniform train load on the span; the total load is 428000 $-$ 184000 = 244000 lbs., and $M_r = 20873333 + 428000 \times 3.8 - 3333333 - 184000 \times 64 = 7390400$ ft. lbs. Hence

$$\frac{b}{2} = \frac{l}{2} - \frac{M_r}{244000} = -0.3 \text{ ft.}$$

As this shows that the resultant of the total load is already left of the centre, we must, for a maximum, have p_{14} on right of centre. If we shift p_{14} a distance 0.15 ft. on right of centre, then, since during the shifting no wheels come on or go off, the resultant will be 0.15 ft. on left. This position then gives a maximum moment.

For this position, we have $y_n = 3.65$, p_9 is 63.85 feet from right end, $M_r = 20873333 + 428000 \times 3.65 - 3333333 - 184000 \times 63.85 = 7353800$ ft. lbs. Hence, the moment at p_{14} is

$$\frac{M_r \times 30.15}{60} - (11223066 - 3333333 - 184000 \times 34) = 2044884 \text{ ft. lbs.}$$

Let us try p_{16} at the centre. For this position $y_n = 30 + 75.4 - 104.3 = 1.1$ ft.; loads $p_1 - p_8$ are off the span; p_9 is distant $30 + 75.4 - 42.7 = 62.7$ feet from right end. The total load is $448000 - 204000 + 4000 \times 1.1 = 248400$ lbs.

$$M_r = 23878666 + 448000 \times 1.1 + \frac{4000\,(1.1)^2}{2} - 4360666 - 204000 \times 62.7 = 7222420 \text{ ft. lbs.,}$$

and

$$\frac{b}{2} = \frac{l}{2} - \frac{M_r}{248400} = 0.93 \text{ ft.}$$

If, now, we should shift p_{16} to the left of the centre, a distance of 0.465 ft., if no load passed off or came on during the shifting, we would have $x = \frac{b}{2}$. But as we shift the train load comes on, and this moves the resultant a little to the right. Let us therefore shift p_{16} a distance of, say, 0.6 ft. $= x$ to left of centre.

For this position $y_n = 30.6 + 75.4 - 104.3 = 1.7$ ft.; loads $p_1 - p_8$ are off; p_9 is distant $30.6 + 75.4 - 42.7 = 63.3$ from right end; total load = 250800 lbs.

$$M_r = 7372180, \quad \frac{b}{2} = \frac{l}{2} - \frac{M_r}{250800} = 0.61 = x.$$

This position, therefore, gives a maximum.

For this position, we have $M_s = 12569466 - 4360666 - 204000 \times 32.7 = 1538000$, and the moment at p_{16} is

$$M = \frac{M_r \times 29.4}{60} - 1538000 = 2074368 \text{ ft. lbs.}$$

Let us try p_{18} at the centre. For this position $y_n = 30 + 79.7 - 104.3 = 5.4$ feet; wheels p_1 to p_{10} are off; p_{10} is distant $30 + 79.7 - 47.4 = 62.3$ feet from right end; the total load is $448000 - 224000 + 4000 \times 5.4 = 245600$;

$$M_r = 23878666 + 448000 \times 5.4 + \frac{4000 \times (5.4)^2}{2} - 5312666 - 224000 \times 62.3 = 7088320,$$

and

$$\frac{b}{2} = \frac{l}{2} - \frac{M_r}{245600} = 1.14 \text{ ft.}$$

If we shift p_{18} a distance $x = 0.75$ on left of centre, we have $y_n = 6.15$, p_{10} distant 63.05 feet from right end. total load = 248600 lbs.

$M_r = 23878666 + 448000 \times 6.15 + \frac{4000\,(6.15)^2}{2} - 5312666 - 224000 \times 63.05 = 7273645$, $\frac{b}{2} = \frac{l}{2} - \frac{M_r}{248600} = 0.74$, or very nearly $= x$.

For the moment at this point, we have $M_s = 14024666 - 5312666 - 224000 \times 32.3 = 1476800$, and

$$M = \frac{M_r \times 29.25}{60} - M_s = 2069102.$$

We see, therefore, that the greatest maximum is for p_{16}, a distance 0.6 on left of the centre, and is 2074368 ft. lbs. at this point.

RECAPITULATION.—We see that in applying our diagram for concentrated load system, we have four criterions in all, for determining the position of the system which gives the maximum shear and moment at any point. Two of these criterions are for shear and two for moments.

Shear.—In the case of *horizontal chords* we have found, page 90, the criterion for position for maximum shear,

$$\frac{P_n + wy_n}{N} \overset{=}{>} P_x, \quad \ldots \ldots \ldots \quad (1)$$

which holds good whatever the character of the web system.

The maximum shear itself is given by

$$\textbf{Shear} = \frac{M_r}{l} - \frac{M_x}{p}. \quad \ldots \ldots \ldots \quad (2)$$

If, however, the chords *are inclined*, then, as we have just shown, page 217, the criterion for position of maximum shears,

$$\frac{P_n + wy_n}{N} \overset{=}{>} P_x \frac{d + np.}{d} \quad \ldots \ldots \ldots \quad (3)$$

The maximum shear itself is given in this case by

$$\textbf{Shear} = \frac{1}{d + (n + 1)p}\left[\frac{M_r d}{l} - \frac{M_x}{p}(d + np)\right] \quad \ldots \ldots \quad (4)$$

which also holds good whatever the character of the web system.

Moments.—For moments, no matter whether the chords are horizontal or inclined, we have for the criterion giving the position for maximum moment, page 92,

$$\frac{z}{l}(P_n + wy_n) \overset{=}{>} P_x, \quad \ldots \ldots \ldots \quad (5)$$

which holds good for both chords *only when vertical and diagonal bracing is used.*

But if the braces are all inclined, then this criterion holds only for the unloaded chord, while for the loaded chord we have the criterion, page 216,

$$\frac{z}{l}(P_n + wy_n) \overset{=}{>} P_2 + \frac{P_3}{p} e.$$

In the first case the maximum moment itself is given by

$$M = \frac{z}{l} M_r - M_x.$$

In the second case by

$$M = \frac{z}{l} M_r - P_2 (x_2 + e) - \frac{P_3 x_3}{p} e.$$

Finally, for the maximum moment in a plate girder, the system must be so placed that the wheel which causes the maximum *is as far on one side of the centre of the span as the centre of gravity of the total load is on the other.*

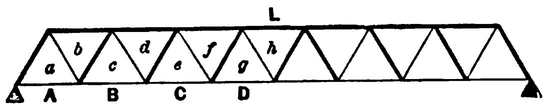
The preceding covers the entire theory of using our diagram. We will now proceed to give illustrations of its use.

APPLICATION TO THE CASES OF CHAPTERS III. AND IV., SECTION II.

WE can now give the solution of the cases of Chapters III. and IV., Section II., pages 95–136, by means of a concentrated load system, as specified and explained here and on pages, 89, *et, seq.* This method, as we have said, is the present practice, and the student should be thoroughly familiar with it. He should prepare a diagram, to a scale of 20 feet to an inch, as directed on page 87, *et seq.*, and refer to it constantly in checking our results.

EXAMPLE I. WARREN GIRDER.—Let us take the example of page 97, $l = 80$ feet, $d = 10$ feet = depth, $N = 8$, live load on bottom chord, single track, two trusses.

Fig. 88



Dead load strains.—Let the panel dead load be 11400 lbs., of which 9000 acts at the lower chord panel points, and 2400 at the upper.

The strains due to dead load may be found by any of the methods of Chapter III., page 97. The method of moments is perhaps the most convenient. Let us adopt it.

We have, for the reaction, $R = 41100$ lbs. For the top chords, therefore,

$Lb \times 10 = 41100 \times 10 - 2400 \times 5,$ $\quad Lb = +\ 39900$ lbs.

$Ld \times 10 = 41100 \times 20 - 2400\ (5 + 15) - 9000 \times 10,$ $\quad Ld = +\ 68400$ "

$Lf \times 10 = 41100 \times 30 - 2400\ (5 + 15 + 25) - 9000\ (10 \times 20),$ $\quad Lf = +\ 85500$ "

$Lh \times 10 = 41100 \times 40 - 2400\ (5 + 15 + 25 + 35) - 9000\ (10 + 20 + 30),$ $\quad Lh = +\ 91200$ "

For the bottom chords,

$Aa \times 10 = -\ 41100 \times 5,$ $\quad Aa = -\ 20550$ lbs.

$Bc \times 10 = -\ 41100 \times 15 + 2400 \times 10 + 9000 \times 5,$ $\quad Bc = -\ 54750$ "

$Ce \times 10 = -\ 41100 \times 25 + 2400\ (10 + 20) + 9000\ (5 + 15),$ $\quad Ce = -\ 77550$ "

$Dg \times 10 = -\ 41100 \times 35 + 2400\ (10 + 20 + 30) + 9000\ (5 + 15 + 25),$ $\quad Dg = -\ 88950$ "

The sec $\theta = 1.117$, and we have for the bracing

$La = +\ 41100 \times 1.117 = +\ 45909$ lbs., $\quad ab = -\ (41100 - 2400)\ 1.117 = -\ 43228$ lbs.

$bc = +\ (41100 - 11400)\ 1.117 = +\ 33175$ lbs., $\quad cd = -\ (41100 - 13800)\ 1.117 = -\ 30494$ "

$de = +\ (41100 - 22800)\ 1.117 = +\ 20441$ " $\quad ef = -\ (41100 - 25200)\ 1.117 = -\ 17760$ "

$fg = +\ (41100 - 34200)\ 1.117 = +\ 7707$ " $\quad gh = -\ (41100 - 36600)\ 1.117 = -\ 5026$ "

One half these results should be taken for each truss.

Live Load Strains.—Having prepared a diagram according to the instructions of page 87 *et seq.*, the student should carefully check the following results :

It should be noted that for the braces we multiply the maximum shear, as given by our diagram, by 1.117. We should take *one half* the results as given for one truss, single track. If we had double track the results, as given, would be the correct strains for one truss, without dividing. Our total results are as follows :

Live Load—Braces.

	Total Load.	M_r.	M_s.	Max. Shear.	
$z = 10$, p_2 at point	342000	14919666	108800	175616	$\begin{cases} La = +\ 196163 \text{ lbs.} \\ ab = -\ 196163 \text{ “} \end{cases}$
$z = 20$, p_2 “	291200	10305786	128000	116022	$\begin{cases} bc = +\ 129596 \text{ “} \\ cd = -\ 129596 \text{ “} \end{cases}$
$z = 30$, p_2 “	240000	7728666	128000	83808	$\begin{cases} de = +\ 93613 \text{ “} \\ ef = -\ 93613 \text{ “} \end{cases}$
$z = 40$, p_2 “	224000	5447066	128000	55288	$\begin{cases} fg = +\ 61757 \text{ “} \\ gh = -\ 61757 \text{ “} \end{cases}$
$z = 50$, p_2 “	184000	3443733	128000	30246	$\begin{cases} fg = -\ 33785 \text{ “} \\ gh = +\ 33785 \text{ “} \end{cases}$

All these shears are greater than for uniform train load alone, page 102.

We see that *fg* and *gh* must be counterbraced for the difference $33785 - 7707 = 26078$ lbs.

For the unloaded chord we apply the diagram, as directed, page 92, and give our results for the student to check.

Live Load—Top Chord.

	Total Load.	M_r.	M_s.	Max. Moment.	
$z = 10$, p_8 at point	342000	14919666	108800	1756158	$Lb = +\ 175616$ lbs.
$z = 20$, p_{14} “	332400	13293220	579200	2744105	$Ld = +\ 274410$ “
$z = 30$, p_{18} “	328400	12990420	1538000	3333407	$Lf = +\ 333341$ “
$z = 40$, p_{18} “	328400	12934286	2965866	3501277	$Lh = +\ 350128$ “

For the loaded chord we must find the position by the criterion given, page 216. In the present case $\frac{e}{p} = \frac{1}{2}$, and our criterion may be written

$$(P_n + wy_n) \quad \frac{80}{z}\left(P_1 + \frac{P_2}{2}\right).$$

Let us try for the maximum moment at the first upper apex on left of centre, that is, the point of moments for *Dg*.

Set one marker at 80 feet and the other at 40 feet, or centre of span, and apply to the diagram. Since $z = 35$, $\frac{80}{z} = \frac{16}{7}$.

When p_{14} is at centre of span, we have $y_n = 6.9$, $P_n = 304000$, $P_1 = 96000$, $P_2 = 51200$. Hence $P_n + wy_n = 331600$, and $\frac{16}{7}\left(P_1 + \frac{P_2}{2}\right) = 277943$.

If p_{14} is moved just a little to right, the total load is unchanged, but P_1 becomes 76800. Hence

$$\frac{16}{7}\left(P_1 + \frac{P_2}{2}\right) = 307200.$$

We see that 331600 is greater than both these results, therefore we try for p_{18} at centre.

We have for this position $P_n + wy_n = 328400$, and for the two values of $\frac{16}{7}\left(P_1 + \frac{P_2}{2}\right)$, 290743 and 320000. As 328400 is greater than both these, we try for p_{18} at centre.

For this position $y_n = 15.4$, $P_n + wy_n = 325600$, and the two values of $\frac{16}{7}\left(P_1 + \frac{P_2}{2}\right)$ are

303543 and 332800. Since 325600 is less than the first and greater than the second, this position gives a maximum.

For this position $M_r + 12738853$, $P_8(x_8 + e) = 2011600$, $\frac{P_8}{2}x_8 = 163200$, hence

$$M = \frac{35}{80}M_r - 2011600 - 163200 = 3398448.$$

In the same way we get the following results:

Live Load—Lower Chord.

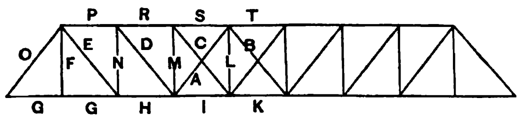
	M_r.	$P_8(x_8 + e)$.	$\frac{P_8}{2}x_8$.	Maximum Moment.	
$z = 5$, p_{18} at 10 ft. from left end	14919666	0	54400	878079	$Aa = -$ 87808 lbs.
$z = 15$, p_{16} at 20 "	13441220	198400	163200	2158628	$Bc = -$ 215863 "
$z = 25$, p_{18} at 30 "	12800320	825600	163200	3011300	$Ce = -$ 301130 "
$z = 35$, p_{18} at 40 "	12738853	2011600	163200	3398448	$Dg = -$ 339845 "

All these moments are greater than for uniform train load alone.

Example 2. Pratt Truss.—As an illustration, let us take the Pratt Truss given in Plate 22, at the end of this work.

Span $= 153$ feet $= l$, number of panels $N = 9$, panel length $p = 17$ feet, depth $= 26$ feet, $\tan\theta = 0.654$, $\sec\theta = 1.195$.

We adopt for the live load the system of our diagram, instead of that specified on Plate 22, and for dead load 1800 lbs. per foot, of which 500 is for the upper chord and 1300 is for lower chord.



Our dead load panel weights are, then, 8500 lbs. at each upper apex, and 22100 lbs. at each lower apex—total, 30600 lbs. This dead load is greater than that for which the truss was actually designed, but our live load is much larger than that assumed by the Bridge Company, and hence we should have heavier trusses.

Dead Load Strains.—We give the results of calculation, according to the above data, in order that the student may check them. We shall adopt for the flanges the method of coefficients, page 101, as requiring the least work.

We have, then, for the chords,

$$P = 7 \times 30600 \times 0.654 = +\ 140086 \text{ lbs.} \qquad H = -\ 140086 \text{ lbs.}$$

$$R = 9 \times 30600 \times 0.654 = +\ 180111 \text{ "} \qquad I = -\ 180111 \text{ "}$$

$$T = S = 10 \times 30600 \times 0.654 = +\ 200124 \text{ "} \qquad K = -\ 200124 \text{ "}$$

$$G = -\ 4 \times 30600 \times 0.654 = -\ 80049 \text{ "}$$

For the web members

$$O = +\ 4 \times 30600 \times 1.195 = +\ 146268 \text{ lbs.} \qquad E = -\ 3 \times 30600 \times 1.195 = -\ 109701 \text{ lbs.}$$

$$D = -\ 2 \times 30600 \times 1.195 = -\ 73134 \text{ "} \qquad C = -\ 30600 \times 1.195 = -\ 36567 \text{ "}$$

$$F = -\ 22100 \text{ "} \qquad N = +\ 2 \times 30600 + 8500 = +\ 69700 \text{ "}$$

$$M = +\ 30600 + 8500 = +\ 39100 \text{ "} \qquad L = +\ 8500$$

$$A = B = 0.$$

Live Load Strains.—Applying our diagram we have the following results:

	M_r	M_z	Shear		
p_4 at $z =$ 17 ft.	50118746	590400	292843	$O = + 349947$ lbs.	
p_4 at $z =$ 34 ft.	37377086	304800	226364	$E = - 270505$ "	
p_3 at $z =$ 51 ft.	28509886	304800	168409	$D = - 201249$ "	$N = + 168409$ lbs.
p_3 at $z =$ 68 ft.	20798533	304800	118008	$C = - 141019$ "	$M = + 118008$ "
p_2 at $z =$ 85 ft.	12774906	12800	75966	$B = - 90779$ "	$L = + 75966$ "
p_2 at $z =$ 102 ft.	7968666	12800	44552	$C = + 53239$ "	$F = - 89153$ "

Since the dead load strain in C is $- 36567$, the counter A is strained

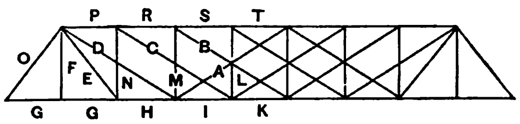
$- 53239 + 36567 = - 16672$ lbs. Half these values for one truss.

For the flanges, we have

	M_r	M_z	M		
p_4 at $z =$ 17 ft.	50118746	590400	4978460	$G = - 191479$ lbs.	
p_{16} at $z =$ 34 ft.	48221353	2206333	8509514	$H = - 327289$ "	$P = + 327289$ lbs.
p_{15} at $z =$ 51 ft.	47776606	5108186	10817349	$I = - 416051$ "	$R = + 416051$ "
p_{14} at $z =$ 68 ft.	50017886	10083866	12146305	$K = - 467165$ "	$S = T = + 467165$ "

Half of these values for a single truss, if there are two trusses.

EXAMPLE 3. DOUBLE INTERSECTION PRATT TRUSS.—Let us take the same span as before, $l = 153$ feet, $N = 9$, $p = 17$ feet, depth $= 26$ feet. For O and E tan $\theta = 0.654$, sec $\theta = 1.195$. For the other ties, tan $\theta = 0.765$, sec $\theta = 1.7$. Let us take the same dead load as before, *viz.*, 8500 lbs. at each upper apex, and 22100 lbs. at each lower apex. Total, 30600 lbs.



Dead Load Strains.—We must use for dead load the method of coefficients, page 116.

We have for the flanges,

$P = 6 \times 30600 \times 0.654 + 30600 \times 0.765 = + 143483$ lbs. $\qquad I = - 143483$ lbs.

$R = S = T = 6 \times 30600 \times 0.654 + 2 \times 30600 \times 0.765 = + 166892$ lbs. $\qquad K = - 166892$ "

$G = - 4 \times 30600 \times 0.654 = - 80049$ lbs.

$H = - 6 \times 30600 \times 0.654 = - 120074$ "

For the web members,

$A = B = 0$, $\qquad F = - 22100$ lbs., $\qquad E = - 2 \times 30600 \times 1.195 = - 73134$ lbs.

$O = + 4 \times 30600 \times 1.195 = + 146268$ lbs., $\qquad D = - 30600 \times 1.7 = - 52080$ lbs. $= C$.

$L = M = + 8500$ lbs., $\qquad N = + 30600 + 8500 = + 39100$ "

Live Load Strains.—For all multiple systems the strains are indeterminate, as it is impossible to say how much in practice each system will take. For this reason such systems are avoided, and probably no more double systems will be erected.

The accurate method of finding the strains for live load, for any panel or brace, would be to find by diagram the position of the system which gives the maximum moment or shear, and then for this position find the actual loads which take effect at each apex, and find the strain for this loading.

As this is exceedingly tedious, and the indeterminate character of the strains in practice renders such accuracy delusive, we adopt the following method, as being simpler and sufficiently accurate:

For the Braces.—Find the maximum shear for any brace by our diagram, as usual. Then find that uniform load which would give the same shear at the same point. Divide this load into apex loads, and calculate the brace for this loading.

If w is the uniform moving load per foot, coming on from the right and reaching to a distance z from the left end, then the shear due to this load is $\frac{w\,(l-z)^2}{2l}$. We may take the distance z as extending to the middle of the panel in front of the point.

If the maximum shear determined by diagram is S, then we have for w,

$$w = \frac{2lS}{(l-z)^2}.$$

If p is the panel length, we have the apex load

$$P = \frac{2plS}{(l-z)^2}.$$

Taking this apex load at each apex from right end up to the brace, we find the strain in the brace for this loading.

In the present case we have the following values:

		S	z	P		
p_4 at	17 feet from left,	292843 lbs.	8.5 feet.	72957 lbs.	$O = +\ 348734$ lbs.	
p_3 "	34 " "	226364 "	25.5 "	72436 "	$E = -\ 153886$ "	
p_3 "	51 " "	168409 "	42.5 "	71748 "	$D = -\ 162631$ "	
p_3 "	68 " "	118008 "	59.5 "	70219 "	$C = -\ 119272$ "	$N = +\ 70219$ lbs.
p_2 "	85 " "	75966 "	76.5 "	65816 "	$B = -\ 74591$ "	$M = +\ 43877$ "
p_3 "	102 " "	44552 "	93.5 "	65464 "	$A = -\ 49461$ "	$L = +\ 29095$ "
						$F = -\ 89153$ "

Thus, for O we have eight panel loads of 72957 lbs., and hence

$$R = 291828, \text{ and } O = 291828 \times 1.195 = +\ 348734 \text{ lbs.}$$

For E we have the panel load 72436 lbs., and since four of these loads are on the system to which E belongs, we have

$$R = \left(\frac{7}{9} + \frac{5}{9} + \frac{3}{9} + \frac{1}{9}\right) 72436 = 128775, \text{ and } E = 128775 \times 1.195 = 153886 \text{ lbs.}$$

For C we have the panel load 70219 lbs., and since three of these loads act on the system to which C belongs,

$$R = \left(\frac{5}{9} + \frac{3}{9} + \frac{1}{9}\right) 70219 = 70219, \text{ and } C = 70219 \times 1.7 = 119272 \text{ lbs.}$$

The shear for C is the compression for N.

For the Chords.—We find by our diagram the maximum moment for any panel, at the nearest

centre of moments. Then find what uniform load over the whole girder would give the same moment at this point. Divide this load into apex loads, and find the strain in the panel for this loading, by coefficients, in the usual manner. Each panel is thus found for its own equivalent uniform load.

If we denote the uniform load per foot by u, then, if z is the distance from the left end to any centre of moments, the moment at this point is $\frac{uz}{2}(l-z)$. If we denote the maximum moment as found by diagram by M, we have the equivalent uniform load $u = \frac{2M}{z(l-z)}$. If p is the panel length, the apex load is

$$P = \frac{2pM}{z(l-z)}.$$

In the present case we have the following values:

	M	z	P		
p_4 at 17 feet from left,	4978460	17	73212 lbs.	$G = -$ 191522 lbs.	
p_{16} " 34 " "	8509514	34	71508 "	$H = -$ 280597 "	
p_{15} " 51 " "	10817349	51	70701 "	$I = -$ 331517 "	$P = +$ 331517 lbs.
p_{14} " 68 " "	12146305	68	71449 "	$K = -$ 389682 "	$R = +$ 389682 "

$$R = S = T = + 389682 \text{ lbs.}$$

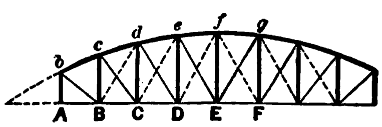
Half of these values to be taken for one truss. The same method applies to lattice truss of three or more systems, or to the Post Truss.

In view of what has preceded, there should now be no difficulty in finding the strains for concentrated load system for any of the trusses with horizontal chords given in Chapter IV., page 110. We shall not, therefore, give here examples of the Baltimore or of the Kellogg and Fink Trusses. The latter are entirely antiquated, and no more built. For long spans, instead of a double-system Pratt, some modification of the Baltimore Truss is used, generally with inclined chords.

EXAMPLE 4. INCLINED CHORDS.—Let us take as an example of inclined chords the case already given, page 130.

Here the span $l = 120$ feet, number of panels $N = 8$, panel length $p = 15$ feet, bracing vertical and inclined. Height, 20 feet at centre, 10 feet at ends, apices of upper chord in a parabola. We have already found for this case the lever arms of the various members, page 131.

Fig. 111



Lower chord,	AB	BC		CD	DE
lever arms,	10	14.375		17.5	19.375 feet.
Upper chord,	bc	cd		de	ef
lever arms,	13.8	17.13		19.22	19.98 feet.
Inclined braces,	bB	cC		dD	eE
lever arms,	27.33	58.33		117.69	379.53 feet.
Vertical braces,	bA	cB	dC	eD	fE
lever arms,	34.285	49.285	84	155	480 feet.

For the distance d, on the left of A, at which the upper panels intersect the lower chord, we have,

panel,	bc	cd	de	ef
	$d =$ 34.285	54	110	420 feet.

Let the dead load be 1150 lbs. per foot on lower chord, and 350 lbs. per foot on upper chord, or 17250 lbs. at each lower apex, and 5250 lbs. at each upper apex; total, 22500 lbs.

Dead Load Strains.—Making use of our lever arms, and the method of moments, we have $R = 78750$ lbs.

$AB = 0$,

$BC \times 14.375 = -78750 \times 15$, $BC = -82173$ lbs.

$CD \times 17.5 = -78750 \times 30 + 22500 \times 15$, $CD = -115714$ "

$DE \times 19.375 = -78750 \times 45 + 22500\,(15 + 30)$, $DE = -130645$ "

$bc \times 13.8 = +78750 \times 15$, $bc = +85600$ lbs.

$cd \times 17.13 = +78750 \times 30 - 22500 \times 15$, $cd = +118797$ "

$de \times 19.22 = +78750 \times 45 - 22500\,(15 + 30)$, $de = +131698$ "

$ef \times 19.98 = +78750 \times 60 - 22500\,(15 + 30 + 45)$, $ef = +135135$ "

$bA = +78750$ lbs.

$bB \times 27.33 = -78750 \times 34.285$, $bB = -98790$ lbs.

$cC \times 58.33 = -78750 \times 54 + 22500 \times 69$, $cC = -46288$ "

$dD \times 117.69 = -78750 \times 110 + 22500\,(125 + 140)$, $dD = -22941$ "

$eE \times 379.53 = -78750 \times 420 + 22500\,(435 + 450 + 465)$, $eE = -7114$ "

$cB \times 49.285 = +78750 \times 34.285 - 17250 \times 49.285$, $cB = +37532$ lbs.

$dC \times 84 = +78750 \times 54 - 22500 \times 69 - 17250 \times 84$, $dC = +14893$ "

$eD \times 155 = +78750 \times 110 - 22500\,(125 + 140) - 17250 \times 155$, $eD = +170$ "

$fE \times 480 = +78750 \times 420 - 22500\,(435 + 450 + 465) - 17250 \times 480 - eE \times 379.53$, $fE = -6000$ "

Half these results to be taken for a single truss if there are two trusses.

Live Load Strains.—For the criterion giving the position of load for maximum shear we have, in this case, page 217,

$$\frac{P_n + wy_n}{N} \gtreqless P_n \frac{d + np}{d},$$

and for the maximum shear,

$$\text{Shear} = \frac{1}{d + (n+1)p}\left[\frac{M_r d}{l} - \frac{M_n}{p}(d + np)\right].$$

This shear is the strain in a post, and multiplied by the sec θ gives the strain in the brace, or

$$\text{Strain} = \frac{1}{y}\left[\frac{M_r d}{l} - \frac{M_n}{p}(d + np)\right],$$

where y is the lever arm for the brace.

We have in the present case the following results :

	M_r	M_z	d	y	
p_4 at 15 feet from left,	30231280	326400	34.285	27.33	$\begin{cases} bB = -\ 288740 \text{ lbs.} \\ bA = +\ 160115 \text{ ''} \end{cases}$
p_2 " 30 " "	21130133	128000	54	58.33	$\begin{cases} cC = -\ 152920 \text{ ''} \\ cB = +\ 106188 \text{ ''} \end{cases}$
p_2 " 45 " "	15206066	128000	110	117.69	$\begin{cases} dD = -\ 108286 \text{ ''} \\ dC = +\ \ 82220 \text{ ''} \end{cases}$
p_2 " 60 " "	10305786	128000	420	379.53	$\begin{cases} eE = -\ \ 84610 \text{ ''} \\ eD = +\ \ 66880 \text{ ''} \end{cases}$
p_2 " 75 " "	6567066	128000	540	379.53	$eE = +\ \ 64730$ "
p_2 " 90 " "	3480533	128000	230	117.69	$dD = +\ \ 37831$ "

We see that the counter strains for $fD = -(64730 - 7114) = -57616$ lbs., and for $eC = -(37831 - 22941) = -14890$ lbs.

For the criterion giving the position of load for maximum moment, we have in this case for both chords

$$\frac{z}{l}(P_n + wy_n) \overset{=}{>} P_m,$$

and for the moment

$$M = \frac{M_r z}{l} - M_m.$$

We find the moments, therefore, precisely as so often illustrated in preceding examples. This moment must be divided by the lever arm of the panel to get the strain.

We shall leave the results to be found by the student. If we had inclined bracing we should use the above criterion and formula for M for the unloaded chord only, and for the loaded chord should have the criterion

$$\frac{z}{l}(P_n + wy_n) \overset{=}{>} P_2 + \frac{P_3}{p}e.$$

$$M = \frac{M_r z}{l} - P_2(x_2 + e) - \frac{P_3 x_3}{p}e.$$

Application to Skew Spans.—There are two cases of skew spans : 1st, where the end floor beams are supported by both trusses; 2d, where the end floor beams rest on the masonry, or are attached at one end to the end foot of one truss. The skew in no way affects the conditions for finding the positions of the system for maximum shear and moment, but in finding these positions the loading *is to be taken along the centre line.* Since the panels on the centre line are not necessarily equal, we cannot put in general $Np = l$, and hence the condition for maximum shear is to be written

$$\frac{(P_n + wy_n)\,p}{l} \overset{=}{>} P_m, \text{ or } \overset{=}{>} \frac{d + np}{d},$$

according as the chords are horizontal or inclined, where p is the *length of the panel on the centre line*, for which the shear is required, and l is the length of centre line or span.

The condition for maximum moment for the chords is

$$\frac{(P_n + wy_n)\,z}{l} \overset{=}{>} P_m, \text{ or } \overset{=}{>} P_2 + \frac{P_3}{p}e,$$

according as the bracing is vertical and inclined, or all inclined, page 219, where we take in like manner $z =$ the distance from *left end of centre line* to the point in question, *on centre line.*

The positions being thus determined, the shear and moment must be found for each truss as follows :

CASE 1. *When the end floor beams are supported by both trusses.*—For the moment at the right end of Truss A, Fig. (d), when the position of the load system on the centre line has been found as directed, we must take the uniform train load wy_n as concentrated at the centre of y_n, and then find its moment with reference to end of Truss A. Its lever arm is, therefore, $y_n - \frac{s}{2} - \frac{y_n}{2} = \frac{y_n - s}{2}$. Hence, for moment at right end of Truss A, we have, if we denote the skew by s,

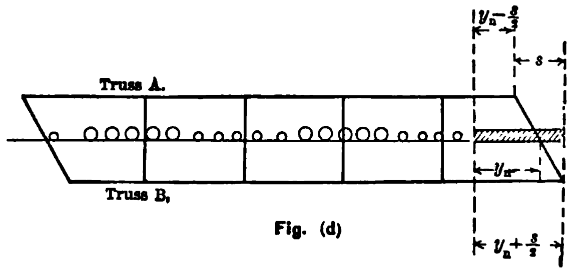


Fig. (d)

$$M_r = M_n + P_n\left(y_n - \frac{s}{2}\right) + wy_n\left(\frac{y_n - s}{2}\right),$$

and in like manner, for Truss B,

$$M_r = M_n + P_n\left(y_n + \frac{s}{2}\right) + wy_n\left(\frac{y_n + s}{2}\right).$$

The shear is now given by

$$S = \frac{M_r}{l} - \frac{M_z}{p},$$

and the moment by

$$M = \frac{M_r}{l}z - M_e,$$

where p is the actual panel length of *truss itself*, and z is the distance *on the truss* from left end to point in question.

It must be distinctly remembered, that while $\frac{z}{l}$ is a ratio on the centre line in getting the *position* for maximum moment, it is a ratio *on the truss*, in getting the moment itself. Also, that $\frac{p}{l}$ is always a ratio on the centre line.

CASE 2. *When the end floor beam rests on the masonry, or is attached to the end foot of one truss.*—In this case all loads between a and the right end come directly upon the masonry at b, and have no effect whatever upon Truss A.

We have, therefore, for Truss A,

$$M_r = M_n + P_n\left(y_n - \frac{s}{2}\right) + \frac{w}{2}\left(y_n - \frac{s}{2}\right)^2,$$

while we have for Truss B

$$M_r = M_n + P_n\left(y_n + \frac{s}{2}\right) + wy_n\left(\frac{y_n + s}{2}\right).$$

We see at once that Truss B is exactly the same as in Case 1. But Truss A is the same as a square span of length cd so far as shear and moments are concerned, but in determining *positions* its length is ab, and we simply consider all loads on centre line, between a and right end, as non-existing.

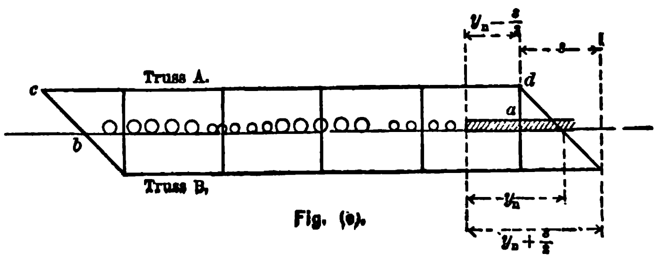
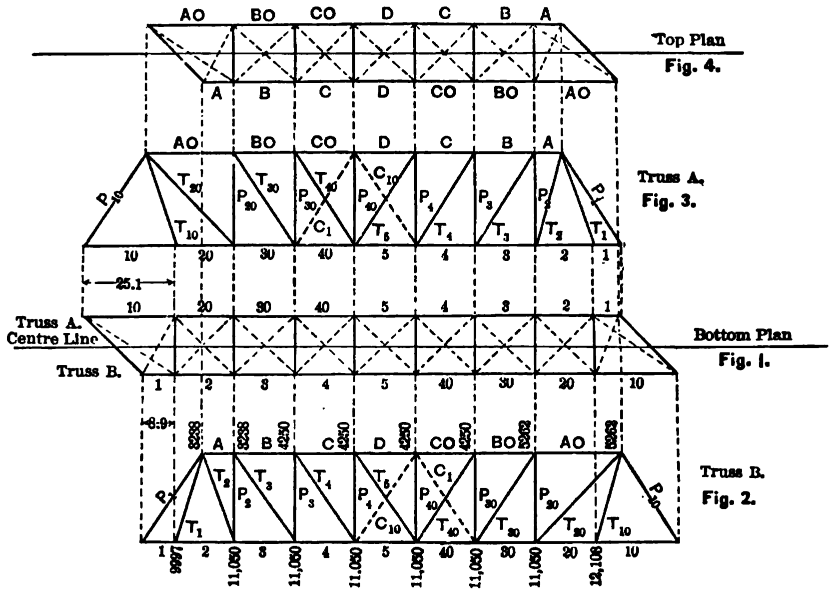


Fig. (e).

Truss B we treat exactly as in Case 1. Since the forward end of Truss A is the same as the rear end of Truss B, we have only to compute one end of each truss.

In case the skew is just one panel length, the strains at each end of Truss B will be equal for both chords and web.

EXAMPLE 5. SKEW SPAN.—Let us take the span at 153 feet, with 9 panels of 17 feet each on the centre line; two Pratt Trusses, depth 26 feet, width between trusses 16.25 feet. Skew $= 45°$ right end forward, or $s = 16.25$ feet.



In Fig. 1 we have represented the bottom plan with the wind bracing. In Fig. 2 we have the elevation of Truss B, and in Fig. 3 the elevation of Truss A. It will be noted that the end posts of both trusses have the same inclination. In Fig. 4 we have the top plan with wind bracing. At the right end of Truss B and left end of Truss A the post at the end of T_{10} is omitted. This is done to save material. The post at the foot of T_1 is, however, retained in Truss B and A. This is necessary in order that the top wind bracing may be as shown in Fig. 4.

The student should study carefully these different Figs., and observe the notation adopted, members on one side of the centre being denoted by letters and numbers, and on the other side by the same letters and numbers, with o annexed. All posts or struts are denoted by P, all tension braces by T, counters by C, lower flanges by numbers, upper flanges by letters.

Thus B is second upper panel from left end of Truss B, and Bo, the corresponding panel on right end. Truss A is the same as Truss B turned round. The strains in the left half of Truss A are the same as for the right half of Truss B.

Dead Load Strains.—Let us take the track at 400 lbs. per lineal foot, and the cross girders, stringers, floor, etc., at 380 lbs. per lineal foot. Then we have $\frac{780}{2} = 390$ lbs. per lineal foot for each truss, *applied along the centre line.*

This gives $17 \times 390 = 6630$ lbs. at every lower apex of Truss B, if the bridge is a through span.

Suppose the weight of the trusses themselves and the wind bracing is 1020 lbs. per lineal foot. This gives 510 lbs. per lineal foot for each truss, of which we assume 250 lbs. for the upper chord and 260 lbs. for the lower chord, *applied along the truss itself.*

We have, then, at the first and second upper apices $(8.5 + 4.45)\ 250 = 3238$ lbs. At the last and next to last upper apex $(8.5 + 12.55)\ 250 = 5262$ lbs. At all the other upper apices $17 \times 250 = 4250$ lbs.

At the first lower apex, we have $(8.5 + 4.45)\ 260 = 3367$ lbs., at the last lower apex $(8.5 + 12.55)\ 260 = 5473$ lbs., at all the other lower apices, $17 \times 260 = 4420$ lbs.

Taking both these loadings, we have the apex loads given in Fig. 2, Truss B, and the dead load strains are the strains due to these loads.

For T_1 and T_{10} we have $\tan\theta = 0.312$, $\sec\theta = 1.048$. For T_2 we have $\tan\theta = 0.342$, $\sec\theta = 1.057$. For T_{20}, we have $\tan\theta = 0.965$, $\sec\theta = 1.39$. For all other inclined members $\tan\theta = 0.654$, $\sec\theta = 1.195$.

The left reaction for Truss B is given by

$$R \times 153 = 9997 \times 144.1 + 11050\,(127.1 + 110.1 + 93.1 + 76.1 + 59.1 + 42.1) + 12103 \times 25.1 + 3238\,(136 + 127.1) + 4250\,(110.1 + 93.1 + 76.1 + 59.1) + 5262\,(42.1 + 17).$$

Hence $R = 65062$ lbs. The right-hand reaction is 57338 lbs.

We have, therefore, for the dead load strains,

$P_1 = 65062 \times 1.195 = +77749$ lbs.	$T_1 = -9997 \times 1.048 = -10477$ lbs.
$T_2 = -51827 \times 1.057 = -54781$ "	$T_3 = -37539 \times 1.195 = -44859$ "
$T_4 = -22239 \times 1.195 = -26575$ "	$P_2 = 37539 + 3238 = +40777$ "
$P_3 = 22239 + 4250 = +26489$ "	$T_5 = -6939 \times 1.195 = -8292$ "
$P_4 = 6939 + 4250 = +11189$ "	$P_{40} = +4250$
$T_{40} = -8361 \times 1.195 = -9991$ "	$P_{30} = 8361 + 4250 = +12611$ "
$T_{30} = -23661 \times 1.195 = -28275$ "	$P_{20} = 23661 + 5262 = +28923$ "
$T_{20} = -39973 \times 1.39 = -55562$ "	$T_{10} = -12103 \times 1.048 = -12684$ "
$P_{10} = 57338 \times 1.195 = +68519$ "	

The strains are the same in the members of Truss A, which are denoted by the same letters.

$A \times 26 = 65062 \times 25.9 - 9997 \times 17 - 3238 \times 8.9$	$A = +57167$ lbs.
$B \times 26 = 65062 \times 42.9 - 9997 \times 34 - 3238 \times 25.9 - 14288 \times 17$	$B = +81723$ "
$C \times 26 = 65062 \times 59.9 - 9997 \times 51 - 3238 \times 42.9 - 14288 \times 34 - 15300 \times 17$	$C = +96274$ "
$D \times 26 = 65062 \times 76.9 - 9997 \times 68 - 3238 \times 59.9 - 14288 \times 51 - 15300\,(17 + 34)$	$D = +100820$ "
$Co \times 26 = 57338 \times 76.1 - 12103 \times 51 - 5262 \times 59.1 - 16312 \times 34 - 15300 \times 17$	$Co = +100787$ "
$Bo \times 26 = 57338 \times 59.1 - 12103 \times 34 - 5262 \times 42.1 - 16312 \times 17$	$Bo = +95307$ "
$Ao \times 26 = 57338 \times 42.1 - 12103 \times 17 - 5262 \times 25.1$	$Ao = +79850$ "

$3 = -57167$ lbs. $4 = -81723$ lbs. $5 = -96274$ lbs. $40 = -95307$ lbs.
$30 = -79850$ "

$1 \times 26 = -65062 \times 17$	$1 = -42540$ lbs.
$2 \times 26 = -65062 \times 17 + 9997 \times 8.1$	$2 = -39426$ "
$20 \times 26 = -57338 \times 17 - 12103 \times 8.1$	$20 = -41261$ "
$10 \times 26 = -57338 \times 17$	$10 = -37490$ "

Live Load Strains.—In the present case $s = 16.25$ feet. For Truss B, we have, page 228,

$$M_r = M_n + P_n\left(y_n + \frac{s}{2}\right) + wy_n\left(\frac{y_n + s}{2}\right),$$

and for the criterion for position for maximum shear, since the chords are horizontal,

$$(P_n + wy_n)\frac{p}{l} \underset{>}{=} P_n,$$

where p is to be taken *on the centre line.*

The maximum shear itself is given by

$$S = \frac{M_r}{l} - \frac{M_n}{p},$$

where p is the panel length *on the truss itself.*

Applying our diagram, we have the following results for Truss B. We take, of course, one-half of the shear for one truss.

Position on Centre Line.	y_n	M_r	M_n	$\frac{S}{2}$	
p_4 at 17 feet from left,	48.2	55309226	590400	147580	$P_1 = +\ 176358$ lbs.
p_3 at 34 " "	26.9	41879598	304800	127896	$T_2 = -\ 135186$ "
p_3 at 51 " "	9.9	32460238	304800	97114	$P_2 = +\ 97114$ " $T_3 = -\ 116051$ "
p_3 at 68 " "	5.4	24103333	304800	69804	$P_3 = +\ 69804$ " $T_4 = -\ 83416$ "
p_2 at 85 " "	0.6	15548346	128000	47046	$P_4 = +\ 47046$ " $T_5 = -\ 56220$ "
p_2 at 102 " "	4.3	9888666	128000	28551	$T_{40} = +\ 34118$ "
p_2 at 119 " "	4.8	5706933	128000	14885	$T_{30} = +\ 17787$ "

T_{40} must be counterbraced for $34118 - 9991 = 24127$ lbs., and this is, therefore, the tension in C_1.

As the dead load tension in T_{30} is greater than 17787, T_{30} does not need to be counterbraced.

For Truss A, we have

$$M_r = M_n + P_n\left(y_n - \frac{s}{2}\right) + wy_n\left(\frac{y_n - s}{2}\right).$$

Hence, for Truss A, we have

Position on Centre Line.	y_n	M_r	M_n	$\frac{S}{2}$	
p_4 at 17 feet from left,	48.2	44928266	590400	135063	$P_{10} = +\ 161400$ lbs.
p_3 at 34 " "	26.9	32871346	304800	98457	$T_{20} = -\ 136855$ ".
p_3 at 51 " "	9.9	24562306	304800	71304	$P_{20} = +\ 71304$ " $T_{30} = -\ 85208$ "
p_3 at 68 " "	5.4	17493733	304800	48204	$P_{30} = +\ 48204$ " $T_{40} = -\ 57604$ "
p_2 at 85 " "	0.6	10001466	12800	28918	$P_{40} = +\ 28918$ " $T_5 = +\ 34557$ "
p_2 at 102 " "	4.3	6024666	12800	15923	$T_4 = +\ 19028$ "

T_5 must be counterbraced for $34557 - 8292 = 26265$ lbs., and this is the tension in C_{10}. As the dead load tension in T_4 is greater than 19028, T_4 does not need to be counterbraced.

Finally, we have, for the greatest load concentration which can come at the foot of T_1 or T_{10} when p_4 is at the foot, 45620 lbs. Hence,

$$T_1 = -\ 45620 \times 1.048 = -\ 48810 \text{ lbs., and } T_{10} = -\ 45620 \times 1.39 = -\ 63412 \text{ lbs.}$$

For the chords we can find panels 1 and 10 by simply multiplying the maximum end shears already found by $\tan\theta$. Hence, we have,

$$1 = -\ 147580 \times 0.654 = -\ 96517 \text{ lbs.} \qquad 10 = -\ 135063 \times 0.654 = -\ 88331 \text{ lbs.}$$

For panel 2 we have already found the maximum end shear for p_4 at first lower apex of Truss B, 147580 lbs., and the concentration at this point 45620 lbs. Hence

$$2 \times 26 = -\ 147580 \times 17 + 45620 \times 8.1, \text{ or } 2 = -\ 82282 \text{ lbs.}$$

For panel 20 we have found the maximum end shear for p_4 at first lower apex of Truss A, 135063 lbs., and the concentration at this point 45620 lbs. Hence

$$20 \times 26 = -135063 \times 17 - 45620 \times 8.1, \text{ or } 20 = -102522 \text{ lbs.}$$

For the other panels we have the following results:

Position on centre line,	y_n	M_r	M_s	M	
p_{15} at 34 feet from left,	90.1	54751373	2206333	7062033	$A = +135808$ lbs. $3 = -135808$ "
p_{14} " 34 " "	85.9	40695413	1633733	9564151	$Ao = +183930$ " $30 = -183930$ "
p_{15} " 51 " "	73.1	53759226	5108186	9965458	$B = +191643$ " $4 = -191643$ "
p_{14} " 51 " "	68.9	43984866	5556505	11433740	$Bo = +219880$ " $40 = -219880$ "
p_{14} " 68 " "	51.9	55209026	10083866	11530629	$C = +221740$ " $5 = -221740$ "
p_{14} " 68 " "	51.9	44826746	10083866	12212292	$Co = +234850$ "

The case which we have solved is that of Fig. (d), page 228. The student should find no difficulty with the case of Fig. (e), page 228, or for the case where the skew is " right forward " and " left back," or *vice versa*, instead of, as in this case, " right and left forward."

SKEW SPAN ON CURVES.—This is perhaps the most complicated case which can arise.

The curve of the track will cause little or no difference in the dead load strains. As to the live load, our diagram will be practically unaffected, and is to be used as in the preceding case to find positions giving maximum shear and moment at any point. But when these positions are known we must find the wheel load concentration for any position *at each floor beam*, where it is crossed by the centre line of the track, and then find the portion of each such concentration which goes to each truss, *according to the point at which this concentration acts on each floor beam.*

This involves considerable tedious figuring in order to determine maximum positions. But with this explanation and the work of the preceding case fully understood, there should be no difficulty in solving any such example.

PIVOT SPAN.—We have shown on page 165 how to find the strains in every member for an apex load of 40 tons at each apex, and given a Table from which the maximum strains can be found for such loading.

If we divide the results of this table by 40 we shall have the strains due to a moving load of *one ton per apex*. Let a Table be drawn up giving the strains due to this load.

If now we wish to apply our system of concentrated loads as given by our diagram, so as to find the maximum strain in any member, as, for instance, *gh*, we can proceed as follows:

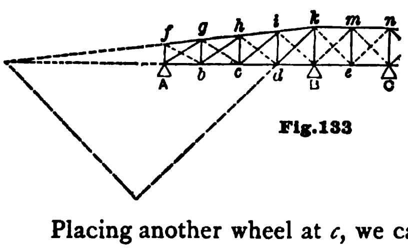


Fig.133

Place a wheel at *c*, and find the load concentrations *in tons*, which act at each panel point, due to this position. Then from the Table we have drawn up, giving the strains in each member for a load of *one ton* at each apex, we can easily find the strain in *gh*, due to the given concentrations.

Placing another wheel at *c*, we can proceed as before. A few trials will give that position which gives the maximum strain in *gh*.

So for any other member.

The same method can be used for the case of the continuous girder.

CHAPTER II.

THEORY OF FLEXURE.

COEFFICIENT OF ELASTICITY.—If a weight P acts upon a piece whose area of cross section is A, and elongates or compresses it by a small amount, λ, we know, from experiment, that within certain limits, twice, three times or four times that weight will produce a displacement of 2λ, 3λ, 4λ, etc. The limit up to which this law of proportionality of force to displacement holds true is called the *elastic limit.* Practically, then, within this limit, *the displacement is directly as the force.* We say "practically," because theoretically there is no such precise limit. In practice, however, it is not difficult to fix by experiment that point beyond which the displacements sensibly diverge from the above law. No material should be strained in use beyond this limit. Hence, in all practical cases we consider the law as sensibly correct.

If we were to assume this law as strictly true for *all* values of the displacement, and if the original length of the piece is L, then the force per unit of area $\frac{P}{A}$ which causes the elongation λ, if it is $\frac{L}{\lambda}$ times as great will cause an elongation of $\lambda \frac{L}{\lambda}$, or of L. This force we call the *coefficient of elasticity.* It is always denoted by E. Hence

$$E = \frac{PL}{A\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The coefficient of elasticity, then, *is that force per unit of area which would elongate a perfectly elastic body* BY ITS OWN LENGTH.

It is, then, a purely theoretical force. But as the law of perfect elasticity which it presupposes holds good practically within certain limits, if we make experiments well within those limits, measuring P, L, A and λ, we can find very accurately what force would cause the elongation L, if the law of proportionality of elongation to applied force held true without limits. Such experiments have been made, and the values of E for different materials are to be found in any work upon the strength of materials.

The value of E thus determined is an accurate measure of the elasticity of any material, since, other things being the same, it depends directly upon the amount of stretch caused by a given weight. It varies, of course, with different materials, and within certain limits even with the same material, due to processes of manufacture, etc. Thus, the coefficient of elasticity of iron, varies with the kind, whether wrought or cast, with the shape, whether in bars or rods, etc.

In any particular case, however, we may consider it as a constant. Iron produced at the same establishment, submitted to the same processes at the same time, ought to be identical in all its properties. In preliminary estimates it is allowable to take a mean value as given by experiments upon the same kind of material as that considered. It is, therefore, assumed, in the Theory of Flexure, that E is a constant quantity for each material. Thus we may take for wrought iron, in general, when special experiments are not at hand, $E = 25,000,000$ lbs. per square inch.

Considering then E as a constant, we have from (1)

$$\lambda = \frac{PL}{EA} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

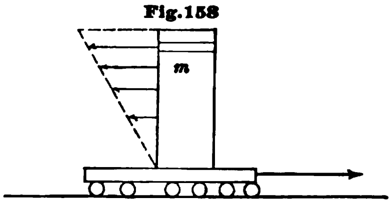
From this equation we can always compute the elongation (or compression) due to a given load, when the dimensions of the piece are known. Or inversely, knowing the elongation (or compression) we can compute with considerable accuracy the force which produces it.

The coefficient of elasticity is thus of great importance and service in all discussions of the strength of materials.

Moment of Inertia.—This is a convenient term for a quantity which occurs so often in the applications of the theory of flexure as to make a special term for it desirable.

The moment of a force is the product of the force by its lever arm. But it often happens, especially in the theory of flexure, that we have to do with forces which are themselves dependent in magnitude upon their lever arms, and vary directly with them. The moment of such a force would be a function of the square of the lever arm. Hence the expression "moment of inertia" as a general term for the moment of all such forces.

Fig. 158

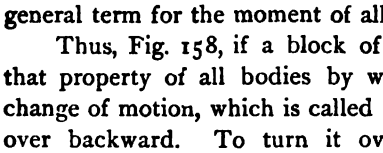


Thus, Fig. 158, if a block of wood standing on end is suddenly pulled forward, by virtue of that property of all bodies by which they offer resistance to change of motion, which is called "*inertia*," the block may fall over backward. To turn it over, however, requires force. Upon every portion of the block there must be a force acting which depends upon the mass of that portion. But a force near the top acts evidently with more effect to cause rotation, than one nearer the bottom. The rotation force depends, then, not only upon the mass but also upon the distance from the bottom. If x is the distance of any small section of the block from the bottom, and m is its mass, then the force at this point is mx. The moment of this force is then mx^2. The moment of inertia of the whole block is, then, the sum of all the elementary reactions, each multiplied by the square of its distance from the bottom. This sum measures the rotative effect just as the sum of the simple moments would, if the applied forces were all equal. That is, it gives the force which acting at a unit's distance would have the same effect. The product mx^2 is called the "moment of inertia."

The same term is applied to all other cases where the forces vary according to the same law; as would be the case if the block sustained the pressure of water, for instance.

The moment of inertia with respect to any axis, then, is a general term for *the algebraic sum of the products obtained by multiplying the mass of every element of a body by the square of the distance of that element from the axis.*

Fig. 159

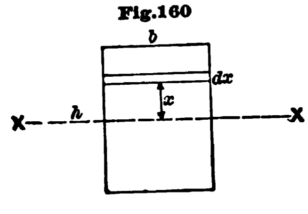


In the theory of flexure, where a body is bent, as is shown in Fig. 159, we assume that the upper fibres are extended, and the lower ones compressed. There is then a line of fibres at the centre which is neither extended nor compressed. This is called the "*neutral axis.*" Above and below this axis the forces of extension or compression are directly proportional to their distance. Their moments will then be proportional to the square of the distance. The case is then precisely similar to that of Fig. 158, and the *moment of inertia* of the cross section with reference to the axis through the centre of gravity will give us that force which acting at a unit's distance would produce the same effect as all the fibre forces which act in reality.

We see thus the meaning of the term, and how it enters into our discussion.

Determination of Moment of Inertia.—By the aid of the calculus we can readily determine the moment of inertia for all the most usual cross sections. As we assume that the fibres at equal distances on each side of the neutral axis are elongated or compressed equally, the neutral axis passes through the centre of gravity of the cross section. We should find the moment of inertia, therefore, of all cross sections *with respect to an axis through the centre of gravity.*

Fig. 160



Rectangle.—Let the breadth, Fig. 160, be b, and height h. Suppose a strip at a distance x from the axis XX through the centre of gravity. The mass of this strip will be proportional to its volume. Its volume will be equal to its area if the section is one unit in thickness. The area is bdx. The moment of inertia of the strip is then

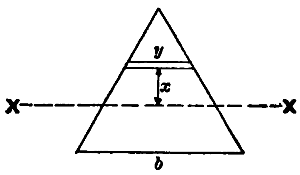
$$b\,x^2\,dx.$$

Integrating this between $+\frac{h}{2}$ and $-\frac{h}{2}$ we have

$$I = \int_{-\frac{h}{2}}^{+\frac{h}{2}} b\,x^2\,dx = \frac{bh^3}{12}.$$

Triangle.—Let the base of the triangle, Fig. 161, be b and the height h, and take the axis XX through the centre of gravity, or $\frac{2}{3}h$ below the apex.

Fig. 161



Take a strip at a distance x from the axis. The length of this strip y is from similar triangles, given by the proportion

$$\tfrac{2}{3}h - x : y :: h : b, \quad \text{or } y = \frac{(\frac{2}{3}h - x)\,b}{h}.$$

The area of the strip is, then,

$$y\,dx = \frac{\frac{2}{3}hbdx - bxdx}{h} = \tfrac{2}{3}bdx - \frac{bxdx}{h}.$$

The moment of inertia of the strip is

$$yx^2\,dx = \tfrac{2}{3}bx^2\,dx - \frac{bx^3\,dx}{h}.$$

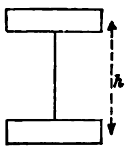
Integrating between $+\frac{2}{3}h$ and $-\frac{1}{3}h$, we have

$$I = \int_{-\frac{1}{3}h}^{+\frac{2}{3}h} \tfrac{2}{3}bx^2\,dx - \frac{bx^3\,dx}{h} = \frac{bh^3}{36}.$$

Radius of Gyration.—If in any case we divide the moment of inertia by the area of cross section, we obtain the square of the distance from the axis to that point at which, if the entire mass were concentrated, the moment of inertia would be the same as that of the cross section itself. This distance we call the "*radius of gyration.*" The value of $\frac{I}{A}$ is then, in general, the square of the radius of gyration. Thus for the rectangle above, the area is bh, hence $\frac{I}{A} = \frac{h^2}{12} = \left(\frac{h}{2\sqrt{3}}\right)^2$ The radius of gyration for the rectangle is then $\frac{h}{2\sqrt{3}}$. In the same way for the triangle, the area is $\frac{bh}{2}$. The radius of gyration is, then, $\frac{h}{3\sqrt{2}}$.

If in any case the radius of gyration is known, we have only to multiply its square by the area in order to obtain the moment of inertia of the cross section.

Fig. 162



In the case of a braced girder, where most of the material is in the flanges, Fig. 162, the radius of gyration may be taken as approximately $\frac{h}{2}$, or half the depth of the girder. If we denote the total flange area by F, the moment of inertia of the cross-section is then $F\frac{h^2}{4}$.

We give below the moment of inertia I for various cross sections, for horizontal axis through the centre of gravity. We also give the area F of the cross section as well as the distance e of the outer fibre from the neutral axis.

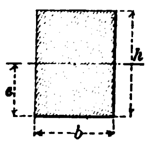
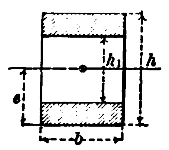
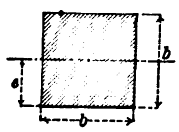
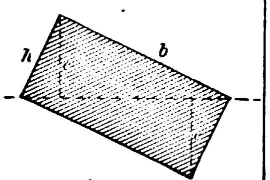
Reduction of moment of inertia.—If we denote by I, the moment of inertia of any cross-section with reference to an axis passing through the centre of gravity, then the moment of inertia of the same cross-section with reference to a parallel axis which does not pass through the centre of gravity, is given by

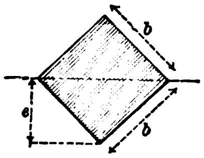
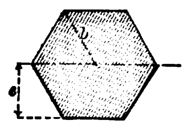
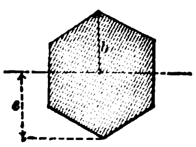
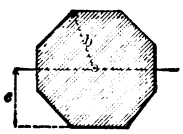
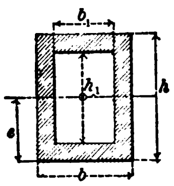
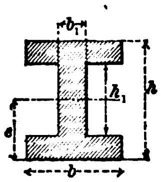
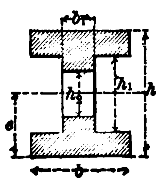
$$I' = I + Fd^2,$$

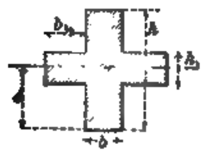
where d is the distance between the two axes. That is, *the moment of inertia of a cross-section, with reference to an eccentric axis, is equal to the moment of inertia with reference to a parallel axis passing through the centre of gravity, plus the product of the area of the cross-section into the square of the distance between the two axes.*

This principle is demonstrated in all Text Books of Mechanics, and enables us to find at once the moment of inertia with reference to any axis, when the moment of inertia with reference to a parallel axis through the centre of gravity is known.

We give below the moment of inertia I, for various cross-sections, such as are likely to occur in practice, for horizontal axis through the centre of gravity. The student would do well to check them by computation. We also give the area F of the cross-section, as well as the distance e of the outer fibre from the neutral axis.

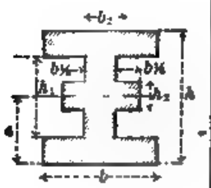
	Axis of x $\begin{cases} F = bh, \\ e = \frac{h}{2}, \\ I = \frac{1}{12} bh^3. \end{cases}$ Axis of y $\begin{cases} e = \frac{b}{2}, \\ I = \frac{1}{12} hb^3. \end{cases}$
	Axis of x $\begin{cases} F = b(h - h_1), \\ e = \frac{h}{2}, \\ I = \frac{b(h^3 - h_1^3)}{12}. \end{cases}$ Axis of y $\begin{cases} e = \frac{b}{2}, \\ I = \frac{(h - h_1) b^3}{12}. \end{cases}$
	$F = b^2$, $e = \frac{b}{2}$, $I = \frac{b^4}{12}$.
	$F = bh$, $e = \frac{bh}{\sqrt{b^2 + h^2}}$, $I = \frac{b^3h^3}{6(b^2 + h^2)}$.

	$F = b^2,$ $e = \dfrac{b}{\sqrt{2}},$ $I = \dfrac{b^4}{12}.$
	$F = 2.598\ b^2 = \dfrac{3\ b^2 \sqrt{3}}{2},$ $e = 0.866\ b = \dfrac{b \sqrt{3}}{2},$ $I = 0.5413\ b^4 = \dfrac{5\ b^4 \sqrt{3}}{16}.$
	$F = 2.598\ b^2 = \dfrac{3\ b^2 \sqrt{3}}{2},$ $e = b,$ $I = 0.5413\ b^4 = \dfrac{5\ b^4 \sqrt{3}}{16}.$
	$F = 2.828\ b^2 = 2\ b^2 \sqrt{2}.$ $e = 0.924\ b = \dfrac{b}{2}\sqrt{2 + \sqrt{2}} = b \cos 22\tfrac{1}{2}°,$ $I = 0.638\ b^4 = \dfrac{b^4}{6}(1 + 2\sqrt{2}).$
	$F = bh - b_1 h_1,$ Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}(bh^3 - b_1 h_1^3). \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2} \\ I = \frac{1}{12}(hb^3 - h_1 b_1^3). \end{cases}$
	$F = bh - (b - b_1)\ h_1,$ Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}[bh^3 - (b - b_1) h_1^3]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}[(h - h_1)\ b^3 + h_1 b_1^3]. \end{cases}$
	$F = b\,(h - h_1) + b_1\,(h_1 - h_2),$ Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}[b\,(h^3 - h_1^3) + b_1\,(h_1^3 - h_2^3)]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}[(h - h_1)\ b^3 + (h_1 - h_2)\ b_1^3] \end{cases}$



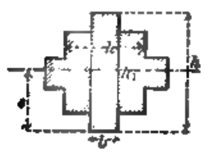
$$F = bh + b_1h_1,$$

Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}(bh^3 + b_1h_1^3). \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b + b_1}{2}, \\ I = \frac{1}{12}[hb^3 + h_1(b + b_1)^3 - h_1b^3]. \end{cases}$



$$F = bh - (b - b_1)h_1 + b_2h_2,$$

Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \frac{1}{12}[bh^3 - (b - b_1)h_1^3 + b_2h_2^3]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}[(h - h_1)b^3 + (h_1 - h_2)b_1^3 + h_2(b_1 + b_2)^3]. \end{cases}$



$$F = bh + (h_1 - b)h_1 + (h - h_1)b,$$

$$e = \frac{h}{2},$$

$$I = \tfrac{1}{12}[bh^3 + (h_1 - b)h_1^3 + (h - h_1)b^3].$$

$$F = 3bh_1,$$

Axis of x $\begin{cases} e = \dfrac{h}{2}, \\ I = \dfrac{b}{12}[9h_1^3 + 6hh_1(h - 2h_1)]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{3}{12}h_1b^3. \end{cases}$

$$F = 4bh_2,$$

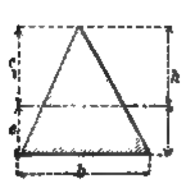
Axis of x $\begin{cases} e = \dfrac{h}{2}, \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{4}{12}h_2b^3. \end{cases}$

$- 2h_2) + 6h_1h_2(h_1 - 2h_2)].$

$h_2) + b_2(h_2 - h_3) + b_3h_3,$

$\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}[(h - h_1)b^3 + (h_1 - h_2)b_1^3 + (h_2 - h_3)b_2^3 + h_3b_3^3]. \end{cases}$

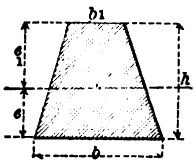
$(h_1^3 - h_2^3) + b_2(h_2^3 - h_3^3) + b_3h_3^3].$



$$F = \frac{bh}{2},$$

$e_1 = \frac{2}{3}h,\quad e = \frac{1}{3}h,$ $\qquad e_1 = h,\quad e = 0,$ $\qquad e_1 = 0,\quad e = h,$

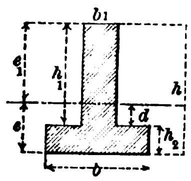
$I = \dfrac{bh^3}{36}.$ $\qquad I = \dfrac{bh^3}{12}.$ $\qquad I = \dfrac{bh^3}{4}.$



$$F = \frac{b + b_1}{2}\,h,$$

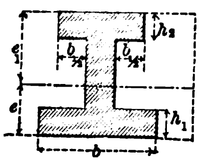
$$e = \frac{b + 2\,b_1}{b + b_1}\,\frac{h}{3},\quad e_1 = \frac{2\,b + b_1}{b + b_1}\,\frac{h}{3},$$

$$I = \frac{b^2 + 4\,bb_1 + b_1^2}{b + b_1}\,\frac{h^3}{36}.$$



$$F = b_1h_1 + bh_2,$$

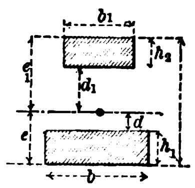
Axis of x $\begin{cases} e = \dfrac{bh_2^2 + b_1h_1\,(h + h_2)}{2\,[bh - (b - b_1)\,h_1]}, \\ I = \frac{1}{3}\,[b\,(e^3 - d^3) + b_1\,(d^3 + e_1^3)]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}\,(h_1b_1^3 + h_2b^3). \end{cases}$



$$F = bh_1 + b_1h_2 + \delta\,h,$$

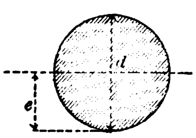
Axis of x $\begin{cases} e = \dfrac{\delta h^2 + 2\,b_1h_2h + bh_1^2 - b_1h_2^2}{2\,(\delta h + bh_1 + b_1h_2)}, \\ I = \frac{1}{3}\,[(b + \delta)\,e^3 - b\,(e - h_1)^3 + (b_1 + \delta)\,e_1^3 - b_1\,(e_1 - h_2)^3]. \end{cases}$

Axis of y $\begin{cases} e = \dfrac{b + \delta}{2}, \\ I = \frac{1}{12}\,[h_2\,(b_1 + \delta)^3 + (h - h_1 - h_2)\,\delta^3 + h_1\,(b + \delta)^3]. \end{cases}$



$$F = bh_1 + b_1h_2,$$

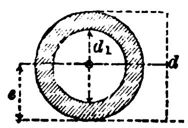
Axis of x $\begin{cases} e = \dfrac{b_1h_2\,(2\,h - h_2) + bh_1^2}{2\,(bh_1 + b_1h_2)}, \\ I = \dfrac{b}{3}\,[e^3 - d^3] + \dfrac{b_1}{3}\,[e_1^3 - d_1^3]. \end{cases}$ Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = \frac{1}{12}\,(h_2b_1^3 + h_1b^3). \end{cases}$



$$F = \frac{\pi}{4}\,d^2,\quad \pi = 3.1416,$$

$$e = e_1 = \frac{d}{2},$$

$$I = \frac{\pi}{64}\,d^4 = 0.0491\,d^4.$$



$$F = \frac{\pi}{4}\,(d^2 - d_1^2),$$

$$e = e_1 = \frac{d}{2},$$

$$I = 0.0491\,(d^4 - d_1^4).$$

$$F = \frac{\pi}{4} bh,$$

$$e = e_1 = \frac{h}{2},$$

$$I = \frac{\pi}{64} bh^3 = 0.0491\, bh^3.$$

$$F = \frac{\pi}{4} (bh - b_1h_1),$$

$$e = e_1 = \frac{h}{2},$$

$$I = 0.0491\, (bh^3 - b_1h_1^3).$$

$$F = \frac{\pi r^2}{2},$$

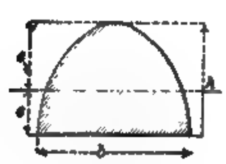
$$e_1 = 0.5765\, r, \quad e = 0.4244\, r,$$

$$I = 0.1098\, r^4.$$

$$F = \frac{\pi r^2}{2},$$

$$e = e_1 = r,$$

$$I = 0.3927\, r^4.$$



$$F = \tfrac{2}{3}\, bh,$$

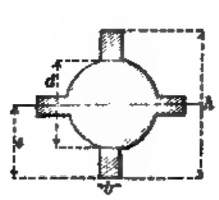
$$e = \tfrac{3}{5}\, h, \quad e_1 = \tfrac{2}{5}\, h,$$

$$I = \tfrac{8}{175}\, bh^3 = \tfrac{12}{175}\, Fh.$$

$$F = \tfrac{2}{3}\, bh,$$

$$e = e_1 = \frac{h}{2},$$

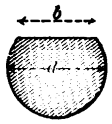
$$I = \tfrac{1}{30}\, bh^3 = \tfrac{1}{20}\, Fh^2.$$



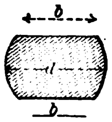
$$F = \frac{\pi}{4} d^2 + 2b(h - d),$$

$$e = e_1 = \frac{h}{2},$$

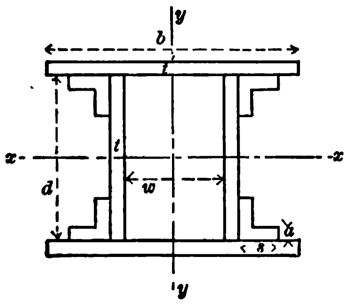
$$I = \tfrac{1}{12}\left[\frac{3\pi}{16} d^4 + b(h^3 - d^3) + b^3(h - d)\right].$$



$$\begin{cases} b = \dfrac{d}{3}, \quad e = 0.476\, d, \quad F = 0.779\, d^2, \\ I = 0.048\, d^4, \\ b = \dfrac{d}{2}, \quad e = 0.447\, d, \quad F = 0.763\, d^2, \\ I = 0.044\, d^4. \end{cases}$$



$$\begin{cases} b = \dfrac{d}{3}, \quad e = e_1 = 0.471\, d_1, \quad F = 0.714\, d^2, \\ I = 0.047\, d^4, \\ b = \dfrac{d}{2}, \quad e = e_1 = 0.433\, d, \quad F = 0.711\, d^2, \\ I = 0.043\, d^4. \end{cases}$$



$$F = 2\, bt' + 4\,(2\, sa - a^2) + 2\, dt,$$

Axis of x

$$\begin{cases} e = d + t', \\ I = \dfrac{bt'^3}{6} + bt'\dfrac{(d+t')^2}{2} + \dfrac{(s+t)\,d^3}{6} - \left[\dfrac{(s-a)(d-2a)^3 + a(d-2s)^3}{6}\right]. \end{cases}$$

If one plate is replaced by latticing, $I = \dfrac{bt'^3}{12} + bt'\dfrac{(d+t')^2}{4} + \text{etc.}, \quad e = d + \dfrac{t'}{2},$

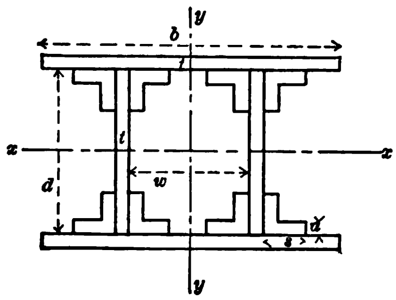
$$F = bt' + \text{etc.}$$

Axis of y

$$\begin{cases} e = \dfrac{b}{2}, \\ I = \dfrac{t'b^3}{6} + \dfrac{a(w+2t+2s)^3}{6} + \dfrac{(s-a)\,(w+2t+2a)^3}{6} + \dfrac{(d-2s)(w+2t)^3}{12} \\ \qquad - \dfrac{dw^3}{12}. \end{cases}$$

If one plate is replaced by latticing, $I = \dfrac{t'b^3}{12} + \text{etc.} \quad F = bt' + \text{etc.}$

If both plates are replaced by latticing, $t' = 0$.



$$F = 2bt' + 8\,(2sa - a^2) + 2dt,$$

Axis of x

$$\begin{cases} e = d + t', \\ I = \dfrac{bt'^3}{6} + bt'\dfrac{(d+t')^2}{2} + \dfrac{(2s+t)d^3}{6} - \left[\dfrac{(s-a)(d-2a)^3 + a(d-2s)^3}{3}\right]. \end{cases}$$

If one plate is replaced by latticing, $I = \dfrac{bt'^3}{12} + bt'\dfrac{(d+t')^2}{4} + \text{etc.} \quad e = d + \dfrac{t'}{2},$

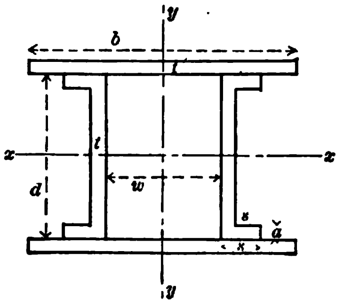
$$F = bt' + \text{etc.}$$

Axis of y

$$\begin{cases} e = \dfrac{b}{2}, \\ I = \dfrac{t'b^3}{6} + \dfrac{a[(w+2t+2s)^3 - (w-2s)^3]}{6} \\ \qquad + \dfrac{(s-a)\,[(w+2t+2a)^3 - (w-2a)^3]}{6} + \dfrac{(d-2s)\,[(w+2t)^3 - w^3]}{12}. \end{cases}$$

If one plate is replaced by latticing, $I = \dfrac{t'b^3}{12} + \text{etc.} \quad F = bt + \text{etc.}$

If both plates are replaced by latticing, $t' = 0$.



$$F = 2bt' + 4sa + 2dt,$$

Axis of x:
$$e = d + t',$$
$$I = \frac{bt'^3}{6} + bt'\frac{(d+t')^2}{2} + \frac{(s+t)d^3 - s(d-2a)^3}{6}.$$

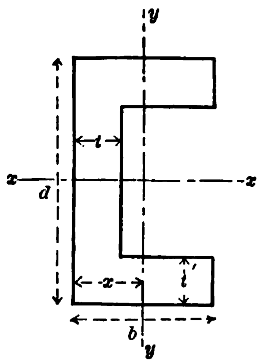
If one plate is replaced by latticing, $I = \frac{bt'^3}{12} + bt'\frac{(d+t')^2}{4} + \text{etc.}$ $\quad e = d + \frac{t'}{2}$.

$$F = bt' + \text{etc.}$$

Axis of y:
$$e = \frac{b}{2},$$
$$I = \frac{t'b^3}{6} + \frac{2a(w+2t+2s)^3 + (d-2a)(w+2t)^3 - dw^3}{12}.$$

If one plate is replaced by latticing, $I = \frac{t'b^3}{12} + \text{etc.}$ $\quad F = bt' + \text{etc.}$

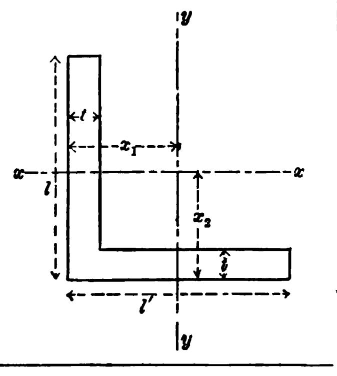
If both plates are replaced by latticing, $t' = 0$.



$$F = dt + 2(b-t)t',$$

Axis of x:
$$e = \frac{d}{2},$$
$$I = \frac{bd^3 - (b-t)(d-2t')^3}{12}.$$

Axis of y:
$$e = b - x, \quad x = \frac{b^2d - (b^2 - t^2)(d - 2t')}{2F},$$
$$I = \frac{2t'(b-x)^3 + dx^3 - (d-2t')(x-t)^3}{3}.$$

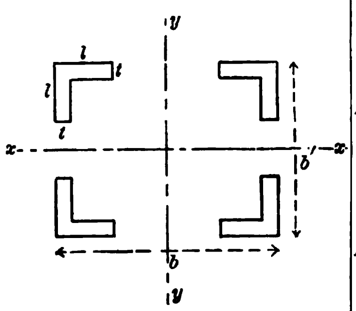


$$F = (l + l' - t)t,$$

$$x_1 = \frac{ll'^2 - (l'^2 - t^2)(l - t)}{2F}, \qquad x_2 = \frac{l^2l' - (l' - t)(l^2 - t^2)}{2F},$$

Axis of x:
$$e = l - x_2,$$
$$I = \frac{t(l-x_2)^3 + l'x_2^3 - (l'-t)(x_2-t)^3}{3},$$

Axis of y:
$$e = l' - x_1,$$
$$I = \frac{t(l'-x_1)^3 + lx_1^3 - (l-t)(x_1-t)^3}{3},$$

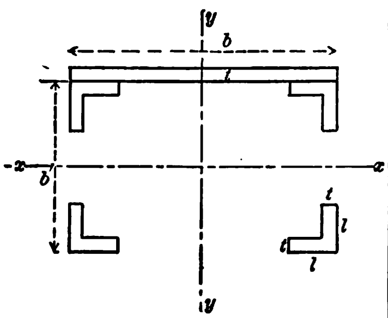


$$F = 4(2lt - t), \qquad x_1 = \frac{l^2 - (l^2 - t^2)(l - t)}{2F} = x_2 = x,$$

Axis of x:
$$e = \frac{b'}{2},$$
$$I_1 = \frac{4[t(l-x)^3 + lx^3 - (l-t)(x-t)^3]}{3} + F\left(\frac{b'}{2} - x\right)^2.$$

Axis of y:
$$e = \frac{b}{2},$$
$$I_2 = \frac{4[t(l-x)^3 + lx^3 - (l-t)(x-t)^3]}{3} + F\left(\frac{b}{2} - x\right)^2.$$

The angles are connected by latticing.



$$F = 4\,(2lt - t^2) + bt. \qquad I_1 \text{ and } I_2 \text{ as above.}$$

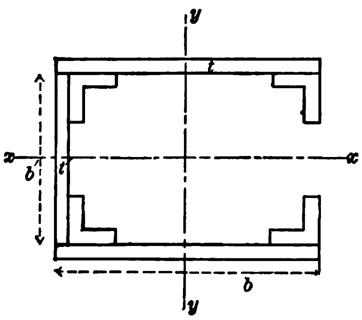
Axis of x $\begin{cases} e = \dfrac{b'}{2} + t, \\ I = I_1 + \dfrac{bt}{4}\left[\dfrac{t^2}{3} + (b' + t)^2\right]. \end{cases}$

If there are two plates, above and below, $I_3 = I_1 + \dfrac{bt}{2}\left[\dfrac{t^2}{3} + (b' + t)^2\right].$

$$F = 4(2lt - t^2) + 2bt.$$

Axis of y $\begin{cases} e = \dfrac{b}{2}, \\ I = I_2 + \dfrac{tb^3}{12}. \end{cases}$

If there are two plates, $I_4 = I_2 + \dfrac{tb^3}{6}.\quad F = 4\,(2lt - t^2) + 2bt.$



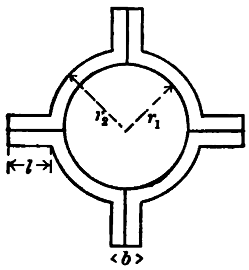
$$F = 4(2lt - t^2) + 2bt + b't'. \quad I_3 \text{ and } I_4 \text{ as above.}$$

Axis of x $\begin{cases} e = \dfrac{b'}{2} + t, \\ I = I_3 + \dfrac{t'(b' + 2t)^3}{12}. \end{cases}$

For four plates, $I = I_3 + \dfrac{t'(b' + 2t)^3}{6}.\quad F = 4\,(2lt - t^2) + 2\,(bt + b't').$

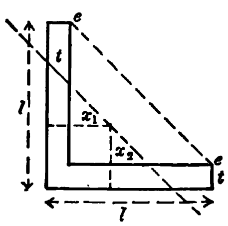
Axis of y $\begin{cases} e = \dfrac{b}{2} + t, \\ I = I_4 + \dfrac{t'(b' + 2t)}{4}\left[\dfrac{t'^2}{3} + (b + t')^2\right]. \end{cases}$

For four plates, $I = I_4 + \dfrac{t'(b' + 2t)}{2}\left[\dfrac{t'^2}{3} + (b + t')^2\right].$



$$F = \pi\,(r_2^2 - r_1^2) + 4\,bl.$$

$$I = \frac{\pi(r_2^4 - r_1^4)}{4} + 2bl\left(r_2 + \frac{l}{2}\right)^2.$$

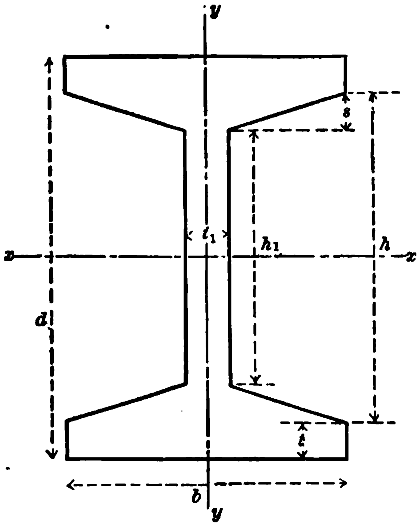


Axis through centre of gravity, parallel to ee.

$$x_1 = \frac{ll'^2 - (l'^2 - t^2)\,(l - t)}{2F}, \quad x_2 = \frac{l^2l' - (l' - t)\,(l^2 - t^2)}{2F},$$

$$F = (l + l' - t)\,t.$$

$$I = \frac{2x_2^4 - 2\,(x_2 - t)^4 + t\left[l' - \left(2x_2 - \dfrac{t}{2}\right)\right]^3}{3}.$$

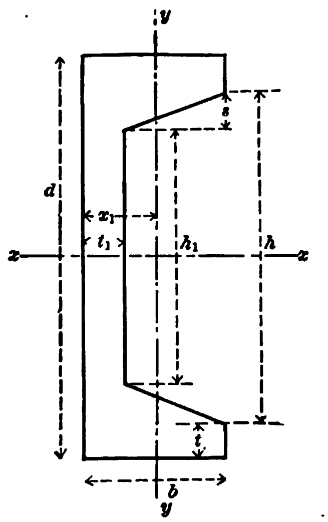


$$F = 2bt + ht_1 + s(b - t_1),$$

Let r = batter $= \dfrac{2s}{b - t_1}$,

Axis x, $I = \dfrac{bd^3 - \dfrac{1}{4r}(h^4 - h_1^4)}{12}$.

Axis y, $I = \dfrac{2tb^3 + h_1 t_1^3}{12} + \dfrac{r(b^4 - t_1^4)}{48}$.

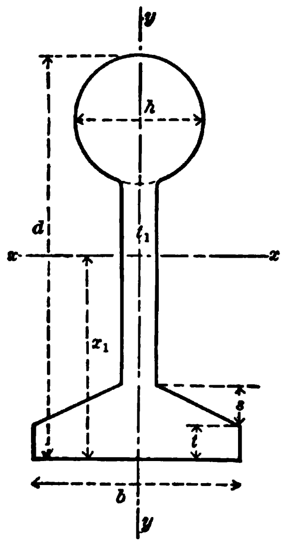


$$F = 2bt + ht_1 + s(b - t_1), \qquad r = \text{batter} = \frac{s}{b - t_1},$$

$$x_1 = \frac{b^2 t + \frac{1}{2} h t_1^2 + \frac{1}{3} s(b - t_1)(b + 2t_1)}{F}.$$

Axis x, $I = \dfrac{bd^3 - \dfrac{1}{8r}(h^4 - h_1^4)}{12}$.

Axis y, $I = \dfrac{2tb^3 + h_1 t_1^3 + \frac{1}{2} r(b^4 - t_1^4)}{3} - Fx_1^2$.



Let area of circular head $= a$,

$$F = a + (d - h)t_1 + (b - t_1)(t + \tfrac{1}{2}s),$$

$$x_1 = \frac{a(2d - h) + t_1(d - h)^2 + bt^2 + s(b - t_1)(t + \frac{1}{3}s)}{2F}.$$

Let $r = \dfrac{2s}{b - t_1}$,

Axis x, $I = a\left[\dfrac{h^2}{16} + \left(d - \dfrac{h}{2}\right)^2\right] + \dfrac{t_1(d - h)^3}{3} + \dfrac{(t + s)^4 - t^4}{6r} - Fx_1^2$.

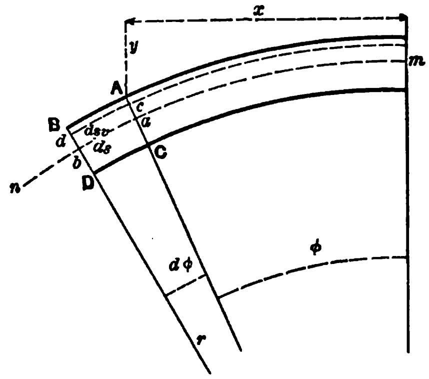
Axis y, $I = \dfrac{\frac{3}{4} ah^3 + t_1^3(d - h - t - s) + tb^3 + \dfrac{r}{8}(b^4 - t_1^4)}{12}$.

The last 13 cases in this Table have been taken from "The Elasticity and Resistance of the Materials of Engineering," by Prof. Wm. H. Burr, New York, John Wiley & Sons, 1883. The Table comprises all cross sections ordinarily met with in practice. For any complex cross section not given in the Table, we may proceed as follows: First find the centre of gravity. This may be found by means of the equilibrium polygon, according to the method given on page 302. Or we may cut the cross section, carefully drawn to scale out of manilla paper or cardboard, and find the centre of gravity by balancing the cross section upon a knife edge in two different positions.

We may then divide the cross section up into rectangles, triangles, trapezoids, etc., and find the sum of the moments of inertia of each with reference to the required axis, through the centre of gravity. For this purpose we find the moment of inertia of each of the rectangles, triangles, etc., into which the cross section is divided, with reference to a parallel axis through its own centre of gravity. Let this be I. Then if the area is a, and the distance between the parallel axes is d, the moment of inertia required for each rectangle, triangle, etc., will be $I' = I + ad^2$. Or the graphical method of page 302 may be employed.

CHANGE OF SHAPE OF THE AXIS.—Let Fig. 163 represent a beam deflected from its original straight line by outer forces. Let the two sections AC and BD be consecutive sections, parallel before flexure, and remaining plane after. Let the length of the axis ma be s, then $ba = ds$. Let $d\varphi$ be the length of the very small arc at distance unity between the sections after flexure.

Fig. 163



If the deflection is small, s will be approximately equal to x, and ds to dx. The elongation of any fibre at a distance v from the neutral axis is $vd\varphi$, if $d\varphi$ is the length of arc at distance unity. The force corresponding to this elongation is from (1), if A is unity and $dx = L$,

$$T = \frac{Evd\varphi}{dx}.$$

If da is the cross section of any fibre, then the whole force of extension of any fibre is

$$\frac{Evd\varphi da}{dx}.$$

The moment of this force is

$$\frac{Ev^2 d\varphi da}{dx}.$$

The integral of this between $+\frac{h}{2}$ and $-\frac{h}{2}$ will give the sum of the moments of all the fibres, which must be equal to the moment M of all the outer forces. Hence

$$M = E\frac{d\varphi}{dx}\int_{-\frac{h}{2}}^{+\frac{h}{2}} v^2 da.$$

But, as we have just seen, the integral is the moment of inertia I of the cross section with reference to the axis through the centre of gravity. Hence

$$M = \frac{EId\varphi}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Since φ is always a very small arc, it may be taken as equal to its tangent, or equal to $\frac{dy}{dx}$.

Hence

$$\varphi = \frac{dy}{dx}, \text{ and } \frac{d\varphi}{dx} = \frac{d^2y}{dx^2}.$$

Therefore

$$M = EI\frac{d^2y}{dx^2}.$$

But from similar triangles we have $vd\varphi : v :: dx : r$, where r is the radius of curvature. Hence

$$\frac{vd\varphi}{v} = \frac{dx}{r}, \quad \text{or } \frac{d\varphi}{dx} = \frac{1}{r}.$$

Hence,

$$M = \frac{EI}{r},$$

also since $T = E\frac{d\varphi}{dx}v$, we have from (3)

$$M = \frac{TI}{v}.$$

Therefore,

$$M = \frac{EI}{r} = EI\frac{d^2y}{dx^2} = \frac{TI}{v} \quad \text{. . . . .} \quad (4)$$

where T is the strain in any fibre at a distance v from the neutral axis.

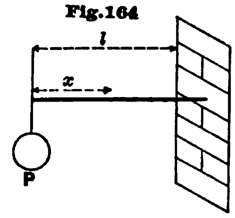
Equation (4) is our fundamental equation.

ASSUMPTIONS UPON WHICH THE THEORY OF FLEXURE IS BASED.—A close examination of the foregoing will reveal the assumptions which lie at the bottom of our theory. Thus we have assumed, first, that the coefficient of elasticity is constant. Second: That fibres at equal distances above and below the neutral axis are equally strained, and hence that the neutral axis passes through the centre of gravity of the cross section. Third: That the deflection is very small, and hence dx can be put for ds and $\frac{dy}{dx}$ for φ. Fourth: That any two plane sections remain plain after flexure. Fifth: That the elastic limit is not exceeded. Upon these assumptions our theory rests. The comparison of its results with experiment shows them to be practically allowable, so long as the limit of elasticity is not exceeded.

CASE I.—BEAM FIXED AT ONE END AND LOADED AT THE OTHER—CONSTANT CROSS SECTION.—We shall always consider a moment positive when it causes compression in the upper fibres, or, considering always the forces *on the left* of the centre of moments, rotation in the direction of the hands of a watch is positive. The distance from the left end to any point is denoted by x.

The complete discussion consists in finding the change of shape of the beam and its deflection at any point, and the breaking weight, or the load it will carry before breaking, both for constant cross section and for uniform strength, as well as the proper shape for uniform strength.*

Fig. 164



(*a*.) *Deflection and Change of Shape*.—In any case, in order to find the change of shape, we have only to find the moment at any point of the outer forces, and equate this moment to $EI\frac{d^2y}{dx^2}$, according to (4). Integrating then

* A beam is said to be of uniform strength when it is so proportioned that the outer forces cause the same strain per unit of cross section at all points.

twice, regarding I as a constant, since the cross section is supposed constant, we shall obtain an equation giving the relation between x and y. The discussion of any case thus reduces to a simple method, consisting of a repetition in each case of the same process.

Thus in the present case, Fig. 164, the moment of the outer forces is $M = -Px$. From (4) then, we have

$$EI\frac{d^2y}{dx^2} = -Px,$$

Integrating once we have

$$EI\frac{dy}{dx} = -\frac{Px^2}{2} + C_1,$$

where C_1 is the constant of integration. Since the beam is fixed horizontally at the right end, the tangent to the curve of deflection must be horizontal at that end. When $x = l$, then $\frac{dy}{dx} = 0$, hence $C_1 = \frac{Pl^2}{2}$, and

$$EI\frac{dy}{dx} = -\frac{Px^2}{2} + \frac{Pl^2}{2}.$$

Integrating again

$$EIy = -\frac{Px^3}{6} + \frac{Pl^2x}{2} + C_2.$$

Since the deflection at the fixed end is zero, for $x = l, y = 0$, and hence $C_2 = -\frac{Pl^3}{3}$, and

$$EIy = -\frac{Px^3}{6} + \frac{Pl^2x}{2} - \frac{Pl^3}{3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This equation gives the deflection at any point. The deflection at the end is evidently greatest, making then $x = 0$, we have the maximum deflection,

$$\Delta = -\frac{Pl^3}{3\,EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If the cross section is rectangular, $I = \frac{bh^3}{12}$, and

$$\Delta = -\frac{4\,Pl^3}{Ebh^3}.$$

(*b.*) *Breaking Weight.*—In order to find the breaking weight, we have from equation (4), page 246,

$$M = \frac{TI}{v},$$

where T is the tensile strain in any fibre distant v from the neutral axis. For symmetrical cross sections,* $v = \frac{h}{2}$ and T is the tensile strain in the outer fibre, and hence $M = \frac{2\,TI}{h}$. For $v = -\frac{h}{2}$ we have the compressive strain in the outer fibre upon the other side. Calling this C we have

* If the two outer fibres are at different distances from the neutral axis, we must put for v its greatest value, so as to get P on the safe side.

$M = \frac{2\,CI}{h}$. Theoretically T and C should be equal. Thus, in the present case, the greatest moment is Pl. Hence, for symmetrical cross section,

$$Pl = \frac{2\,TI}{h} = \frac{2\,CI}{h},$$

or,

$$P = \frac{2\,TI}{hl} = \frac{2\,CI}{hl} \qquad (7)$$

For a rectangular cross section $I = \frac{bh^3}{12}$, and

$$P = \frac{Tbh^2}{6\,l} = \frac{Cbh^2}{6\,l}.$$

Theoretically, then, if we know the tensile strength T or the crushing strength C of the material, either of these should give us the breaking strength P. This, however, is not the case in practice. The crushing strength is not always equal to the tensile strength. Moreover, the theory from which the formula is deduced is based upon the assumption that the elastic limit is not exceeded. But at the breaking point this limit is exceeded. The breaking weight is then really given by

$$P = \frac{2\,RI}{hl}, \qquad (8)$$

where R must be determined for each kind of material by experiments made at the breaking point. We may call R, then, the modulus of breaking strength *for flexure*. It evidently varies with the material, and for the same material varies with the shape of cross section. In the Table which follows, R is determined by experiments *with rectangular cross sections only*. It is found by experiment that R is always intermediate between T and C.* If then experiments are not at hand, we may take that value of T or C which is the *smallest*, and if the cross section is not symmetrical, that value of v which is the *greatest*. We shall thus always be on the safe side.

We give in the following Table the values of C, T, R and E, for all materials of usual occurrence, in pounds per square inch. We also give the value of the shearing strength S and the average weight per cubic foot.

The authority quoted is given in the second column, and the name of the experimenter, when known, is indicated by one of the following abbreviations: B = Barlow, Bv = Bevan, C = Clark, D = Denison, F = Fairbairn, G = Grant, H = Hodgkinson, Hl = Hill, K = Kirkaldy, K C = Keystone Bridge Co., M = Moore, Mu = Muschenbroeck, Re = Rennie, Ro = Rondelet, T = Tredgold, Wd = Wade, Wi = Wilkinson. The table is an extension of that given by J. D. Crehore, C. E., "Mechanics of the Girder,"—Wiley & Sons, 1886.

MATERIAL.	Authority	C Lbs. per sq. in. compressive strength.	T Lbs. per sq. in. tensile strength.	R Lbs. per sq. in. cross-breaking strength by rupture.	S Lbs. per sq. in. shearing strength.	E Lbs. per sq. in. coefficient of elasticity.	Weight in lbs. per cubic ft.
CAST IRON.							
Average	Wood....	96,000	16,000	36,000	....	17,000,000	
Cannon specimens.....	Lanza....	84,500 to 175,000 Wd	20,148 to 28,805 Wd	....	....	....	
Mean of 9 specimens...	Stoney...	105,945 H	16,720 H	37,695 H	....	....	
Mean of 16 specimens..	Stoney...	86,284 H	15,298 H	....	....	12,000,000	
Bars less 1 inch wide...	Stoney...	....	....	45,696 C	....	....	
Bars 3 inches wide.....	Stoney...	....	....	30,240 C	....	....	

* For rectangular cross sections we have $\frac{Cbx}{2} \times \frac{2}{3}x = \frac{Tb(h-x)}{2} \times \frac{2(h-x)}{3} = \frac{Rb}{2} \times \frac{h}{2} \times \frac{2}{3} \cdot \frac{h}{2}$; or,

$$R = \frac{4\,C}{\left(1 + \sqrt{\frac{C}{T}}\right)^2} = \frac{4\,T}{\left(1 + \sqrt{\frac{T}{C}}\right)^2}$$

MATERIAL.	Authority	*C* Lbs. per sq. in. compressive strength.	*T* Lbs. per sq. in. tensile strength.	*R* Lbs. per sq. in. cross-breaking strength by rupture.	*S* Lbs. per sq. in. shearing strength.	*E* Lbs. per sq. in. coefficient of elasticity.	Weight in lbs. per cubic foot.
CAST IRON (*Cont.*).							
Bars small round	Stoney			26,880 C			450
Circular tubes	Stoney			38,304 C			
Square tubes	Stoney			45,965 C			
Various qualities	Lanza	82,000 to 145,000	13,400 to 29,000	30,000 to 43,500	16,000 to 24,740	14 to 29,000,000	
Average	Bovey	100,000	15,000		18,000	17,000,000	
Average market value	Bovey	76,000	12,000		18,000		
Very good	Bovey		22,000 to 27,000				
Average	Weisbach		18,500			14,220,000	
Average	Rankine	112,000	16,500	38,250		17,000,000	
WROUGHT IRON.							
Bars rolled	Wood		57,557				
Angle iron	Wood	30,000	54,729	33,000		24,000,000	
Plates, lengthways	Wood		50,737				
Plates, crossways	Wood		46,171				
Bars, new	Stoney			51,341 C			
Bars, previously strain'd	Stoney			74,995 C			480
Bars, new, round	Stoney			30,240 C			
Boiler tubes, welded	Stoney			70,291 C			
Circular tubes, riveted	Stoney			43,814 C			
Rolled I beams	Stoney			61,824 C			
T iron, flange up	Stoney			53,760 C			
T iron, flange down	Stoney			51,475 C			
Average	Stoney	40,320	57,555 K	52,567 C		24,000,000	
Bars and Bolts	Rankine	36,000	60,000				
Bars and Bolts	Rankine	40,000	70,000			29,000,000	
Plates	Rankine		51,000				
Plates, double riveted	Rankine		35,700				
Plates, single riveted	Rankine		28,600				
Hoops, best-best	Rankine		64,000				480
Wire	Rankine		70,000				A bar one square inch in cross section and 3 feet long weighs 10 lbs.
Wire	Rankine		100,000			25,300,000	
Wire ropes	Rankine		90,000			15,000,000	
Plate beams	Rankine			42,000			
Mean of 113 tests	Lovett		50,915				
Mean of 27 tests	Lovett					27,300,000	
Low average	Bovey	32,000			40,000		
Bar average	Bovey	26,000 to 66,000	40,000 to 52,000	33,000 to 58,000	29,000 to 42,000	29,000,000	
Market bars, full size	Bovey		41,000 to 44,000				
Market bars, prepared	Bovey		44,000 to 46,000				
L, T, and other sections	Bovey		44,766				
Plate, average	Bovey		41,000 to 44,733				
Plate, prepared	Bovey		42,000				
Plates, punched	Bovey				45,000 to 54,000		
Iron wire	Bovey		62,000 to 89,000			25,300,000	
STEEL.							
Bessemer, hammered	Stoney	225,568 F	83,391 F	128,083 K		31,000,000	
Bessemer, rolled	Stoney		71,658 K	115,181 K			
Crucible, hammered	Stoney		85,546 K	147,840 K			
Crucible rolled	Stoney		68,589 K	118,272 K			
Cast, not hardened	Stoney	198,944 Wd					
Cast, low temper	Stoney	354,544 Wd					
Cast, mean temper	Stoney	391,985 Wd					490
Cast, high temper	Stoney	372,598 Wd					
6 eye bars ¾″ round	Lanza		73,150 K C			28,210,000	
6 rolled and annealed	Lanza		69,470 K C			29,210,000	
Bars	Rankine		100,000			29,000,000	
Bars	Rankine		130,000			42,000,000	
Plates, average	Rankine		80,000				
Plates	Lanza		77,840 to 86,330 Hl				
Plates, L and T bars	Bovey	60,000 to 80,000	60,000 to 80,000				
Bessemer, average	Bovey		56,000	80,000 to 129,000	48,000	30,000,000	
WOOD.							
Alder	Stoney	6,831 H	13,900 Mu				50
Apple	Bovey		17,600	5,300 to 7,000			50
Ash	Stoney	9,363 H	16,700 Bv	12,156 B	1,250	1,525,000	43 to 53
Ash	Rankine	9,000	17,000 B	13,000		1,600,000	47
Beech	Rankine	11,500	9,360	10,500		1,350,000	43 to 53
Beech	Stoney	9,363 H	11,500 B	9,366 B			
Beech	Stoney		17,300 Mu				
Birch, American	Stoney	11,663 H		12,366 B		1,645,000	45 to 49
Birch, English	Stoney	6,402 H	15,000 Bv	11,568 B			
Box	Stoney	8,000	20,000 B	14,670 T		1,800,000	64
Box	Rankine	10,300	20,000				
Cedar, American	Stoney	5,000	10,000	4,596 D		486,000	35 to 47
Cedar, Lebanon	Rankine	5,860	11,400	7,400		486,000	
Chestnut, Spanish	Stoney	5,060	13,300 Ro		616	1,140,000	35 to 41
Chestnut	Rankine	5,350	11,500	10,660		1,140,000	
Deal, Christiana	Stoney		12,900 Bv	9,372 B			43
Deal, red	Stoney	6,586 H					
Deal, white	Stoney	7,293 H					

MATERIAL.	Authority	*C* Lbs. per sq. in. compressive strength.	*T* Lbs. per sq. in. tensile strength.	*R* Lbs. per sq. in cross-breaking strength by rupture.	*S* Lbs. per sq. in. shearing strength.	*E* Lbs. per sq. in. coefficient of elasticity.	Weight in lbs. per cubic foot.
WOOD (*Cont.*).							
Elm	Rankine	10,300	14,000	7,850	1,250	1,020,000	34 to 37
Elm	Stoney	10,331 H	14,400 Bv				
Elm, English	Stoney			4,692 B			
Fir, spruce	Stoney	6,819 H	9,000	8,076 M	420	1,800,000	29 to 32
Fir, red pine	Rankine	5,375	12,000	7,100		1,460,000	
Fir, red pine	Rankine	6,200	14,000	9,540		1,900,000	
Fir, larch	Rankine	5,570	9,000	5,000		900,000	
Fir, larch	Rankine		10,000	10,000		1,360,000	
Hemlock	Stoney			6,852 D	480		47
Larch	Stoney	5,568 H	10,220 Ro	8,010 B	860 to 1,520	1,360,000	32 to 38
Lignum Vitae	Rankine	8,920	11,800	12,000		1,000,000	41 to 83
Locust	Rankine	4,500	16,000	11,200	1,070		58
Locust	Stoney		20,100 Mu	20,580 B			
Mahogany	Rankine	6,600	8,000	7,600		1,255,000	53
Mahogany	Rankine	8,200	21,800	11,500			
Mahogany	Stoney	8,198 H	8,000 B			3,000,000	
Mahogany	Stoney		16,500 Bv	10,314 M			
Maple	Stoney		17,400 Bv	10,164 D			49
Maple	Rankine	8,150	10,600				
Oak, European	Rankine	7,700	10,000	8,700	2,680 to 4,460	1,200,000	49 to 58
Oak, European	Rankine	10,000	19,800	13,600	6,960	1,750,000	
Oak, American red	Rankine	6,000	10,250	10,600		2,150,000	61
Oak, English	Stoney	10,058 H	10,000 B	10,164 B			49 to 58
Oak, English	Stoney	5,780 to 8,980	19,800 Bv				
Oak, French	Stoney		13,950 Ro	8,898 M			
Oak, Quebec	Stoney	5,982 H					
Oak, American red	Stoney			10,122 D			61
Oak, American white	Stoney			10,458 B			
Pine, American red	Stoney	7,518 H	2,400 to 7,200	9,162 B	440 to 720	1,960,000	34
Pine, American pitch	Stoney	6,000	7,650 Mu	10,362 B		1,252,000	41 to 58
Pine, American white	Stoney		2,600 to 6,600	7,374 D	440	2,300,000	36
Pine, American yellow	Stoney	5,445 H	4,400 to 10,600	7,110 B	454	1,600,000	32
Pine, Norway	Stoney		14,300 Bv			3,000,000	
Pine, Norway	Stoney		7,287 Bv			2,350,000	
Poplar	Bovey	2,760 to 4,560	5,360 to 6,400			763,000	23 to 26
Sycamore	Rankine	6,320	13,000	9,600		1,040,000	36 to 43
Sycamore	Stoney	7,082 H	13,000 Bv				
Teak	Stoney	12,101 H	15,000 Bv	12,648 B		2,100,000	41 to 52
Teak, Indian	Rankine	12,000	15,000	12,000 to 19,000		2,400,000	
Walnut	Stoney	7,227 H	8,130 Mu	8,000			38 to 57
Walnut	Stoney	6,400	7,800 Bv				
Willow	Stoney	6,128 H	14,000 Bv	3,300 to 4,700		1,400,000	24 to 35
Willow	Rankine	5,400 to 2,600	9,000 to 12,500	6,600			
STONE.							
Granite	Stoney	3,173 to 13,440 Wi		456 to 2,442 Wi			168
Granite	Rankine	4,000 to 11,000					
Limestone	Stoney	3,050 F to 18,043 Wi	670 to 2,800	1.698 to 2,484 Wi			96
Marble	Stoney	200,160 Wi to 3,216 Re	551 H to 722 Bv	1,252 H to 2,697 H			96
Sandstone	Stoney	2,185 to 7,884	1,054 to 1,261	2,010 to 5,142 Re			150
Sandstone	Rankine	2,200 to 5,500		1,100 to 2,360			
Slate	Rankine	17,344	9,600 to 12,800	5,000 to 7,370		1,300,000 to 1,600,000	175
Bricks, pale red	Stoney	562 Re					150
Bricks, red	Stoney	808 Re					
Bricks, fire	Stoney	1,717 Re					
Bricks, Gault clay	Stoney	2,240 G					
Bricks, ordinary	Rankine		280 to 300				125
Lime, mortar	Stoney	618 Ro	51				100
Portland cement	Stoney	5,984 G	358 G				80
Plaster of Paris	Stoney		71 Ro				144
Roman cement, 2 years	Stoney		546 G				80
Roman cement, 3 years	Stoney		604 G				
Roman cement, 4 years	Stoney		632 G				
Roman cement, 5 years	Stoney		627 G				
Roman cement, 6 years	Stoney		666 G				
Roman cement, 7 years	Stoney		709 G				

In using our formulæ, all dimensions should be in inches, if T, C, R, E are in lbs. or tons per *square inch*, and the result P will then be in lbs. or tons. If the dimensions are all taken in feet, T, C, R and E must be taken in lbs. or tons per *square foot*.

CASE 2.—BEAM FIXED AT ONE END AND LOADED AT THE OTHER—UNIFORM STRENGTH.—Suppose the cross section or I is not constant as before, but varies in such a manner that at every point of every cross section T or C is constant. The beam is then of uniform strength throughout.

We have from (4), for the outer fibre,

$$Px = \frac{2\,TI}{h}, \text{ or } T = \frac{Phx}{2\,I}.$$

For a rectangular cross section, for instance,

$$T = \frac{6\,Px}{bh^2}.$$

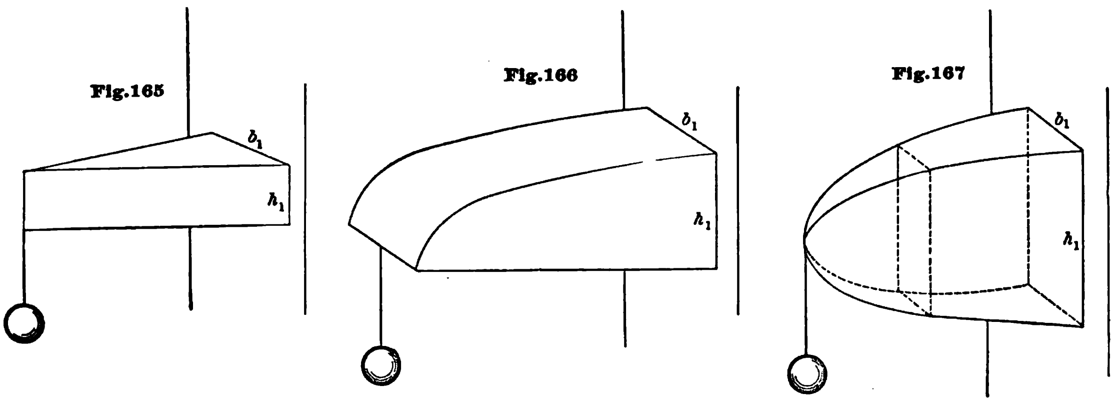
This, then, gives the value of T at any point distant x from the end. Suppose the breadth and height at the fixed end are denoted by b_1 and h_1. Then,

$$T = \frac{6\,Pl}{b_1h_1^2}.$$

Now since T is required to be constant, we have,

$$\frac{6\,Px}{bh^2} = \frac{6\,Pl}{b_1h_1^2}, \quad \text{or} \quad \frac{bh^2}{b_1h_1^2} = \frac{x}{l} \quad \dots\dots\dots\dots \quad (9)$$

If the height is constant, then $h = h_1$, and we have the breadth at any point $b = b_1\frac{x}{l}$. That is, the breadth varies as the ordinates to a straight line, as shown in Fig. 165. If, on the other hand, the breadth is constant, $b = b_1$, and we have $h^2 = h_1^2\frac{x}{l}$. That is, the height varies as the ordinates to a parabola, as shown in Fig. 166.



If both b and h vary, but the cross section at all points is similar, we have,

$$\frac{b_1}{h_1} = \frac{b}{h}, \quad \text{or}\ b = \frac{b_1h}{h_1},$$

and hence substituting in (9), $h^3 = h_1^3\frac{x}{l}$, which is the equation of a cubic parabola. The breadth varies according to the same law, as shown in Fig. 167.

(*a*). *Deflection and Change of Shape.*—Since I is no longer constant, we have in the present case, from (4),

$$\frac{d^2y}{dx^2} = -\frac{Px}{EI} = -\frac{Px}{E \times \frac{bh^3}{12}},$$

where b and h are variable, as we have just seen. If, as in Fig. 165, the height is constant and always equal to h_1, then, as we have seen, $b = b_1\frac{x}{l}$.

Hence for rectangular cross section,

$$\frac{d^2y}{dx^2} = -\frac{12\,Pl}{E\,h_1^3 b_1}.$$

Integrating this, since for $x = l$, $\frac{dy}{dx} = 0$, we have

$$\frac{dy}{dx} = -\frac{12\,Plx}{E\,h_1^3 b_1} + \frac{12\,Pl^2}{E\,h_1^3 b_1}.$$

Integrating again, since for $x = l$, $y = 0$, we have

$$y = -\frac{6\,Plx^2}{E h_1^3 b_1} + \frac{12\,Pl^2x}{E h_1^3 b_1} - \frac{6\,Pl^3}{E h_1^3 b_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

This equation gives the deflection at any point for a beam, as shown in Fig. 165.

The greatest deflection will be at the end, and is equal to

$$\varDelta = \frac{6\,Pl^3}{E\,h_1^3 b_1}.$$

The deflection for a beam of the same length with constant cross section, we have already found to be $\frac{4\,Pl^3}{E\,b_1 h_1^3}$ for rectangular cross section. We see, then, that other things being the same, the beam of uniform strength deflects $\frac{3}{2}$ as much as the beam of constant cross section.

In similar manner we find for constant breadth, Fig. 166,

$$y = 2\,\varDelta_0\left[1 - 3\,\frac{x}{l} + 2\sqrt{\left(\frac{x}{l}\right)^3}\,\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

$$\varDelta = 2\,\varDelta_0 = \frac{8\,Pl^3}{E\,b_1 h_1^3},$$

where $\varDelta_0$ stands for the deflection of the beam of constant cross section, or $\frac{4\,Pl^3}{E\,b_1 h_1^3}$.

For similar cross sections, Fig. 167, we have,

$$y = \tfrac{9}{5}\,\varDelta_0\left[1 - \frac{5}{2}\frac{x}{l} + \frac{3}{2}\sqrt[3]{\left(\frac{x}{l}\right)^5}\,\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

$$\varDelta = \tfrac{9}{5}\,\varDelta_0 = \tfrac{36}{5}\,\frac{Pl^3}{E\,b_1 h_1^3}.$$

If we call the volume of the beam of constant cross section V, then in the first case, Fig. 105, the volume $V_1 = \frac{1}{2}\,V$; in the second, Fig. 166, $V_2 = \frac{2}{3}\,V$; in the third, Fig. 167, $V_3 = \frac{3}{5}\,V$, or

$$V : V_2 : V_3 : V_1 = 30 : 20 : 18 : 15.$$

The maximum deflections, as we see, are as

$$2\,\varDelta_0, \quad \tfrac{9}{5}\,\varDelta_0, \quad \tfrac{3}{2}\,\varDelta_0, \quad \text{or as 20, 18 and 15.}$$

That is, the deflections at the ends for a beam of uniform strength in the three cases ***are as the volumes.***

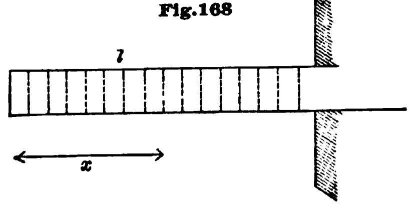
(*b*) *Breaking Strength.*—We have, just as in the case of constant cross-section,

$$P = \frac{TI_1}{vl}, \text{ or } \frac{CI_1}{vl},$$

where I_1 is the moment of inertia of the cross section at the fixed end $= \frac{1}{12} b_1 h_1^3$ for rectangular cross section.

The breaking weight is evidently the same as for beam of constant cross section, if the weight of beam itself be disregarded. The only difference is, that more material is required in the latter case.

Fig. 168



CASE 3.—BEAM AS BEFORE, FIXED AT ONE END—UNIFORM LOAD—CONSTANT CROSS SECTION.—If p is the load per unit of length, we have for the moment at any point distant x from the free end, Fig. 168, from (4),

$$EI\frac{d^2y}{dx^2} = -px \times \frac{x}{2} = -\frac{px^2}{2}.$$

Integrating once, since for $x = l, \frac{dy}{dx} = 0$, we have

$$EI\frac{dy}{dx} = -\frac{px^3}{6} + \frac{pl^3}{6}.$$

Integrating again, since for $x = l, y = 0$, we have

$$EIy = -\frac{px^4}{24} + \frac{pl^3x}{6} - \frac{pl^4}{8} \quad \ldots\ldots\ldots\ldots \quad (13)$$

The deflection at the end, then, is

$$\varDelta = \frac{pl^4}{8EI},$$

or only $\frac{3}{8}$ as great as for an equal load at the end.

For the breaking weight, we have, since the greatest moment is at the fixed end and equal to $\frac{pl^2}{2}$, from (4),

$$\frac{pl^2}{2} = \frac{TI}{v}, \text{ or } \frac{CI}{v}, \qquad \text{hence } pl = \frac{2\,TI}{vl}, \text{ or } \frac{2\,CI}{vl},$$

taking always whichever value of T or C is the least, or twice as much as for an equal weight at the end.*

CASE 4.—BEAM FIXED AT ONE END—UNIFORM LOAD—CONSTANT STRENGTH.—We have the moment at any point $\frac{px^2}{2}$. Putting this equal to $\frac{2\,TI}{h}$, we find $T = \frac{phx^2}{4\,I}$, or for rectangular cross section $T = \frac{3\,px^2}{bh^2}$. If b_1 and h_1 are the breadth and height at the fixed end, then since T must be constant,

$$\frac{3\,px^2}{bh^2} = \frac{3\,pl^2}{b_1h_1^2}, \text{ or } \frac{bh^2}{b_1h_1^2} = \frac{x^2}{l^2} \quad \ldots\ldots\ldots\ldots \quad (14)$$

* If the two outer fibres are at different distances from the neutral axis, we must put for v its greatest value so as to get P on the safe side.

If the height is constant $h = h_1$, and $b = b_1\left(\frac{x}{l}\right)^2$. This is the equation of a parabola, as

Fig. 169

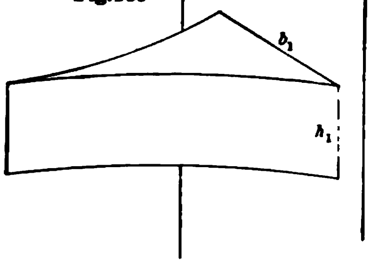
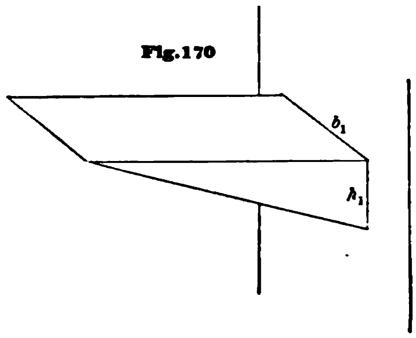


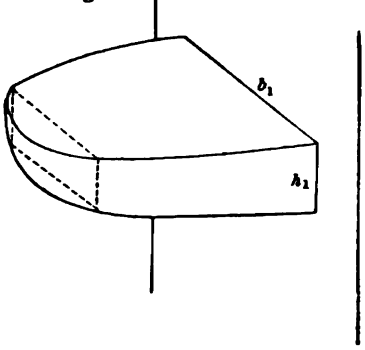
Fig. 170



shown in Fig. 169. If the breadth is constant, $b = b_1$, and (14) becomes $h = h_1\frac{x}{l}$. This is the equation of a straight line, as shown in Fig. 170.

For similar cross sections we have $\frac{b_1}{h_1} = \frac{b}{h}$, or $b = \frac{b_1 h}{h_1}$. Hence (14) becomes $h^3 = h_1^3\frac{x^2}{l^2}$. This is the equation of a cubic parabola. The shape of the beam is, therefore, as shown in Fig. 171.

Fig. 171



CHANGE OF SHAPE.—We have from (4),

$$\frac{d^2y}{dx^2} = \frac{px^2}{2EI},$$

or for rectangular cross section,

$$\frac{d^2y}{dx^2} = \frac{6px^2}{Ebh^3}.$$

For constant height we have, as we have seen, $b = b_1\frac{x^2}{l^2}$, and $h = h_1$. Hence

$$\frac{d^2y}{dx^2} = \frac{6pl^2}{Eb_1h_1^3}.$$

Integrating, since for $x = l$, $\frac{dy}{dx} = 0$, we have

$$\frac{dy}{dx} = \frac{6pl^2x}{Eb_1h_1^3} - \frac{6pl^3}{Eb_1h_1^3}.$$

Integrating again, since for $x = l$, $y = 0$, we have

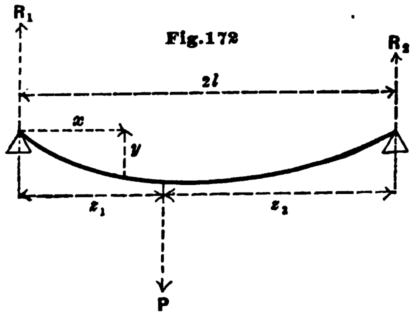
$$y = \frac{3pl^2x^2}{Eb_1h_1^3} - \frac{6pl^3x}{Eb_1h_1^3} + \frac{3pl^4}{Eb_1h_1^3} \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (15)$$

The deflection at the end is, then,

$$\Delta = \frac{3pl^4}{Eb_1h_1^3},$$

or twice as much as for a beam of constant cross-section. In a similar manner we can easily find the deflection in the cases of Figs. 170 and 171.

CASE 5.—BEAM SUPPORTED AT BOTH ENDS—CONSTANT CROSS SECTION—CONCENTRATED LOAD.—Let the weight P be distant from the left end, Fig. 172, by a distance z_1 and from the right end by a distance z_2. Let the distance of any point from the left end be x.



The upward reaction at the left support is by moments $R_1 \times 2l = P \times z_2$, or $R_1 = \frac{Pz_2}{2l}$. The moment at any point between the left end and the weight, or when $x < z_1$,

$$M = + R_1 x = + \frac{Pz_2 x}{2l}.$$

For any point to the right of P, or when $x > z_1$,

$$M = + R_1 x - P(x - z_1) = \frac{Pz_2 x}{2l} - P(x - z_1).$$

The greatest moment is evidently at the point of application of the load, or when $x = z_1$. Hence the maximum moment is $\frac{P z_1 z_2}{2l}$.

(*a.*) *Breaking Weight.*—From (4) we have

$$M = \frac{Pz_1 z_2}{2l} = \frac{TI}{v}, \quad \text{or } P = \frac{2TIl}{v z_1 z_2};$$

where for T we must put R when known by experiment, or that value of T or C which is the smallest, and for v the distance from the neutral axis to the outer fibre.*

For rectangular cross section $I = \frac{bh^3}{12}$, and hence $P = \frac{Rbh^2 l}{3 z_1 z_2}$. For a load in the middle $z_1 = z_2 = l$, and $P = \frac{4RI}{hl}$, or 4 times as great as for a beam of the same length fixed at one end and free at the other end.

(*b.*) *Change of Shape.*—From (4) we have

$$\text{when } x < z_1, \quad EI\frac{d^2y}{dx^2} = \frac{Pz_2 x}{2l}; \qquad \text{when } x > z_1, \quad EI\frac{d^2y}{dx^2} = \frac{Pz_1}{2l}(2l - x).$$

Integrating, we have

$$EI\frac{dy}{dx} = \frac{Pz_2 x^2}{4l} + C_1; \qquad EI\frac{dy}{dx} = \frac{Pz_1}{2l}\left(2lx - \frac{x^2}{2}\right) + C_2.$$

For $x = z_1$ these two values of $\frac{dy}{dx}$ are equal, and hence, since $z_2 = 2l - z_1$, we have $C_2 = C_1 - \frac{Pz_1^2}{2}$.

We thus have the two equations

$$EI\frac{dy}{dx} = \frac{Pz_2 x^2}{4l} + C_1; \qquad \text{and } EI\frac{dy}{dx} = \frac{Pz_1}{2l}\left(2lx - \frac{x^2}{2}\right) - \frac{Pz_1^2}{2} + C_1,$$

both containing the same constant C_1.

* If the two outer fibres are at different distances from the neutral axis, we must put for v the greatest of the two values, so as to get P on the safe side.

Integrating again we have

when $x < z_1$, $EIy = \frac{Pz_2x^3}{12\,l} + C_1x + C_3$; when $x > z_1$, $EIy = \frac{Pz_1}{2\,l}\left(lx^2 - \frac{x^3}{6}\right) - \frac{Pz_1^2x}{2} + C_1x + C_4$.

In the first of these equations, when $x = 0$, $y = 0$; hence $C_3 = 0$. When $x = z_1$, y in one equals y in the other, hence $C_4 = \frac{Pz_1^3}{6}$. For $x = 2\,l$, y in the second equation is zero, hence $C_1 = -\frac{Pz_1z_2}{12\,l}(4\,l - z_1)$.

Substituting these constants we have, when

$$x < z_1,\quad y = \frac{Pz_2x}{12\,EIl}(x^2 - 4\,lz_1 + z_1^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\text{when } x > z_1,\quad y = \frac{Pz_1(2\,l - x)}{12\,EIl}(z_1^2 - 4\,lx + x^2) \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The deflection at the load is, therefore, for $x = z_1$,

$$y = -\frac{Pz_1^2z_2^2}{6\,EIl}.$$

If we insert the value of C_1 in the value for $\frac{dy}{dx}$ and place $\frac{dy}{dx} = 0$, we find for the value of x *which makes the deflection a maximum*,

$$x = \sqrt{\tfrac{1}{3}(4\,l - z_1)\,z_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The greatest deflection is not at the weight, therefore, except when the weight is in the middle. Inserting this value of x in the value for y, we have for the maximum deflection

$$\varDelta = \frac{Pz_1z_2(4\,l - z_1)}{54\,EIl}\sqrt{3\,z_1(4\,l - z_1)}.$$

If the load is in the middle of the beam, we have $z_1 = z_2 = l$, and the equation of the curve of deflection is

$$y = \frac{P\,x}{12\,EI}(3\,l^2 - x^2).$$

The deflection at the weight in this case is found by making $x = l$, or

$$\varDelta = \frac{Pl^3}{6\,EI},$$

or only $\frac{1}{16}$th as much as for a beam of the same length fixed at one end and loaded at the other end.

(*c.*) *Uniform Strength.*—The change of shape and form for uniform strength may be easily found, precisely as on page 252, for a beam fixed at one end and loaded at the other end.

If the weight, for instance, is at the centre of the beam, the deflection is greatest at the weight. Each half of the beam may then be considered as a beam of the length l, fixed horizontally at one end and with an upward force $\frac{P}{2}$ at the other. Each half of the beam should then have the shape of Figs. 165, 166, or 167, according as the height or breadth is constant, or the cross sections are similar.

Thus, Fig. 173 shows the shape of a beam of uniform strength, for constant height, weight in the middle.

Fig. 174, for constant breadth, weight in the middle.

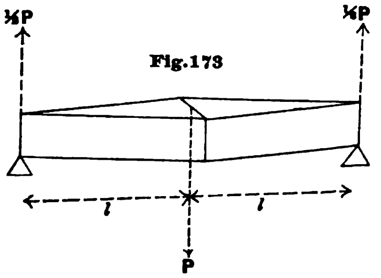


Fig.173

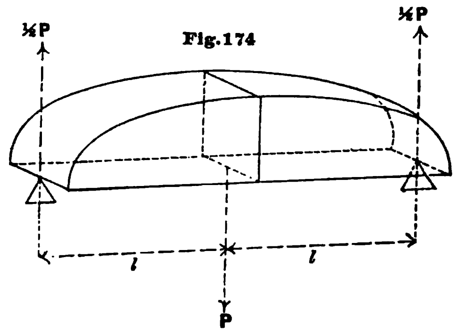


Fig.174

Fig. 175, for similar cross sections, weight in the middle.

In each of these cases, the deflection is the same as for a beam whose length is l, fixed at one end horizontally and with an upward force of $\frac{P}{2}$ at the other. The deflection in each case is given by (10), (11) and (12), where for P we must insert $\frac{P}{2}$.

When the weight P is placed at any point, we have only to find the point at which the deflection is greatest, or that point for which $\frac{dy}{dx} = 0$. This point we may consider as the fixed end of a beam, whose length is the distance to each of the other ends, the force at the extremity being the reaction.

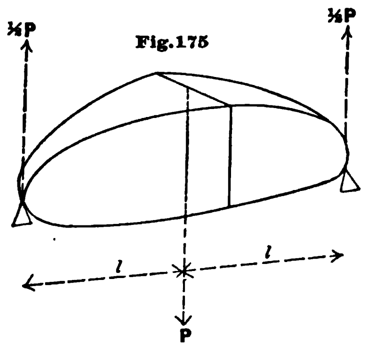


Fig.175

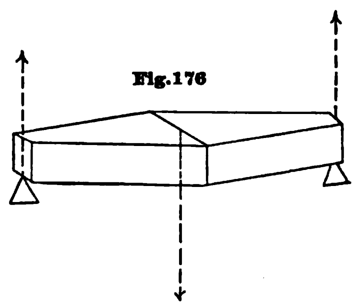


Fig.176

Equations (10), (11) and (12), will then give the deflection, when we put for l the length of each portion, and for P the reaction at the end.

The method of page 252 must be followed in each case. Owing to the shear, Figs. 173, 174

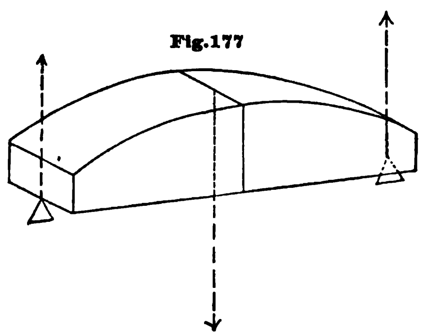


Fig.177

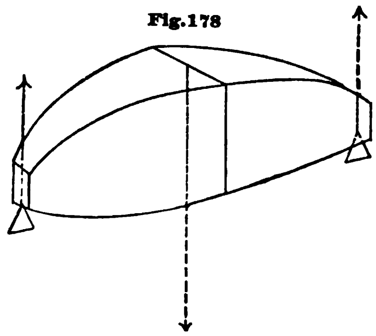
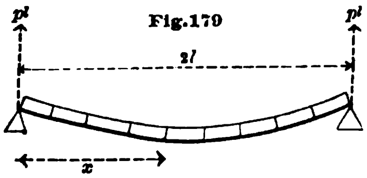


Fig.178

and 175 cannot end in a line as shown, but cross section enough should be allowed at the ends to resist the shear at those points, as shown in Figs. 176, 177 and 178.

CASE 6.—BEAM SUPPORTED AT BOTH ENDS—CONSTANT CROSS SECTION—UNIFORM LOAD.—



For a load p per unit of length, the entire load is $2\,pl$, Fig. 179. The reaction at each end is pl. The moment at any point is

$$M = plx - \frac{px^2}{2}.$$

This is evidently greatest at the centre, or when $x = l$. Hence

$$\max M = \frac{pl^2}{2}.$$

For the breaking weight then, from (4)

$$\frac{pl^2}{2} = \frac{TI}{v} \quad \text{or } 2\,pl = \frac{4\,TI}{vl} \quad \text{or } \frac{4\,CI}{vl} \qquad (19)$$

or four times as much as for a beam of the same length loaded uniformly and fixed at one end.

For the change of shape, we have from (4),

$$EI\frac{d^2y}{dx^2} = plx - \frac{px^2}{2}.$$

Integrating once, since for $x = l$, $\frac{dy}{dx} = 0$, we have

$$EI\frac{dy}{dx} = \frac{plx^2}{2} - \frac{px^3}{6} - \frac{pl^3}{3}.$$

Integrating again, since for $x = 0, y = 0$,

$$EIy = \frac{plx^3}{6} - \frac{px^4}{24} - \frac{pl^3x}{3},$$

or

$$y = \frac{px}{24\,EI}\left(4\,lx^2 - x^3 - 8\,l^3\right) \qquad (20)$$

This is greatest at the centre, or for $x = l$. Hence the maximum deflection is

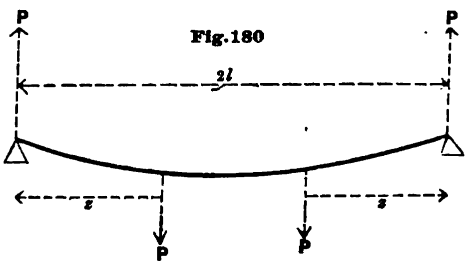
$$\varDelta = \frac{5\,pl^4}{24\,EI},$$

or only $\frac{5}{128}$ of a beam of the same length fixed at one end and uniformly loaded.

For uniform strength, since the deflection is greatest at the centre, we can consider each half of the beam as a beam fixed horizontally at one end and with an upward force at the other equal to pl.

For rectangular cross section each half will then be as shown in Figs. 173, 174 and 175. The deflection in each case may be found as in equation (15). The same method applies easily to any other form of cross section.

CASE 7.—BEAM SUPPORTED AT BOTH ENDS—CONSTANT CROSS SECTIONS—WITH TWO EQUAL AND SYMMETRICALLY PLACED LOADS.—Let the beam, Fig. 180, support two weights P, P, placed at equal distances z from each end. The reaction at each support is then P, and the greatest moment is evidently at the centre and equal to Pz.



For the breaking weight we have, then,

$$Pz = \frac{TI}{v}, \quad \text{or } \frac{CI}{v}, \quad \text{or } P = \frac{TI}{vz}, \quad \text{or } \frac{CI}{vz}.$$

For rectangular cross section, $I = \frac{1}{12} bh^3$, and $v = \frac{h}{2}$, hence

$$P = \frac{Rbh^2}{6z}.$$

For change of shape, we have, from (4),

$$\text{when } x < z, \quad EI\frac{d^2y}{dx^2} = Px, \qquad \text{when } x > z, \quad EI\frac{d^2y}{dx^2} = Pz.$$

Integrating, we have,

$$EI\frac{dy}{dx} = \frac{Px^2}{2} + C_1, \qquad EI\frac{dy}{dx} = Pzx + C_2.$$

In the second of these equations, when $x = l$, $\frac{dy}{dx} = 0$; hence $C_2 = -Pzl$. When $x = z$, $\frac{dy}{dx}$ in the first is the same as $\frac{dy}{dx}$ in the second, hence $C_1 = \frac{Pz^2}{2} - Pzl$. Hence

$$EI\frac{dy}{dx} = \frac{Px^2}{2} + \frac{Pz^2}{2} - Pzl, \qquad EI\frac{dy}{dx} = Pzx - Pzl.$$

Integrating again, since for $x = 0$ in the first of these equations $y = 0$, we have

$$EIy = \frac{Px^3}{6} + \frac{Pz^2x}{2} - Pzlx, \qquad EIy = \frac{Pzx^2}{2} - Pzlx + C_3,$$

when $x = z$, y in the first is the same as y in the second, hence $C_3 = \frac{Pz^3}{6}$.

The deflection for any point on the left of the first weight is given by

$$y = \frac{Px}{6EI}(x^2 + 3z^2 - 6zl),$$

and for any point between the weights,

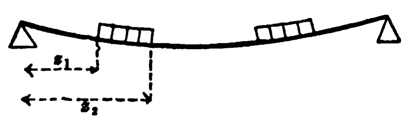
$$y = \frac{Pz}{6EI}(z^2 + 3x^2 - 6xl) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The maximum deflection is at the centre and equal to

$$\Delta = \frac{Pz}{6EI}(z^2 - 3l^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

If the loads are uniformly distributed, instead of being concentrated as shown in Fig. 181, we can put pdz in the place of P. Equation (22) then becomes

Fig.181



$$\Delta = \int \frac{pzdz}{6EI}(z^2 - 3l^2).$$

If we integrate this between the limits z_2 and z_1, we have for the deflection at the centre,

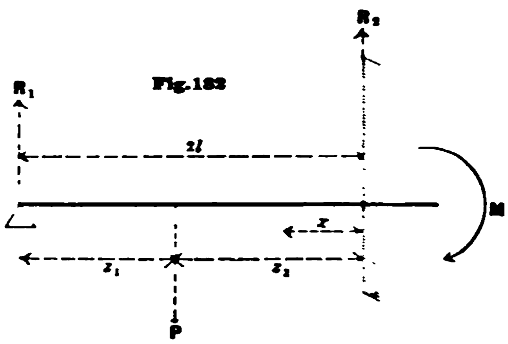
$$\Delta = \frac{p}{24\,EI}\left[(z_2^4 - z_1^4) - 6\,l^2\,(z_2^2 - z_1^2)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

When the load covers the whole beam, $z_2 = l$, and $z_1 = 0$, and

$$\Delta = \frac{5\,pl^4}{24\,EI},$$

as already found.

CASE 8.—BEAM SUPPORTED AT ONE END AND FIXED AT THE OTHER—CONSTANT CROSS SECTION—CONCENTRATED LOAD.—Let the beam be fixed horizontally at the right end, Fig. 182. At this end, then, we have not only a vertical reaction R_2, but also a negative moment M, which causes the beam to be horizontal. At the left end we have only the reaction R_1. Let the weight P be distant from the left end by a distance z_1, and from the right end by a distance z_2. Then from (4), taking x *from the fixed end*,



$$\text{when } x > z_2,\ EI\,\frac{d^2y}{dx^2} = R_1\,(2\,l - x),$$

$$\text{when } x < z_2,\ EI\,\frac{d^2y}{dx^2} = R_1\,(2\,l - x) - P\,(z_2 - x).$$

Integrating we have

$$EI\,\frac{dy}{dx} = 2\,R_1\,lx - \frac{R_1x^2}{2} + C_1, \qquad EI\,\frac{dy}{dx} = 2\,R_1\,lx - \frac{R_1x^2}{2} - Pz_2x + \frac{Px^2}{2} + C_2.$$

When $x = 0$, $\frac{dy}{dx}$ in the second equation is zero, and hence $C_2 = 0$. When $x = z_2$, $\frac{dy}{dx}$ is the same in both. Hence $C_1 = -\frac{Pz_2^2}{2}$. Inserting these values of C_1 and C_2, and integrating again, we have

$$EIy = R_1lx^2 - \frac{R_1x^3}{6} - \frac{Pz_2^2x}{2} + C_3, \qquad EIy = R_1lx^2 - \frac{R_1x^3}{6} - \frac{Pz_2x^2}{2} + \frac{Px^3}{6} + C_4.$$

When $x = 0$, $y = 0$, in the second equation, and hence $C_4 = 0$. When $x = z_2$, y is equal in both, hence $C_3 = \frac{Pz_2^3}{6}$. When $x = 2\,l$, y in the first equation is zero. Hence $R_1 = -\frac{P\,(z_2^3 - 6\,z_2^2\,l)}{16\,l^3}$. Substituting these values of C_4, C_3 and R_1, we have for the deflection at any point between the weight and the supported end,

$$y = -\frac{P}{6\,EI}\left[\frac{z_2^2\,(6\,l - z_2)\,(6\,l - x)\,x^2}{2\,l^3} - (3\,x - z_2)\,z_2^2\right] \quad . \quad . \quad . \quad . \quad (24)$$

Putting the values of $\frac{dy}{dx} = 0$, we have for the point at which the deflection is a maximum,

$$x = 2\,l - 2\,l\sqrt{\frac{2\,l - z_2}{6\,l - z_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Substituting this value of x in the value of y above, we have for the maximum deflection

$$\Delta = -\frac{Pz_2^2}{6\,EI}\,(2\,l - z_2)\sqrt{\frac{2\,l - z_2}{6\,l - z_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

When the weight is in the middle, $R_1 = \frac{5}{16} P$, and

$$\Delta = -\frac{Pl^3}{6\,EI} \times \frac{1}{\sqrt{5}}, \quad \text{or} \ \frac{1}{16\sqrt{5}}$$

as much as for a beam of same length, fixed at one end and loaded at the other, and only $\frac{1}{\sqrt{5}}$ as much as for beam of same length simply supported at both ends.

Breaking Weight.—Since we know R_1, we can find the moment at any point. Rupture will occur where the moment is greatest, that is, either at the fixed end or at the weight.

The moment at the weight is $R_1\,(2\,l - z_2)$. The moment at the fixed end is $2\,R_1 l - Pz_2$. Now as R_1 is always less than P, we see at once that the moment at the weight is greatest. We have, then from (4),

$$R_1\,(2\,l - z_2) = \frac{Pz_2^2\,(6\,l - z_2)}{16\,l^3}\,(2\,l - z_2) = \frac{TI}{v}, \quad \text{or} \ \frac{CI}{v}.$$

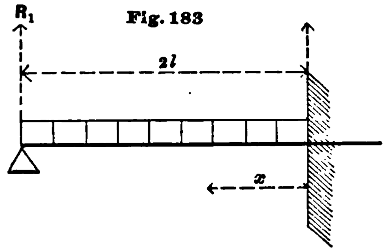
Hence the breaking weight is for symmetrical cross section,

$$P = \frac{32\ TIl^3}{hz_2^2\,(6\,l - z_2)\,(2\,l - z_2)} \quad \ldots \ldots \ldots \ldots \quad (27)$$

If the weight is in the middle, $P = \frac{32\ TI}{5\,hl}$, or $\frac{4}{5}$ths as much as for the same beam supported at the ends.

CASE 9.—BEAM FIXED AT ONE END AND SUPPORTED AT THE OTHER—CONSTANT CROSS SECTION—UNIFORM LOAD.—In this case, Fig. 183, the moment at any point is

Fig. 183



$$EI\,\frac{d^2y}{dx^2} = R_1\,(2\,l - x) - \frac{p\,(2\,l - x)^2}{2}.$$

Integrating twice and determining the constants by the conditions that for $x = 0$, $\frac{dy}{dx} = 0$, and $y = 0$, we easily obtain $R_1 = \frac{3}{8}\,pl$, and

$$\frac{dy}{dx} = \frac{px}{48\,EI}\,(24\,l^2 - 30\,lx + 8\,x^2) \quad \ldots \ldots \ldots \ldots \quad (28)$$

$$y = \frac{px^2}{48\,EI}\,(2\,l - x)\,(6\,l - 2\,x) \quad \ldots \ldots \ldots \ldots \quad (29)$$

Putting (28) equal to zero, we find for the point at which the deflection is a maximum,

$$x = \frac{15 - \sqrt{33}}{8}\,l, \quad \text{or} \ x = 1.157\,l.$$

The maximum deflection itself is then

$$\Delta = -\frac{39 + 55\sqrt{33}}{16^3}\,\frac{pl^4}{EI} = -\,0.0864\,\frac{pl^4}{EI}.$$

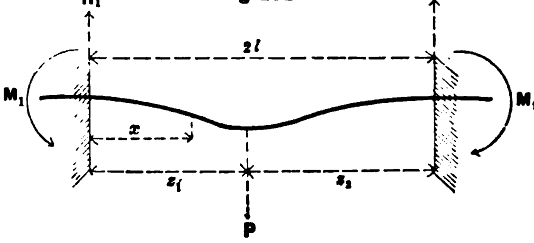
For the breaking weight we have, since the greatest moment is at the fixed end and equal to $\frac{pl^2}{2}$,

$$\frac{pl^2}{2} = \frac{TI}{v}, \quad \text{or} \ 2\,pl = \frac{4\,TI}{vl}.$$

The strength is then $\frac{3}{2}$ as great as for the same load in the middle, but no greater than for beam of same length and load supported at both ends.

CASE 10.—BEAM FIXED AT BOTH ENDS—CONSTANT CROSS SECTION—CONCENTRATED LOAD. —Let z_1 be the distance from the left end to the weight, Fig. 184, and z_2 the distance from the right end to the weight. Let the reaction at the left end be R_1 and the moment at the left end M_1. Let x be measured from the left end.

Fig. 184



Then we have from (4)

$$\text{when } x < z_1, \quad EI\frac{d^2y}{dx^2} = R_1x + M_1,$$

$$\text{when } x > z_1, \quad EI\frac{d^2y}{dx^2} = R_1x - P(x - z_1) + M_1.$$

Integrating we have

$$EI\frac{dy}{dx} = R_1\frac{x^2}{2} + M_1x + C_1\,; \qquad EI\frac{dy}{dx} = R_1\frac{x^2}{2} - P\frac{x^2}{2} + Pz_1x + M_1x + C_2.$$

If $x = 0$, $\frac{dy}{dx}$ in the first equation equals zero, and $C_1 = 0$. For $x = z_1$, $\frac{dy}{dx}$ is the same in both equations, and hence $C_2 = -\frac{Pz_1^2}{2}$. For $x = 2\,l$, $\frac{dy}{dx}$ in the second equation is zero, and hence

$$4\,M_1l = Pz_1^2 - 4\,Pz_1l - 4\,R_1l^2 + 4\,Pl^2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

Integrating again, after substituting the values of C_1 and C_2,

$$EIy = R_1\frac{x^3}{6} + M_1\frac{x^2}{2} + C_3\,; \qquad EIy = R_1\frac{x^3}{6} - \frac{Px^3}{6} + \frac{Pz_1x^2}{2} + M_1\frac{x^2}{2} - \frac{Pz_1^2x}{2} + C_4.$$

For $x = 0$, y in the first equation is zero, and hence $C_3 = 0$.
For $x = z_1$, y in both equations is the same, hence $C_4 = \frac{Pz_1^3}{6}$.
For $x = 2\,l$, $y = 0$ in the second equation, and hence

$$12\,M_1l^2 = 6\,Pz_1^2l - 12\,Pz_1l^2 - 8\,R_1l^3 + 8\,Pl^3 - Pz_1^3 \quad . \; . \; . \; . \; . \quad (31)$$

Equations (30) and (31) contain two unknown quantities, M_1 and R_1. Eliminating M_1, we have

$$R_1 = \frac{4\,Pl^3 + Pz_1^3 - 3\,Pz_1^2l}{4\,l^3}.$$

Or

$$R_1 = P\frac{z_2^2\,(3\,z_1 + z_2)}{8\,l^3}, \text{ and } R_2 = P\frac{z_1^2\,(3\,z_2 + z_1)}{8\,l^3} \quad . \; . \; . \; . \; . \; . \quad (32)$$

Eliminating R_1 we have

$$M_1 = -P\frac{z_1z_2^2}{4\,l^2}, \text{ and } M_2 = -P\frac{z_2z_1^2}{4\,l^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (33)$$

Substituting these values, we have

$$\frac{dy}{dx} = \frac{Pz_2^2x}{16\,EIl^3}\left[4\,lz_1 - (3\,z_1 + z_2)\,x\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$y = \frac{Pz_2^2x^2}{48\,EIl^3}\left[6\,lz_1 - (3\,z_1 + z_2)\,x\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The point at which the deflection is a maximum is then

$$x = \frac{4\,lz_1}{3\,z_1 + z_2},$$

and the maximum deflection is

$$\varDelta = \frac{2\,Pz_1^3z_2^2}{3\,EI\,(3\,z_1 + z_2)^2}.$$

This expression will be itself a maximum when $z_1 = z_2$, or $z_1 = l$. That is, the greatest deflection is at the weight when the weight is in the middle. This deflection is

$$\varDelta = \frac{Pl^3}{24\,EI},$$

or only $\frac{1}{4}$th as much as for beam supported at the ends.

Breaking Weight.—The greatest moment is easily shown to be at the nearest end, and equal to

$$\frac{Pz_1z_2^2}{4\,l^2} \text{ or } \frac{Pz_2z_1^2}{4\,l^2}.$$

This is a maximum for $z_1 = \frac{2}{3}\,l$. That is, the greatest moment at the end occurs when the load is distant one third of the length from that end.

The value of this greatest moment is $\frac{8\,Pl}{27}$. Hence from (4)

$$\frac{8\,Pl}{27} = \frac{TI}{v}, \quad \text{or } P = \frac{27\,TI}{8\,vl},$$

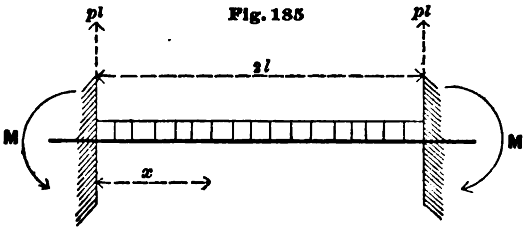
or $\frac{27}{16}$ times as great as for a beam supported at the ends. If the weight is in the middle, we have

$$\frac{Pl}{4} = \frac{TI}{v}, \quad \text{or } P = \frac{4\,TI}{vl},$$

or twice as much as the same beam simply supported at the ends.

Case 11.—Beam Fixed at both Ends—Constant Cross Section—Uniform Load.—In this case, Fig. 185, the reaction at each end is pl.

Fig. 185



We have then from (4)

$$EI\,\frac{d^2y}{dx^2} = plx - \frac{px^2}{2} + M.$$

Integrating, since for $x = 0$, $\frac{dy}{dx} = 0$

$$EI\,\frac{dy}{dx} = \frac{plx^2}{2} + Mx - \frac{px^3}{6}.$$

When $x = 2\,l$, $\dfrac{dy}{dx}$ also equals zero, hence $M = -\dfrac{pl^2}{3}$.

Inserting this value of M and integrating again

$$EIy = \frac{plx^3}{6} - \frac{px^4}{24} - \frac{pl^2x^2}{6}.$$

Since for $x = 0$ $y = 0$, the constant is zero.
The deflection at any point is then

$$y = \frac{px^2}{24\,EI}(4\,lx - x^2 - 4\,l^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

This is greatest at the centre, or for $x = l$. The greatest deflection is then

$$\varDelta = \frac{pl^4}{24\,EI}.$$

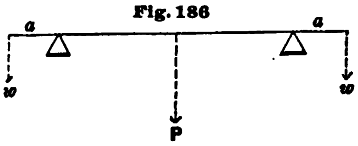
The greatest moment is easily proved to be at the end. Hence the breaking weight

$$\frac{pl^2}{3} = \frac{TI}{v}, \quad \text{or } 2\,pl = \frac{6\,TI}{vl}.$$

In the beam fixed at one end and supported at the other as well as in the beam fixed at both ends, the moment at a fixed end is negative, *i.e.*, causes tension in the upper fibres.

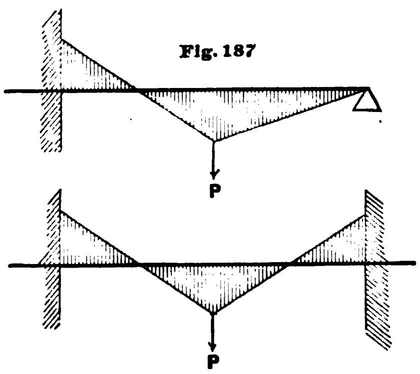
The beam may be fixed either by letting it into the wall or by prolonging it beyond the support and suspending a weight from the end, as shown in Fig. 186. In this case, the moment at the end being found as above, we can easily find the weight w, if the prolongation a is given, or the prolongation a if the weight w is given. Thus wa must equal the moment at the end.

Fig. 186



From the fixed end the moment decreases to a point where the moment is zero. Past this point the moment becomes positive, and in the case of the beam supported at one end, increases gradually to a maximum and then decreases to zero at the supported end. In the beam fixed at both ends, it increases to a maximum, then decreases to zero, then changes sign and becomes negative again and increases to the other end, as shown in Fig. 187. These points at which the moments become zero are ***points of inflection***, because here the moment changes sign, *i.e.*, the curvature changes from convex to concave or the reverse. They can be easily found by finding the values of x which make the expression for the moments zero.

Fig. 187



Thus for a beam fixed at one end and supported at the other, uniform load, the inflection point is at a distance from the fixed end of $x = \frac{1}{4}$ the length. For both ends fixed, we make

$$M_x = plx - \frac{px^2}{2} - \frac{pl^2}{3}$$

equal to zero and find $x = 0.42262\,l$, and $1.5774\,l$ where l is the half length.

The curve of moments in any case may be determined graphically according to the principles of Chap. IV. page 34, or by a discussion of the equation of moments.*

* Examples for practice illustrative of the foregoing, will be found at the end of Part I., page 306, and the student is earnestly recommended to solve them.

Deflection of Open Work or Framed Girders.—A framed girder whose members have been properly designed to resist the stresses which act upon them, may be considered as a girder of uniform strength.

The equation on page 246,

$$EI\frac{d^2y}{dx^2} = \frac{TI}{v}, \text{ or } E\frac{d^2y}{dx^2} = \frac{T}{v}$$

here becomes, since we can put $v = \frac{h}{2}$,

$$E\frac{d^2y}{dx^2} = \frac{2T}{h}, \quad \ldots \ldots \ldots \ldots \ldots \quad (1)$$

where h is the height, and T is the outer fibre strain. For T we may take in any case the mean of the unit stresses allowed in the top and bottom chords. By integrating (1) twice between the proper limits in any given case, we may determine the deflection.

1. *Framed semi-girder, of constant depth.*—Thus for semi-girder of constant depth, we have

$$E\frac{d^2y}{dx^2} = -\frac{2T}{h}, \text{ where } h \text{ is constant.}$$

Integrating, and making $\frac{dy}{dx} = 0$, for $x = 0$, we have

$$E\frac{dy}{dx} = -\frac{2Tx}{h}.$$

Integrating again, and making $y = 0$, for $x = 0$, we have

$$Ey = -\frac{Tx^2}{h}, \text{ or } y = -\frac{Tx^2}{Eh}, \quad \ldots \ldots \ldots \ldots \quad (2)$$

which gives the deflection y for any value of x from the fixed end.

Example.—*Suppose a semi-girder of wrought iron ($E = 25000000$), whose height, centre to centre, is 20 feet, length 50 feet, and unit stress in the top chord, 8000 lbs., and in the bottom chord 10000 lbs. per square inch.*

$$\text{Then, } T = \tfrac{1}{2}(8000 + 10000) = 9000,$$

and the deflection at the end, $x = l$, taking l and h in inches, is

$$\Delta = \frac{9000 \times 600^2}{240 \times 25000000} = 0.54 \text{ inch.}$$

2. *Framed semi-girder of variable depth.*—In this case h will also vary with x. The depth usually varies as the ordinate to a straight line, or a parabola or circle.

If the upper chord slopes uniformly from the fixed end where the depth is h_1 to the free end where it is h_0, we have for the depth h at any point, if l is the length,

$$h = h_1 - \frac{h_1 - h_0}{l}x.$$

Hence

$$dh = \frac{h_0 - h_1}{l}dx, \quad \text{and } dx^2 = \left(dh\frac{l}{h_0 - h_1}\right)^2.$$

We have then from (1)

$$E\frac{d^2y\,(h_0 - h_1)^2}{dh^2 l^2} = \frac{2T}{h}.$$

Integrating, since $\frac{dy}{dh} = 0$, for $h = h_1$, we have

$$\frac{E(h_0 - h_1)^2}{2Tl^2}\frac{dy}{dh} = \text{nat. log}\frac{h}{h_1}.$$

Integrating again, since $y = 0$ for $h = h_1$, we have

$$\frac{E\,(h_0 - h_1)^2}{2\,Tl^2}y = h\left(\text{nat. log}\,\frac{h}{h_1} - 1\right) + h_1,$$

or,
$$y = \frac{2\,Tl^2}{E\,(h_0 - h_1)^2}\left[h_1 - h\left(2.302585\,\log\frac{h_1}{h} + 1\right)\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

In similar manner, if the chords are one or both parabolic, we have $h = h_1 - \frac{h_1 - h_0}{l^2}x^2$, and by two integrations, putting $m^2 = \frac{h_1}{h_1 - h_0}$,

$$y = \frac{2.302585\;Tl}{m\,(h_1 - h_0)\,E}[(ml+x)\log(ml+x) + (ml-x)\log(ml-x) - 2\,ml\log ml] \quad . \quad (4)$$

If the chords are circular, the deflection will be not sensibly different from that for parabolic chords.

For framed or open work girders supported at both ends, the same formulæ apply as for semi-girders, except that we must take the origin of x at the centre of the span, and put $\frac{1}{2}l$ for l in (2), (3), and (4), l being in all cases the span.

EXAMPLE.—*Suppose a framed wrought iron girder,* $E = 25000000$, *height at centre* 25 *feet, span* = 200 *feet, supported at the ends. Unit stress in upper chord* 8000, *in lower chord* 10000 *lbs. per sq. inch.*

We have here $T = \frac{1}{2}(8000 + 10000) = 9000$.

If the height is constant, we have from (2), for the central deflection, making $x = \frac{l}{2}$,

$$\Delta = \frac{Tl^2}{4\,Eh} = \frac{9000 \times 2400^2}{4 \times 25000000 \times 300} = 1.728 \text{ inches.}$$

If the span is a truncated bowstring, the depth at centre $h_1 = 25$ feet, and at ends $h_0 = 15$ feet, then from (4) we have, since $m^2 = \frac{h_1}{h_1 - h_0} = 2.5$, $m\,(\frac{1}{2}l) = 1897.366$, $m\,(\frac{1}{2}l) + x = 3097.366$, $m\,(\frac{1}{2}l) - x = 697.366$, $\log\frac{1}{2}ml = 3.2781512$, $\log(\frac{1}{2}ml + x) = 3.4909925$, $\log(\frac{1}{2}ml - x) = 2.8434608$;

for the central deflection,

$$\Delta = \frac{2.302585 \times 9000 \times 1200}{1.58114 \times 120 \times 25000000}(10812.88 + 1982.93 - 12439.70) = 1.86695 \text{ inches.}$$

If the chords slope uniformly, $h_1 = 25$, $h_0 = 15$, we have for the central deflection, from (3),

$$\Delta = \frac{2 \times 9000 \times 2400^2}{4 \times 25000000 \times 120}\left[300 - 180\left(2.302585\,\log\frac{300}{180} + 1\right)\right] = 2.0197 \text{ inches.}$$

For framed or open work girders fixed horizontally at both ends, the points of contrary flexure may be taken at half way between the load if concentrated, or the centre of gravity of the loading if distributed, and the fixed ends. The formulæ for semi-girders then apply here also, as the beam is divided by the points of contrary flexure into a semi-girder at each end, and a girder supported at the ends in the middle.

EXAMPLE.—*Suppose a framed wrought iron girder* ($E = 25000000$) 100 *feet long, top chord sloping uniformly, height at left end* $h_0 = 15$ *feet, and at right end* $h_1 = 25$ *feet, to be fixed horizontally at both ends. The unit stress in upper chord* 8000 *and in lower chord* 10000 *lbs. per sq. inch. Load of* 200000 *lbs. at centre.*

We have $T = \frac{1}{2}(8000 + 10000) = 9000$.

The points of contrary flexure are at 25 feet from each end. The case becomes then, a semi-girder of 25 feet and a girder supported at the ends of 50 feet.

For the semi-girder, we have height at fixed end $h_1 = 15$ feet, and at free end $h = h_0 = 15 + \frac{10}{4} = 17.5$ feet. Hence, from (3), the deflection is

$$y = \frac{2 \times 9000 \times 300^2}{25000000 \times 30^2} \left[180 - 210 \left(2.302585 \log \tfrac{6}{7} + 1\right)\right] = 0.171 \text{ inch.}$$

For the girder supported at the ends, we have $h_1 = 20$ feet, and $h = h_0 = 17.5$ feet, and hence the deflection is

$$y = \frac{2 \times 9000 \times 300^2}{25000000 \times 30^2} \left[240 - 210 \left(2.302585 \log \tfrac{8}{7} + 1\right)\right] = 0.14 \text{ inch.}$$

The total deflection at centre is then 0.311 inch.

If the girder is fixed at one end and supported at the other, the point of contrary flexure may be taken between the weight and fixed end, or between the centre of gravity of the loading and fixed end, making the distance from it to the weight or centre of gravity of the loading, equal to the distance of the latter from the free end. We thus have a girder supported at ends and a semi-girder. The one extends from the free end to the point of contrary flexure. The other from this point to the free end.

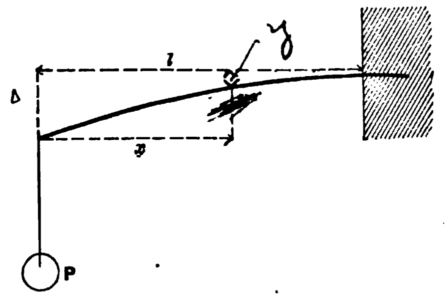
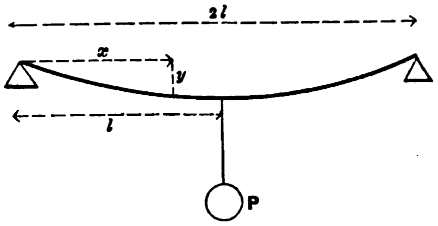
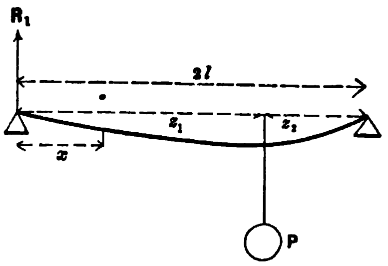
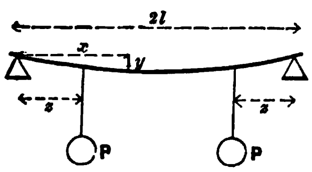
Equations (2), (3), and (4) then, will if properly applied, solve all cases of deflection of framed or open work girders.

Pages 265 to 266, have been condensed from "Mechanics of the Girder," by John Davenport Crehore, Wiley & Sons, 1886. The reader will there find the preceding method of determining the deflection of framed girders applied at length, and illustrated by numerous examples. The preceding will be found sufficient for all practical purposes.

Mr. Crehore, in the work cited, has also shown that for properly framed girders (*i. e.* of *uniform strength*), of the same central height and length, "*the total deflection is* NEARLY *in the inverse ratio of the areas of the figures of the girders.* This is exactly the ratio of the deflection in case of the girder of uniform height, and of that sloping uniformly; *viz.*, ratio of areas, $\frac{2}{1}$; ratio of deflections, $\frac{1}{2}$. We may therefore, without appreciable error, employ this principle in finding the total deflection of open or framed girders of uniform strength and variable height."

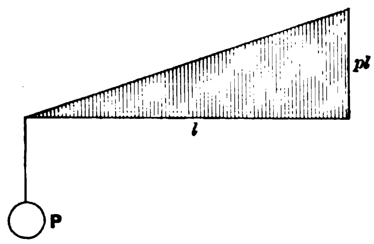
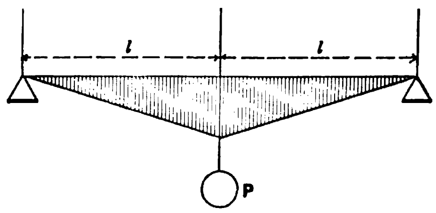
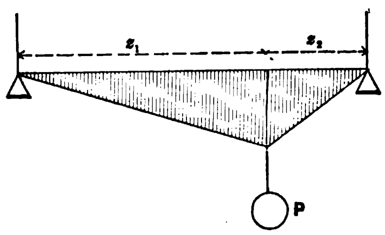
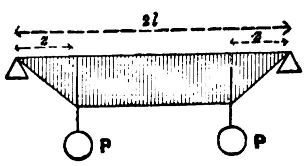
We give below a recapitulation of our results, as well as some

BEAMS OF CONSTANT

CASE.	MOMENT.	EQUATION OF ELASTIC LINE.
	$M_x = - Px,$ max. $M = - Pl.$	$y = \frac{P}{6\,EI}[2\,l^3 - 3\,l^2 x + x^3],$ $\Delta = \frac{Pl^3}{3\,EI}$ at end.
	when $x < l,$ $M_x = \frac{P}{2}x,$ max. $M = \frac{Pl}{2}.$	$y = \frac{Px}{12\,EI}[3\,l^2 - x^2],$ $\Delta = \frac{Pl^3}{6\,EI}$ at centre.
	$x < z_1,$ $M_x = R_1 x = \frac{Pz_2 x}{2\,l},$ $x > z_1,$ $M_x = \frac{Pz_2 x}{2\,l} - P(x - z_1)$ max. $M = \frac{Pz_1 z_2}{2\,l}.$	$x < z_1,$ $y = \frac{Pz_2 x}{12\,EIl}[4\,lz_1 - z_1^2 - x^2],$ $x > z_1,$ $y = \frac{Pz_1(2\,l - x)}{12\,EIl}[4\,lx - x^2 - z_1^2],$ $\Delta = \frac{Pz_1 z_2(4\,l - z_1)}{54\,EIl}\sqrt{3\,z_1(4\,l - z_1)},$ max. deflection occurs at $x = \sqrt{\frac{1}{3}(4\,l - z_1)z_1}.$
	$x < z,$ $M_x = Px,$ $x > z,$ $M_x = Pz =$ max. $M.$	$x < z,$ $y = \frac{Px}{6\,EI}[6\,lz - 3\,z^2 - x^2],$ $x > z,$ $y = \frac{Pz}{6\,EI}[6\,lx - 3\,x^2 - z^2],$ $\Delta = \frac{Pz}{6\,EI}[3\,l^2 - z^2]$ at centre.

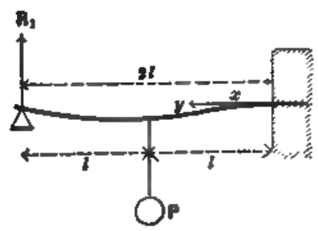
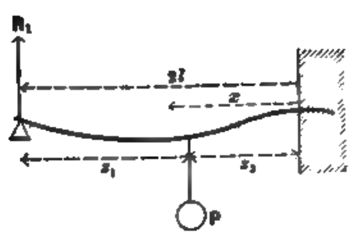
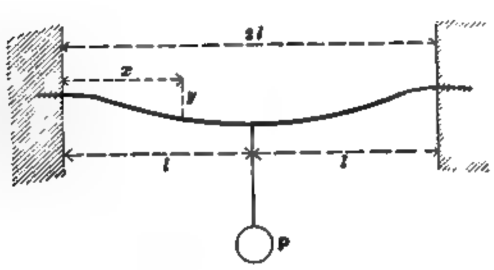
others which the student can now readily demonstrate.

CROSS SECTION.

BREAKING WEIGHT.	RELATIVE STRENGTH.	GRAPHIC ILLUSTRATION OF MOMENTS.
$P = \frac{TI}{vl}$, or $\frac{CI}{vl}$.	1	
$P = \frac{2\,TI}{vl}$, or $\frac{2\,CI}{vl}$.	4	
$P = \frac{2\,TIl}{vz_1\,z_2}$. In general, either T or C, whichever is the least, is to be put for T in all formulæ for breaking weight.	$4\,\frac{l^2}{z_1\,z_2}$.	
$P = \frac{TI}{vz}$.	$2\,\frac{l}{z}$	

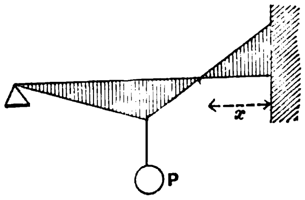
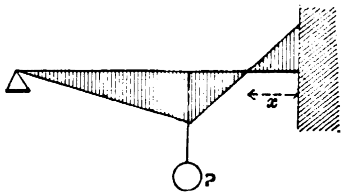
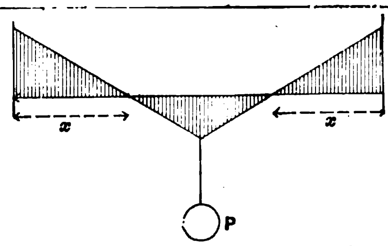
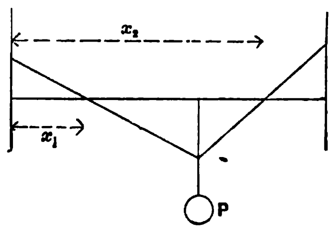
We give below a recapitulation of our results, as well as some

BEAMS OF CONSTANT

CASE.	MOMENT.	EQUATION OF ELASTIC LINE.
	$R_1 = \frac{5}{16} P$, $x > l$, $M_x = \frac{5}{16} P (2l - x)$, $x < l$, $M_x = \frac{P}{16} (11x - 6l)$, max. $M = \frac{3}{8} Pl$.	$x < l$, $y = \frac{Px^2}{96 EI} [18l - 11x]$, $x > l$, $y = \frac{P}{96 EI} [5x^3 - 30lx^2 + 48 l^2 x - 16 l^3]$ $\Delta = \frac{1}{6} \frac{Pl^3}{\sqrt{5} EI}$, Max. deflection occurs at $x = 2l\left(1 - \frac{1}{\sqrt{5}}\right)$.
	$R_1 = P \frac{6 z_2^2 l - z_2^3}{16 l^3}$, $x < z_2$, $M_x = R_1 (2l - x)$ $- P (z_2 - x)$, $x > z_2$, $M_x = R_1 (2l - x)$, max. $M = R_1 (2l - z_2)$.	$x < z_2$ $y = \frac{P}{6 EI} [R_1 x^3 - 6 R_1 lx^2$ $+ 3 P z_2 x^2 - Px^3]$, $x > z_2$, $y = \frac{1}{6 EI} [R_1 x^3 - 6 R_1 lx^2$ $+ 3 P z_2^2 x - P z_2^3]$, $\Delta = \frac{P z_2^2}{6 EI} (2l - z_2) \sqrt{\frac{2l - z_2}{6l - z_2}}$, Max. deflection occurs at $x = 2l - 2l \sqrt{\frac{2l - z_2}{6l - z_2}}$.
	$x < l$, $M_x = \frac{P}{4} (2x - l)$, $x > l$, $M_x = \frac{P}{4} (3l - 2x)$, max. $M = -\frac{Pl}{4}$.	$x < l$, $y = \frac{Px^2}{24 EI} [3l - 2x]$, $x > l$, $y = \frac{P}{24 EI} [2x^3 + 12 l^2 x$ $- 4 l^3 - 9 lx^2]$, $\Delta = \frac{Pl^3}{24 EI}$.
	$R_1 = P \frac{z_2^2 (3 z_1 + z_2)}{8 l^3}$, $M_1 = -P \frac{z_1 z_2^2}{4 l^2}$, $x < z_1$, $M_x = R_1 x + M_1$, $x > z_1$, $M_x = R_1 x$ $- P (x - z_1) + M_1$, max. $M = \frac{P z_1 z_2^2}{4 l^2}$ at end.	$x < z_1$, $y = \frac{P x^2 z_2^2}{48 E I l^3} [6 z_1 l - (3 z_1 + z_2) x]$ $x > z_1$, $y = \frac{P x^2 z_2^2}{48 E I l^3} \Big[\frac{8 l^3 (x - z_1)^3}{x^2 z_2^3}$ $+ 6 z_1 l - (3 z_1 + z_2) x\Big]$. $\Delta = \frac{2 P z_1^3 z_2^2}{3 EI (3 z_1 + z_2)^2}$ when $x < z_1$. Max. deflection occurs at $x = \frac{4 l z_1}{3 z_1 + z_2}$.

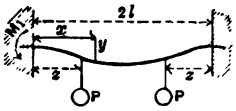
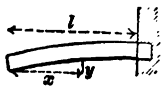
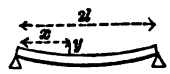
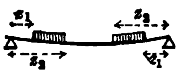
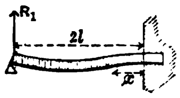
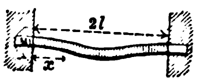
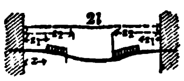
others, which the student can now readily demonstrate.—*Continued.*

CROSS SECTION.

BREAKING WEIGHT.	RELATIVE STRENGTH.	GRAPHIC ILLUSTRATION OF MOMENTS.
$P = \frac{16\ TI}{5\ vl}$.	6.4.	 Distance of point of inflection $x = \frac{6}{11}\ l$.
$P = \frac{16\ TIl^3}{vz_2^2(6l - z_2)(2l - z_2)}$.	$\frac{32\ l^4}{z_2^2(6l - z_2)(2l - z_2)}$.	 Distance of point of inflection $x = \frac{Pz_2 - 2\ R_1 l}{P - R_1}$.
$P = \frac{4\ TI}{vl}$.	8.	 Distance to point of inflection $x = \frac{l}{2}$.
$P = \frac{27\ TI}{8\ vl}$.	6.75.	 Distance to point of inflection $x_1 = \frac{z_1}{3\ z_1 + z_2}\ 2\ l$. $x_2 = \frac{8\ z_1 l^3 - 2\ z_1 z_2^2 l}{8\ l^3 - z_2^2(3z_1 + z_2)}$.

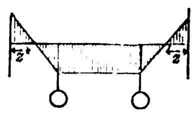
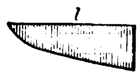
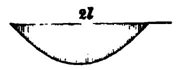
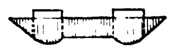
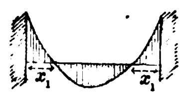
We give below a recapitulation of our results, as well as some

BEAMS OF CONSTANT

CASE.	MOMENT.	EQUATION OF ELASTIC LINE.
	$x < z$, $M_x = Px + \frac{Pz^2}{2l} - Pz$ $x > z$, $M_x = \frac{Pz^2}{2\,l} = \text{max. } M.$	$x < z$, $y = \frac{Px^2}{12\,EIl}[6\,lz - 3\,z^2 - 2\,xl]$, $x > z$, $y = \frac{Pz^2}{12\,EIl}[6\,lx - 3\,x^2 - 2\,zl]$, $\varDelta = \frac{Pz^2}{12\,EI}[3\,l - 2\,z]$ at centre.
	$M_x = -\frac{px^2}{2}$, $\text{max. } M = -\frac{pl^2}{2}.$	$y = \frac{p}{24\,EI}[x^4 - 4\,l^3x + 3\,l^4]$, $\varDelta = \frac{pl^4}{8\,EI}.$
	$M_x = plx - \frac{px^2}{2}$, $\text{max. } M = \frac{pl^2}{2}.$	$y = \frac{px}{24\,EI}[x^3 - 4\,lx^2 + 8\,l^3].$
	$x < z_1$, $M_x = p\,(z_2 - z_1)\,x$, $x > z_2$, $M_x = \frac{p\,(z_2^2 - z_1^2)}{2}.$	$x < z_1$, $y = \frac{px}{6\,EI}[x^2(z_2 - z_1) + (z_2^3 - z_1^3)$ $- 3\,l\,(z_2^2 - z_1^2)]$, $x > z_2$, $y = \frac{p}{24\,EI}[(z_2^2 - z_1^2)(12\,xl - 6\,x^2)$ $- (z_2^4 - z_1^4)].$
	$R_1 = \frac{5}{8}\,pl$, $M_x = \frac{p}{4}(2\,x - l)$ $(2\,l - x)$, $\text{max. } M = -\frac{pl^2}{2}.$	$y = -\frac{px^2}{48\,EI}(2\,l - x)(6\,l - 2\,x)$, $\varDelta = 0.0864\,\frac{pl^4}{EI}$, Max. deflection occurs at $x = 1.157\,l.$
	$M_1 = -\frac{pl^2}{3}$, $M_x = plx - \frac{px^2}{2} - \frac{pl^2}{3}$ $\text{max. } M = -\frac{pl^2}{3}.$	$y = \frac{px^2}{24\,EI}[4\,l^2 + x^2 - 4\,lx]$, $\varDelta = \frac{pl^4}{24\,EI}$ at centre.
	$x < z_1$, $M_x = px\,(z_2 - z_1)$ $+ \frac{p}{6\,l}(z_2^3 - z_1^3)$ $- \frac{p}{2}(z_2^2 - z_1^2)$, $x > z_2$, $M_x = \frac{p}{6\,l}(z_2^3 - z_1^3).$	$x < z_1$, $y = \frac{px^2}{12\,EIl}[3\,l\,(z_2^2 - z_1^2) - (z_2^3 - z_1^3)$ $- 2\,lx\,(z_2 - z_1)$, $x > z_2$, $y = \frac{p}{24\,EIl}[(4\,lx - 2\,x^3)(z_2^3 - z_1^3)$ $- l\,(z_2^4 - z_1^4)].$

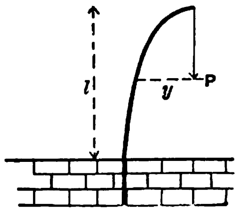
others, which the student can now readily demonstrate.—*Continued.*

CROSS SECTION.

BREAKING WEIGHT.	RELATIVE STRENGTH.	GRAPHIC ILLUSTRATION OF MOMENTS.
$P = \frac{2\,TIl}{vz^2}.$	$4\,\frac{l^2}{z^2}.$	 $x_1 = z - \frac{z^2}{2\,l}.$
$P = \frac{2\,TI}{vl}.$	2	 Curve of moments a parabola.
$P = \frac{4\,TI}{vl}.$	8	 Curve of moments a parabola.
$P = \frac{4\,TI}{v\,(z_2 + z_1)}.$	$8\,\frac{l}{z_2 + z_1}.$	
$P = \frac{4\,TI}{vl}.$	8	 $x_1 = \frac{1}{2}\,l.$ Curve of moments a parabola.
$P = \frac{6\,TI}{vl}.$	12	 $x_1 = 0.42262\,l.$
$P = \frac{12\,TIl}{v\,(z_2^2 + z_2\,z_1 + z_1^2)}.$	$\frac{24\,l^2}{z_2^2 + z_2\,z_1 + z_1^2}.$	

COLUMN FORMULÆ.

COLUMN—FIXED AT ONE END, FREE AT THE OTHER.—



Let l = the length of column,

P = the weight at the free extremity.

Then, for the moment at any point, we have

$$EI\frac{d^2y}{dx^2} = -Py.$$

Multiply both sides by $2dy$ and we have

$$EI\frac{2\,dy\,d^2y}{dx^2} = -2\,Py\,dy.$$

Integrating, we have

$$EI\frac{dy^2}{dx^2} = -Py^2 + C.$$

But when y = the maximum deflection Δ, $\frac{dy}{dx} = 0$, hence $C = P\Delta^2$, and

$$dx = \sqrt{\frac{EI}{P}}\,\frac{dy}{\sqrt{\Delta^2 - y^2}}.$$

Integrating again

$$x = \sqrt{\frac{EI}{P}}\text{ arc sin}\frac{y}{\Delta} + C'.$$

When $y = 0$, $x = 0$, hence $C' = 0$, and we have

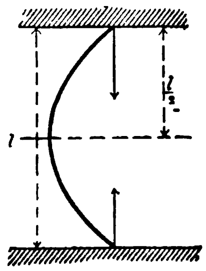
$$y = \Delta \sin x\sqrt{\frac{P}{EI}}.$$

When $x = l$, $y = \Delta$, and hence

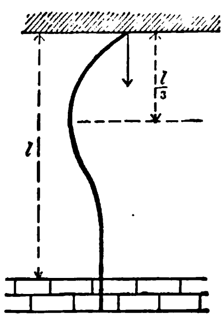
$$l\sqrt{\frac{P}{EI}} = \frac{\pi}{2},$$

or

$$P = EI\frac{\pi^2}{4l^2}.$$

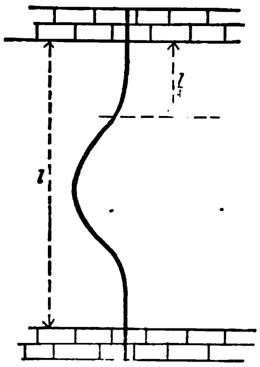


COLUMN—ROUND AT BOTH ENDS.—In this case we see that the strength is the same as that of a column fixed at one end and free at the other, of one half the length. We have, therefore, $P = EI\frac{\pi^2}{l^2}$.



COLUMN—FIXED AT ONE END AND ROUND AT THE OTHER.—In this case we see that the strength is the same as that of a column fixed at one end and free at the other, of one third the length. We have, therefore

$$P = \tfrac{9}{4}\frac{\pi^2 EI}{l^2}.$$



COLUMN—FIXED AT BOTH ENDS.—In this case, the strength is that of a column fixed at one end and free at the other, of only one-fourth the length. Hence

$$P = \frac{4\pi^2 EI}{l^2}.$$

EULER'S FORMULÆ.—The preceding formulæ are all of the form

$$P = \frac{n\pi^2 EI}{l^2},$$

where the value of n depends upon the condition of the ends.

Thus for one end fixed and one end free, $n = \frac{1}{4}$.

" both ends round or free to turn, $n = 1$.

" one end fixed and one end round, $n = \frac{9}{4}$.

" both ends fixed, $n = 4$.

These formulæ are known as Euler's formulæ.

Since for round columns $I = \frac{\pi d^4}{4}$, we see from Euler's formulæ, that the strength of round columns is directly as the fourth power of the diameter, and inversely as the square of the length.

HODGKINSON'S FORMULÆ FOR ROUND COLUMNS.—Hodgkinson found, as the result of his experiments upon round columns, that Euler's formulæ were only approximately correct.

Thus Hodgkinson's empirical formulæ are :

For solid cast-iron cvlindrical columns, round ends, $P = 14.9 \frac{d^{3.5}}{l^{1.63}}$.

" " " " flat ends, $P = 44.2 \frac{d^{3.5}}{l^{1.63}}$.

For solid wrought-iron cylindrical columns, round ends, $P = 42 \frac{d^{3.76}}{l^2}$.

" " " " flat ends, $P = 134 \frac{d^{3.76}}{l^2}$.

RANKINE'S FORMULA.—The foregoing formulæ assume that the load P acts exactly in the longitudinal axis of the column. Let us suppose that this is not the case, but that the load is more or less eccentric, and acts at a distance from the axis of q.

Since the deflection of the column, in all cases where the limit of elasticity is not exceeded, is small, we may consider the elastic curve formed by the axis of the column to be a circle. The radius of curvature for a column fixed at one end and free at the other will then be $\rho = \frac{l^2}{2\Delta}$ approximately.

From the theory of flexure we have (equation 4),

$$\frac{EI}{\rho} = P(\Delta + q), \text{ hence } P(\Delta + q)l^2 = 2EI\Delta.$$

and

$$\Delta = \frac{Pql^2}{2EI - Pl^2}, \text{ or } \Delta + q = \frac{2EIq}{2EI - Pl^2}.$$

Again, from the theory of flexure (equation 4), we have,

$$P(\varDelta + q) = \frac{RI}{v}, \text{ or } R = \frac{P(\varDelta + q)v}{I} = \frac{2PEqv}{2EI - Pl^2},$$

where R is the unit stress in the outer fibre, due to flexure alone.

If μ is the ultimate strength of the material, that is, the unit stress in the outer fibre at the moment of breaking, then $\mu = R + \frac{P}{A}$, or $R = \mu - \frac{P}{A}$.

Hence $$\mu - \frac{P}{A} = \frac{2PEqv}{2EI - Pl^2} \text{ or } \mu A = P\left(1 + \frac{2AEqv}{2EI - Pl^2}\right).$$

or $$P(2EI - Pl^2 + 2AEqv) = \mu A(2EI - Pl^2)$$

Since Pl^2 is small compared to $2EI + 2AEqv$, we may neglect it, and hence if r^2 is the square of the radius of gyration of the cross section,

$$\frac{P}{A} = \frac{2\mu EI}{2E(I + Aqv) + \mu Al^2} = \frac{\mu}{1 + \frac{qv}{r^2} + \frac{\mu l^2}{2Er^2}}.$$

If the load acts in the axis of the column, $q = 0$, and we have for the "crippling strength" of the column per square inch,

$$\frac{P}{A} = \frac{\mu}{1 + c\frac{l^2}{r^2}},$$

where $c = \frac{\mu}{2E}$ is a constant, depending upon the end conditions of the column and upon the material.

In this form, the equation is known as *Rankine's formula*, and is the one most generally used for long struts. We see that the accuracy of this formula depends essentially upon the conditions that the load shall *be applied in the axis*, and the *limit of elasticity shall not be exceeded.*

In order not to exceed the limit of elasticity, the allowable working unit stress must be only a portion of the "crippling strength." The fraction of the crippling strength thus taken is the "factor of safety." The factor usually taken is $\frac{1}{4 + \frac{l}{20d}}$, where l is the length in inches, and d the least dimension of the cross section in inches. If β is the allowable working stress, per square inch, we have then

$$\beta = \frac{1}{4 + \frac{l}{20d}}\left[\frac{\mu}{1 + \frac{cl^2}{r^2}}\right].$$

Euler's formulæ are defective in not taking account of the strength of the material, and Hodgkinson's in their limited range. Under the above conditions, Rankine's formula is preferable.

TREDGOLD'S OR GORDON'S FORMULA.—Since r^2 is in general some function of the depth or d^2, we may write Rankine's formula in the form,

$$\frac{P}{A} = \frac{\mu}{1 + k\frac{l^2}{d^2}},$$

where d is the least depth of cross section, and k is a constant to be determined by experiment.

We see at once that, except in the case of rectangular and circular cross sections, r will not be a simple function of the depth, but will vary with the breadth also. Hence the strict application of Tredgold's formula is limited to rectangular and circular cross sections. For all others k is *not* a constant, and hence Rankine's formula is preferable. The usual values of k and c will be found given in Part II.

COMBINED TENSION AND FLEXURE.—We have from equation (4),

$$M = \frac{RI}{v},$$

for simple flexure, where M is the moment at any cross section in inch lbs., R the allowable stress in lbs. per square inch in the outer fibre, v the distance from the neutral axis to the outer fibre, in inches, and I the moment of inertia of the cross section, all dimensions being taken in inches.

But a piece may sometimes be subjected to flexure, and at the same time to tension. Thus, for instance, the lower chord of a bridge may be subjected to tension through the whole panel length, and at the same time a load may be applied by means of a cross-tie between the panel points.

In such a case, let S be the tensile stress uniformly distributed over the area of cross section A. Then $\frac{S}{A}$ is the unit tensile stress. The combined unit stress on the outer fibres in tension will then be $R + \frac{S}{A}$, and on the outer fibres in compression $R - \frac{P}{A}$, where R is the outer fibre unit stress due to flexure alone. The neutral axis is now no longer at the centre of gravity of the cross section. A strict discussion of the case leads to results of great complexity. If, however, we neglect the deflection, as in all practical cases we may safely do, we may proportion the beam as follows :

Let β be the allowable working unit stress, which must not be exceeded. Then $\beta = R + \frac{S}{A}$, and hence $R = \beta - \frac{S}{A}$.

From equation (4) we have then

$$M = \frac{\left(\beta - \frac{S}{A}\right) I}{v},$$

where M is the maximum moment due to the loading, I the moment of inertia, and v the distance to the outer fibre from the centre. Putting for I its value Ar^2, where r is the radius of gyration of the cross section, we have for the cross section $A = \frac{Mv}{\beta r^2} + \frac{S}{\beta}$.

That is, the required area is that due to flexure alone *plus* that due to the tensile stress.

From these equations we can find the dimensions required in any practical case, for a piece subjected to flexure and tension simultaneously. The value of β is found by dividing the ultimate strength for the material by the factor of safety.

EXAMPLE.—*Suppose a rectangular iron bar which forms the bottom bay of a bridge to be 12 feet long, 2 inches wide, and to be subjected to a longitudinal tension of 20000 lbs. If it supports in addition a load of 5000 lbs. at the centre, what should be the depth, in order that the allowable unit stress shall not exceed 10000 lbs. per square inch?*

Here $M = 2500 \times 6 \times 12 = 180000$, $I = \frac{bd^3}{12} = \frac{d^3}{6}$, $v = \frac{d}{2}$, $\beta = 10000$, $\frac{S}{A} = \frac{20000}{2 \times d} = \frac{10000}{d}$. Hence, $180000 = \left(10000 - \frac{10000}{d}\right)\frac{d^2}{3}$, or $d^2 - d = 54$, $\therefore d = 7.86$ inches.

COMBINED COMPRESSION AND FLEXURE.—This case is exactly similar to the preceding, except that for the allowable unit stress β, we should take the value given by Gordon's formula for long struts, as already given.

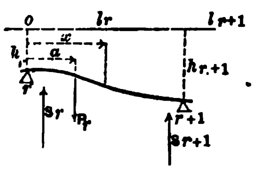
CHAPTER III.

CONTINUOUS GIRDER.

In the following we shall give the complete development of the general formulæ of Chapters VI. and VII. As these formulæ include, as we have seen, all the others as special cases, it is sufficient to show how they are obtained in order to enable the reader to deduce all the others. The notation adopted is the same as that given on page 139.

CONDITIONS OF EQUILIBRIUM.—In the r th span of a continuous girder, whose length is lr, Fig. 188, take a point o vertically above the r th support as the origin of co-ordinates, and the horizontal through o as the axis of abscissas. At a distance x from the left support, conceive a vertical section, and between the support and this section let there be a concentrated load P_r, whose distance from the left support is a.

Fig.188



Now, if the girder is continuous over any number of supports, we have at the support r a moment M_r, and just to the right of support r, a shear S_r.

For any point of the girder the necessary conditions of equilibrium are,

1st. The algebraic sum of all the horizontal forces must be zero.

2d. The algebraic sum of all the vertical forces must be zero.

3d. The algebraic sum of the moments of all the forces must be zero.

Thus for any section x we have from the third condition for the moment M_x at the section x.

$$M_x = M_r + S_r x - P_r (x - a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If in this we make $x = l_r$, M_x becomes M_{r+1}, and we have

$$M_{r+1} = M_r + S_r l_r - P_r (l_r - a).$$

From this we obtain the shear S_r in terms of the moments at the two supports, or

$$S_r = \frac{M_{r+1} - M_r}{l_r} + \frac{P_r}{l_r}(l_r - a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is the same as equation (III.a), page 141.

For an unloaded span the weight P disappears, and we have

$$S_m = \frac{M_{m+1} - M_m}{l_m}.$$

This is equation (IV.), given on page 141.

For the shear just to the left of the right support of loaded span,

$$S'_{r+1} = P - S_r = \frac{M_r - M_{r+1}}{l_r} + \frac{P_r a}{l_r}.$$

This is the same as equation (III.b), given on page 141. For unloaded span, the weight P disappears, and

$$S'_m = \frac{M_{m-1} - M_m}{l_{m-1}}.$$

S'_m is the shear on the left of any support m, and S_m that on the right. The reaction at any support is

$$R_m = S'_m + S_m.$$

These are the formulæ already given in Chapter X., page 141.

EQUATION OF THE ELASTIC LINE.—We can now easily deduce the equation of the elastic line for the continuous girder of constant cross section, or constant moment of inertia.

The differential equation of the elastic line is (page 246),

$$EI\frac{d^2y}{dx^2} = M_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where E is the coefficient of elasticity, and I is the moment of inertia of the cross section.

Inserting the value of M_x as given by (1) we have

$$\frac{d^2y}{dx^2} = \frac{M_r + S_r x - P_r(x-a)}{EI}.$$

We can integrate this expression between the limits $x = 0$, and x, upon the condition that x is always greater than a, that is, *the point considered is always on the right of the weight.* When, therefore, $x = 0$, a must be zero also, and hence $(x - a) = 0$. We must, therefore, take the integral of $P_r(x - a)$ simultaneously between the limits $x = a$, and x, or treat $(x - a)$ as a variable which becomes zero when $x = 0$.

We have, then, integrating once,

$$\frac{dy}{dx} = \frac{2M_r x + S_r x^2 - P_r(x-a)^2}{2EI} + C;$$

where the constant of integration $C = \frac{dy}{dx} = t_r =$ the tangent of the angle which the tangent at the support r to the curve of deflection makes with the horizontal. Hence

$$\frac{dy}{dx} = t_r + \frac{2M_r x + S_r x^2 - P_r(x-a)^2}{2EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

If we take the origin at a distance h_r above the support r, Fig. 204, and integrate again, the constant will be $-h_r$, and hence

$$y = -h_r + t_r x + \frac{3M_r x^2 + S_r x^3 - P_r(x-a)^3}{6EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

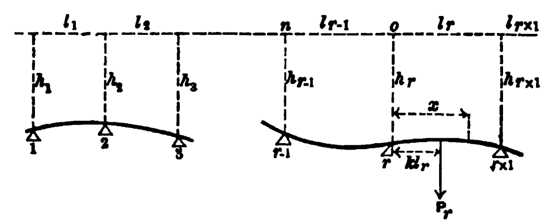
which is the general equation of the elastic curve. If in this we make $x = l_r$, y becomes $-h_{r+1}$. If also we put $\frac{a}{l_r} = k$, or $a = kl_r$, and insert for S_r its value as given by (2), we have for t_r

$$t_r = -\frac{h_{r+1} - h_r}{l_r} - \frac{1}{6EI}\left[2M_r l_r + M_{r+1}l_r + P_r l_r^2(2k - 3k^2 + k^3)\right] \quad . \quad . \quad . \quad (5)$$

We see, therefore, that the equation of the curve of deflection is completely determined when we know M_r and M_{r+1}, the moments at the two supports of the loaded span.

THEOREM OF THREE MOMENTS.—These moments are readily found by applying the "theorem of three moments," which we shall now deduce.

Fig. 189



In Fig. 189 we have represented a portion of a continuous girder, the spans being l_1, l_2, etc., l_r, and the supports 1, 2 . . . . r.

The equation of the elastic line between P_r and the $r + 1$th support is given by (4), and the tangent of the angle which the curve makes with the horizontal is given by (3 *a*). If in (3 *a*) we substitute for S_r its value as given by (2), and for t_r its value from (5), and make at the same time $x = l_r$, then $\frac{dy}{dx}$ becomes t_{r+1}, or the tangent at $r + 1$, and we have

$$t_{r+1} = -\frac{h_{r+1} - h_r}{l_r} + \frac{1}{6EI}[M_r l_r + 2M_{r+1} l_r + P_r l_r^2 (k - k^3)] \quad . \; . \; . \; . \quad (6)$$

Equation (6) gives the tangent of the angle which the tangent to the curve, at the support $r + 1$, makes with the horizontal.

If we were to suppose a weight P_{r-1} in the span l_{r-1} at a distance kl_{r-1} from the support $r - 1$, the origin being taken at n, Fig. 189, instead of at o, and were to find in a similar manner t_r, we should evidently obtain precisely the same equation as (6), only each of the subscripts would be diminished by unity. Hence we can write down at once

$$t_r = -\frac{h_r - h_{r-1}}{l_{r-1}} + \frac{1}{6EI}[M_{r-1} l_{r-1} + 2M_r l_{r-1} + P_{r-1} l_{r-1}^2 (k - k^3)] \quad . \; . \; . \quad (7)$$

If there is no weight in the span l_{r-1}, equation (7) still holds good, only P_{r-1} is zero.

But equation (5) gives us t_r for a weight P_r in the span l_r. If there is no weight in that span P_r is zero. Equating these two values of t_r, we have, generally,

$$M_{r-1} l_{r-1} + 2M_r (l_{r-1} + l_r) + M_{r+1} l_r = -6EI\left[\frac{h_{r-1} - h_r}{l_{r-1}} + \frac{h_{r+1} - h_r}{l_r}\right]$$
$$- P_{r-1} l_{r-1}^2 (k - k^3) - P_r l_r^2 (2k - 3k^2 + k^3) \quad . \; . \; . \; . \quad (8)$$

This is the general form of the theorem of three moments for a girder of constant cross section. It gives the relation between the moments at three consecutive supports, in terms of the spans, the load in the spans and the height of the supports.

The moments at the end supports are of course zero, when the girder is merely supported at the ends. For each of the piers, then, we can write an equation like the above, and thus we have as many equations as there are unknown moments.

DETERMINATION OF MOMENTS—UNIFORM LOAD—SUPPORTS ALL ON A LEVEL—SPANS ALL EQUAL.—When all the supports are in the same horizontal line, the ordinates h_1, h_2, h_3, etc., are all equal. Hence the term involving EI disappears, and we have simply

$$M_{r-1} l_{r-1} + 2M_r (l_{r-1} + l_r) + M_{r+1} l_r = -P_{r-1} l_{r-1}^2 (k - k^3) - P_r l_r^2 (2k - 3k^2 + k^3). \; . \quad (9)$$

If the spans are all equal, we have

$$M_{r-1} + 4M_r + M_{r+1} = -P_{r-1} l (k - k^3) - P_r l (2k - 3k^2 + k^3). \; . \; . \quad (10)$$

If we have the girder loaded from end to end with the load u per unit of length, then $uda = P$.

Substituting this value of P and remembering that $k = \frac{a}{l}$, and integrating between $a = 0$ and $a = l$, we have

$$P_{r-1} l_{r-1}^2 \, (k - k^3) = \frac{u l_{r-1}^3}{4}, \text{ and } P_r l^2 \, (2k - 3k^2 + k^3) = \frac{u l_r^3}{4}.$$

Hence for level supports, spans all equal, and uniform load, our theorem reduces to

$$M_{r-1} + 4 M_r + M_{r+1} = -\frac{ul^2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Let s be the number of spans. Then, applying equation (11) and remembering that M_1 and M_{s+1} are both zero, we can write down the following equations:

$$\left.\begin{array}{ll}
c_2 & 4 M_2 + M_3 = -\frac{ul^2}{2}, \\
c_3 & M_2 + 4 M_3 + M_4 = -\frac{ul^2}{2}, \\
c_4 & M_3 + 4 M_4 + M_5 = -\frac{ul^2}{2}, \\
c_5 & M_4 + 4 M_5 + M_6 = -\frac{ul^2}{2}, \\
c_6 & M_5 + 4 M_6 + M_7 = -\frac{ul^2}{2}, \\
 & \cdot \\
 & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
 & \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
c_{s-1} & M_{s-2} + 4 M_{s-1} + M_s = -\frac{ul^2}{2}, \\
c_s & M_{s-1} + 4 M_s = -\frac{ul^2}{2}.
\end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The solution of these equations can be best effected by the method of indeterminate coefficients. Thus we multiply the first equation by a number c_2, whose value we shall hereafter determine, so as to satisfy desired conditions. The second we multiply by c_3, the third by c_4, the r th by c_{r+1}, etc., the index of c corresponding always to that of M in the middle term. Having performed these multiplications, add the equations and arrange according to the coefficients of M_2, M_3, etc. We thus obtain the equation

$$\left.\begin{array}{l}
(4 c_2 + c_3) M_2 + (c_2 + 4 c_3 + c_4) M_3 + (c_3 + 4 c_4 + c_5) M_4 + \ldots\ldots\ldots \\
+ (c_{s-2} + 4 c_{s-1} + c_s) M_{s-1} + (c_{s-1} + 4 c_s) M_s = -\frac{ul^2}{2} (c_2 + c_3 + \ldots . c_s)
\end{array}\right\} \quad . \quad . \quad . \quad (13)$$

Now suppose we wish to determine M_s. We have only to require that such relations shall exist among the multipliers c, that all the terms except the last in the above equation shall disappear. We have, then, for the conditions which these multipliers must satisfy,

$$4c_2 + c_3 = 0, \quad c_3 + 4c_4 + c_5 = 0, \quad c_{s-2} + 4c_{s-1} + c_s = 0,$$
$$c_2 + 4c_3 + c_4 = 0, \quad c_4 + 4c_5 + c_6 = 0, \text{ etc.}$$

Assuming $c_2 = 1$, we find, therefore,

$$c_3 = -4, \quad c_4 = +15, \quad c_5 = -56, \quad c_6 = +209, \quad c_7 = -780, \quad c_8 = +2{,}911, \text{ etc.}$$

The numbers, as we see, change sign alternately, and each is equal to four times the preceding, minus the one next preceding it.

From equations (12) we see now that

$$M_3 = c_3 M_2 - \frac{ul^2}{2},$$

$$M_4 = -4M_3 - M_2 - \frac{ul^2}{2} = -4c_3 M_2 + 2ul^2 - M_2 - \frac{ul^2}{2} = c_4 M_2 + \frac{3ul^2}{2},$$

$$M_5 = -4M_4 - M_3 - \frac{ul^2}{2} = -4c_4 M_2 - 6ul^2 - c_3 M_2 + \frac{ul^2}{2} - \frac{ul^2}{2} = c_5 M_2 - \frac{12ul^2}{2}.$$

In similar manner,

$$M_6 = c_6 M_2 + \frac{44ul^2}{2}, \quad M_7 = c_7 M_2 - \frac{165ul^2}{2}, \text{ etc.}$$

But $1, -3, +12, -44, +165$, etc., are the algebraic sums of c_2, $c_2 + c_3$, $c_2 + c_3 + c_4$, $c_2 + c_3 + c_4 + c_5$, etc., respectively. Hence we have in general for the moment at any support,

$$M_m = c_m M_2 - \frac{ul^2}{2}(c_2 + \ldots c_{m-1}) \quad \ldots \ldots \ldots \quad (14)$$

Now from equation (13), since all the terms except the one containing M_s are zero, we have

$$M_s = -\frac{\frac{ul^2}{2}(c_2 + c_3 + \ldots c_s)}{c_{s-1} + 4c_s}.$$

But since according to the law of the numbers denoted by c, $c_{s-1} + 4c_s + c_{s+1} = 0$, we have $c_{s-1} + 4c_s = -c_{s+1}$. Hence

$$M_s = \frac{ul^2(c_2 + c_3 + \ldots c_s)}{2c_{s+1}}.$$

If the spans are all equal, the supports horizontal, and the load uniform over the whole girder, the moment at the support s must be the same as the moment at the support 2, or $M_s = M_2$. Hence

$$M_2 = \frac{ul^2(c_2 + c_3 + \ldots . c_s)}{2c_{s+1}} \quad \ldots \ldots \ldots \quad (15)$$

Equation (14) then becomes

$$M_m = c_m \frac{ul^2(c_2 + c_3 + \ldots . c_s)}{2c_{s+1}} - \frac{ul^2}{2}(c_2 + \ldots c_{m-1}) \quad \ldots \ldots \quad (16)$$

If we write down the values

$$\begin{aligned} c_2 &= 1, \\ 4c_2 + c_3 &= 0, \\ c_2 + 4c_3 + c_4 &= 0, \\ c_3 + 4c_4 + c_5 &= 0, \text{ etc.,} \end{aligned}$$

we see that the sum is in general

$$6(c_2 + c_3 + \ldots c_m) + 5c_{m+1} + c_{m+2} = 1.$$

Hence the sum of the first m numbers is

$$(c_2 + c_3 + \ldots \ldots c_m) = \tfrac{1}{6}(1 - 5c_{m+1} - c_{m+2}) \quad \ldots \ldots \quad (17)$$

Applying this formula for the sum of the numbers to equation (16), we have

$$(c_2 + \ldots c_s) = \tfrac{1}{6}(1 - 5c_{s+1} - c_{s+2}),$$

$$(c_2 + \ldots c_{m-1}) = \tfrac{1}{6}(1 - 5c_m - c_{m+1}).$$

Hence

$$M_m = \frac{ul^2}{12c_{s+1}}[c_m(1 - 5c_{s+1} - c_{s+2}) - c_{s+1}(1 - 5c_m - c_{m+1})],$$

or, after reducing,

$$M_m = \frac{ul^2}{12c_{s+1}}[c_m(1 - c_{s+2}) - c_{s+1}(1 - c_{m+1})] \quad \ldots \ldots \ldots \quad (18)$$

This is the formula given on page 147.

Determination of the Moments—Supports all on Level—Concentrated Load—Spans all Different.—When all the supports are on a level, the term involving EI in the theorem of three moments disappears, and we have

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -P_{r-1}l_{r-1}^2(k - k^3) - P_r l_r^2(2k - 3k^2 + k^3).$$

Now let s be the number of spans, and let a single load P be placed in the r th span.

From the above theorem, since M_1 and M_{s+1} are zero, we may write down the following equations:

$$\left.\begin{aligned} 2M_2(l_1 + l_2) + M_3 l_2 &= 0, \\ M_2 l_2 + 2M_3(l_2 + l_3) + M_4 l_3 &= 0, \\ \cdots\cdots\cdots & \cdots\cdots \\ \cdots\cdots\cdots & \cdots\cdots \\ M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -P_r l_r^2(2k - 3k^2 + k^3) &= -A, \\ M_r l_r + 2M_{r+1}(l_r + l_{r+1}) + M_{r+2}l_{r+1} = -P_r l_r^2(k - k^3) &= -B, \\ \cdots\cdots\cdots & \cdots\cdots \\ \cdots\cdots\cdots & \cdots\cdots \\ M_{s-2}l_{s-2} + 2M_{s-1}(l_{s-2} + l_{s-1}) + M_s l_{s-1} &= 0, \\ M_{s-1}l_{s-1} + 2M_s(l_{s-1} + l_s) &= 0, \end{aligned}\right\} \quad \ldots \quad (19)$$

We can best solve these equations by the method of indeterminate coefficients.

Thus we multiply the first equation by c_2, the second by c_3, and so on, the index of c corresponding always to that of the middle term. Having performed these multiplications, add the equations and arrange according to the coefficients of M_2, M_3, etc. We thus have the equation

$$
\begin{aligned}
&[2\,c_2\,(l_1 + l_2) + c_3\,l_2]\,M_2 + [c_2\,l_2 + 2\,c_3\,(l_2 + l_3) + c_4\,l_3]\,M_3 + \ldots, \\
&+ [c_{r-1}l_{r-1} + 2\,c_r\,(l_{r-1} + l_r) + c_{r+1}l_r]\,M_r + \ldots, \\
&+ [c_{s-2}l_{s-2} + 2\,c_{s-1}\,(l_{s-2} + l_{s-1}) + c_s\,l_{s-1}]\,M_{s-1}, \\
&+ [c_{s-1}l_{s-1} + 2\,c_s\,(l_{s-1} + l_s)]\,M_s = -\,Ac_r - Bc_{r+1} \quad \ldots \ldots \quad (20)
\end{aligned}
$$

Now suppose we wish to determine M_s. We have only to impose such conditions upon the multipliers that all the terms in the first member of the above equation, except the last, shall disappear. We have then evidently, for the conditions which the multipliers must satisfy,

$$
\begin{aligned}
2\,c_2\,(l_1 + l_2) + c_3 l_2 &= 0, \\
c_2 l_2 + 2\,c_3\,(l_2 + l_3) + c_4 l_3 &= 0, \\
\ldots\ldots\ldots\ldots& \\
\ldots\ldots\ldots\ldots& \\
c_{r-1}l_{r-1} + 2\,c_r\,(l_{r-1} + l_r) + c_{r+1}l_r &= 0, \\
\ldots\ldots\ldots\ldots& \\
c_{s-2}l_{s-2} + 2\,c_{s-1}\,(l_{s-2} + l_{s-1}) + c_s l_{s-1} &= 0, \text{ etc.,}
\end{aligned}
$$

while for M_s we have at once

$$
M_s = -\,\frac{Ac_r + Bc_{r+1}}{c_{s-1}l_{s-1} + 2\,c_s\,(l_{s-1} + l_s)} = \frac{Ac_r + Bc_{r+1}}{c_{s+1}l_s} \quad \ldots \ldots \quad (21)
$$

If in similar manner we multiply the *last* of equations (19) by the number d_2 the last but one by d_3, the r th by d_{s-r+1}, etc., then add, and make all terms except that containing M_2 equal to zero, we should have the conditions

$$
\begin{aligned}
2\,d_2\,(l_s + l_{s-1}) + d_3 l_{s-1} &= 0, \\
d_2 l_{s-1} + 2\,d_3\,(l_{s-1} + l_{s-2}) + d_4 l_{s-2} &= 0, \\
\ldots\ldots\ldots\ldots& \\
d_{s-r+1}l_r + 2\,d_{s-r+2}(l_r + l_{r-1}) + d_{s-r+3}l_{r-1} &= 0, \\
\ldots\ldots\ldots\ldots& \\
d_{s-2}l_3 + 2\,d_{s-1}\,(l_3 + l_2) + d_s l_2 &= 0,
\end{aligned}
$$

while for the moment M_2 we have

$$
M_2 = -\,\frac{Ad_{s-r+2} + Bd_{s-r+1}}{d_{s-1}l_2 + 2\,d_s\,(l_2 + l_1)} = \frac{Ad_{s-r+2} + Bd_{s-r+1}}{d_{s+1}l_1} \quad \ldots \ldots \quad (22)
$$

The values of M_2 and M_s are thus given in terms of the quantities A and B and c and d.

A and B depend simply upon the load and its position in the rth span. Thus $A_r = P_r l_r^2\,(2\,k - 3\,k^2 + k^3)$, $B_r = P_r l_r^2\,(k - k^3)$, as for the multipliers c and d, they depend only upon the lengths of the

spans, and need only satisfy the conditions above. Hence assuming $c_1 = 0$, $c_2 = 1$, and $d_1 = 0$, $d_2 = 1$, we can deduce the proper values for all the others. Thus

$$c_1 = 0, \qquad d_1 = 0,$$

$$c_2 = 1, \qquad d_2 = 1,$$

$$c_3 = -2\frac{l_1 + l_2}{l_2}, \qquad d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}},$$

$$c_4 = -2c_3\frac{l_2 + l_3}{l_3} - c_2\frac{l_2}{l_3}, \qquad d_4 = -2d_3\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\frac{l_{s-1}}{l_{s-2}},$$

$$c_5 = -2c_4\frac{l_3 + l_4}{l_4} - c_3\frac{l_3}{l_4}, \qquad d_5 = -2d_4\frac{l_{s-2} + l_{s-3}}{l_{s-3}} - d_3\frac{l_{s-2}}{l_{s-3}},$$

$$c_6 = -2c_5\frac{l_4 + l_5}{l_5} - c_4\frac{l_4}{l_5}, \text{ etc.}, \qquad d_6 = -2d_5\frac{l_{s-3} + l_{s-4}}{l_{s-4}} - d_4\frac{l_{s-3}}{l_{s-4}}, \text{ etc.}$$

Now from equations (19) we see at once by examination, that $M_3 = c_3M_2$, $M_4 = c_4M_2$, $M_5 = c_5M_2$, etc., or generally when $m < r + 1$

$$M_m = c_mM_2 = \frac{c_m}{d_{s+1}l_1}(A_rd_{s-r+2} + B_rd_{s-r+1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Also taking the same equations in reverse order, $M_{s-1} = d_3M_s$, $M_{s-2} = d_4M_s$, etc., or generally, when $m > r$,

$$M_m = d_{s-m+2}M_s = \frac{d_{s-m+2}}{c_{s+1}l_s}(A_rc_r + B_rc_{r+1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

These are the equations I. and II., given on page 141.

Uniform Load.—The above equations (23) and (24), give the moment at any support for a concentrated load in any span. For a uniform load over the whole of any one span, we have only to give a different value to A and B.

Thus, for several concentrated loads, we should have

$$A_r = \Sigma P_rl_r^2\,(2k - 3k^2 + k^3). \qquad B_r = \Sigma P_rl_r^2\,(k_r - k_r^3).$$

For a uniform load over the whole span l_r, let w be the load per unit of length, then

$$\Sigma P = \int_0^{l_r} wda, \quad \text{or since } a = kl_r,\ \Sigma P = \int_0^1 wl_rdk.$$

Inserting this in place of ΣP, and integrating, we have

$$A = \tfrac{1}{4}\,wl_r^3.$$

In similar way we find

$$B = \tfrac{1}{4}\,wl_r^3.$$

These are the values of A and B given on page 142.

Equations (23) and (24) hold good, therefore, both for uniform and concentrated loading, for any number of spans of any lengths, provided only the supports are all on a level, and only one span is loaded.

UNIFORM LOAD OVER ENTIRE LENGTH OF GIRDER.—Let the uniform load, u, per unit of length extend over the entire girder, covering all the spans. Then the general theorem of three moments (page 280) becomes

$$M_{r-1}l_{r-1} + 2\,M_r\,(l_{r-1} + l_r) + M_{r+1}l_r = -\frac{u}{4}\,(l^3_{r-1} + l_r^3).$$

Applying this theorem and remembering that M_1 and M_{s+1} are zero, we have the following equations·

$$\left.\begin{aligned}
2\,M_2\,(l_1 + l_2) + M_3 l_2 &= -\frac{u}{4}\,(l_1^3 + l_2^3),\\
M_2 l_2 + 2\,M_3\,(l_2 + l_3) + M_4 l_3 &= -\frac{u}{4}\,(l_2^3 + l_3^3),\\
M_3 l_3 + 2\,M_4\,(l_3 + l_4) + M_5 l_4 &= -\frac{u}{4}(l_3^3 + l_4^3),\\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad&\cdot\quad\cdot\\
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad&\cdot\quad\cdot\\
M_{s-2}l_{s-2} + 2\,M_{s-1}\,(l_{s-2} + l_{s-1}) + M_s l_{s-1} &= -\frac{u}{4}(l^3_{s-2} + l^3_{s-1}),\\
M_{s-1}l_{s-1} + 2\,M_s\,(l_{s-1} + l_s) &= -\frac{u}{4}(l^3_{s-1} + l_s^3).
\end{aligned}\right\}\quad\cdots\quad (1)$$

Let us solve these equations precisely as we have equations (19) on page 283. Thus if we multiply the first by c_2, the second by c_3, and so on, and then add and arrange according to the coefficients of M_2, M_3, etc., we have

$$[2\,c_2\,(l_1 + l_2) + c_3\,l_2]\,M_2 + [c_2 l_2 + 2\,c_3\,(l_2 + l_3) + c_4 l_3]\,M_3$$
$$+ \;.\;.\;.\;.\; + [c_{s-2}l_{s-2} + 2\,c_{s-1}\,(l_{s-2} + l_{s-1}) + c_s l_{s-1}\,]\,M_{s-1}$$
$$+ [c_{s-1}l_{s-1} + 2\,c_s\,(l_{s-1} + l_s)\,]\,M_s = -\frac{u}{4}\,[(l_1^3 + l_2^3)\,c_2 + (l_2^3 + l_3^3)\,c_3 + \;.\;.\;.\;(l^3_{s-1} + l_s^3)\,c_s].$$

If we give these numbers c, such values as shall make each term in the left of this equation, except the one containing M_n, equal to zero, we shall have evidently the same values for c as given on page 285.

If, in similar manner, we multiply the last of equations (1) by d_2, the next by d_3, and so on, and add and arrange as before, we should have

$$[2\,d_2\,(l_s + l_{s-1}) + d_3 l_{s-1}\,]\,M_s + [d_2 l_{s-1} + 2\,d_3\,(l_{s-1} + l_{s-2}) + d_4 l_{s-2}]\,M_{s-1}$$
$$+ \;.\;.\;.\;.\;.\; + [d_{s-2}l_3 + 2\,d_{s-1}\,(l_3 + l_2) + d_s l_2]\,M_3$$
$$+ [d_{s-1}l_2 + 2\,d_s\,(l_2 + l_1)\,]\,M_2 = -\frac{u}{4}\,[(l^3_{s-1} + l_s^3)\,d_2 + (l^3_{s-2} + l^3_{s-1})\,d_3 + \;.\;.\;.\;.\;(l_1^3 + l_2^3)\,d_s].$$

If we give these numbers d, such values as shall make all the terms on the left except the one

containing M_2, equal to zero, we shall have evidently the same values for d as on page 284. If then we take these values for d, we have

$$M_2 = -\frac{\frac{u}{4}\left[(l_{s-1}^3 + l_s^3)\,d_2 + \;.\;.\;.\;.\;(l_1^3 + l_2^3)\,d_s\right]}{d_{s-1}l_2 + 2\,d_s(l_2 + l_1)} \quad . \;.\;.\;.\;.\;. \quad (2)$$

But from equation (1) we see that

$$M_3 = -\frac{u}{4\,l_2}(l_1^3 + l_2^3) + c_3\,M_2,$$

or

$$M_3 = \frac{u}{4}\,b_3 + c_3\,M_2, \qquad \text{where } b_3 = -\frac{l_1^3 + l_2^3}{l_2}.$$

In similar manner,

$$M_4 = \frac{u}{4}\,b_4 + c_4\,M_2, \qquad \text{where } b_4 = -\frac{l_2^3 + l_3^3}{l_3} - 2\,b_3\,\frac{l_2 + l_3}{l_3},$$

$$M_5 = \frac{u}{4}\,b_5 + c_5\,M_2, \qquad \text{where } b_5 = -\frac{l_3^3 + l_4^3}{l_4} - 2\,b_4\,\frac{l_3 + l_4}{l_4} - b_3\,\frac{l_3}{l_4},$$

$$M_6 = \frac{u}{4}\,b_6 + c_6\,M_2, \qquad \text{where } b_6 = -\frac{l_4^3 + l_5^3}{l_5} - 2\,b_5\,\frac{l_4 + l_5}{l_5} - b_4\,\frac{l_4}{l_5},$$

or generally

$$M_m = \frac{u}{4}\,b_m + c_m\,M_2, \qquad \text{where } b_m = -\frac{l_{m-2}^3 + l_{m-1}^3}{l_{m-1}} - 2\,b_{m-1}\,\frac{l_{m-2} + l_{m-1}}{l_{m-1}} - b_{m-2}\,\frac{l_{m-2}}{l_{m-1}}.$$

Inserting the value of M_2, as given by equation (2), we have

$$M_m = \frac{u}{4}\left[b_m - \frac{c_m\left[(l_{s-1}^3 + l_s^3)\,d_2 + (l_{s-2}^3 + l_{s-1}^3)\,d_3 + \;.\;.\;.\;.\;(l_1^3 + l_2^3)\,d_s\right]}{d_{s-1}\,l_2 + 2\,d_s\,(l_2 + l_1)}\right].$$

This is the formula given on page 147 of the Text.

FORMULÆ FOR THE TIPPER.*—The expressions for the moments and shears in this case, already given on page 161, may also be easily deduced. The solution is tedious by reason of lengthy reductions, but the process of deduction is simple.

The construction in this case is indicated by Fig. 130, page 161. We suppose, as shown there, a weight P upon the first span. Under the action of this weight the beam deflects, and one centre support falls and the other rises an equal amount. Thus if we take the level line as reference, $h_2 = -h_3$. Moreover, the reactions at these two supports must always be equal.

We have, then, $h_2 = -h_3$, and calling the supports 1, 2, 3 and 4, we have from (2), page 278, since $M_1 = M_4 = 0$, and $l_1 = l_3$,

$$\left.\begin{aligned}
R_1 &= S_1 = S_r = \frac{M_2}{l_1} + P\,(1 - k), \\
R_2 &= S'_2 + S_2 = -\frac{M_2}{l_1} + Pk + \frac{M_3 - M_2}{l_2}, \\
R_3 &= S'_3 + S_3 = \frac{M_2 - M_3}{l_2} - \frac{M_3}{l_1}, \\
R_4 &= S'_4 = \frac{M_3}{l_1}.
\end{aligned}\right\} \quad . \;.\;.\;.\;.\;.\;.\;. \quad (25)$$

---

* The formulæ for this case were first given by Clemens Herschel, C. E. "Continuous Revolving Draw Bridges," Little, Brown & Co., 1875.

These reactions will evidently be known if we can determine the moments. Let

$$Y_r = 6\,EI\left[\frac{h_{r-1} - h_r}{l_{r-1}} + \frac{h_{r+1} - h_r}{l_r}\right].$$

Then the general equation of these moments as given by equation (8) becomes, when we neglect P_r, that is, suppose only the first span loaded,

$$M_{r-1}\,l_{r-1} + 2\,M_r\,(l_{r-1} + l_r) + M_{r+1}\,l_r = -\,Y_r - P_{r-1}l_{r-1}^2\,(k - k^3).$$

This expresses a relation between the moments at three consecutive supports, for load between the first two. Let $r - 1 = 1$, or $r = 2$. Then since $M_1 = M_4 = 0$, we have

$$2\,M_2\,(l_1 + l_2) + M_3\,l_2 = -\,Y_2 - Pl_1^2\,(k - k^3) = R \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where R stands for convenience for the expression on the right. Let $r - 1 = 2$, or $r = 3$. Then the weight disappears, and since $l_1 = l_3$,

$$M_2\,l_2 + 2\,M_3\,(l_2 + l_1) = -\,Y_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

From (27) we have

$$M_2 = \frac{-\,Y_3 - 2\,M_3\,(l_2 + l_1)}{l_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

But since R_2 must always equal R_3, we have from (25),

$$\frac{M_2 - M_3}{l_1} + \frac{2\,M_2 - 2\,M_3}{l_2} = Pk \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Substituting (28) in (26), we have

$$M_3 = \frac{-\,Rl_2 - 2\,Y_3\,(l_1 + l_2)}{3\,l_2^2 + 8\,l_2\,l_1 + 4\,l_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Substituting (28) in (29) we have

$$M_2 = \frac{-\,l_1\,l_2^2\,Pk - Y_3\,(l_2 + 2\,l_1)}{3\,l_2^2 + 8\,l_2 l_1 + 4\,l_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

From (30) and (31) we have, then,

$$Y_2 + R = l_1\,l_2\,Pk.$$

Insert this in the value of R given by (26), and we have

$$Y_2 - Y_3 = -\,Pl_1^2\,(k - k^3) - Pl_1\,l_2 k = -\,P\,(l_1^2 k - l_1^2 k^3 + l_1\,l_2\,k).$$

Now in the present case $h_1 = 0$, $h_4 = 0$, and $h_2 = -\,h_3$, and since also $l_3 = l_1$, we have

$$Y_2 = 6\,EI\left[\;\frac{2\,h_3}{l_2} + \frac{h_3}{l_1}\right],$$

$$Y_3 = 6\,EI\left[-\frac{2\,h_3}{l_2} - \frac{h_3}{l_1}\right].$$

That is, $Y_2 = -\,Y_3$.

Hence, from our equations above,

$$Y_2 = -\frac{P}{2}\left[l_1^2 k - l_1^2 k^3 + l_1 l_2 k\right],$$

$$Y_3 = \frac{P}{2}\left[l_1^2 k - l_1^2 k^3 + l_1 l_3 k\right].$$

Substituting these values in (27) and (29), we can obtain at once M_2 and M_3, which, finally substituted in equation (25), will give us the reactions as already given on page 161, where we put nl in place of l_2.

The student will do well to deduce the equations of page 161, and thus check their accuracy.

General Formula; all Spans Different, all Supports out of Level; Constant Moment of Inertia.*—The general theorem of three moments, already deduced, is, Eq. (8),

$$M_{r-1}l_{r-1} + 2M_r(l_{r-1} + l_r) + M_{r+1}l_r = -Y_r - A_r - B_{r-1}.$$

For all spans loaded and all supports out of level, we have the series of equations

$$\left.\begin{array}{l}
2M_2(l_1 + l_2) + M_3 l_2 = -Y_2 - A_2 - B_1, \\
M_2 l_2 + 2M_3(l_2 + l_3) + M_4 l_3 = -Y_3 - A_3 - B_2, \\
M_3 l_3 + 2M_4(l_3 + l_4) + M_5 l_4 = -Y_4 - A_4 - B_3, \\
\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
M_{m-2}l_{m-2} + 2M_{m-1}(l_{m-2} + l_{m-1}) + M_m l_{m-1} = -Y_{m-1} - A_{m-1} - B_{m-2}, \\
M_{m-1}l_{m-1} + 2M_m(l_{m-1} + l_m) + M_{m+1}l_m = -Y_m - A_m - B_{m-1}, \\
M_m l_m + 2M_{m+1}(l_m + l_{m+1}) + M_{m+2}l_{m+1} = -Y_{m+1} - A_{m+1} - B_m, \\
\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\
M_{s-2}l_{s-2} + 2M_{s-1}(l_{s-2} + l_{s-1}) + M_s l_{s-1} = -Y_{s-1} - A_{s-1} - B_{s-2}, \\
M_{s-1}l_{s-1} + 2M_s(l_{s-1} + l_s) = -Y_s - A_s - B_{s-1}.
\end{array}\right\} \quad . \quad . \quad . \quad . \quad (a)$$

Multiply the first of equations (a) by b_2, the next by b_3, etc. Arrange the products according to the coefficients of M_2, M_3, etc., and add the resulting equations, and we have

$$\begin{aligned}
&[2b_2(l_1 + l_2) + b_3 l_2]\,M_2 + [b_2 l_2 + 2b_3(l_2 + l_3) + b_4 l_3]\,M_3 + \cdots \\
&+ [b_{m-1}l_{m-1} + 2b_m(l_{m-1} + l_m) + b_{m+1}l_m]\,M_m + \cdots \\
&+ [b_{s-1}l_{s-1} + 2b_s(l_{s-1} + l_s)]\,M_s = -\sum_{r=s}^{r=1}(Y_r + A_r + B_{r-1})\,b_r.
\end{aligned}$$

If we give such values to the multipliers b, that the coefficient of every M shall be zero, except the coefficient of M_m, we shall have

$$M_m = -\frac{\sum_{r=s}^{r=1}(Y_r + A_r + B_{r-1})\,b_r}{Z_m}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where

$$Z_m = b_{m-1}l_{m-1} + 2b_m(l_{m-1} + l_m) + b_{m+1}l_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

* The method of demonstration here given was first used by C. H. Lindenberger, *Journal of Franklin Institute*, December, 1888.

It is required to find the values of b. The coefficient of any M in general is

$$Z_n = b_{n-1}l_{n-1} + 2b_n(l_{n-1} + l_n) + b_{n+1}l_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For all values of n less than m, the values of c, already found, page 285, will make the coefficient of M_n equal to zero. We have, then, for $n < m$ or $= m$, $b_n = c_n$.

Now the values of d, already found, page 285, are counted from the bottom, and d_{s-n+2} corresponds, therefore, to the same place in the series as b_n. These values of d make the coefficients of M zero also. But d_2 has been assumed equal to 1. If it were taken equal to 2, all the d's would be twice as great; if 3, three times, etc. If then we take it equal to b_s, every d will be b_s times greater, and instead of d_{s-n+2}, we shall have $b_s d_{s-n+2}$. Hence $b_n = b_s d_{s-n+2}$, for $n = m$, or $> m$. But when $n = m$, b_n and c_m are identical. Hence $c_m = b_s d_{s-m+2}$, or $b_s = \frac{c_m}{d_{s-m+2}}$. Substituting this value of b_s, we have, when $n > m$,

$$b_n = \frac{c_m}{d_{s-m+2}} d_{s-n+2}.$$

We have, therefore, from (3),

$$Z_m = c_{m-1}l_{m-1} + 2c_m(l_{m-1} + l_m) + \frac{c_m}{d_{s-m+2}} d_{s-m+1}l_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

From the law of the multipliers c and d, page 284, we have also,

$$\left.\begin{aligned} c_{m-1}l_{m-1} + 2c_m(l_{m-1} + l_m) + c_{m+1}l_m = 0 \\ d_{s-m+1}l_{m-1} + 2d_{s-m+2}(l_{m-1} + l_m) + d_{s-m+3}l_m = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Subtract the first of these from (5), and we have

$$\frac{Z_m d_{s-m+2}}{l_m} = c_m d_{s-m+1} - c_{m+1}d_{s-m+2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Multiply the first of equations (6) by d_{s-m+2}, and the second by c_m, and subtract, and we have

$$l_{m-1}(c_{m-1}d_{s-m+2} - c_m d_{s-m+3}) + l_m(c_{m+1}d_{s-m+2} - c_m d_{s-m+1}) = 0 \quad . \quad . \quad . \quad . \quad (8)$$

Comparing this with (7) we see that the first term is equal to $Z_{m-1}d_{s-m+3}$, and the second term is $- Z_m d_{s-m+2}$. We have, therefore, the general relation

$$Z_m d_{s-m+2} = Z_{m-1}d_{s-m+3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Since this relation holds generally, we may write

$$Z_m d_{s-m+2} = Z_{m+1}d_{s-m+1} = Z_{m+2}d_{s-m} = \text{etc.} = Z_s d_2 = Z_s.$$

Hence, we have

$$Z_m d_{s-m+2} = Z_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From (10) and (7) we have at once,

$$Z_s = c_m d_{s-m+1}l_m - c_{m+1}d_{s-m+2}l_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and this value of Z_s holds good generally *for any value of m.*

If we make in (11) $m = s - 1$, we have $Z_s = c_{s-1}l_{s-1} - c_s d_3 l_{s-1}$. Since we have

$$d_3 = -\frac{2(l_{s-1} + l_s)}{l_{s-1}}, \text{ we have by substitution, } Z_s = c_{s-1}l_{s-1} + 2c_s(l_{s-1} + l_s).$$

Again, if we make $m = 2$ in (11), we have $Z_s = d_{s-1}l_2 - c_3 d_s l_2$. Since we have

$$c_3 = -\frac{2(l_1 + l_2)}{l_2}, \text{ we have by substitution, } Z_s = d_{s-1}l_2 + 2d_s(l_1 + l_2).$$

Again, if we make $m = s$ in (11), we have $Z_s = -c_{s+1}l_s$. If we make $m = 1$ in (11), we have $Z_s = -d_{s+1}l_1$. Any of these values of Z_s may be used.

If, now, we insert in (2) the value of Z_m given by (10), we have for the moment at any support,

$$M_m = -\frac{d_{s-m+2}\sum\limits_{r=s}^{r=1}(Y_r + A_r + B_{r-1})\,b_r}{Z_s},$$

where $b_r = c_r$ as long as r is less than m, or equal to m, and $b_r = \dfrac{c}{d_{s-m+2}}\,d_{s-r+2}$, where r is greater than m.

We can therefore write finally for the moment at any support in general,

$$M_m = -\frac{d_{s-m+2}\sum\limits_{r=m}^{r=1}(Y_r + A_r + B_{r-1})\,c_r}{Z_s} - \frac{c_m\sum\limits_{r=s}^{r=m+1}(Y_r + A_r + B_{r-1})\,d_{s-r+2}}{Z_s} \quad . \; . \quad \text{(A)}$$

where, in general, $Z_s = c_m d_{s-m+1} l_m - c_{m+1} d_{s-m+2} l_m$. The values most convenient for use* are $Z_s = c_{s-1}l_{s-1} + 2c_s(l_{s-1} + l_s) = -c_{s+1}l_s = d_{s-1}l_2 + 2d_s(l_1 + l_2) = -d_{s+1}l_1$.

Any of these values of Z_s may be used. The values of Y_r, A_r, and B_r have already been given, as also the values of c and d. We repeat them here for convenience of reference.

For concentrated loads, $A_r = \Sigma P_r l_r^2\,(2k_r - 3k_r^2 + k_r^3)$; $B_r = \Sigma P_r l_r^2\,(k_r - k_r^3)$.

For uniform loading, $A_r = B_r = \frac{1}{4} w_r l_r^3$.

$$Y_r = 6EI\left[\frac{h_{r-1} - h_r}{l_{r-1}} + \frac{h_{r+1} - h_r}{l_r}\right].$$

$$c_1 = 0,\; c_2 = 1,\; c_3 = -2\frac{l_1 + l_2}{l_2},\; c_4 = -2c_3\frac{l_2 + l_3}{l_3} - c_2\frac{l_2}{l_3},\; c_n = -2c_{n-1}\frac{l_{n-2} + l_{n-1}}{l_{n-1}} - c_{n-2}\frac{l_{n-2}}{l_{n-1}},$$

$$d_1 = 0,\; d_2 = 1,\; d_3 = -2\frac{l_s + l_{s-1}}{l_{s-1}},\; d_4 = -2d_3\frac{l_{s-1} + l_{s-2}}{l_{s-2}} - d_2\frac{l_{s-1}}{l_{s-2}},$$

$$d_n = -2d_{n-1}\frac{l_{s-n+3} + l_{s-n+2}}{l_{s-n+2}} - d_{n-2}\frac{l_{s-n+3}}{l_{s-n+2}}.$$

For the shear at the left support of a loaded span,

$$S_r = \frac{M_{r+1} - M_r}{l_r} + q_r;$$

at the right support of a loaded span,

$$S'_{r+1} = \frac{M_r - M_{r+1}}{l_r} + q'_r,$$

where, for concentrated loads, $\quad q_r = \Sigma P_r(1 - k_r), \quad q'_r = \Sigma P_r k_r,$

and for uniform loading, $\quad q_r = q'_r = \frac{1}{2} w_r l_r.$

For unloaded spans,

$$S_m = \frac{M_{m+1} - M_m}{l_m}, \qquad S'_m = \frac{M_{m-1} - M_m}{l_{m-1}}.$$

* The equality of these values was first pointed out by C. H. Lindenberger, *Jour. of Franklin Inst.*, Dec., 1888.

The formulæ of page 291, are all that are needed for the complete solution of any case of continuous girder for constant moment of inertia.

We give in the following pages a series of examples illustrating the use of the general formula (A), which includes all the special cases hitherto discussed.

1. *Let all the spans be equal and unloaded. Find the moment at any support n, due to a change of level of that support.*

Here the numbers c and d are identical, B and A are zero, also $Y_{n-1} = Y_{n+1} = -\frac{Y_n}{2}$, and all other Y's are zero.

From (A) we have for the moment at any support m, on the left of n, or when n is greater than m,

$$M_m = \frac{c_m}{d_{s+1}l}[Y_{n-1}d_{s-n+3} + Y_n d_{s-n+2} + Y_{n+1}d_{s-n+1}]$$

$$= \frac{c_m Y_n}{2c_{s+1}l}[-c_{s-n+3} + 2c_{s-n+2} - c_{s-n+1}].$$

Since, for all spans equal, we have $c_{s-n+3} = -4c_{s-n+2} - c_{s-n+1}$, we have, after inserting this value of c_{s-n+3}, and the value of $Y_n = 6EI\left[\frac{2(h_{n-1} - h_n)}{l}\right]$,

when $n > m$,

$$M_m = \frac{36EI(h_{n-1} - h_n)c_m c_{s-n+2}}{c_{s+1}l^2},$$

which is the equation given on page 152.

For the moment at the support n itself, we have, from (A),

$$M_n = \frac{d_{s-n+2}}{c_{s+1}l}(Y_{n-1}c_{n-1} + Y_n c_n) + \frac{c_n}{c_{s+1}l}Y_{n+1}d_{s-n+1}$$

$$= \frac{c_n}{c_{s+1}l}(Y_{n+1}d_{s-n+1} + Y_n d_{s-n+2} + Y_{n-1}d_{s-n+3}) - \frac{Y_{n-1}}{c_{s+1}l}(c_n d_{s-n+3} - c_{n-1}d_{s-n+2})$$

$$= \frac{Y_n c_n}{2c_{s+1}l}(-c_{s-n+1} + 2c_{s-n+2} - c_{s-n+3}) + \frac{Y_n}{2c_{s+1}l}(c_n d_{s-n+3} - c_{n-1}d_{s-n+2}).$$

But from (11) we have $Z_s = -c_{s+1}l = c_{n-1}d_{s-n+2}l - c_n d_{s-n+3}l$. The second term, therefore, reduces to $\frac{Y_n}{2l}$. Reducing as before, we have,

when $m = n$,

$$M_n = \frac{6EI(h_{n-1} - h_n)}{l^2} + \frac{36EI(h_{n-1} - h_n)c_n c_{s-n+2}}{c_{s+1}l^2}.$$

For the moment at any support on the right of n, we have $m > n$, and from (A),

$$M_m = \frac{d_{s-m+2}}{c_{s+1}l}(Y_{n-1}c_{n-1} + Y_n c_n + Y_{n+1}c_{n+1})$$

$$= \frac{Y_n c_{s-m+2}}{2c_{s+1}l}(-c_{n-1} + 2c_n - c_{n+1}).$$

Since $c_{n+1} = -4c_n - c_{n-1}$, we have, after inserting this value of c_{n+1}, and the value of Y_n, when $n < m$,

$$M_m = \frac{36EI(h_{n-1} - h_n)c_n c_{s-m+2}}{c_{s+1}l^2}.$$

This equation is given on page 152.

If the spans are all different we have, from (A), for the moment at any support on the left of n,

$$M_m = \frac{c_m}{d_{s+1} l_1} \left[Y_{n-1} d_{s-n+2} + Y_n d_{s-n+2} + Y_{n+1} d_{s-n+1} \right].$$

Putting for Y_n, Y_{n-1}, Y_{n+1}, their values, and remembering that $h_{n-1} - h_n = h_{n+1} - h_n$, we have when $n > m$,

$$M_m = \frac{6EIc_m\,(h_{n-1} - h_n)}{d_{s+1} l_1} \left[\frac{d_{s-n+2} - d_{s-n+2}}{l_{n-1}} + \frac{d_{s-n+2} - d_{s-n+1}}{l_n} \right].$$

For the moment at the support n we have,
when $n = m$,

$$M_n = \frac{6EI\,(h_{n-1} - h_n)}{l^2_{n-1}} + \frac{6EI\,c_n\,(h_{n-1} - h_n)}{d_{s+1} l_1} \left[\frac{d_{s-n+2} - d_{s-n+2}}{l_{n-1}} + \frac{d_{s-n+2} - d_{s-n+1}}{l_n} \right].$$

For the moment at any support on the right of n, we have,
when $n < m$,

$$M_m = \frac{6EI\,d_{s-m+2}\,(h_{n-1} - h_n)}{c_{s+1} l_s} \left[\frac{c_n - c_{n-1}}{l_{n-1}} + \frac{c_n - c_{n+1}}{l_n} \right].$$

These formulæ are given on page 152.

2. *Find the general formulæ for a continuous beam of two spans.*

Here $s = 2$, and we have, from (A),

$$M_1 = 0,\ M_3 = 0,\ M_2 = -\frac{Y_2 + A_2 + B_1}{2\,(l_1 + l_2)},\ S_1 = \frac{M_2}{l_1} + q_1,\ S'_2 = -\frac{M_2}{l_1} + q'_1,\ S_2 = -\frac{M_2}{l_2} + q_2,\ S'_3 = \frac{M_2}{l_2} + q'_2.$$

For concentrated loading, $q_1 = \Sigma P_1\,(1 - k_1)$, $q'_1 = \Sigma P_1 k_1$, $q_2 = \Sigma P_2\,(1 - k_2)$, $q'_2 = \Sigma P_2 k_2$.

For uniform loading, $q_1 = q'_1 = \frac{1}{2}\, w_1 l_1$, $q_2 = q'_2 = \frac{1}{2}\, w_2 l_2$.

These formulæ will solve any case of two spans. For example :

A plate girder is continuous over three supports, $l_1 = 30$ feet, $l_2 = 50$ feet, the supports being all on level. The uniform load per foot in the first span is $w_1 = 3000$ lbs., in the second $w_2 = 350$ lbs. What are the moments and reactions ?

Since the supports are all on level, $Y_2 = 0 = Y_1 = Y_3$, and we have,

$$M_1 = 0,\ M_3 = 0,\ M_2 = -\frac{A_2 + B_1}{2\,(l_1 + l_2)}.$$

In the present case

$$A_2 = \frac{w_2 l_2^3}{4},\ B_1 = \frac{w_1 l^3}{4}.$$

Hence,

$$M_2 = -\frac{w_1 l_1^3 + w_2 l_2^3}{8\,(l_1 + l_2)} = -\frac{3000 \times 30^3 + 350 \times 50^3}{8\,(30 + 50)} = -194921.875 \text{ foot lbs.}$$

We have, therefore,

$$R_1 = S_1 = \frac{M_2}{l_1} + \frac{w_1 l_1}{2} = -\frac{194921.875}{30} + \frac{3000 \times 30}{2} = +38502.6 \text{ lbs.}$$

$$R_3 = S_3' = \frac{M_2}{l_2} + \frac{w_2 l_2}{2} = -\frac{194921.875}{50} + \frac{350 \times 50}{2} = +4851.5625 \text{ lbs.}$$

$$S'_2 = -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = +51497.39 \text{ lbs.} \qquad S_2 = -\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = +12648.44 \text{ lbs.}$$

$$R_2 = S'_2 + S_2 = +64145.8 \text{ lbs.}$$

3. *If the centre support is lowered 3 feet below the level of the others, what are the moments and reactions? Let $E = 24000000$ lbs. per sq. in. and $I = 53400$ for dimensions in inches.*

Since $s = 2$, we have, from (A), $M_1 = 0$, $M_3 = 0$.

$$M_2 = \frac{Y_2 + A_2 + B_1}{c_2 l_2} = -\frac{\dfrac{w_1 l_1^3}{4} + \dfrac{w_2 l_2^3}{4} + 6EI\left[\dfrac{h_1 - h_2}{l_1} + \dfrac{h_3 - h_2}{l_2}\right]}{2\,(l_1 + l_2)}.$$

Since the centre support is lowered, h_2 is *greater* than h_1 and h_3, and we have $h_1 - h_2 = h_3 - h_2 = -3$ feet. If we take the span in feet and w_1 and w_2 in lbs. per foot, we must take I in feet and E in lbs. per sq. foot. We must therefore divide the value of I given, by 12^4, and multiply the value of E by 144, or divide the value of EI, as given by 144. We have, therefore,

$$M_2 = -\frac{\dfrac{3000 \times 30^3}{4} + \dfrac{350 \times 50^3}{4} - \dfrac{6 \times 24000000 \times 53400}{144}\left(\dfrac{3}{30} + \dfrac{3}{50}\right)}{2\,(30 + 50)} = +\,53205078.125 \text{ ft. lbs.}$$

$$R_1 = S_1 = \frac{M_2}{l_1} + \frac{w_1 l_1}{2} = +\,1818502.6 \text{ lbs.} \qquad R_3 = S'_3 = \frac{M_2}{l_2} + \frac{w_2 l_2}{2} = +\,1072851.56 \text{ lbs.}$$

$$S'_2 = -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = -\,1728502.6 \text{ lbs.} \qquad S_2 = -\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = -\,1055351.56 \text{ lbs.}$$

$$R_2 = S'_2 + S_2 = -\,2783854.16 \text{ lbs.}$$

If the second support were 3 feet below the first and 3 feet above the third, we would have $h_1 - h_2 = -3$, and $h_3 - h_2 = +3$. So for any differences of level.

4. *How far must the second support be lowered in order that the moment may be zero?*

Since supports 1 and 3 are on level, $h_1 - h_2 = h_3 - h_2$. Since $M_2 = 0$, we have from (A),

$$M_2 = -\frac{\dfrac{w_1 l_1^3}{4} + \dfrac{w_2 l_2^3}{4} + 6EI\left[\dfrac{h_1 - h_2}{l_1} + \dfrac{h_3 - h_2}{l_2}\right]}{2\,(l_1 + l_2)} = 0,$$

or,

$$\frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} + 6EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right] = 0. \quad \therefore\ h_1 - h_2 = -\frac{w_1 l_1^4 l_2 + w_2 l_2^4 l_1}{24\,EI\,(l_1 + l_2)}.$$

Inserting numerical values

$$h_1 - h_2 = -\frac{3000 \times 30^4 \times 50 + 350 \times 50^4 \times 30}{\dfrac{24 \times 24000000 \times 53400}{144}\,(30 + 50)} = -\,0.0045 \text{ ft.} = -\,0.054 \text{ inch.}$$

Therefore a sinking of the second support of only 0.05 inch is sufficient to make M_2 zero.

In this case we have

$$R_1 = S_1 = \frac{w_1 l_1}{2} = +\,45000 \text{ lbs.} \qquad R_3 = S'_3 = +\,\frac{w_2 l_2}{2} = +\,8750 \text{ lbs.}$$

$$S_2' = \frac{w_1 l_1}{2} = +\,45000 \text{ lbs.} \qquad S_2 = \frac{w_2 l_2}{2} = +\,8750 \text{ lbs.} \qquad R_2 = S'_2 + S_2 = +\,53750 \text{ lbs.}$$

5. *How far must the second support be lowered in order that the pressure on the second support may be zero?*

Here we have

$$S'_2 + S_2 = R_2 = 0, \text{ or } -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} - \frac{M_2}{l_2} + \frac{w_2 l_2}{2} = 0.$$

or,

$$M_2 = \frac{w_1 l_1^2 l_2 + w_2 l_1 l_2^2}{2\,(l_1 + l_2)} = +\ 1007812.5 \text{ ft. lbs.}$$

From the general value of M_2 in the preceding case we have

$$-\,w_1 l_1^2 l_2 - w_2 l_1 l_2^2 = \frac{w_1 l_1^3}{4} + \frac{w_2 l_2^3}{4} + 6EI\left[\frac{h_1 - h_2}{l_1} + \frac{h_3 - h_2}{l_2}\right].$$

Hence, since $h_1 - h_2 = h_3 - h_2$,

$$h_1 - h_2 = -\frac{w_1 l_1^4 l_2 + w_2 l_1 l_2^4 + 4w_1 l_1^3 l_2^2 + 4w_2 l_1^2 l_2^3}{24EI\,(l_1 + l_2)} = -\ 0.0611 \text{ ft.} = -\ 0.73 \text{ inch.}$$

Therefore, a sinking of the second support of only 0.7 inch is sufficient to convert the two spans into one long span.

$$R_1 = S_1 = \frac{M_2}{l_1} + \frac{w_1 l_1}{2} = +\ 78593.75 \text{ lbs} \qquad R_3 = S'_3 = \frac{M_2}{l_2} + \frac{w_2 l_2}{2} = +\ 28906.25 \text{ lbs.}$$

$$S'_2 = -\frac{M_2}{l_1} + \frac{w_1 l_1}{2} = +\ 11406.25 \text{ lbs.} \qquad S_2 = -\frac{M_2}{l_2} + \frac{w_2 l_2}{2} = -11406.25 \text{ lbs.}$$

$$R_2 = S'_2 + S_2 = 0.$$

If the spans l_1 and l_2 were equal, and the loading w_1 and w_2 equal, we would have at once $h_1 - h_2 = -\frac{5wl^4}{24EI}$, or the deflection at the centre of a span $2l$, as should be, and $M_2 = +\frac{wl^2}{2}$, as should be.

6. *If we have a concentrated load $P_1 = 90000$ lbs. in the first span, at a distance $\frac{1}{4}l_1$ from the left end, and $P_2 = 18000$ lbs. at a distance $\frac{1}{2}l_2$, what are the moments and reactions?*

This case is precisely like example 2, except in the values of A and B. We have now $k_1 = \frac{1}{4}$, $k_2 = \frac{1}{2}$, $A_2 = P_2 l_2^2\,(2k_2 - 3k_2^2 + k_2^3) = \frac{3}{8}P_2 l_2^2$, $B_1 = P_1 l_1^2\,(k_1 - k_1^3) = \frac{15}{64}P_1 l_1^2$.

From (A) we find, as in example 2, $M_1 = 0$, $M_3 = 0$, and

$$M_2 = -\frac{A_2 + B_1}{2\,(l_1 + l_2)}.$$

Inserting the values of A_2 and B_1, we find easily,

$$M_2 = -\ 224121.094 \text{ ft. lbs.} \qquad R_1 = S_1 = \frac{M_2}{l_1} + P_1\,(1 - k) = +\ 60029.3 \text{ lbs.}$$

$$R_3 = S'_3 = \frac{M_2}{l_2} + P_2 k_2 = +\ 4517.58 \text{ lbs.} \qquad S'_2 = -\frac{M_2}{l_1} + P_1 k_1 = +\ 29970.7 \text{ lbs.}$$

$$S_2 = -\frac{M_2}{l_2} + P_2\,(1 - k_2) = +\ 13482.42 \text{ lbs.} \qquad R_2 = S'_2 + S_2 = +\ 43453.12 \text{ lbs.}$$

7. *If the second support is lowered* 3 *feet in this case*, we have simply to use the values of A_2 and B_1 for this case, in the formulæ of example 3, and we find

$M_2 = +\ 53175878.9$ ft. lbs. $\quad R_1 = S_1 = +\ 1840029.29$ lbs. $\quad R_3 = S'_3 = +\ 1072517.57$ lbs.

$S'_2 = -\ 1750029.29$ lbs. $\quad S_2 = -\ 1054517.57$ lbs. $\quad R_2 = S'_2 + S_2 = -\ 2804546.85$ lbs.

8. *To find the distance the second support must be lowered in order that M_2 may be zero*, we proceed as in example 4, and place $Y_2 + A_2 + B_1 = 0$. Inserting the new values for A_2 and B_1, we find $h_1 - h_2 = -\ 0.01259$ ft. $= -\ 0.1511$ inch.

9. *To find how far the second support must be lowered, in order that the pressure on the second support may be zero.*

We have, as in example 5,

$$-\frac{M_2}{l_1} + P_1k_1 - \frac{M_2}{l_2} + P_2(1 - k_2) = 0. \quad \therefore\ M_2 = \frac{P_1k_1l_1l_2 + P_2(1 - k_2)\,l_1l_2}{l_1 + l_2} = +\ 590625 \text{ ft. lbs.}$$

We have also from (A),

$$M_2 = -\frac{A_2 + B_1 + Y_2}{2(l_1 + l_2)}, \text{ and hence } A_2 + B_1 + Y_2 = -\ 2P_1k_1l_1l_2 - 2P_2(1 - k_2)\,l_1l_2.$$

Putting for Y_2 its value $Y_2 = 6EI\left[\dfrac{h_1 - h_2}{l_1} + \dfrac{h_3 - h_2}{l_2}\right]$, and remembering that $h_1 - h_2 = h_3 - h_2$,

we have, after inserting numerical values, $h_1 - h_2 = -\ 0.04577$ ft. $= -\ 0.55$ in.
$R_1 = S_1 = +\ 87187.5$ lbs. $\quad R_3 = S'_3 = +\ 20812.5. \quad S'_2 = +\ 2812.5. \quad S_2 = -\ 2812.5. \quad R_2 = 0.$

10. *How much must the second support be raised or lowered in order that the reaction at the first support may be any required amount?*

We have, in general, $M_2 = R_1l_1 - q_1l_1$ where $q_1 = P_1(1 - k_1)$ for concentrated load, and $q_1 = \dfrac{w_1l_1}{2}$ for uniform loading.

We have, also,

$$M_2 = -\frac{A_2 + B_1 + Y_2}{2(l_1 + l_2)}, \text{ hence } Y_2 = -\ A_2 - B_1 - 2(l_1 + l_2)(R_1l_1 - q_1l_1).$$

Inserting the value of Y_2, and remembering that $h_1 - h_2 = h_3 - h_2$, we have

$$h_1 - h_2 = -\frac{[A_2 + B_1 + 2(l_1 + l_2)(R_1l_1 - q_1l_1)]\,l_1l_2}{6EI(l_1 + l_2)}.$$

This is a general formula for two spans, whatever the loading.

If we take R_1 zero, we have the amount of elevation of the second support necessary to just lift the left end. For concentrated load in each span, we have

$$h_1 - h_2 = -\frac{2(l_1 + l_2)\,l_1^2l_2\,[R_1 - P_1(1 - k_1)] + P_2(2k_2 - 3k_2^2 + k_2^3)\,l_2^2l_1 + P_1(k_1 - k_1^3)\,l_1^2l_2}{6EI\,(l_1 + l_2)},$$

Inserting numerical values, $k_1 = \dfrac{1}{4}$, $k_2 = \dfrac{1}{2}$, as in example 6, and making $R_1 = 0$, we have $h_1 - h_2 = +\ 0.10117$ ft. $= +\ 1.214$ in. The second support must therefore be *raised* 1.2 in., in order that left end may just touch.

In this case, we have $M_2 = -P_1(1-k_1)l_1 = -2025000$ ft. lbs. $\quad R_1 = 0$.

$$R_3 = S'_3 = \frac{M_2}{l_2} + P_2k_2 = -31500 \text{ lbs.} \qquad S'_2 = -\frac{M_2}{l_1} + P_1k_1 = +90000 \text{ lbs.}$$

$$S_2 = -\frac{M_2}{l_2} + P_2(1-k_2) = +49500 \text{ lbs.} \qquad R_2 = S'_2 + S_2 = +139500 \text{ lbs.}$$

11. *Let a beam of two equal spans have a load, P_1 in the first span and P_2 in the second span, each load being at the centre of its span. Let the second support be lowered by an amount, $h_1 - h_2 = -\dfrac{(P_1+P_2)l^3}{48EI}$. What are the moments, shears, and reactions?*

In this case, $\quad k_1 = k_2 = \frac{1}{2},\ A_2 = \frac{3}{8}P_2l^2,\ B_1 = \frac{3}{8}P_1l^2,$

$$Y_2 = 6EI\left[\frac{2(h_1-h_2)}{l}\right] = -\frac{1}{4}(P_1+P_2)l^2, \text{ and}$$

$$M_2 = -\frac{Y_2 + A_2 + B_1}{4l} = -\frac{3l}{32}(P_1+P_2) + \frac{l}{16}(P_1+P_2) = -(P_1+P_2)\frac{l}{32},$$

$$R_1 = S_1 = \frac{M_2}{l} + \frac{P_1}{2} = \frac{15P_1 - P_2}{32}. \qquad R_3 = S'_3 = \frac{M_2}{l} + \frac{P_2}{2} = \frac{15P_2 - P_1}{32}.$$

$$S'_2 = -\frac{M_2}{l} + \frac{P_1}{2} = \frac{17P_1 + P_2}{32}. \qquad S_2 = -\frac{M_2}{l} + \frac{P_2}{2} = \frac{P_1 + 17P_2}{32}.$$

$$R_2 = S'_2 + S_2 = \frac{18(P_1+P_2)}{32}.$$

12. *A beam of one span is fixed horizontally at the right end. What are the shears and moments?*

Here we have $s = 2,\ l_2 = 0,\ c_1 = 0,\ c_2 = 1,\ d_1 = 0,\ d_2 = 1,\ d_3 = -2,\ h_3 - h_2 = 0,\ A_2 = 0 = B_2,$ $Y_2 = 6EI\left[\dfrac{h_1 - h_2}{l}\right]$.

From (A), $\ M_1 = 0,\quad M_2 = -\dfrac{B_1 + Y_2}{2l},\quad M_3 = 0,\quad S_1 = \dfrac{M_2}{l} + q_1,\quad S'_2 = -\dfrac{M_2}{l} + q'_1.$

For ends on level and uniform loading,

$$Y_2 = 0,\quad B_1 = \frac{1}{4}wl^2,\quad M_2 = -\frac{wl^2}{8},\quad R_1 = S_1 = \frac{3}{8}wl,\quad S'_2 = \frac{5}{8}wl.$$

For ends on level and load P anywhere in the span,

$$M_2 = -\frac{Pl}{2}(k - k^3). \text{ For } P \text{ in centre, } k = \frac{1}{2}, \text{ and } M_2 = -\frac{3Pl}{16},\quad S_1 = \frac{5}{16}P,\quad S'_2 = \frac{11}{16}P.$$

How far must the right end sink in order that the moment may be zero?

Here we have $M_2 = -\dfrac{B_1 + Y_2}{2l} = 0$, or $Y_2 = -B_1 = 6EI\left[\dfrac{h_1 - h_2}{l}\right]$.

Hence $h_1 - h_2 = -\dfrac{B_1 l}{6EI},\ S_1 = q_1,\ S'_2 = q'_1.$

If the loading is uniform, $S_1 = \dfrac{wl}{2} = S'_2. \qquad h_1 - h_2 = -\dfrac{wl^4}{24EI}.$

For concentrated load, $S_1 = P(1-k)\ S_2' = Pk. \qquad h_1 - h_2 = -\frac{Pl^3(k-k^3)}{6EI}.$

For load in centre, $k = \frac{1}{2}$, and $S_1 = \frac{P}{2} = S_2'. \qquad h_1 - h_2 = -\frac{Pl^3}{16EI}.$

How far must the right end rise in order that S_1 may be zero?

Here $\frac{M_2}{l} + q_1 = 0$, or $-\frac{B_1 + Y_2}{2l^2} + q_1 = 0$, or $Y_2 = -B_1 + 2q_1l^2 = 6EI\left[\frac{h_1 - h_2}{l}\right].$

Hence $h_1 - h_2 = \frac{2q_1l^3 - B_1l}{6EI}. \qquad M_2 = -q_1l,\ S'_2 = q_1 + q'_1.$

If load is uniform, $q'_1 = q_1 = \frac{wl}{2},\ B_1 = \frac{wl^3}{4},\ h_1 - h_2 = \frac{wl^4}{8EI},\ M_2 = -\frac{wl^2}{2},\ S'_2 = wl.$

If load is concentrated, $q'_1 = Pk,\ q_1 = P(1-k),\ B_1 = Pl^3(k-k^3),\ h_1 - h_2 = \frac{Pl^3(2 - 3k + k^3)}{6EI}$
$M_2 = -Pl(1-k),\ S'_2 = P.$

13. *Find the general formulæ for a continuous beam of three spans.*

Here $s = 3$, and we have, from (A), $M_1 = 0 = M_4$.

$$M_2 = \frac{d_3(Y_2 + A_2 + B_1)}{d_4l_1} + \frac{Y_3 + A_3 + B_2}{d_4l_1}. \qquad M_3 = \frac{Y_2 + A_2 + B_1 + c_3(Y_3 + A_3 + B_2)}{d_4l_1}.$$

$$S_1 = \frac{M_2}{l_1} + q_1,\ S'_2 = -\frac{M_2}{l_1} + q'_1,\ S_2 = \frac{M_3 - M_2}{l_2} + q_2,\ S'_3 = \frac{M_2 - M_3}{l_2} + q'_2,\ S_3 = -\frac{M_3}{l_3} + q_3,$$
$$S'_4 = \frac{M_3}{l_3} + q'_3.$$

For concentrated loads, $q_1 = \Sigma P_1(1-k_1),\ q'_1 = \Sigma P_1k_1,\ q_2 = \Sigma P_2(1-k_2),\ q'_2 = \Sigma P_2k_2,$
$q_3 = \Sigma P_3(1-k_3),\ q'_3 = \Sigma P_3k_3.$

For uniform loading, $q_1 = q'_1 = \frac{1}{2}w_1l_1,\ q_2 = q'_2 = \frac{1}{2}w_2l_2,\ q_3 = q'_3 = \frac{1}{2}w_3l_3.$

These general formulæ will solve any case of three spans.

14. *A continuous beam of four equal spans, all supports on level, has the second span uniformly loaded. What are the moments and shears?*

Here we have $Y_1 = Y_2 = Y_3 = Y_4 = 0,\quad A_1 = A_3 = A_4 = 0 = B_1 = B_3 = B_4,\quad A_2 = B_2 = \frac{wl^3}{4}.$

Also, $c_1 = d_1 = 0,\ c_2 = d_2 = 1,\ c_3 = d_3 = -4,\ c_4 = d_4 = +15,\ c_5 = d_5 = -56,\ s = 4,\ r = 2.$
We have, therefore, from (A),

$$M_1 = 0,\quad M_5 = 0,\quad M_2 = \frac{c_2}{d_5l}[A_2d_4 + B_2d_3] = -\frac{11wl^2}{224},\quad M_3 = \frac{d_3}{d_5l}[A_2c_2 + B_2c_3] = -\frac{12wl^2}{224},$$
$$M_4 = \frac{d_2}{d_5l}[A_2c_2 + B_2c_3] = +\frac{3wl^2}{224}.$$

$$R_1 = S_1 = \frac{M_2}{l} = -\frac{11wl}{224},\ S'_2 = -\frac{M_2}{l} = +\frac{11wl}{224},\ S_2 = \frac{M_3 - M_2}{l} + \frac{wl}{2} = +\frac{111wl}{224},$$
$$S'_3 = \frac{M_2 - M_3}{l} + \frac{wl}{2} = +\frac{113wl}{224},\ S_3 = \frac{M_4 - M_3}{l} = +\frac{15wl}{224},\ S'_4 = \frac{M_3 - M_4}{l} = -\frac{15wl}{224},$$
$$S_4 = -\frac{M_4}{l} = -\frac{3wl}{224},\ R_5 = S'_5 = \frac{M_4}{l} = +\frac{3wl}{224}.$$

15. *How much should the supports be lowered in order to make all the moments zero?*

Here we have the conditions $(Y_2 + A_2)d_4 + B_2d_3 + Y_3d_3 + Y_4d_2 = 0,\ (Y_2 + A_2)c_2 + B_2c_3 + Y_3d_2 + Y_4d_2 = 0,\ (Y_2 + A_2)c_2 + B_2c_3 + Y_3c_3 + Y_4c_4 = 0.$

Substituting the values of c and d, we find $Y_2 = -A_2$, $Y_3 = -B_3$, $Y_4 = 0$; that is, the fourth support is on level with the third and fifth, and we must have $h_3 = h_4 = h_5$. We have, then, $Y_3 = 6EI\left[\frac{h_2 - h_3}{l} + \frac{h_4 - h_3}{l}\right] = 6EI\left[\frac{h_2 - h_3}{l}\right]$, and hence $h_2 - h_3 = -\frac{B_3 l}{6EI}$. The minus sign shows that the third support is below the second.

We also have $Y_2 = 6EI\left[\frac{h_1 - h_2}{l} + \frac{h_3 - h_2}{l}\right] = -A_2$. Inserting the value of $h_2 - h_3$, we have $h_1 - h_2 = -\frac{(A_2 + B_3)\,l}{6EI}$, and the second support is below the first.

Let $l = 50$ feet, $E = 24000000$ lbs. per square inch, $I = 53400$ for dimensions in inches. Then, if we take dimensions in feet, $EI = \frac{24000000 \times 53400}{144} = 8900000000$. Take $w_2 = 3000$ lbs. per foot.

Then $A_2 = \frac{w_2 l^3}{4} = 118750000 = B_3$, and $h_1 - h_2 = -0.226$ feet $= -2.7$ inches, $h_2 - h_3 = -0.113$ feet $= -1.35$ inches.

In order to make the moment at the second support only equal to zero, we have

$$(Y_2 + A_2)\,d_4 + B_2 d_3 + Y_3 d_2 = 0, \text{ and } Y_2 = 6EI\left[\frac{2\,(h_1 - h_2)}{l}\right], \quad Y_3 = -6EI\left[\frac{h_1 - h_2}{l}\right].$$

Hence,

$$h_1 - h_2 = -\frac{11wl^4}{816EI} = -0.284 \text{ feet} = -3.4 \text{ inches.}$$

16. *A beam continuous over seven spans has a load in every span. Find the moment and shear at the fourth support.*

We have from (A), since $s = 7$,

$$M_4 = \frac{d_5}{d_8 l_1}\left[(Y_2 + A_2 + B_1)\,c_2 + (Y_3 + A_3 + B_2)\,c_3 + (Y_4 + A_4 + B_3)\,c_4\right],$$

$$+ \frac{c_4}{d_8 l_1}\left[(Y_5 + A_5 + B_4)\,d_4 + (Y_6 + A_6 + B_5)\,d_3 + (Y_7 + A_7 + B_6)\,d_2\right],$$

$$M_5 = \frac{d_4}{d_8 l_1}\left[(Y_2 + A_2 + B_1)\,c_2 + (Y_3 + A_3 + B_2)\,c_3 + (Y_4 + A_4 + B_3)\,c_4 + (Y_5 + A_5 + B_4)\,c_5\right],$$

$$+ \frac{c_5}{d_8 l_1}\left[(Y_6 + A_6 + B_5)\,d_3 + (Y_7 + A_7 + B_6)\,d_2\right].$$

$$S_4 = \frac{M_5 - M_4}{l_4} + q_4, \quad q_4 = \Sigma P_4\,(1 - k_4) \text{ for concentrated loads.} \quad q_4 = \frac{w_4 l_4}{2} \text{ for uniform load.}$$

Suppose the supports are all on level, all spans equal, $l = 80$ feet, and only the first, third, and sixth spans are uniformly loaded, with a load $w = 2$ tons per foot.

Then $A_2 = A_4 = A_5 = A_7 = 0$, $B_2 = B_4 = B_5 = B_7 = 0$, $A_1 = A_3 = A_6 = \frac{wl^2}{4} = B_1 = B_3 = B_6$, $c_1 = d_1 = 0$, $c_2 = d_2 = 1$, $c_3 = d_3 = -4$, $c_4 = d_4 = +15$, $c_5 = d_5 = -56$, $c_6 = d_6 = +209$, $c_7 = d_7 = -780$, $c_8 = d_8 = +2911$, and every Y is zero.

$$M_4 = \frac{d_5}{d_8 l}\left[B_1 c_2 + A_3 c_3 + B_3 c_4\right] + \frac{c_4}{d_8 l}\left[A_6 d_3 + B_6 d_2\right] = -\frac{717wl^2}{11644} = -788.18 \text{ ft. tons.}$$

$$M_5 = \frac{d_4}{d_8 l}\left[B_1 c_2 + A_3 c_3 + B_3 c_4\right] + \frac{c_5}{d_8 l}\left[A_6 d_5 + B_6 d_6\right] = + \frac{348 w l^2}{11644} = + 382.55 \text{ ft. tons.}$$

$S_4 = + 14.63$ tons.

Suppose the supports are all on level, all spans equal, $l = 80$ feet, and only the second, fifth, and seventh spans are uniformly loaded, with a load $w = 2$ tons per foot.

Then $A_1 = A_3 = A_4 = A_6 = 0 = B_1 = B_3 = B_4 = B_6$, $A_2 = A_5 = A_7 = \frac{wl^2}{4} = B_2 = B_5 = B_7$,

$$M_4 = \frac{d_5}{d_8 l}\left[A_2 c_2 + B_2 c_3\right] + \frac{c_4}{d_8 l}\left[A_5 d_4 + B_5 d_5 + A_7 d_6\right] = + \frac{348 w l^2}{11644} = + 382.55.$$

$$M_5 = \frac{d_4}{d_8 l}\left[A_2 c_2 + B_2 c_3 + A_5 c_5\right] + \frac{c_5}{d_8 l}\left[B_5 d_5 + A_7 d_6\right] = - \frac{717 w l^2}{11644} = - 788.18 \text{ ft. tons.}$$

$S_4 = - 14.63$ tons.

Suppose a load $P_4 = 20$ tons in the fourth span only.
Here all Y's are zero, and all A's and B's are zero, except

$A_4 = P_4 l^2 (2k - 3k^2 + k^3)$, $B_4 = P_4 l^2 (k - k^3)$, and we have

$$M_4 = \frac{d_5}{d_8 l}\left[A_4 c_4\right] + \frac{c_4}{d_8 l}\left[B_4 d_4\right] = - \frac{15 Pl}{2911}(97k - 168k^2 + 71k^3).$$

$$M_5 = \frac{d_4}{d_8 l}\left[A_4 c_4 + B_4 c_5\right] = - \frac{15 Pl}{2911}(26k + 45k^2 - 71k^3).$$

$$S_4 = \frac{15P}{2911}(71k - 213k^2 + 142k^3) + P(1 - k).$$

Suppose a uniform load w per foot over the whole girder.
Then

$$M_4 = \frac{d_5}{d_8 l}\left[(A_2 + B_1) c_2 + (A_3 + B_2) c_3 + (A_4 + B_3) c_4\right] + \frac{c_4}{d_8 l}\left[(A_5 + B_4) d_4 + (A_6 + B_5) d_5 + (A_7 + B_6) d_6\right];$$

or

$$M_4 = \frac{d_5 A}{d_8 l}\left[2c_2 + 2c_3 + 2c_4\right] + \frac{c_4 A}{d_8 l}\left[2d_4 + 2d_5 + 2d_6\right] = - \frac{12}{142} w l^2.$$

$$M_5 = \frac{d_4}{d_8 l}\left[(A_2 + B_1) c_2 + (A_3 + B_2) c_3 + (A_4 + B_3) c_4 + (A_5 + B_4) c_5\right] + \frac{c_5}{d_8 l}\left[(A_6 + B_5) d_5 + (A_7 + B_6) d_6\right];$$

or,

$$M_5 = \frac{d_4 A}{d_8 l}\left[2c_2 + 2c_3 + 2c_4 + 2c_5\right] + \frac{c_5 A}{d_8 l}\left[2d_5 + 2d_6\right] = - \frac{12}{142} w l^2. \quad S_4 = + \frac{wl}{2}.$$

If the spans are all equal, $l = 80$ feet, uniform load $w = 4000$ lbs. per ft. over the whole girder, how far must the fourth support be lowered below the level of the others in order that the moment at the fourth support may be zero?

Here, we have $M_4 = 0$, $h_3 - h_4 = h_5 - h_4$, all the B's and A's are equal to $\frac{wl^2}{4} = A$;

$$Y_1 = 0, \quad Y_2 = 0, \quad Y_3 = 6EI\left[\frac{h_4 - h_3}{l}\right], \quad Y_4 = 6EI\left[\frac{2(h_3 - h_4)}{l}\right] = - 2Y_3,$$

$$Y_5 = 6EI\left[\frac{h_4 - h_5}{l}\right] = - Y_3, \quad Y_6 = 0.$$

Also, since $M_4 = 0$, we have

$$d_5 [Y_3 c_3 - 2 Y_3 c_4 + A (2c_2 + 2c_3 + 2c_4)] + c_4 [- Y_3 d_4 + A (+ 2d_4 + 2d_3 + 2d_2)] = 0.$$

Hence $Y_3 = 6EI \left[\frac{h_3 - h_4}{l}\right] = \frac{A (2c_2 + 2c_3 + 2c_4) + A (2d_4 + 2d_3 + 2d_2)}{2c_4 d_5 + c_4 d_4 - c_3 d_5} = - \frac{246wl^3}{1395}$, and

$$h_3 - h_4 = - \frac{41wl^4}{1395EI}.$$

If $E = 24000000$ lbs. per square inch, and $I = 53400$ for dimensions in inches, $h_3 - h_4 = -0.541$ ft. $= - 6.5$ inches.

17. *A beam of one span is fixed horizontally at the ends. What are the end moments and shears?*

Here $s = 3$, $l_1 = 0$, $l_3 = 0$, $B_1 = A_1 = B_3 = A_3 = 0$. $c_1 = d_1 = 0$, $c_2 = d_2 = 1$, $c_3 = d_3 = -2$.

We have, from (A),

$$M_2 = - \frac{c_2}{l(d_2 + 2d_3)} [(Y_2 + A_2) d_2 + B_2 d_2 + Y_3] = \frac{- 2 (Y_2 + A_2) + B_2 + Y_3}{3l};$$

$$M_3 = \frac{Y_2 + A_2 - 2B_2 - 2 Y_3}{3l}.$$

If the ends are on level, $Y_2 = Y_3 = 0$, and

$$M_2 = \frac{- 2A_2 + B_2}{3l}, \quad M_3 = \frac{A_2 - 2B_2}{3l}.$$

For concentrated load and ends level,

$$M_2 = - Pl (k - 2k^2 + k^3), \qquad M_3 = - Pl (k^2 - k^3), \qquad S_2 = P (1 - 3k^2 + 2k^3).$$

For uniform load and ends level,

$$M_2 = M_3 = - \frac{wl^2}{12}, \qquad S_2 = \frac{wl}{2}.$$

For uniform load and ends out of level,

$$M_2 = - \frac{2Y_2}{3l} - \frac{wl^2}{12} + \frac{Y_3}{3l}, \qquad M_3 = \frac{Y_2}{3l} - \frac{wl^2}{12} - \frac{2Y_3}{3l}, \qquad S_2 = \frac{Y_3 - Y_2}{l^2} + \frac{wl}{2}.$$

How much must the left end be lowered to make S_2 zero?

Here, we have,

$$\frac{Y_3 - Y_2}{l^2} + \frac{wl}{2} = 0, \quad \text{or, } Y_3 - Y_2 = - \frac{wl^3}{2}.$$

Since $Y_3 = - Y_2$, we have

$$Y_3 = - \frac{wl^3}{4} = 6EI\left[\frac{h_3 - h_2}{l}\right]. \quad \text{Hence, } h_3 - h_2 = - \frac{wl^4}{24EI};$$

$$M_2 = \frac{wl^2}{4} - \frac{wl^2}{12} = + \frac{wl^2}{6}, \quad M_3 = - \frac{wl^2}{4} - \frac{wl^2}{12} = - \frac{wl^2}{3}, \quad S_3' = \frac{M_2 - M_3}{l} + \frac{wl}{2} = wl.$$

How much must the left end be lowered to make $M_2 = 0$?

$$\text{Here } - \frac{2Y_2}{3l} - \frac{wl^2}{12} + \frac{Y_3}{3l} = 0, \quad Y_3 = - Y_2, \quad \text{hence, } Y_3 = - \frac{wl^3}{12} = 6EI\left[\frac{h_3 - h_2}{l}\right],$$

$$\text{and } h_3 - h_2 = - \frac{wl^4}{72EI}. \quad M_3 = - \frac{wl^2}{6}. \quad S_2 = + \frac{wl}{3}. \quad S'_3 = + \frac{2wl}{3}.$$

CHAPTER IV.

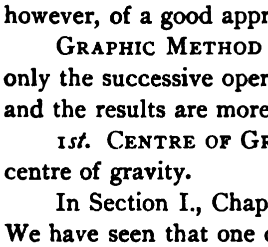
GRAPHIC DETERMINATION OF MOMENT OF INERTIA.

FROM the preceding Chapters we see that when it is required to determine the breaking weight or deflection of beams, or the proper shape for uniform strength, it is necessary that we should know the moment of inertia of the cross section with reference to an axis through its centre of gravity. We have already given, page 234 of this Appendix, the definition and significance of the term "moment of inertia," and have illustrated, by several examples, the method of calculating it. We have also given on page 236, a Table containing the moment of inertia for all the more usual forms of cross section which occur in practice.

Other forms may, however, sometimes occur which are not given in the Table, and the calculation of which is very difficult or tedious. In such case, several practical methods are in use.

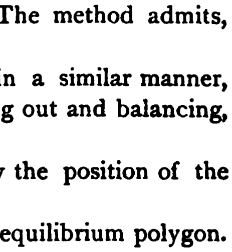
PRACTICAL METHOD OF DETERMINATION.—Thus take for instance, such a cross section as that shown in Fig. 190. It has been suggested in such a case, to find the centre of gravity of the cross section by first cutting out the cross section, drawn to any convenient scale, from a piece of cardboard. Then by balancing the cross section thus cut out upon a knife blade, first along some line XX, and then along some other line as YY, the position of the centre of gravity, C, may be found.

Fig.190



This being done, the cross-section may be ruled off into portions as shown in Fig. 191, and the area and centre of gravity of each of these portions measured and determined. The moment of inertia of the whole cross section is then taken as equal to the sum of the moments of inertia of each of these portions. The moment of inertia of each portion is taken as equal to its area multiplied by the square of the distance of its centre of gravity from the centre of gravity of the whole cross section. This is not, strictly speaking, correct. The method admits, however, of a good approximate determination.

Fig.191



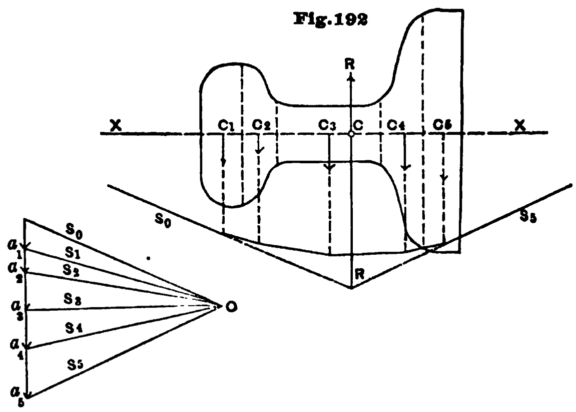
GRAPHIC METHOD OF DETERMINATION.—The graphic method proceeds in a similar manner, only the successive operations are all performed by construction, without cutting out and balancing, and the results are more accurate.

1st. CENTRE OF GRAVITY.—We have first, then, to determine graphically the position of the centre of gravity.

In Section I., Chapter IV., we have already deduced the properties of the equilibrium polygon. We have seen that one of these properties is that if any two of its sides are produced, the point of intersection is a point upon the resultant of all the forces between these two sides (page 39). Thus in Fig. 192, if XX is an axis of symmetry, we know that the centre of gravity must lie somewheres on that line. Now we can divide the cross section up into a number of smaller ones, by vertical lines. The area and centre of gravity of each of these portions must then be found by separate construction, to be given hereafter. Suppose, then, these centres c_1 c_2 . . c_8 found, Fig. 207. Con-

sider the areas $a_1, a_2 \ldots a_5$ of each portion as a force or weight, and lay it off to any convenient scale. Then choose a pole, O, and draw the strings $S_0, S_1, S_2, \ldots S_5$. Considering, then, each force a_1, a_2 etc., as acting at its centre of gravity, we can draw the equilibrium polygon. The intersection of the two outer lines S_0 and S_5, give then a point R in the resultant (page 39). We can, therefore, draw this resultant, RR. Its intersection with the axis of symmetry, XX, will give the position of the centre of gravity, C.

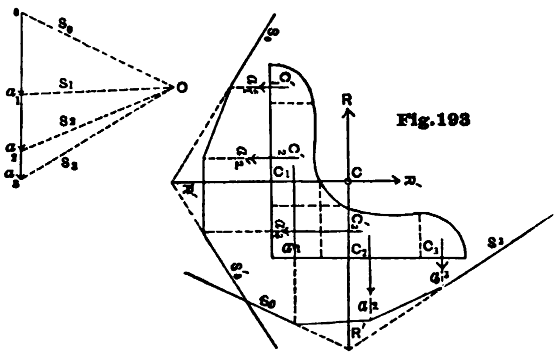
Fig. 192



If there is no axis of symmetry, we have only to divide the cross section into strips by lines at right angles to the first lines of division. The areas and centres of gravity of these strips being found, we take the new areas as forces acting at right angles to the former direction of the area forces, and construct a new force polygon whose lines are respectively perpendicular to the corresponding lines of the first. In other words, we may consider the cross section in Fig. 192, turned round through 90°. We thus find a new resultant. The intersection of this new resultant with that just found, will give the centre of gravity.

In Fig. 193, we have given this construction for a cross section which has no axis of symmetry. Thus dividing the cross section by vertical lines, we find the centres of gravity c_1, c_2, c_3, and consider the area of each portion, a_1, a_2, a_3, as a vertical force acting at each centre of gravity.

Fig. 193



We can then lay off these forces, choose a pole, O, and draw the strings S_0, S_1, S_2, S_3. Constructing the equilibrium polygon and prolonging the sides S_0 and S_3, we obtain the resultant RR. Somewhere in this resultant the centre of gravity must lie.

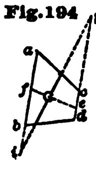
Again, divide the cross section by horizontal lines. Find the centres of gravity c'_1, c'_2, c'_3, and consider the area of each portion a'_1, a'_2, a'_3 as a horizontal force. Lay these forces off, choose, a pole o' and draw the strings, S'_0, S'_1, S'_2, S'_3. Construct the corresponding equilibrium polygon and produce the strings S'_0 and S'_3. We thus find the resultant $R'R'$. The intersection C, of the two resultants, gives the position of the centre of gravity.

Centre of Gravity of Partial Areas.—We thus see that we can find the centres of gravity of any cross section by a general method of construction. But in order to apply this method, we must first find the areas and centres of gravity of the small portions or slices into which the entire cross section is divided. These slices are in general either parallelograms, triangles, or trapezoids.

Area and Centre of Gravity of Parallelogram.—The area of a parallelogram is the product of its base by its altitude. The centre of gravity is at the intersection of its two diagonals.

Area and Centre of Gravity of Triangle.—The area of a triangle is half the product of its base by its altitude. The centre of gravity is at the intersection of the two lines drawn from any two vertices to the middle of the opposite side.

Fig. 194

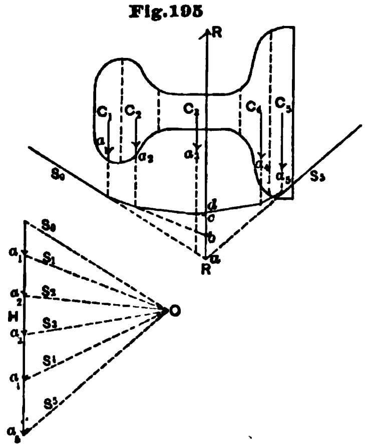


Area and Centre of Gravity of Trapezoid.—The area of a trapezoid, Fig. 194, is equal to its breadth multiplied by half the sum of its two parallel sides. Its centre of gravity may be found as follows: Draw a line, ef, through the centre of cd and ab. Then ef is an axis of symmetry. Prolong ab and make bi equal to cd. Prolong cd till ck is equal to ab. Then join ik. The intersection of ik with ef gives the centre of gravity.

We can thus find the areas and centres of gravity of the small slices into which the entire cross section is divided, as shown in Fig. 192 or 193. We should in general take the lines of division in the case of curved sections so close together that each slice may be treated as a trapezoid. When not limited by this restriction, the fewer the slices taken the better. Thus, in Fig. 192, we have taken narrow slices at the curved ends, but much broader slices where the bounding lines are straight.

2d. MOMENT FORCES.—The centre of gravity of the given cross section being thus easily found by construction, we know from the properties of the equilibrium polygon (page 40), that the pole distance multiplied by the ordinate to the polygon gives the moment of the forces. Thus, in Fig. 195, the distance *ab* cut off from the resultant *RR* by the two strings S_0 and S_1 prolonged, taken to the scale of distance, multiplied by the pole distance *OH* taken to the scale of force, gives the moment of the force a_1 with reference to the line *RR* as an axis. In like manner *bc* multiplied by the pole distance gives the moment of a_2, and *cd* gives the moment of a_3, all with reference to *RR*.

Fig.195



If, then, we find the centre of gravity, as illustrated in Figs. 192 or 193, the segments into which the resultant *R* is divided by the strings prolonged, are proportional to the moment of each area or slice with reference to that resultant as an axis.

If, therefore, we takes these moments *and consider them as forces*, by laying them off to scale and choosing a new pole and drawing a new equilibrium polygon, the new segments thus found, multiplied by the new pole distance, *will give the moments of the moment forces, or the moments of inertia of the slices.*

POINT OF APPLICATION OF THE MOMENT FORCES.—The *moment forces* should not, however, be considered as acting at the centre of gravity of the slices. If *d* is the distance from the centre of gravity of the entire cross section to the centre of gravity of any slice, and *r* is the radius of gyration of the slice, then the moment force should not be considered as acting at the distance *d*, but at the distance $d + \frac{r^2}{d}$. The radius of gyration being that distance at which, if the whole mass of the slice were concentrated, its moment of inertia would be the same as the moment of inertia of the slice itself. We have seen, page 235, that it is found by dividing the moment of inertia by the area.

Thus, the radius of gyration of a rectangle or parallelogram, Fig. 196, is equal to $\frac{b}{2\sqrt{3}}$, where

Fig.196

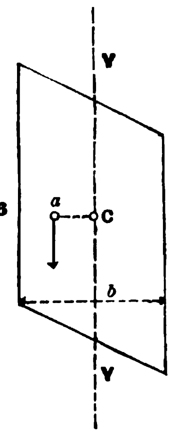


Fig.197

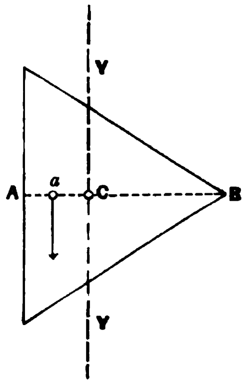
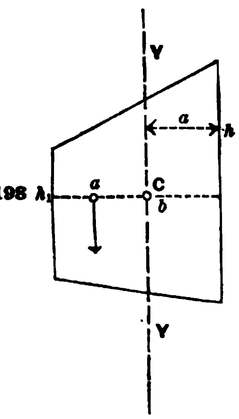


Fig.198



b is the breadth or horizontal distance between the parallel sides, the axis *YY* being vertical. The moment force, therefore, should be considered as acting, not at the centre of gravity *C*, but at a distance $\frac{\overline{aC}^2}{d} = \frac{b^2}{12d}$ from the axis *YY*, as shown in the figure.

The radius of gyration of a triangle, Fig. 197, is $\frac{h}{3\sqrt{2}}$, where h is the distance AB, and the axis YY through the centre of gravity is vertical. The moment force should, therefore, be taken as acting not at the centre of gravity C, but at a distance $\frac{h^2}{18d}$ from the axis YY.

The radius of gyration of a trapezoid, Fig. 198, is equal to

$$\frac{b\sqrt{h^2 + 4hh_1 + h_1^2}}{3(h + h_1)\sqrt{2}},$$

where h and h_1 are the lengths of the two parallel sides.

The moment force should be taken, therefore, as acting at a distance $\frac{b^2(h^2 + 4hh_1 + h_1^2)}{18d(h + h_1)}$ from this axis.

In any case, the distance $\frac{\overline{aC}^2}{d}$ is laid off to the left of YY, if the axis through the centre of gravity of the entire cross section lies on the right of YY, and *vice versa*.

EXAMPLE.—We can now give an example showing the entire process of finding graphically the moment of inertia of any cross section.

Suppose a cross section such as Fig. 199. It is required to find its moment of inertia with reference to the vertical axis YY passing through its centre of gravity.

1st. TO FIND THE CENTRE OF GRAVITY.—Draw the axis of symmetry XX, if there is one. Then divide the cross section by vertical lines into straight lined slices. In our Fig. 199 we have one triangle, one trapezoid and two rectangles. Find the centres of gravity c_1, c_2, c_3 and c_4 of these areas, as already instructed, and also the areas a_1, a_2, a_3 and a_4, themselves (page 303). Consider these areas as vertical forces acting at the corresponding centres of gravity, as shown in Fig. 199.

Fig. 199

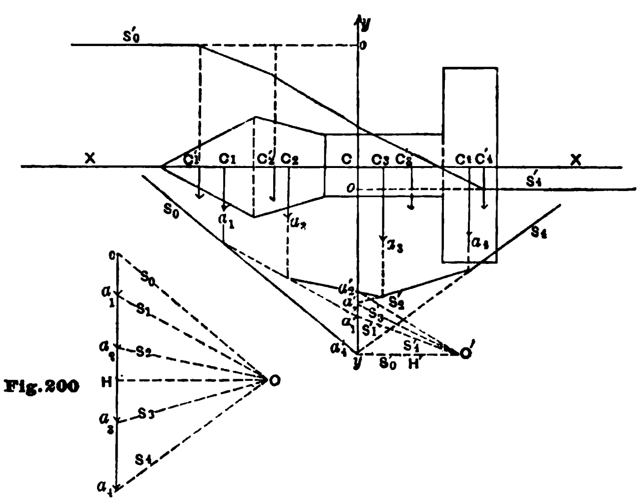


Fig. 200

Next form the force polygon, Fig. 200, by laying off the area forces to any convenient scale, as 10 sq. inches to an inch for instance. Choose a pole O, and draw the strings S_0, S_1 . . S_4. Now draw the corresponding equilibrium polygon in Fig. 199. The intersection Y of its extreme strings S_0 and S_4, gives a point on the resultant YY which passes through the centre of gravity. The intersection C of YY with XX gives the centre of gravity of the cross section.

If there were no axis of symmetry the process should be repeated, considering all the area forces as acting horizontally. We should thus obtain a second resultant at right angles to YY.

2d. TO FIND THE MOMENT OF INERTIA.—The segment Ya_1' cut off from YY taken to the scale of length adopted for the Figure, multiplied by the pole distance H taken to the scale adopted for the areas in Fig. 200, gives the moment of the area a_1 with reference to YY. In the same way $a_1'\,a_2 \times H$ gives the moment of the second area, $a_2'\,a_3' \times H$ gives the moment for the third area, and $a_3'\,a_4'$ gives the moment for the fourth area.

Now consider these moments as if they were forces. Choose a new pole O', and new pole distance H', and draw the strings S_0', S_1' . . S_4'.

Find the radius of gyration of each area (page 235), and consider the moment forces as acting at

c_1', c_2', c_3', c_4', the distance $c_1' c_1$ being equal to the square of the radius of gyration of the first area, divided by d_1, etc. Observe that this distance is laid off on the left of the centre of gravity for the first two areas, and on the right for the other two. That is, the centre of gravity is always on the same side of the new centres as the axis YY.

Consider the moment forces as acting at these new centres, and construct the new equilibrium polygon. Produce its extreme strings to intersection oo with YY. Measure oo to the scale of length adopted for the entire Figure. Then $\overline{oo} \times H \times H'$ is the moment of inertia, where H is measured to the scale of area forces adopted for Fig. 200, and H' to the scale of length of Fig. 199.

In similar manner the moment of inertia of any cross section may be found, with reference to any desired axis. The lines of division should always be parallel to that axis, and should be so close together that the boundary between may be considered straight, and no closer.

EXAMPLES.*

The student who has carefully studied this work, should be able to solve easily and accurately the following examples:

1. A wrought iron tie-rod, 30 feet long and 4 sq. ins. in area of cross section, is subjected to 40000 lbs. tension. What is the unit stress? If the coefficient of elasticity is 30000000 lbs. per sq. in., what is the elongation?

Unit stress = 10000 lbs. per sq. in. Elongation = 0.01 ft.

2. An iron bar, 10 ft. in length, stretches .012 ft. under a unit stress of 25000 lbs. per sq. in. What is E?

$E = 20833333$ lbs. per sq. in.

3. A rectangular timber tie is 12 ins. deep and 40 ft. long. If $E = 1200000$ lbs. per sq. inch, find the proper thickness of the tie, so that its elongation under a pull of 270000 lbs. may not exceed 1.2 ins.

Thickness = 7.5 ins.

4. A roof tie-rod, 142 feet in length and 4 sq. ins. in sectional area, is subjected to a stress of 80000 lbs. If $E = 30000000$ lbs. find the elongation of the rod.

Elongation = 1.136 ins.

5. The length of a cast iron pillar is diminished from 20 ft. to 19.97 ft. under a given load. Find the compressive unit stress, E being 17000000 lbs. per sq. in.

Unit stress = 25500 lbs. per sq. in.

6. A wrought iron bar, 2 sq. ins. sectional area, has its ends fixed between two immovable blocks when the temperature is at 60° F. Taking the coefficient of expansion at 0.000006944 per unit of length, for one degree, what pressure will be exerted upon the blocks when the temperature is 100° F.?

Pressure = 0.00055552 E.
If $E = 30000000$ lbs. per sq. in., Pressure = 16665.6 lbs.

7. The dead load of a bridge is 5 tons, and the live load 10 tons per panel, the corresponding factors of safety being 3 and 6. Find the *compound* factor of safety.

Factor = 5.

8. The dead load upon a short hollow cast iron pillar, with a sectional area of 20 sq. ins., is 50 tons. If the compression is not to exceed 0.0015 of the length, find the greatest live load to which the pillar can be subjected, E being 17000000 lbs. per sq. in.

Live load = 410000 lbs. = 205 tons.

9. A steel suspension rod, 30 ft. long and ½ sq. in. sectional area, carries 3500 lbs of the roadway and 3000 lbs. of the live load. Determine the *gross* load and also the extension of the rod, E being 35000000 lbs.

Gross load = 6500 lbs. Extension = 0.133 inch.

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* These examples have been compiled from Prof. Bovey's "Applied Mechanics," Stoney's "Theory of Strains," Wood's "Strength of Materials," and Weisbach's "Mechanics of Engineering."

10. A beam 40 ft. long carries a load of 20000 lbs. Find the shearing force at 15 ft. from one end, and also the maximum bending moment of the beam :—

(*a*) When the beam is supported at the ends and loaded in the middle.
(*b*) When it is supported at the ends and loaded uniformly.
(*c*) When it is fixed at one end and loaded at the other.
(*d*) When it is fixed at one end and loaded uniformly.

(*a*) Shear = 10000 lbs. Maximum moment = 200000 ft. lbs. at middle.
(*b*) Shear = 2500 lbs. Maximum moment = 100000 ft. lbs. at middle.
(*c*) Shear = 20000 lbs. Maximum moment = 800000 ft. lbs. at end.
(*d*) Shear = 7500 lbs. Maximum moment = 400000 ft. lbs. at end.

Draw the curves of shearing force and bending moment.

11. Discuss the effect produced in each of the cases of Question (10): *first*, when a single weight of 2000 lbs. passes over the beam ; *second*, when a train weighing 2000 lbs. per lineal ft. moves across the beam.

Draw the curves of shearing force and bending moment.

12. A beam 20 ft. in length rests upon two supports and carries a weight of 10 tons at 5 ft. from one end. Find the maximum bending moment.

Maximum moment at weight = 37.5 ft. tons.

Draw the curves of shearing force and bending moment.

13. A uniform rigid bar weighs W lbs., and is supported by two strings attached to its ends. Find the tensions in the strings and the inclination of the bar when the strings are inclined to the vertical at angles of 60° and 30° respectively.

Tensions = 0.5 W and 0.866 W.
Inclination of bar with horizontal = 30°.
Compression in bar = 0.5 W.
Vertical components of string tensions = 0.25 W and 0.75 W.
Horizontal component of string tensions = 0.433 W.

Solve by diagram and calculation.

14. A car of weight W for a 4 ft. 8½ in. gauge, is 33 ft. long, 6 ft. deep, and its bottom is 2 ft. 6 ins. above the rails. Find the additional weight thrown upon the leeward rails, when the wind blows upon the side of the car with a pressure of 20 lbs. per sq. ft. Find the minimum wind pressure that will blow the car over.

Additional weight = 4625.84 lbs.
Minimum pressure = 0.428 W.

15. What is the breadth and depth of the strongest rectangular beam which can be cut from a cylindrical log of diameter D?

$$\text{Breadth} = D\sqrt{\tfrac{1}{3}}. \quad \text{Depth} = D\sqrt{\tfrac{2}{3}}.$$

16. A round beam and a square beam are equal in length and equally loaded. Find the ratio of the diameter to the side of the square, so that the two beams may be of equal strength.

$$\frac{\text{diameter}}{\text{side}} = 2\sqrt[3]{\frac{2}{3\pi}}.$$

17. Compare the relative strengths of a cylindrical beam and the strongest rectangular and square beams that can be cut from it.

$$\frac{\text{Strength of cylindrical}}{\text{Strongest rectangular}} = \frac{9\pi\sqrt{3}}{32} = 1.53. \qquad \frac{\text{Strength of cylindrical}}{\text{Strongest square}} = \frac{3\pi\sqrt{2}}{8} = 1.66.$$

18. Compare the relative strengths of a solid square beam to that of the solid inscribed cylinder.

$$\frac{\text{Strength of square}}{\text{Strength of cylinder}} = \frac{16}{3\pi} = 1.7.$$

19. Compare the strength of a square beam with its sides vertical, to that of the same beam with one diagonal vertical.

$$\frac{\text{Strength side vertical}}{\text{Strength diagonal vertical}} = \sqrt{2} = 1.414.$$

20. A beam of yellow pine, 14 ins. wide, 15 ins. deep, and resting upon supports 10 ft. 9 ins. apart, was just able to bear a weight of 34 tons at the centre. What weight will a beam of the same material, 3 ft. 9 ins. between the supports and 5 ins. square bear ?

3.86 tons.

21. Determine the form of a beam of uniform strength, for constant depth and for constant breadth.

(1) When the beam rests upon two supports and is uniformly loaded.
(2) When the beam rests upon two supports and is loaded at the centre.
(3) When the beam is fixed at one end and loaded at the other.
(4) When the beam is fixed at one end and uniformly loaded.
(5) When the beam in cases (1) and (4) carries an additional weight at the centre and end respectively.

22. Compare the strengths of two rectangular beams of equal length, the breadth and depth of one, being respectively equal to the depth and breadth of the other.

The strengths are directly as the breadths, and inversely as the depths.

23. A cast iron beam 4 ins. square rests upon supports 6 ft. apart. Determine the breaking weight at the centre, taking $R = 30000$ lbs. per sq. in.

Breaking weight $= 17777\frac{7}{9}$ lbs.

24. A yellow pine beam, 14 ins. wide, 15 ins. deep, and resting upon supports 10 ft. 6 ins. apart, broke down under a uniformly distributed load of 60.97 tons. Find the coefficient of rupture R.

$R = 3658.2$ lbs.

25. A cast iron rectangular girder rests upon supports 12 ft. apart, and carries a weight of 2000 lbs. at the centre. If the breadth is one-half the depth, find the sectional area of the girder, so that the inch stress in the metal may nowhere exceed 4000 lbs.

Area = 18 sq. ins., depth = 6 ins., breadth = 3 ins.

26. A wrought iron bar, 4 ins. deep, $\frac{3}{4}$ in. wide, and rigidly fixed at one end, gave way when loaded with 1568 lbs. at the free end, at a point 2 ft. 8 ins. from the load. Find R.

$R = 25088$ lbs.

27. A wrought iron bar, 2 ins. wide and 4 ins deep, rests upon supports 12 ft. apart. Determine the uniformly distributed load which the bar will safely carry in addition to its own weight, if $R = 50000$ lbs. and factor of safety is 4. A bar of iron 3 ft. long and one square inch in cross section is assumed to weigh 10 lbs.

Weight = 3384 lbs.

28. Find the length of a beam of ash 6 ins. square, which would break of its own weight when supported at the ends, the weight of the timber being 30 lbs. per cubic ft. and $R = 7000$ lbs. per sq. in.

Length $= 149\frac{2}{3}$ ft.

29. A railway girder 50 ft. in the clear and 6 ft. deep, carries a uniformly distributed load of 50 tons. Find the maximum shearing stress at 20 ft. from one end, when a train weighing $1\frac{1}{4}$ tons per lineal foot crosses the girder.

Also, find the minimum theoretic thickness of the web, 4 tons being the safe shearing inch stress of the metal.

Shear = 16.25 tons. Thickness = 0.056 in.

30. A cast iron semi-girder, 8 ft. long and 12 ins. deep, carries a uniformly distributed load of 16000 lbs. Find the area of the top flange at the fixed end, neglecting the web, so that the inch stress may not exceed 3000 lbs.

Area = 21.3 sq. inches.

31. A cast iron girder, $27\frac{1}{2}$ ins. deep, rests upon supports 26 ft. apart. Its bottom flange is 16 ins. wide and 3 ins. deep. Neglecting the web, find the breaking weight at the centre, the tearing inch stress of cast iron being 15000 lbs.

Weight = 253846 lbs.

32. The lattice bridge at the Boyne Viaduct is in three spans, continuous. Each side span is 140 ft. 11 ins. long, and 22 ft. 3 ins. deep. The permanent load supported by one main girder of a side span is 0.68 ton per running foot, and the sectional area of its lower flange over the centre pier is 127 sq. ins. On one occasion an extraordinary load in the centre span depressed it to such an extent as to raise the ends of the side spans off the abutments, thus forming each side span into a semi-girder. What was the compressive inch stress in the lower flange at the pier?

Inch stress = 2.4 tons.

33. A semi-girder, 44.7 ft. long, and 22.25 ft. deep, supports a uniformly distributed load of 1.82 tons per foot, and a weight of 161.6 tons in addition at the extremity. What is the inch stress on the net section of the tension flange at the point of support, neglecting the web, the gross area being 132.6 ins., but reduced by rivet holes to the extent of $\frac{7}{9}$ths?

Inch stress = 3.94 tons.

34. A girder, 50 ft. long and 4 ft. deep, supports a uniformly distributed load of 32 tons. Find the stress in either flange at 9 feet from one end, neglecting the web.

Stress = 29.5 tons.

35. A piece of teak, 2 ins deep, and $1\frac{15}{16}$ ins. wide, is fixed at one extremity. Find the weight which if hung at 2 ft. from the point of attachment will break it by crushing the fibres of the lower side, assuming that the crushing strength for teak is considerably less than its tearing strength, and equal to 12000 lbs. per square inch.

Weight = 646 lbs.

36. The effective length and depth of a cast iron girder were $27\frac{1}{3}$ ft. and 18 ins. respectively, and its bottom flange was 10 ins. wide and $1\frac{1}{2}$ ins. deep. The girder failed under a weight of $29\frac{1}{2}$ tons at the centre. Find the maximum inch stress in the bottom flange, neglecting the web.

Stress = 8.96 tons.

37. A cylindrical beam 2 ins. in diameter, 60 inches long, and weighing $\frac{1}{4}$ lb. per cubic inch, deflects $\frac{3}{5}$ in. under a weight of 3000 lbs. at the centre. Find E.

$E = 28929144$.

38. A rectangular beam, 5 ft. long, 3 ins. wide, and 3 ins. deep, is deflected $\frac{1}{16}$ in. by a weight of 3000 lbs. applied at the middle. Find E.

$E = 20000000$.

39. A joist, whose length is 16 ft., width 2 ins., depth 12 ins., and coefficient of elasticity 1600000 lbs., is deflected $\frac{1}{2}$ in. by a weight in the middle. Find the weight, neglecting the weight of the beam.

Weight = 1562 lbs.

40. An iron rectangular beam, whose length is 12 ft., breadth $1\frac{1}{2}$ ins., coefficient of elasticity 24000000 lbs., has a weight of 10000 lbs. suspended at the middle. Find the depth in order that the deflection may be $\frac{1}{480}$th of the length.

Depth = 8.8 in.

41. A rectangular wooden beam, 6 ins. wide and 30 ft. long, is supported at its ends. The coefficient of elasticity is 1800000 lbs. The weight of a cubic foot of the beam is 50 lbs. Find the depth that it may deflect 1 inch from its own weight.

Depth = 6.5 ins.

How deep must it be to deflect $\frac{1}{400}$th of its length?

Depth = 6.8 ins.

42. Required the depth of a rectangular beam which is supported at its ends, and so loaded at the middle that the elongation of the lowest fibre shall equal $\frac{1}{1400}$th of its original length.

$$\text{Depth} = \sqrt{\frac{2100\,Pl}{Eb}}.$$

43. Required the radius of curvature at the middle point of a wooden beam, when the load is 3000 lbs., the length 10 ft., breadth 4 ins., depth 8 ins., and $E = 1000000$ lbs.

Radius = 1896 inches.

44. Let the beam be of iron, supported at its ends. Let the breadth be 1 in., depth 2 ins., length 8 ft., and E 25000000 lbs. Required the radius of curvature at the middle when the deflection is [illegible]th of an inch.

Radius = 3840 inches.

45. A beam whose depth is 8 ins., and length 8 feet, is supported at its ends, and sustains 500 lbs. per foot. Find its breadth so that it shall have a factor of safety of $\frac{1}{10}$th, R being 14000 lbs.

Breadth = $3\frac{3}{4}$ ins.

46. A beam, whose length is 12 ft., breadth 2 ins., and depth 5 ins., is supported at its ends. Find the weight uniformly distributed, it will sustain, the coefficient of safety being $\frac{1}{4}$ and $R = 80000$ lbs.

Weight = 9259 lbs.

47. A wooden beam, whose length is 12 ft., is supported at its ends. Find its breadth and depth so that it shall sustain one ton uniformly distributed over its whole length, R being 15000 lbs., the coefficient of safety $\frac{1}{6}$th, and the depth 4 times the breadth.

Breadth = 2.08 ins.
Depth = 8.32 ins.

48. A wrought iron beam 12 ft. long, 2 ins. wide, and 4 ins. deep, is supported at its ends. The material weighs $\frac{1}{4}$ lb. per cubic inch. Taking R at 54000 lbs., find what weight uniformly distributed it will sustain.

Without the weight of the beam, 16000 lbs.
With the weight of the beam, 15712 lbs.

49. A beam is fixed at one end. Length 20 ft., breadth $1\frac{1}{2}$ ins. $R = 40000$ lbs. If the weight of the material is $\frac{1}{4}$ lb. per cubic inch, find the depth so that it may sustain its own weight and 500 lbs. at the free end.

Depth = 4.05 inches.

50. The breadth of a beam is 3 ins., depth 8 ins., weight of a cubic ft. 50 lbs., $R = 12000$ lbs. Find the length so that it will break from its own weight when supported at the ends.

Length = 175.27 feet.

51. If a beam 6 ft. long, $1\frac{1}{2}$ ins. wide and 4 ins. deep is supported at its ends, and loaded at the middle so as to produce a deflection of $\frac{3}{4}$ inch, find the greatest inch strain on the fibres, taking $E = 25000000$ lbs. Also find the load.

Strain = 86805 lbs.
Load = 19290 lbs.

52. For the same beam, if the greatest fibre strain is 12000 lbs. per sq. in. find the greatest deflection.

Deflection = 0.103 inch.

53. What should be the size of a square wooden beam of 12 feet span, which sustains a load of 300 lbs. at the centre, and has at the same time a longitudinal tension of 2000 lbs.; the maximum working unit stress being taken at 1000 lbs. per square inch.

Size = 4.02 inches.

54. A rectangular oak beam 1 foot deep and $\frac{1}{2}$ ft. wide, and 15 feet long, is fixed horizontally at one end and is free at the other end. Let the weight of the beam itself be 54 pounds per cubic foot. Suppose it sustains a uniform load of 100 pounds per foot of length extending over only 4 feet of the beam, beginning at 5 feet from the fixed end; also a weight of 100 pounds placed at 11 feet from the fixed end. Let $E = 2000000$ lbs. per square inch. What is the total deflection at the free end?

Deflection due to weight of beam = 0.17085 inch.
" " uniform load = 0.12627 "
Deflection due to the weight = 0.0684 inch.
Total deflection = 0.36553 inch.

55. If the same beam is loaded with 5 equal weights of 100 lbs. each. at intervals of 3 feet, what is the deflection at the free end, and at the third loaded point from the fixed end?

Total deflection at the free end = 0.27 inch.
Total deflection at the third point = 0.12555 inch.

56. Same beam of oak, supported at the two ends. What is the central deflection due to its own weight?

Deflection = 0.001483 foot.

57. A beam of pine weighing 40 lbs. per cubic foot, $18\frac{1}{2}$ inches deep, 15 inches wide, $12\frac{1}{2}$ feet long, is supported at the ends, and has a weight of 17935 lbs. placed at 48 inches from one end. What is the deflection at centre and point of application of weight? $E = 1680000$ lbs. per sq. in.

Deflection at centre due weight of beam = 0.0032 inch.
Deflection at centre for weight added = 0.078617 inch.

Deflection at 48 inches due weight of beam = 0.0027 inch.
Deflection at 48 inches due weight added = 0 07185 inch.

58. A wrought iron 15 inch I beam, whose moment of inertia is 691, has a length of 30 feet. $E = 24000000$. If supported at the ends, and a uniform load of 75 lbs. per inch covers the first 10 feet, what is the deflection at the end of the load?

Deflection = 0.23444 inch.

What is the deflection at the centre of the beam?

Deflection = 0.24421 inch.

What is the deflection 10 feet from the unloaded end?

Deflection = 0.19537 inch.

Where is the point of greatest deflection? and what is the greatest deflection?

At 13.1676 feet. Greatest deflection = 0.24847 inch.

If the beam's own weight is 5.573 lbs. per inch, what is the deflection at centre?

Deflection = 0.07349 inch.

If the same 10 foot load is moved along to the centre, what is the deflection at the centre?

Deflection = 0.50063 inch.

If the uniform load of 75 lbs. per inch covers the whole span, what is the central deflection?

Deflection = 0.98305 inch.

If the same beam is half loaded with 75 pounds per inch, what is the deflection at centre? What is the maximum deflection? and at what point is it?

Deflection = 0.494525 inch.
Max. deflection = 0.49855 inch.
Within the loaded part and 14.48 inches from centre of beam.

If the same beam has 3 weights of 4500 lbs. each, placed at intervals of 60 inches, beginning at one end, what is the deflection at the centre?

Deflection = 0.6154 inch.

If there are 8 weights, each equal to 3000 lbs., at intervals of 40 inches, what is the central deflection?

Deflection = 0.97926 inch.

59. Suppose the same beam as in 58 to be fixed horizontally at both ends, and loaded uniformly with 75 lbs. per inch. What is the deflection 10 feet from either end? At the centre?

Deflection = 0.1563 inch.
At centre = 0.19781 inch.

If only one end is fixed, the other supported, what is the deflection at 10 feet? At centre? At 20 feet? What is the maximum deflection? Where is it?

Deflection at 10 feet = 0.39074 inch.
Deflection at centre = 0.39563 inch.
Deflection at 20 feet = 0.27352 inch.
Maximum deflection = 0.41018 inch.
At 151.7524 inches from supported end.

60. Same beam, fixed horizontally at both ends, with a concentrated load of 27000 lbs. If the load is in the centre, what is the deflection at half way between the centre and either end? What is centre deflection? Where are the points of contrary flexure?

Deflection = 0.19781 inch.
Centre deflection = 0.39562 inch.
At 90 inches from each end.

If the load is 7.5 feet from the left end, where and what is the maximum deflection?

Maximum deflection = 0.2136 inch.
At 12 feet from left end.

If only the right end is fixed and the other supported, and the load of 27000 lbs. is at the centre, what are the deflections at the quarter points? The centre? And what is the maximum deflection?

At the quarter points, deflection = 0.5316 inch and 0.3091 inch.
Central deflection = 0.69234 inch.
Maximum deflection = 0.70732 inch at $2l\sqrt{\frac{1}{5}}$ from supported ends.

61. Same beam as in 58, fixed horizontally at both ends, has 3 weights of 4500 lbs. each, placed at intervals of 60 inches, beginning at the left end. What is centre deflection?

Deflection = 0.13187 inch.

If 2 other equal weights of 4500 lbs. each are added at the same interval of 60 inches, what is the central deflection due to these last two weights?

Deflection = 0.06594 inch.

Suppose the fifth weight removed, what is the deflection at the fourth weight? At the third weight? And second weight?

Fourth weight, deflection = 0.13748 inch.
Third weight, " = 0.18072 inch.
Second weight, " = 0.1458 inch.

What are the end moments due to these four weights? and where are the points of contrary flexure?

$M = -750000$ inch-pounds.
$M_1 = -600000$ inch-pounds.
74.806 and 275.294 inches.

PART II.

DETERMINATION OF DIMENSIONS AND DESIGNING OF DETAILS.

II. DETERMINATION OF DIMENSIONS.

CHAPTER I.

ULTIMATE STRENGTH.—LIMIT OF ELASTICITY.—OLD AND NEW METHODS OF DIMENSIONING.

IN Part I. we have learned how to find the strains in the various pieces of any framed structure due to the action of assumed outer forces. In Chapter VIII. of this Part we shall see how to estimate the intensity of these outer forces, viz.: snow and wind load, live and dead load.

It is evident that having then properly assumed our outer forces, and then having calculated the resulting strains in the pieces, as directed in Part I., only one-half of our problem is solved. The other half is to properly determine the cross section of any piece in order that it may resist the strain that comes upon it. This is, in fact, the most important part of our problem, as upon it depends the safety and efficiency of the structure.*

Its proper solution requires a thorough knowledge of the strength of materials. This is in itself a subject for special treatises, and we cannot attempt to include it here. Much of what is necessary to be known has been given in the Appendix, page 233. We shall content ourselves, therefore, in the present Chapter, with giving the results of the best modern practice as regards wood, iron and steel, referring the student to other works which treat of the subject specially for fuller information. This part of our problem is still in process of development, as our knowledge of materials is continually being increased by experiment, and the student will therefore bear in mind that the practice of to-day may be modified by future knowledge.

ULTIMATE STRENGTH AND LIMIT OF ELASTICITY.—The smallest quiescent load per square inch which causes rupture of a piece, we call the *breaking load*, or the *ultimate strength* of the material.

It is found by experiment that if a piece of any material be subjected to pure tension or pure compression, the change of length is, within certain limits, very nearly proportional to the load. That is, a double load causes a double elongation or compression, three times the original load causes three times the original elongation or compression, and so on.

* In the words of Theodore Cooper, "A successful bridge engineer, from the American point of view, must be something more than a mere calculator of strains. That is the most elementary part of the duty, and does not come within the province of designing. After the selection of the skeleton form and relative proportion of panels, depths, and widths of spans, a very moderate knowledge of mechanical mathematics would enable any one to determine the strains in an American bridge. He must, in addition to his knowledge as to the effects of varying forms and proportions, have a full knowledge of the capacity of his forms and their connections, and also of the practical processes of manufacture and erection. He must know how his design can be made and put together, and whether it is so harmonized in all its parts and connections that each part may do its full duty under all possible conditions of service.

"In addition to knowing all the elements that make up a perfect design, he must have the instinct of designing or the power of adapting his knowledge to any individual case, in order to obtain the best or desired result.

"Then experience, observation, and a sharp competition with men of like knowledge and instinct, will give him his position as a bridge engineer."—*Trans. Am. Soc. C. E.*, July, 1889.

This law is not exactly true, but within certain limits is approximately so. Thus, for any material, the curve denoting the relation between change of length and acting load, is within these limits approximately a straight line. This limit is called the "*limit of elasticity.*" We may, therefore, define the limit of elasticity as that point at which the law of proportionality of change of length to acting force ceases to hold good. The load corresponding to this point will evidently be much less than the breaking load.

We give in the following Table a few mean values of the ultimate strength and limit of elasticity for wood, iron and steel. These values will of course vary considerably with the quality of the material, mode of manufacture, etc. In any special case the only reliable knowledge for the engineer to build upon is actual experiment. Such values as we give are useful only for preliminary calculations. Much more detailed knowledge may be found in those works which treat specially of the strength of the materials, as well as in the Appendix, page 248, and the student should read and constantly refer to the specifications, page 455.

TABLE OF ULTIMATE STRENGTH AND LIMIT OF ELASTICITY IN POUNDS PER SQUARE INCH.

	ULTIMATE STRENGTH.			LIMIT OF ELASTICITY.			COEFF. OF ELASTICITY
	Comp.	*Tens.*	*Shear.*	*Comp.*	*Tens.*	*Shear.*	
WOOD,							
Oak, parallel to fibre........	10,000	11,400	1,140	2,570	3,000	300	1,070,000
" transverse to fibre......	5,000	700	2,300	......	......	......	....
Pine, parallel to fibre........	8,600	10,000	860	......	3,000	[illegible]	....
" transverse to fibre......	3,000	640	1,860	......	......	......	....
Beech, parallel to fibre.......	9,400	14,300	940	......	2,300	......	....
" transverse to fibre.....	5,000	1,000	......	......	......	......	....
IRON,							
Cast iron	100,000	18,600	15,000	21,400	10,700	8.600	14,000,000
Wrought iron..............	60,000	57,000	45,700	20,000	20,000	16,000	28,700,000
Plate iron...................	43,000	47,000	37,000	20,000	20,000	16,000	26,000,000
Wire......................	......	86,000	......	......	31,400	......	31,300,000
STEEL,							
Soft steel..................	80,000	71,400	57,000	28.600	28,600	23,000	29,000,000
Plate......................	70,000	71,400	57,000	36,000	36,000	28,600	....
Wire......................	.....	130,000	......	......	64,300	......	....
Hard......................	107,000	107,000	85,700	38,600	38,600	30,860	32,000,000
Cast steel—soft	143,000	114,000	91,400	71,400	53,600	40,000	34,000,000
" " hard............	......	143,000	114,300	......	95,000	76,140	....
" " wire	......	160,000	......	......	......	......	43,000,000

ALLOWABLE STRESS PER SQUARE INCH—FACTOR OF SAFETY.—The limit of elasticity marks the point beyond which the material should never be strained. In practice the working strain should be well within this limit, say ½ or ⅔ds of it at most for quiescent loads.

When this limit is not known, it is sometimes customary to take a certain fraction of the ultimate strength as the safe load as, for instance, ⅕th or ⅙th. In such case we call 5 or 6 the "*factor of safety*," that is, it will take five or six times the working load to break the piece. Evidently the ultimate load divided by the factor of safety ought to give a result well within the limit of elasticity. Also we may evidently take this factor less for quiescent loads than for intermittent and oft-repeated loading accompanied by shock.

If β is the allowable stress per square inch, and μ is the ultimate strength, and n the factor of safety, then we have

$$\beta = \frac{\mu}{n}.$$

We give in the following Table the factor of safety n according to good practice:

TABLE OF FACTOR OF SAFETY.

MATERIAL.	TEMPORARY CONSTRUCTIONS.	BUILDINGS IN GENERAL.	BRIDGE AND ROOF CONSTRUCTIONS.	MACHINES AND STRUCTURES SUBJECT TO SHOCK.
Wood	6	9	10	15
Cast iron	...	6	7	10
Wrought iron	3	4		
Iron plate	...	4		
Ordinary steel	....	....	5 to 6	7 to 8
Bessemer steel	....	....		
Cast steel	....	....		
Stone	10	20	30	35

We have accordingly for the allowable stress in pounds per square inch for average materials, $\beta = \frac{\mu}{n}$, the following Table:

TABLE OF ALLOWABLE STRESS IN POUNDS PER SQUARE INCH.

MATERIAL.	TEMPORARY CONSTRUCTION.			BUILDINGS IN GENERAL.			BRIDGE AND ROOF CONSTRUCTIONS.			MACHINES AND STRUCTURES SUBJECT TO SHOCK.		
	Tens.	Comp.	Shear.	Tens.	Comp.	Shear.	Tens.	Comp.	Shear.	Tens.	Comp.	Shear.
Oak } Direction of fibre	1,860	1,710	210	1,300	1,143	143	1,143	1,000	114	860	714	86
Pine } Direction of fibre	1,710	1,430	140	1,143	1,000	100	1,000	860	86	714	600	60
Cast iron	4,300	10,700	3,400	3,600	8,600	2,860	2,860	7,140	2,140	1,860	4,300	1,430
Wrought Iron	17,100	17,100	14,300	14,300	14,300	11,430	11,430	11,430	9,140	7,140	7,140	5,710
Iron plate	....	....	....	11,430	11,430	8,600	10,000	10,000	8,000	4,300	4,300	3,430
Iron wire	....	....	....	....	....	....	17,140	....	....	11,430	....	....
Ordinary steel (soft)	...	....	....	....	...	....	14,300	14,300	11,430	10,000	10,000	8,000
Steel (hard)	....	....	....	....	....	....	21,430	21,430	17,140	14,300	14,300	11,430
Cast steel	....	....	....	....	....	....	28,600	28,600	23,000	20,000	20,000	15,710

Under temporary constructions we include scaffoldings, arch centreings, etc., as well as trusses for quiescent loading. Under constructions in general, such structures as are subjected to but little shock and whose load can be exactly determined.

From *A Manual of Rules, Tables and Data for Mechanical Engineers*, by D. K. Clark, London, 1877, page 625, we extract the following:

"The elastic strength of materials, cast iron excepted, is, in general terms, half of its ultimate or breaking strength. For cast iron, though there is no already defined elastic limit, the same measure may be adopted. If a working load of half the elastic strength, or one-fourth of the ultimate strength, be accepted, equal range for fluctuation within the elastic limit is provided. But, as bodies of the same material are not uniform in strength, it is necessary to observe a lower limit than a fourth where the material is exposed to great or to sudden variations of load."

CAST IRON.—Stoney recommends one-fourth of the ultimate tensile strength, for dead weights; one-sixth for cast-iron bridge girders; one-eighth for frame posts and machinery. In compression, free from flexure, according to Stoney, cast iron will bear 8 tons per square inch; for cast-iron arches, 3 tons per square inch; for cast-iron pillars, supporting dead loads, one-sixth of the ultimate strength; for pillars subjected to vibration from machinery, one-eighth; and for pillars subjected to shocks from heavy loaded wagons and the like, one-tenth, or even less where the strength is exerted in resistance to flexure.

WROUGHT IRON.—For bars and plates, 5 tons per square inch of net section is taken as the safe working tensile stress; for bar iron of extra quality 6 tons. In compression, where flexure is prevented, 4 tons is the safe limit; in small sizes, 3 tons. For wrought-iron columns, subjected to shocks, Stoney allows a sixth of the calculated breaking weight; with quiescent loads, one-fourth. For machinery, an eighth to a tenth is usually practised; and for steam boilers, a fourth to an eighth.

Mr. Roebling says, "Long experience has proved, beyond the shadow of a doubt, that good iron, exposed to a tensile strain not above one-fifth of the ultimate strength, and not subjected to strong vibration or torsion, may be depended upon for a thousand years.*

STEEL.—A committee of the British Association recommended a maximum working tensile stress of 9 tons per square inch. Mr. Stoney recommends, for mild steel, a fourth of the ultimate strength, or 8 tons per square inch. The limit for compression must be regulated very much by the nature of the steel, and whether it be unannealed or annealed. Probably a limit of 9 tons per square inch, the same as the limit for tension, would be the safe maximum for general purposes. In the absence of experience, Mr. Stoney recommends that, for steel pillars, an addition not exceeding 50 per cent. should be made to the safe load for wrought-iron pillars of the same dimensions.

TIMBER.—One-tenth of the ultimate stress is an accepted limit. Timber piles have, in some situations, borne permanently one-fifth of their ultimate compressive strength.

FOUNDATIONS.—According to Professor Rankine, the maximum pressure on foundations in firm earth is from 17 lbs. to 23 lbs. per square inch; and he says that, on rock, it should not exceed one-eighth of the crushing load.

MASON WORK.—Mr. Stoney says that the working load on rubble masonry, brick-work or concrete, rarely exceeds one-sixth of the crushing weight of the aggregate mass; and that this seems to be a safe limit. In an arch, the calculated pressure should not exceed one-twentieth of the crushing pressure of the stone.

ROPES.—For round ropes, the working load should not exceed a seventh of the ultimate strength, and for flat ropes, one-ninth.

Professor Rankine gives the following data as factors of strength:

	Dead Load.	Live Load.
Factors of safety for perfect materials and workmanship	2	4
For good ordinary materials and workmanship:		
Metals	3	6
Timber	4 to 5	8 to 10
Masonry	4	8

A *dead load* on a structure is one that is put on by imperceptible degrees, and that remains steady; such as the weight of the structure itself.

A *live load* is one that is put on suddenly, or is accompanied with vibration; such as a swift train travelling over a railway bridge, or a force exerted in a moving machine."

ALLOWABLE STRESS FOR WROUGHT-IRON BRIDGE MEMBERS.—Evidently, the allowable stress per square inch, even for the same material, must be varied according to the mode of action of the stress, whether quiescent, or intermittent, etc.

In bridge construction the quality of the iron used is carefully covered by specifications stating in detail the tests it must satisfy. We refer the student to the specifications at the end of this work for information as to current practice on this point.

For wrought iron, which shows an ultimate strength of 52,000 lbs. per square inch and stretches 18 per cent. in a distance of 8 inches, the allowable *tensile* stresses adopted by our leading railroads are about as follows: †

* *Engineering*, August, 1867.

† Specifications vary in these values. In any case the designer must be governed by the specifications adopted.

	β Lbs. per square inch.
On lateral bracing	15,000
On solid rolled beams, used as floor beams and stringers	10,000
On bottom chords and main diagonals	10,000
On counter rods and long verticals	8–9,000
On bottom flanges of riveted floor beams, net section	8,000
On bottom flanges of riveted longitudinal plate girders, over 20 feet long	8,000
On bottom flanges of riveted longitudinal plate girders, under 20 feet long	7,000
On floor beam hangers and other members liable to sudden loading	5–6,000

The allowable *compressive* stresses are as follows:

	β Lbs. per square inch.
On rolled beams used as floor beams and stringers	10,000
On riveted plate girders used as floor beams	6,000
On riveted longitudinal plate girders over 20 feet	6,000
On riveted longitudinal plate girders not over 20 feet	5,000

The formula for BEAMS will be found on page 268, *et seq.*

LONG PIECES IN COMPRESSION.—In general, when the length of a piece is more than ten times its least dimension, it is called a "long piece." When such a long piece is in compression, it is subject to flexure, and requires more material than would be necessary for the compressive strength alone. The formula in general use for finding the ultimate or crippling strength in pounds per square inch, is Gordon's, as deduced in the Appendix, page 276:

$$P = \frac{\mu}{1 + c\frac{l^2}{r^2}} \quad \left\{ \begin{array}{l} \textit{For all cross sections in general} \\ \qquad \textit{except hollow round.} \end{array} \right.$$

where l = length of strut in inches, and r = *least radius of gyration of the cross section* in inches. This formula is a modification of that known as "*Rankine's* formula," as deduced from Hodgkinson's experiments upon long struts, and is intended to apply in general to all forms of cross section *except hollow round.* For hollow round cross sections we put the *exterior diameter d in place of r.*

The value of μ depends upon the material, and the value of c upon the end conditions of the strut.*

			Flat ends.	*Both ends pinned.*	*One end flat, one end pinned.*
Thus for WROUGHT IRON,		$\mu = 40000$,	$c = \frac{1}{36000}$,	$\frac{2}{36000}$,	$\frac{1}{24000}$

For CAST IRON, the crippling strength may be taken at twice as much as for wrought iron.

For *hollow cylindrical struts* of WROUGHT IRON,	$r = d$,	$\mu = 40000$,	$c = \frac{1}{4500}$,	$\frac{1}{2250}$,	$\frac{1}{3000}$
of CAST IRON,	$r = d$,	$\mu = 80000$,	$c = \frac{1}{800}$,	$\frac{1}{400}$,	$\frac{3}{1600}$
For rectangular struts of WOOD,	$r = d$,	$\mu = 5600$,	$c = \frac{1}{550}$,	$\frac{1}{275}$,	$\frac{1.5}{550}$

* Other values will be found in general specifications, page 455. The values given here are recommended.

FACTOR OF SAFETY FOR LONG STRUTS.—The preceding formulæ will enable us to find the ultimate or "crippling strength" in pounds per square inch, for struts of any cross section and length, of wood or iron.

In practice, only a portion of the crippling strength is taken as the "*safe stress.*" This portion is called the "*factor of safety.*" For *quiescent loads* (buildings, etc.), this factor is taken at 4 for wrought iron and 6 for cast iron and wood struts.

For *variable loads* (bridges, etc.), a sliding factor of safety is used equal to $4+\frac{l}{20\,d}$, for all WROUGHT IRON struts of any cross section except hollow round, and $7+\frac{l}{20\,d}$ for all CAST IRON struts of any cross section except hollow round.

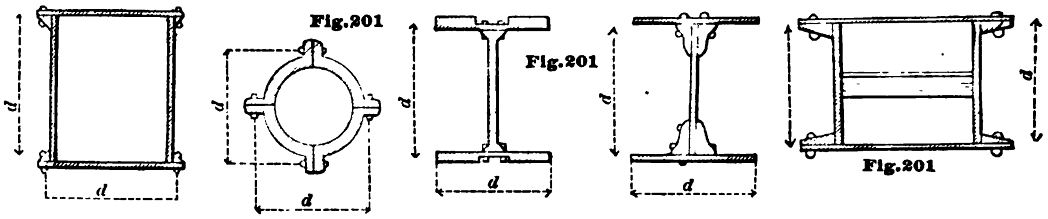
For *hollow round cross sections*, we have $3+\frac{l}{10\,d}$ for WROUGHT IRON, and $6+\frac{l}{10\,d}$ for CAST IRON.

For WOOD we have $6+\frac{l}{10\,d}$.

In all these expressions, l = length in inches and d = least dimension of the rectangle which encloses the given cross section.

SPECIAL FORMS OF CROSS SECTION.—The forms of wrought iron column in general use in American bridge construction are as follows:

A SQUARE COLUMN. B PHŒNIX COLUMN. C AMERICAN COLUMN. D COMMON COLUMN E COMMON CHORD SECTION.



For these forms, the following special formulæ are recommended by C. Shaler Smith for *wrought iron*; where d is the least dimension of the rectangle enclosing the cross section, and l is the length, both in inches.

	A.	*B.*	*C.*	*D.*	*E.*
Flat ends,	$\frac{38500}{1+\frac{l^2}{5820\,d^2}}$,	$\frac{42500}{1+\frac{l^2}{4500\,d^2}}$,	$\frac{36500}{1+\frac{l^2}{3750\,d^2}}$,	$\frac{36500}{1+\frac{l^2}{2700\,d^2}}$,	The pin being so placed that the moment of inertia is, as near as practicable, equal on both sides of same, use formula for square column.
One pin end,	$\frac{38500}{1+\frac{l^2}{3000\,d^2}}$,	$\frac{40000}{1+\frac{l^2}{2250\,d^2}}$,	$\frac{36500}{1+\frac{l^2}{2250\,d^2}}$,	$\frac{36500}{1+\frac{l^2}{1500\,d^2}}$,	
Two pin end,	$\frac{37500}{1+\frac{l^2}{1900\,d^2}}$,	$\frac{36600}{1+\frac{l^2}{1500\,d^2}}$,	$\frac{36500}{1+\frac{l^2}{1750\,d^2}}$,	$\frac{36500}{1+\frac{l^2}{1200\,d^2}}$,	

The safe working stress is found by dividing the "crippling stress," as determined by the above formula, by $4+\frac{l}{20\,d}$, where l is length in inches, and d is least dimension of enclosing rectangle.

To these we may add, for *open latticed channel struts*, consisting of two channel bars, latticed at sides, the distance between the channels being not less than their depth:

$$P = \underset{\textit{Flat ends.}}{\frac{38500}{1+\frac{l^2}{4880\,d^2}}}, \qquad \underset{\textit{One pin end.}}{\frac{38500}{1+\frac{l^2}{3260\,d^2}}}, \qquad \underset{\textit{Two pin end.}}{\frac{38500}{1+\frac{l^2}{2440\,d^2}}},$$

also, for single I bars,

$$P = \underset{\textit{Flat ends.}}{\frac{38500}{1+\frac{l^2}{1720\,w^2}}}, \qquad \underset{\textit{One pin end.}}{\frac{38500}{1+\frac{l^2}{1150\,w^2}}}, \qquad \underset{\textit{Two pin end.}}{\frac{38500}{1+\frac{l^2}{860\,w^2}}},$$

where w is the width of the flange at top and bottom.

OLD METHOD OF DIMENSIONING.—The method of dimensioning still customary with many engineers is as follows:

Let F be the cross-section of the piece, max. B the greatest stress which ever comes upon it, and β the allowable stress per square inch. Then

$$F = \frac{\text{max. } B}{\beta}.$$

Max. B is found by calculation of strains, β is taken in accordance with the preceding remarks, varying with the action of the stress, whether quiescent or intermittent.

If β is the nth part of the ultimate strength of the piece it is said to have a factor of safety of n, or n-fold security. This, however, is not really the case except for quiescent loading. For intermittent loading, especially accompanied by shocks, the factor of safety is really less.

The above method gives the cross section for pure tension or pure compression. If a piece is sometimes in compression and sometimes in tension, it is customary to take the area equal to the sum of the area which would be required for each stress separately, or $F = \frac{\text{max. tens.} + \text{max. comp.}}{\beta}$ provided the piece is so short that the compression does not cause flexure as in the case of long struts. In this latter case we have

$$F = \frac{\text{max. tens.}}{\beta} + \frac{\text{max. comp.}}{\text{column strength}},$$

where "column strength" is to be found from the formula already given for long struts, viz.:

$$\beta = \frac{1}{4 + \frac{1}{20}\frac{l}{d}}\left(\frac{\mu}{1 + c\frac{l^2}{r^2}}\right).$$

For combined flexure and tension or compression, we have the formula deduced in the Appendix, page 277:

$$F = \frac{M\nu}{\beta r^2} + \frac{S}{\beta},$$

where M is the maximum moment due to flexure, ν the distance to outer fibre from

centre, S the tensile or compressive stress, r the radius of gyration of the cross section, and β the allowable working stress for tension or compression as given on page 319, or as found from the formula for "column strength" according to whether flexure is to be feared or not.

NEW METHOD.—We have called the method just explained the "old method," not because it is in any sense antiquated, for it is still used by many if not most of our best engineers, but in order to distinguish it from a later method, based upon the experimental results of Wöhler and Spangenberg, and developed mainly by Weyrauch, Launhardt and Winkler. This method we shall therefore call the "new method." It affords a more satisfactory and rational means of allowing for the effect of oft-repeated stress than the "old method," where such allowance is made simply by an arbitrary change in the value of β, which resembles a *guess*, based upon experience, to be sure, but liable to vary considerably with different engineers. In 1858, Wöhler called attention to the necessity of experiments made with oft-repeated stress, in order to obtain a more rational basis for a method of dimensioning. In the years 1859–1870 he made many very careful experiments, under the appointment of the Prussian Minister of Public Works, upon tension, flexure and torsion. In these experiments, the specimens were rapidly strained and released within fixed limits, by means of an apparatus driven by a steam engine, and the number of deformations registered. In the years 1871–1873, these experiments were continued by Prof. Spangenberg at Berlin.

From these experiments the following conclusions were drawn:

1. Rupture may be caused not only by a stress equal to the so-called "breaking load" once and gradually applied, but by a very much smaller stress than this, if it is often enough repeated.

2. The injurious effect of repeated vibration or change of stress is least near the position of zero strain, and increases as the deformation departs from this position and approaches the allowable limits for quiescent load.

3. When the maximum stress is less than a given amount, depending upon the material, rupture will take place only after an infinite number of repetitions.

4. This given amount is less for alternating stress (alternately compression and tension) than for repeated stress of one kind only, and less for repeated stress of one kind only, than for quiescent stress.

LAUNHARDT'S FORMULA.—Let us now seek to determine the allowable stress β per unit of area, from the given working strength.

According to Wöhler's conclusions, the number of repetitions may be greater the less the loading, so that when the loading sinks to a certain amount, rupture will take place only after an infinite number of repetitions. If then, we denote the stress per unit of area, for which, after removal, the piece would always return to its originally unstrained condition, by η, then η will correspond to the unit load for an infinite number of repetitions. If, however, the unit stress is greater than η, then, for an infinite number of repetions, the piece will not continue to return to the unstrained condition, but will have eventually a certain set or residual strain. Such a stress, greater than η, which would therefore eventually cause rupture, if applied a sufficient number of times, we call the "working stress," and denote it by a, while the stress η we call the "primitive safe stress," "safe," because it admits of an infinite number of repetitions, and "primitive," because at each repetition the load is wholly removed and the piece returns to its primitive unstrained condition.

Now let the "working stress" a, as above defined, consist of two parts, a portion σ which always acts, and which we may call the "residual stress," and a portion δ which

may be repeated an infinite number of times without rupture, the piece after each repetition returning to the residual stress. We may then call δ the "safe stress" simply, while η is the "primitive safe stress." Then we have the relation $\delta = a - \sigma$, and hence,

$$a = \delta + \sigma \quad \ldots \quad (1)$$

We see then that the working stress a is some function of δ, or in general,

$$a = k\delta, \quad \ldots \quad (2)$$

where k denotes some unknown coefficient.

In order to determine k, we have for the limiting values of a, when the residual strain $\sigma = 0$, $a = \eta = \delta$; when the difference $\delta = 0$, $a = \sigma = \mu =$ ultimate strength for quiescent load.

Thus ultimate strength and primitive safe strength are special cases of working strength.

Since now, for $\delta = 0$, $a = \mu$, we see from (2) that for this limit, $k = \infty$. Since also for $\delta = \eta$, we have $a = \delta$, we see from (2) that for this limit $k = 1$.

These conditions satisfy perfectly the expression which Launhardt gives, viz.,

$$k = \frac{\mu - \eta}{\mu - a}.$$

This expression we have still to test by experiment for intermediate values, of course before we can accept it finally as correct. Assuming its correctness at present, we have from (2):

$$a = \frac{\mu - \eta}{\mu - a}\,\delta.$$

Or putting for δ its value from (1):

$$a = \frac{\mu - \eta}{\mu - a}(a - \sigma).$$

Reducing:

$$a = \eta\left(1 + \frac{\mu - \eta}{\eta}\frac{\sigma}{a}\right) \quad \ldots \quad (3)$$

If we denote by min. B the least and by max. B the greatest stress on the piece, then we have evidently

$$\frac{\sigma}{a} = \frac{\text{min. } B}{\text{max. } B};$$

and hence the working stress

$$a = \eta\left[1 + \frac{\mu - \eta}{\eta}\frac{\text{min. } B}{\text{max. } B}\right] \quad \ldots \quad (4)$$

This is Launhardt's formula. We see from equation (1) that it manifestly holds good only for the case where min. B and max. B have the same sign, that is, only for repeated tension or repeated compression. Also in the latter case it is understood that there is no tendency to flexure.

The value of η for compression has not yet been satisfactorily determined. We therefore take the values of μ and η the same for compression as for tension, a practice which seems justified by certain observations, and which, as regards μ, has always been the custom heretofore.

We have yet to show that Launhardt's expression for the coefficient k holds good for intermediate values of η and μ. For this purpose, we solve (3) with reference to a, and obtain

$$a = \frac{\eta}{2} + \sqrt{\left(\frac{\eta}{2}\right)^2 + \sigma(\mu - \eta)},$$

where we must have + before the radical, because a must be positive and greater than η. According to the method of loading and the kind of material, μ and η vary, as also a, for any given σ. Hence, in order that an experiment may possess any value, the results must all be obtained in the same manner and with the same material. The results best suited for comparison are beyond doubt those obtained by Wöhler with Krupp cast-steel, and it may be said for Launhardt's formula that it agrees excellently well with them. Thus Wöhler found $\mu = 1{,}100$ centners, $\eta = 500$ centners, hence

$$a = 250 + \sqrt{62{,}500 + 600\,\sigma}.$$

Below we give the comparison of the formula with the experimental results of Wöhler:

For σ =	0	250	400	600	1,100
a by experiment =	500	700	800	900	1,100
a by formula =	500	711	800	900	1,100.

According to previous views, the single quiescent stress of 1,100 is that necessary for rupture, but we see from the above that *all stresses down to* 500 *may cause rupture, if repeated often enough.*

WEYRAUCH'S FORMULA.—It frequently happens that a piece may be subjected to alternate compression and tension. Since the formula of Launhardt no longer holds good in such case, we must deduce one which does. Such a formula is Weyrauch's. Wöhler has shown that the working strength is much less than when the repeated stress is always of one kind. He has also investigated the case in which the opposite stresses are equal. The strength in this case we call the "*vibration safe strength*," and denote it by η'. Thus if the stress in one direction is zero, η' becomes η, the primitive safe strength. Here then are two limits given.

Let now, a piece of one square unit cross section be subjected to alternate compression and tension. Then for any value a for the greater of these two stresses, there will be a corresponding value a' for the less, so that for the greatest number of alternations which can ever occur between $\pm\, a$ and $\mp\, a'$, the material remains uninjured. The difference of the stresses is then

$$\left.\begin{aligned} \delta &= a + a' \\ \text{or} \qquad a &= \delta - a' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5)$$

where simply numerical values are inserted without regard to sign or character of stress.

Now, according to Wohler's law, a decreases as δ increases; and, in general, a is a function of δ. We can, therefore, put

$$a = k\delta \quad \ldots\ldots\ldots\ldots \quad (6)$$

But from (5) we have

$$\text{when } a' = 0, \qquad a = \eta = \delta,$$

$$\text{when } a = a', \qquad a = \eta' = \tfrac{1}{2}\delta.$$

We have also, from (6),

$$\text{when } a = \eta, \qquad k = 1,$$
$$\text{when } a = \eta', \qquad k = \tfrac{1}{2}.$$

These conditions are satisfied by

$$k = \frac{\eta - \eta'}{2\,\eta - \eta' - a},$$

hence

$$a = \frac{\eta - \eta'}{2\,\eta - \eta' - a}\,\delta,$$

or since

$$a = \frac{\eta - \eta'}{2\,\eta - \eta' - a}\,(a + a'),$$

we have

$$a = \eta\left[1 - \frac{\eta - \eta'}{\eta}\,\frac{a'}{a}\right].$$

If now, for any piece, max. B is the greatest stress whether of tension or compression, and max. B' the greatest stress of the opposite kind (less than max. B), we have

$$\frac{a'}{a} = \frac{\text{max. } B'}{\text{max. } B}.$$

and hence

$$a = \eta\left[1 - \frac{\eta - \eta'}{\eta}\,\frac{\text{max. } B'}{\text{max. } B}\right] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

This is Weyrauch's formula. All quantities are simply to be inserted numerically without reference to their signs before insertion.

The primitive safe strength is η, the vibration safe strength η', and a the working strength in the direction of the greatest of the two stresses, max. B. Since η is not yet known for compression, we may, as in Launhardt's formula, for the present, use its value for tension, which is rather too small if any thing.

In many constructions, the alternations occur between the limits a and a' for primitive stress of zero. In others, we have a previous stress of σ, in most cases due to the dead weight. However we may conceive it to be, the action of each complete alternation must be similar, nor can it be changed by the long continued action of σ, which lies far within the elastic limits.

If then generally, we denote by φ the ratio of the two limiting stresses of a piece, the less to the greater, without reference tó sign, our formulæ become:

For repeated stress of one kind only

$$a = \eta\left[1 + \frac{\mu - \eta}{\eta}\,\varphi\right]$$

For repeated stresses of alternate kinds

$$a = \eta\left[1 - \frac{\eta - \eta'}{\eta}\,\varphi\right]$$

LIST OF LITERATURE.

We give for the benefit of the student a list of some of the more important works treating of the subject of the preceding Chapter :

STONEY, BINDON B.—"Theory of Strains in Girders and Similar Structures." London : Longmans, Green & Co., 1869.

WOOD, DE VOLSON.—" Treatise on the Resistance of Materials." New York : John Wiley & Sons, 1871.

WEYRAUCH.—" Strength and Determination of the Dimensions of Structures of Iron and Steel." Translated by A. J. Du Bois. New York : John Wiley & Sons, 1877.

OTT, KARL VON.—" Vorträge über Baumechanik." Prag : 1880.

SPANGENBERG.—" Ueber das Verhalten der metalle bei wiederholten Anstrengungen." Erbkamms Ztschr. für Bauwesen, 1874 and 1875. Also separate reprint by *Ernst* and *Korn*. Berlin, 1875.

WÖHLER.—Ztschr. für Bauwesen, 1860, 1863, 1866, 1870. Also reprint by *Ernst* and *Korn*, under the title, "Die Festigkeits Versuche mit Eisen und Stahl." Berlin, 1870.

LAUNHARDT—" Die Inanspruchnahme des Eisens." Ztschr. d. Hannöv, Arch. u. Ing. Vereins, 1873.

WINKLER.—" Wahl der Zulässigen Inanspruchnahme der Eisen Constructionen." Wien, 1877.

NEW METHOD FOR THE DETERMINATION OF THE ALLOWABLE UNIT STRESS.—As soon as we have determined the maximum strains in any piece by statical calculation, as detailed in Part I., we can find from the preceding equations, as soon as the proper values of μ, η, and η' are known, that stress a per unit of area, which will cause rupture only after an infinite number of repetitions. These values of μ, η, and η' for various materials, will presently be given in the Recapitulation which follows.

It must, of course, be understood that thus far flexure has not been considered, that is, all struts are supposed very short, and the equations above apply therefore to pure compression or tension. No account has also been taken of those prejudicial influences which do not admit of precise estimation, such as sudden shocks, impact of moving loads, lack of homogeneity of materials, action of the atmosphere, rust, changes of temperature, etc. Of these, impact may be included by properly modifying the values of $\frac{\mu - \eta}{\eta}$ and $\frac{\eta - \eta'}{\eta}$ in the above formulæ, and the others may be allowed for by means of a factor of safety.

If, then, min. B is the constant steady tension or compression, and max. B the greatest total stress, and n the factor of safety, we have for the allowable stress β, per unit of cross section, for ***repeated stress of one kind only***,

$$\beta = \frac{\eta}{n}\left[1 + \frac{\mu - \eta}{\eta}\,\frac{\text{min. } B}{\text{max. } B}\right], \quad \ldots \ldots \ldots \ldots \quad \text{(I.)}$$

and for ***alternating stress of opposite kinds***,

$$\beta = \frac{\eta}{n}\left[1 - \frac{\eta - \eta'}{\eta}\,\frac{\text{max. } B'}{\text{max. } B}\right], \quad \ldots \ldots \ldots \quad \text{(II.)}$$

where max. B is the greatest stress, whether of tension or compression, and max. B' the greatest stress of opposite kind, *less* than max. B. That is, the greatest of the two maximum stresses is always to be put *in the denominator*.

The difference, then, between the old and new methods, is, that while in the former a portion of the ultimate strength is taken, in the latter, a portion of the "working strength" is taken as the allowable stress. This portion is constant for the new method,

and the allowable stress varies according to the action of the repeated loading, while to accommodate the old method to such action, the factor of safety, or the allowable unit stress, is rather arbitrarily chosen, and varies greatly in individual practice.

NEW METHOD—APPLICATION TO LONG STRUTS.—The method just given applies to pieces in pure compression or tension, but does not take into account the extra material required for stiffening, in the case of long struts. This may easily be done, in a method similar to that adopted in the old method. Thus we have for

repeated compression, taking flexure into account,

$$\text{allowable stress} = \frac{\beta}{1 + c\frac{l^2}{r^2}} = \frac{\eta}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 + \frac{\mu - \eta}{\eta} \cdot \frac{\text{min. } B}{\text{max. } B}\right], \quad . \quad . \quad \text{(III.)}$$

and for

alternating stress, taking flexure into account,

$$\text{allowable stress} = \frac{\beta}{1 + c\frac{l^2}{r^2}} = \frac{\eta}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 - \frac{\eta - \eta'}{\eta} \frac{\text{max. } B'}{\text{max. } B}\right], \quad . \quad . \quad \text{(IV.)}$$

where max. B is the *greatest* of the two stresses.

In these equations, c has the same value as in the old method, l is the length in inches, and r the least radius of gyration of cross section in inches.

RECAPITULATION—OLD AND NEW METHODS OF DIMENSIONING.—VALUES OF $\frac{\eta}{n}$, $\frac{\mu - \eta}{\eta}$, AND $\frac{\eta - \eta'}{\eta}$.

OLD METHOD.

Let F be the cross section of the piece, max. B the greatest stress which can ever come upon it, and β the allowable stress per square inch. Then for simple tension or compression, when flexure is not to be apprehended,

$$F = \frac{\text{max. } B}{\beta}.$$

The customary values of β for the various pieces we have to deal with, for simple tension and compression (without flexure), are given on page 319. These values are different for different pieces, in order to allow for the effect of repetition, shock, etc.

If the piece is subjected to *alternating stress, i. e.*, sometimes tension and sometimes compression, then if flexure is not to be guarded against, we have

$$F = \frac{\text{max. tension} + \text{max. compression}}{\beta}.$$

The values of β being taken from page 319.

If the piece is so long that flexure has to be guarded against, that is, in general when $\frac{l}{r}$ is greater than 30, or $\frac{l}{d}$ is greater than 10, we have

$$F = \frac{\text{max. compression}}{\beta_1} \quad \text{or} \quad \frac{\text{max. compression}}{\beta_1} + \frac{\text{max. tension}}{\beta},$$

where β is as before, given on page 319, and β_1 is given by Gordon's formula,

$$\beta_1 = \frac{1}{4 + \frac{1}{20}\frac{l}{d}}\left[\frac{\mu}{1 + c\frac{l^2}{r^2}}\right],$$

l being the length, d the least dimension, and r the least radius of gyration of cross section in inches, and μ being taken as given on page 319.

For a piece in longitudinal *tension* and at the same time acting like a beam to support a transverse load, we have (Appendix, page 277),

$$F = \frac{M\nu}{\beta r^2} + \frac{S}{\beta},$$

where β is given on page 319, S is the longitudinal tension in lbs., and M the greatest moment in inch lbs. due to the transverse load, ν is the distance from the neutral axis to the outer fibre, and r the radius of gyration with reference to the neutral axis, in inches.

For a piece in longitudinal *compression* and at the same time acting like a beam to support a transverse load,

$$F = \frac{M\nu}{\beta_1 r^2} + \frac{S}{\beta_1},$$

where S is the longitudinal compression, and β_1 is given by Gordon's formula.

NEW METHOD.

By the *new method*, we have in *all cases*,

$$F = \frac{\text{max. } B}{\beta} \quad \text{or} \quad \frac{\text{max. } B}{\beta_1},$$

but instead of the values of β and β_1 used in the old method, we have,

For repeated stress of one kind, without flexure,

$$\beta = \frac{\eta}{n}\left[1 + \frac{\mu - \eta}{\eta}\,\frac{\text{min. } B}{\text{max. } B}\right].$$

For repeated stress of one kind, with flexure,

$$\beta_1 = \frac{\eta}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 + \frac{\mu - \eta}{\eta}\,\frac{\text{min. } B}{\text{max. } B}\right].$$

The values of $\frac{\eta}{n}$ and $\frac{\mu - \eta}{\eta}$ will be given presently for different materials. Min. B is the steady stress, if any, acting all the time upon the piece; max. B, the greatest total stress (*including* min. B and also any repeated stress), which acts upon the piece.

For alternating stress, without flexure,

$$\beta = \frac{\eta}{n}\left[1 - \frac{\eta - \eta'}{\eta}\,\frac{\text{max. } B'}{\text{max. } B}\right].$$

For alternating stress, with flexure,

$$\beta_1 = \frac{\eta}{n\left(1 + c\frac{l^2}{r^2}\right)}\left[1 - \frac{\eta - \eta'}{\eta}\,\frac{\text{max. } B'}{\text{max. } B}\right],$$

where max. B is always the *greatest* of the two opposite maximum stresses.

For a piece subjected to longitudinal tension and at the same time acting as a beam to sustain a load, we have as before,

$$F = \frac{Mv}{\beta r^2} + \frac{S}{\beta},$$

or if subjected to longitudinal compression,

$$F = \frac{Mv}{\beta_1 r^2} + \frac{S}{\beta_1},$$

where β and β_1 are as just given above.

The values of c, μ, and $\frac{1}{1 + c\frac{l^2}{r^2}}$ in all these formulæ have been given on pages 319.

It is unnecessary to repeat them here. Finally, for the values of $\frac{\eta}{n}$, $\frac{\mu - \eta}{\eta}$, and $\frac{\eta - \eta'}{\eta}$ to be used in the *new* method, we have,

	$\frac{\eta}{n}$	$\frac{\mu-\eta}{\eta}$	$\frac{\eta-\eta'}{\eta}$
Wood	400	2	$\frac{1}{2}$
* Wrought iron, double rolled (links or rods), in tension.	7500	1	$\frac{1}{2}$
Wrought iron plates in tension	7000	1	$\frac{1}{2}$
Wrought iron in compression	6500	1	$\frac{1}{2}$
Cast iron	10000	$\frac{4}{3}$	$\frac{2}{5}$
Ordinary steel (soft)	9530	$\frac{3}{2}$	$\frac{2}{5}$
Soft cast steel	17870	1	$\frac{7}{15}$
Iron wire rope	11400	$\frac{3}{2}$	
Steel wire rope	26700	1	

For *shear*, for iron and steel, we may take $\frac{4}{5}\beta$ as the allowable stress, where β is to be found as above.

* The values for wrought iron are those adopted by Joseph M. Wilson, C. E., in his specifications. "Specifications for Strength of Iron Bridges," *Trans. Am. Soc. Civil Engineers*, June, 1886, also page 455.

Prof. Merriman has deduced ("*Mechanics of Materials*," Wiley & Sons, 1885) the single formula, both for repeated stress of one kind and for alternating stress also,

$$\beta = \frac{\eta}{n}\left[1 + \frac{\mu - \eta'}{2\eta}R + \frac{\mu + \eta' - 2\eta}{2\eta}R^2\right],$$

where R stands for the ratio of the least limiting stress to the greatest limiting stress, or what we have called $\frac{\text{min. } B}{\text{max. } B}$ for repeated stress of one kind, and $\frac{\text{max. } B'}{\text{max. } B}$ for alternating stress, only *regard is paid to the character or sign* of the stress. Thus, if both limiting stresses are tension or both compression, R is *positive;* if one is tension and the other compression, R is *negative*. With this understanding, the single formula of Prof. Merriman replaces Launhardt's and Weyrauch's.

To apply it to long struts we have simply to put $\dfrac{\eta}{n\left(1 + c\dfrac{l^2}{r^2}\right)}$ in place of $\dfrac{\eta}{n}$.

The values of the coefficients are as follows:

	$\frac{\eta}{n}$	$\frac{\mu - \eta'}{2\eta}$	$\frac{\mu + \eta' - 2\eta}{2}$
Wood	400	$\frac{5}{4}$	$\frac{3}{4}$
Wrought iron, double rolled (links or rods), in tension	7500	$\frac{3}{4}$	$\frac{1}{4}$
Wrought iron plates in tension	7000	$\frac{3}{4}$	$\frac{1}{4}$
Wrought iron in compression	6500	$\frac{3}{4}$	$\frac{1}{4}$
Cast iron	10000	$1\frac{1}{3}$	$\frac{7}{15}$
Ordinary steel (soft)	9530	$\frac{19}{20}$	$\frac{11}{20}$
Soft cast steel	17870	$\frac{11}{15}$	$\frac{4}{15}$
Iron wire rope	11400	$\frac{3}{4}$	$\frac{3}{4}$
Steel wire rope	26700	$\frac{1}{2}$	$\frac{1}{2}$

THE STRAIGHT-LINE FORMULA.—In the *Transactions of the American Society of Civil Engineers* for July, 1886, Thomas H. Johnson, C. E., has given formulæ for long struts which seem more rational and to accord better with the results of experiment than any others hitherto proposed. He found that above a certain ratio of $\frac{l}{r}$, given by $\frac{l}{r} = \sqrt{\frac{3Ea}{\mu}}$, the formula of Euler holds good, viz.: $P = \frac{aEr^2}{l^2}$, where a is π^2 for round ends, $\frac{5}{3}\pi^2$ for hinged ends, $\frac{5}{2}\pi^2$ for flat ends. Below this value of $\frac{l}{r}$ we have $P = \mu - c\frac{l}{r}$, or a *straight line* where $c = \sqrt{\frac{4}{aE}\left(\frac{\mu}{3}\right)^3}$, P being the crippling load in lbs. per sq. inch, μ the ultimate crushing strength in lbs. per sq. inch, for very short specimens, l the length in inches, and r = radius of gyration of section in inches.

As the limiting value of $\frac{l}{r}$ is very large, in nearly all practical cases the straight-line formula applies.

The accord of these formulæ with all the results of experiment is most satisfactory, and it would seem that both for accuracy and simplicity they must soon replace the modified Gordon formulæ now in use, as also the so-called "new method," based upon Launhardt and Weyrauch.

Since P is the crippling load in lbs. per sq. inch, we must use a factor of safety. For quiescent loads, therefore, we take 4 for wrought iron and 6 for cast iron and wood. For variable loads, the sliding factor $4 + \frac{l}{20d}$, where d is the least dimension in inches of the bounding rectangle of the section, and l is the length in inches.

Mr. Johnson has given the following values for different materials:

JOHNSON'S STRAIGHT-LINE FORMULA FOR VARIOUS MATERIALS AND END BEARINGS.

$$\text{Factor of safety,} \begin{cases} \text{quiescent loading} & \begin{cases} \text{wrought iron} = 4 \\ \text{cast iron} = 6 \end{cases} \\ \text{variable loading } 4 + \dfrac{l}{20d} \end{cases}$$

l = length in inches, d = least dimension in inches, of cross-section.
P = crippling load in lbs. per sq. inch.
μ = ultimate crushing strength in lbs. per sq. inch for very short specimens.
r = radius of gyration of cross-section in inches.

For round ends $a = \pi^2$, for hinged ends $a = \frac{5}{3}\pi^2$, for flat ends $a = \frac{5}{2}\pi^2$.

MATERIAL.	E in lbs.	μ lbs.	END BEARING.	$P = \mu - \frac{l}{r}\sqrt{\frac{4}{aE}\left(\frac{\mu}{3}\right)^3}$ when $\frac{l}{r} < \sqrt{\frac{3Ea}{\mu}}$	$\frac{l}{r} = \sqrt{\frac{3Ea}{\mu}}$	$P = \frac{Ear^2}{l^2}$ when $\frac{l}{r} > \sqrt{\frac{3Ea}{\mu}}$
Wrought Iron.	27000000	42000	Flat,	$P = 42000 - 128\frac{l}{r}$	218.1	$P = 656090000\frac{r^2}{l^2}$
			Hinged,	$P = 42000 - 157\frac{l}{r}$	178.1	$P = 444150000\frac{r^2}{l^2}$
			Round,	$P = 42000 - 203\frac{l}{r}$	138.1	$P = 266490000\frac{r^2}{l^2}$
Mild Steel (Carbon = 0.12).	27000000	52500	Flat,	$P = 52500 - 179\frac{l}{r}$	195.1	$P = 666090000\frac{r^2}{l^2}$
			Hinged,	$P = 52500 - 220\frac{l}{r}$	159.3	$P = 444150000\frac{r^2}{l^2}$
			Round,	$P = 52500 - 284\frac{l}{r}$	123.3	$P = 266490000\frac{r^2}{l^2}$
Hard Steel (Carbon 0.36).	27000000	80000	Flat,	$P = 80000 - 337\frac{l}{r}$	158	$P = 666090000\frac{r^2}{l^2}$
			Hinged,	$P = 80000 - 414\frac{l}{r}$	129	$P = 444150000\frac{r^2}{l^2}$
			Round,	$P = 80000 - 534\frac{l}{r}$	99.9	$P = 266490000\frac{r^2}{l^2}$
Cast Iron.	16000000	80000	Flat,	$P = 80000 - 438\frac{l}{r}$	121.6	$P = 394720000\frac{r^2}{l^2}$
			Hinged,	$P = 80000 - 537\frac{l}{r}$	99.3	$P = 263200000\frac{r^2}{l^2}$
			Round,	$P = 80000 - 693\frac{l}{r}$	77	$P = 157920000\frac{r^2}{l^2}$
Oak.	1200000	5400	Flat,	$P = 5400 - 28\frac{l}{r}$	128.1	$P = 29604000\frac{r^2}{l^2}$

Theodore Cooper, C. E., has adopted the straight-line formula in his specifications,* but varies somewhat the constants employed.

He makes the limit of length of any compression member 45 times its least width. Within this limit, for *wrought iron* he gives the following formulæ, for *allowable* compression per square inch of cross-section.

For *chords*,

$$\beta = 8000 - 30\frac{l}{r} \text{ for live load strains.}$$

$$\beta = 16000 - 60\frac{l}{r} \text{ for dead load strains.}$$

For all posts,

$$\beta = 7000 - 40\frac{l}{r} \text{ for live load strains.}$$

$$\beta = 14000 - 80\frac{l}{r} \text{ for dead load strains.}$$

$$\beta = 10500 - 60\frac{l}{r} \text{ for wind strains.}$$

For lateral struts,

$$\beta = 9000 - 50\frac{l}{r} \text{ for assumed initial strain.}$$

TABLES FOR LONG STRUTS.—To lessen the labor of computation, we shall now give a number of tables, from which we can find for any ratio of $\frac{l}{r}$ or $\frac{l}{d}$ the crippling stress, in accordance with the formulæ already given. As the factor of safety is given by itself, and the crippling strength by itself, the working stress for any desired factor of safety can be obtained if desired. The tables for "Square," "Phœnix," "American," and "Common" columns (page 336) were given by C. Shaler Smith in *Trans. Am. Soc. of Civil Engrs.*, for October, 1880.

The values of $\frac{1}{1 + c\frac{l^2}{r^2}}$ or $\frac{1}{1 + c\frac{l^2}{d^2}}$, to be used in the formulæ for the *new method*, may be easily taken from these tables, by dividing the crippling strength given in the table by the value of μ or ultimate strength taken in any case.

The straight-line formulæ of Johnson or Cooper require no Tables, are easily applied, and are coming into general use. We have thus three methods for finding the crippling strength for long struts—the old method, by means of the following Tables, the new method, also by use of the following Tables and the formulæ already given, and the "straight-line formulæ," which will probably eventually replace the other two.

* *General Specifications for Iron and Steel Railroad Bridges and Viaducts.* 1888, Engineering News Publishing Company, also page 455.

TABLE I.

STRENGTH OF WROUGHT IRON STRUTS OF ANY CROSS SECTION—EXCEPT HOLLOW ROUND. (*For Cast Iron, take twice the tabular values.*)

l = length in inches.
r = least radius of gyration in inches.
d = least dimension of rectangle enclosing the given cross section, in inches.

Factor of safety $\begin{cases} \text{for wrought iron} = 4 + \dfrac{l}{20d} \\ \text{for cast iron} = 7 + \dfrac{l}{20d} \end{cases}$ Intermittent Loading; $\begin{cases} \text{for wrought iron} = 4 \\ \text{for cast iron} = 6 \end{cases}$ Quiescent Loading.

STRUT. *Flat ends.*	STRUT. *Pin and flat ends.*	STRUT. *Pin ends.*
$\dfrac{40000}{1 + \dfrac{l^2}{36000r^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{24000r^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{18000r^2}}$

$\frac{l}{r}$.	*Crippling Strength in tons (2000 lbs.) per square inch.*			$\frac{l}{r}$.	*Crippling Strength in tons (2000 lbs.) per square inch.*		
	Flat ends.	Pin and flat.	Pin ends.		Flat ends.	Pin and flat.	Pin ends.
30	19.510	19.275	19.050	90	16.325	14.955	13.800
32	19.455	19.180	18.925	92	16.195	14.785	13.605
34	19.380	19.080	18.795	94	16.060	14.620	13.415
36	19.305	18.975	18.655	96	15.925	14.450	13.230
38	19.230	18.865	18.515	98	15.790	14.285	13.040
40	19.150	18.750	18.365	100	15.650	14.120	12.855
42	19.065	18.630	18.215	102	15.515	13.950	12.675
44	18.980	18.510	18.060	104	15.380	13.790	12.495
46	18.890	18.380	17.895	106	15.245	13.625	12.315
48	18.795	18.250	17.730	108	15.105	13.460	12.135
50	18.700	18.115	17.560	110	14.970	13.300	11.960
52	18.600	17.975	17.390	112	14.835	13.135	11.785
54	18.500	17.835	17.210	114	14 695	12.975	11.615
56	18.400	17.690	17.035	116	14.560	12.815	11.445
58	18.290	17 540	16.850	118	14.425	12.655	11.275
60	18.180	17.390	16.665	120	14.285	12.500	11.110
62	18.070	17.240	16.480	122	14.150	12.355	10 950
64	17.955	17.035	16 300	124	14.015	12.190	10.785
66	17.840	16.930	16.110	126	13.880	12.035	10.625
68	17.725	16.770	15.930	128	13.745	11.885	10.470
70	17.605	16.610	15.725	130	13.610	11.735	10.315
72	17.485	16.450	15.535	132	13.475	11.590	10.165
74	17.360	16.285	15.340	134	13.345	11.440	10.010
76	17.255	16.120	15.150	136	13.210	11.295	9.865
78	17.110	15.955	14 955	138	13.080	11.150	9.720
80	16 980	15.790	14.760	140	12.950	11.010	9.575
82	16.855	15.620	14.565	142	12.820	10.870	9.435
84	16.720	15.455	14.375	144	12.690	10.730	9.295
86	16.590	15.290	14.180	146	12.560	10.590	9.155
88	16.460	15.120	13.985	148	12.435	10.455	9.020

NEW METHOD.—*For repeated compression:* For wrought iron, $\beta = \dfrac{6500}{1 + c\dfrac{l^2}{r^2}}\left[1 + \dfrac{\text{min. } B}{\text{max. } B}\right]$; fo· cast iron $\beta = \dfrac{10000}{1 + c\dfrac{l^2}{r^2}}\left[1 + \dfrac{4}{3}\dfrac{\text{min. } B}{\text{max. } B}\right]$. The crippling strength in pounds, divided by 40000, gives the value of $\dfrac{1}{1 + c\dfrac{l^2}{r^2}}$ for wrought iron, and divided by 80000 for cast iron.

TABLE II.

Strength of Hollow Cylindrical Cast and Wrought Iron Struts.

l = length in inches.
d = least diameter in inches.

Factor of safety $\begin{cases} \text{for wrought iron} = 3 + \dfrac{l}{10d} \\ \text{for cast iron} \quad = 6 + \dfrac{l}{10d} \end{cases}$ Intermittent Loading; $\begin{matrix} \text{for wrought iron} = 4 \\ \text{for cast iron} \quad = 6 \end{matrix}\Bigg\}$ Quiescent Loading

Cast Iron.			Wrought Iron.		
Flat ends.	*Pin and flat.*	*Pin ends.*	*Flat ends.*	*Pin and flat.*	*Pin ends.*
$\dfrac{80000}{1 + \dfrac{l^2}{800d^2}}$	$\dfrac{80000}{1 + \dfrac{3l^2}{1600d^2}}$	$\dfrac{80000}{1 + \dfrac{l^2}{400d^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{4500d^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{3000d^2}}$	$\dfrac{40000}{1 + \dfrac{l^2}{2250d^2}}$

$\frac{l}{d}$	*Crippling Strength in tons (2000 lbs.) per sq. in.* Flat ends.	Pin and flat.	Pin ends.	$6 + \frac{l}{10d}$	$\frac{l}{d}$	*Crippling Strength in tons (2000 lbs.) per sq. in.* Flat ends.	Pin and flat.	Pin ends.	$3 + \frac{l}{10d}$
10	35.555	33.685	32.000	7.	10	19.565	19.354	19.150	4
11	34.745	32.603	30.710	7.1	11	19.476	19.225	18.980	4.1
12	33.895	31.496	29 412	7.2	12	19.380	19.084	18.797	4.2
13	33.024	30.375	28.120	7.3	13	19.276	18.933	18.603	4.3
14	32.128	29 250	26.846	7.4	14	19.165	18.774	18.397	4.4
15	31.220	28.132	25.600	7.5	15	19.048	18.605	18.182	4.5
16	30.303	27 027	24.390	7.6	16	18.924	18.427	17.957	4.6
17	29.385	25.943	23.223	7.7	17	18.793	18.270	17.724	4.7
18	28.470	24.883	22.099	7 8	18	18.657	18.051	17.483	4.8
19	27.563	23.854	21.025	7.9	19	18.514	17.852	17.235	4.9
20	26.667	22.858	20.000	8.0	20	18.367	17.647	16.981	5.0
21	25.786	21.895	19.025	8.1	21	18.215	17.436	16.723	5.1
22	24.922	20.970	18.100	8.2	22	18.058	17.222	16.459	5.2
23	24.078	20.082	17.223	8.3	23	17.896	17.000	16.193	5.3
24	23.256	19.230	16.393	8.4	24	17.731	16.778	15.924	5.4
25	22.456	18 418	15.625	8.5	25	17.561	16 552	15.652	5.5
26	21.680	17.640	14.870	8.6	26	17.388	16.322	15.379	5.6
27	20.928	16.900	14.171	8 7	27	17.212	16.090	15.106	5.7
28	20.202	16 195	13.513	8.8	28	17.032	15.856	14.832	5 8
29	19.500	15.523	12.893	8.9	29	16.851	15.621	14.558	5.9
30	18.823	14.884	12.306	9.0	30	16.666	15.385	14.286	6.0
31	18.172	14.276	11.756	9.1	31	16.481	15.148	14.014	6.1
32	17.544	13 698	11.236	9.2	32	16.292	14.911	13.745	6.2
33	16.940	13.149	10.745	9.3	33	16.103	14.674	13.477	6.3
34	16.360	12.628	10.283	9.4	34	15.912	14.437	13.212	6.4
35	15.803	12.133	9.846	9.5	35	15.720	14.202	12.950	6.5
36	15.267	11.662	9.434	9.6	36	15.528	13.966	12 690	6.6
37	14.754	11.215	9.044	9.7	37	15.335	13.733	12.435	6.7
38	14.260	10.789	8.677	9.8	38	15 141	13.501	12.182	6.8
39	13.787	10.385	8.329	9.9	39	14.947	13.271	11.933	6.9
40	13.333	10.000	8.000	10.0	40	14.754	13.043	11.688	7.0

New Method.—*For repeated compression:* For wrought iron $\beta = \dfrac{6500}{1 + c\dfrac{l^2}{d^2}}\left[1 + \dfrac{\text{min. } B}{\text{max. } B}\right]$; for cast iron $\beta = \dfrac{10000}{1 + c\dfrac{l^2}{d^2}}\left[1 + \dfrac{4}{3}\dfrac{\text{min. } B}{\text{max. } B}\right]$. The crippling strength in pounds, divided by 40000 or 80000 gives the value of $\dfrac{1}{1 + c\dfrac{l^2}{d^2}}$ for wrought or cast iron.

TABLE III.

STRENGTH OF RECTANGULAR TIMBER STRUTS.

l = length in inches.
d = least side in inches.

Flat ends.	*Pin and flat.*	*Pin ends.*
$\dfrac{5600}{1+\dfrac{l^2}{550d^2}}$	$\dfrac{5600}{1+\dfrac{1.5\,l^2}{550d^2}}$	$\dfrac{5600}{1+\dfrac{l^2}{275d^2}}$

Factor of safety $6+\dfrac{l}{10d}$ for intermittent loading and 6 for quiescent loading.

$\dfrac{l}{d}$	*Crippling Strength in lbs. per square inch.*			$6+\dfrac{l}{10d}$	$\dfrac{l}{d}$	*Crippling Strength in lbs. per square inch.*			$6+\dfrac{l}{10d}$
	Flat.	Pin and flat.	Pin.			Flat.	Pin and flat.	Pin.	
12	4440	4020	3680	7.2	30	2120	1620	1310	9
13.2	4250	3800	3430	7.32	31.2	2020	1530	1230	9.12
14.4	4070	3580	3190	7.44	32.4	1930	1450	1160	9.24
15.6	3880	3370	2970	7.56	33.6	1830	1370	1100	9.36
16.8	3700	3160	2760	7.68	34.8	1750	1300	1040	9.48
18	3520	2970	2570	7.8	36	1670	1230	980	9.6
19.2	3350	2790	2390	7.92	37.2	1590	1170	930	9.72
20.4	3190	2620	2230	8.04	38.4	1520	1120	880	9.84
21.6	3040	2470	2080	8.16	39.6	1450	1060	840	9.96
22.8	2890	2320	1940	8.28	40.8	1390	1010	790	10.08
24	2740	2180	1810	8.4	42	1330	960	760	10.2
25.2	2600	2050	1690	8.52	43.2	1270	920	720	10.32
26.4	2470	1930	1580	8.64	44.4	1220	880	690	10.44
27.6	2350	1820	1490	8.76	45.6	1170	840	650	10.56
28.8	2230	1720	1400	8.88	46.8	1120	800	620	10.68

NEW METHOD.—*For repeated compression:* $\beta = \dfrac{400}{1+c\dfrac{l^2}{d^2}}\left[1+2\dfrac{\text{min. } B}{\text{max. } B}\right]$. Crippling strength in pounds divided by 5600 gives $\dfrac{1}{1+c\dfrac{l^2}{d^2}}$.

TABLE IV.

STRENGTH OF WROUGHT IRON STRUTS.

SQUARE COLUMN. Fig. 201, page 320.			PHŒNIX COLUMN. Fig. 201, page. 320.		
Flat ends.	*Pin and flat.*	*Pin ends.*	*Flat ends.*	*Pin and flat.*	*Pin ends.*
$\dfrac{38500}{1+\dfrac{l^2}{5820d^2}}$	$\dfrac{38500}{1+\dfrac{l^2}{3000d^2}}$	$\dfrac{37500}{1+\dfrac{l^2}{1900d^2}}$	$\dfrac{42500}{1+\dfrac{l^2}{4500d^2}}$	$\dfrac{40000}{1+\dfrac{l^2}{2250d^2}}$	$\dfrac{36600}{1+\dfrac{l^2}{1500d^2}}$

l = length in inches.
d = least dimension in inches.

Factor $4 + \dfrac{l}{20d^2}$

$\dfrac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\dfrac{l}{20d}$	$\dfrac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\dfrac{l}{20d}$
	Flat ends.	Pin and flat.	Pin ends.			Flat ends.	Pin and flat.	Pin ends.	
15	18.533	17.907	16.899	4.75	15	20.238	18.182	15.913	4.75
16	18.438	17 730	16.641	4.80	16	20 106	17.948	15.619	4.80
17	18.340	17.552	16.384	4.85	17	19.967	17.713	15.327	4.85
18	18.235	17.374	16.127	4.90	18	19.823	17.479	15.034	4.90
19	18.126	17.180	15.869	4.95	19	19.672	17.230	14.741	4.95
20	18 012	16.986	15.613	5.00	20	19.515	16.981	14.447	5.00
21	17.893	16.783	15.339	5.05	31	19.353	16.723	14.142	5.05
22	17.772	16.576	15.063	5.10	22	19.187	16.459	13.835	5.10
23	17.646	16.365	14.784	5.15	23	19.014	16.193	13 529	5.15
24	17.517	16.150	14.504	5.20	24	18.838	15.924	13.222	5.20
25	17.384	15.931	14.222	5.25	25	18.658	15.652	12.917	5.25
26	17.246	15.710	13 940	5.30	26	18.474	15.379	12.615	5.30
27	17.106	15 487	13.659	5.35	27	18.287	15.106	12.315	5.35
28	16.965	15.262	13.379	5.40	28	18.096	14.832	12.018	5.40
29	16.820	15.035	13.101	5.45	29	17.904	14.554	11.726	5.45
30	16.672	14.808	12.825	5.50	30	17.712	14.286	11.437	5.50
31	16.522	14.577	12.552	5.55	31	17.511	14.014	11.154	5.55
32	16.370	14.352	12.281	5.60	32	17.311	13 745	10.875	5.60
33	16.216	14.123	12.014	5 65	33	17.105	13.474	10.602	5.65
34	16.060	13.897	11.751	5.70	34	16.907	13.212	10.335	5.70
35	15.903	13.668	11.491	5 75	35	16.703	12.949	10.073	5.75
36	15.743	13.443	11.236	5.80	36	16.498	12 691	9.818	5.80
37	15.582	13.212	10.985	5.85	37	16.292	12.435	9.568	5.85
38	15.422	12.995	10.738	5.90	38	16.072	12.182	9.324	5.90
39	15.261	12.779	10.497	5 95	39	15.866	11.933	9.086	5.95
40	15.099	12.555	10.260	6 00	40	15.676	11.689	8.854	6.00
45	14.281	11.493	9.149	6.25	45	14.655	10.527	7 787	6.25
50	13.466	10.503	8.163	6.50	50	13.661	9.474	6.862	6.50
55	12.666	9.585	7.292	6 75	55	12.708	8.531	6.066	6.75
60	11.894	8.750	6.529	7.00	60	11.806	7.675	5 383	7.00

NEW METHOD.—*For repeated compression:* $\beta = \dfrac{6500}{1 + c\dfrac{l^2}{d^2}}\left[1 + \dfrac{\text{min. } B}{\text{max. } B}\right]$. Crippling strength in pounds divided by the numerators of the respective formulæ, as given at the top of the Table, gives the value of $\dfrac{1}{1 + c\dfrac{l^2}{d^2}}$.

TABLE V.

Strength of Wrought Iron Struts.

American Column.
Fig. 201, page 320.

Common Column.
Fig. 201, page 320.

American Column			Common Column		
Flat ends.	*Pin and flat.*	*Pin ends.*	*Flat ends.*	*Pin and flat.*	*Pin ends.*
$\dfrac{36500}{1+\dfrac{l^2}{3750d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{3000d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{1900d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{2700d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{1500d^2}}$	$\dfrac{36500}{1+\dfrac{l^2}{1200d^2}}$

l = length in inches.
d = least dimension in inches.
Factor $4 + \dfrac{l}{20d}$.

$\dfrac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\dfrac{l}{20d}$	$\dfrac{l}{d}$	*Crippling Strength in tons* (2000 *lbs.*) *per square inch.*			$4+\dfrac{l}{20d}$
	Flat ends.	Pin and flat.	Pin ends.			Flat ends.	Pin and flat.	Pin ends.	
15	17.217	16.591	16.171	4.75	15	16.847	15.869	15.333	4.75
16	17.084	16.377	15.908	4.80	16	16.669	15.582	15.004	4.80
17	16.944	16.163	15.645	4.85	17	16.486	15.295	14.675	4.85
18	16.799	15.949	15.382	4.90	18	16.295	15.008	14.346	4.90
19	16.647	15.723	15.119	4.95	19	16.098	14.707	14.017	4.95
20	16.491	15.496	14 854	5.00	20	15.895	14.407	13.688	5.00
21	16.329	15.259	14.577	5.05	21	15.688	14.104	13.317	5.05
22	16.164	15.019	14.296	5.10	22	15.476	13.798	13.005	5.10
23	15.994	14.776	14.014	5.15	23	15.260	13.492	12.666	5.15
24	15.820	14.531	13.731	5.20	24	15.041	13.187	12.331	5.20
25	15.643	14.283	13.448	5.25	25	14.819	12 883	12.000	5.25
26	15.413	14.034	13.165	5.30	26	14 596	12.581	11.674	5.30
27	15.279	13.784	12.883	5.35	27	14.370	12.282	11.353	5.35
28	15.094	13.534	12.605	5.40	28	14.143	11.986	11.039	5.40
29	14.907	13.285	12.327	5.45	29	13.916	11.694	10.730	5.45
30	14.718	13.036	12.052	5.50	30	13.688	11.406	10.428	5.50
31	14.527	12.788	11.781	5.55	31	13.459	11.124	10.134	5.55
32	14.336	12.542	11.513	5 60	32	13.232	10.846	9.847	5.60
33	14.143	12.295	11.249	5.65	33	13 005	10.574	9.568	5.65
34	13.949	12 056	10.990	5.70	34	12.779	10.307	9.296	5.70
35	13.756	11.818	10.736	5.75	35	12.554	10.046	9.031	5.75
36	13.563	11.580	10.485	5.80	36	12.435	9.791	8.774	5.80
37	13.370	11.347	10.240	5 85	37	12.316	0.542	8.525	5.85
38	13.177	11.116	9.999	5.90	38	12.197	9.298	8.283	5.90
39	12.984	10.889	9.764	5.95	39	12.078	9.062	8.049	5.95
40	12.792	10.666	9.534	6.00	40	11.959	8.831	7.822	6.00
45	11.851	9.605	8.460	6.25	45	10 429	7.766	6.791	6.25
50	10.950	8.645	7.515	6.50	50	9.476	6.844	5 919	6.50
55	10.102	7.785	6.689	6.75	55	8.607	6.050	5.184	6.75
60	9.311	7.019	5.969	7.00	60	7.822	5.369	4 563	7 00

New Method.—*For repeated compression:* $\beta = \dfrac{6500}{1 + c\dfrac{l^2}{d^2}}\left[1 + \dfrac{\text{min. } B}{\text{max. } B}\right]$. Crippling strength in pounds divided by 36,500 gives the value of $\dfrac{1}{1 + c\dfrac{l^2}{d^2}}$.

CHAPTER II.

CROSS-SECTIONING, DETERMINATION OF DIMENSIONS.

A. TENSION MEMBERS.

WE shall make constant use of the Tables and methods of Chapter I., in finding the proper cross section and size of the various members which we have to design. The student, before proceeding further, should read over carefully the general specifications at the end, look over all the plates and illustrations of various members given in the following pages, and familiarize himself at all times and at every opportunity with the way in which various members are made up and put together, by careful examination of actual structures.

CARNEGIE'S POCKET BOOK OF SHAPES.—The various members we have to design are made up, broadly speaking, of I bars and channel bars, combined with plates and rectangular bars by means of pins and rivets. When the requisite area of cross section has been found, according to the principles of the preceding chapter, it might seem that any dimensions which would give the required area would be equally good. But such is not the case. Eye bars, channels, plates, angle irons, etc., are produced by the various mills and rolling companies of *certain sizes*. These sizes are sufficiently numerous and cover a range sufficiently great to answer all practical requirements. But if in our design we specify sizes which are not rolled, such requirements evidently cannot be filled without the expense of making new rolls for the special purpose. We are limited, therefore, in our choice to the sizes actually produced by the various mills and rolling companies, and must choose such sizes as can be readily ordered and bought in the market.

For the purpose of facilitating such choice, the various mills issue for the use of their patrons, "Pocket Books," which, besides much valuable miscellaneous information, contain detailed lists of all the various sizes of iron which they roll, giving for each size and shape the weight per foot, area of section, dimensions, moment of inertia of cross section, radius of gyration of cross section, etc., etc. In short, all the information that can be desired in order to facilitate designing is given. Among the best of such pocket books are those of the PENCOYD IRON WORKS, John Wiley & Sons, and the UNION IRON MILLS, Carnegie Bros. & Co.,* Pittsburg, Pa. These works are readily procured, and all our future calculations will be based upon the Tables given by the latter. The student who would intelligently read what follows, should always have "Carnegie's Pocket Book" within reach. All our references to it refer to the edition of 1889. We shall, whenever necessary, simply refer to this edition by page, and thus avoid the incorporation in this work of extensive Tables. The principles of designing once understood, the reader can patronize any company whose Tables furnish him with the necessary information.

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* *Pocket Companion of Useful Information and Tables Appertaining to the Use of Wrought Iron, as manufactured by Carnegie Bros. & Co.* Edited by C. L. Strobel, C. E., Pittsburg, Pa., 1889. This book is indispensable to students wishing to read this section. Designers will also find a very serviceable help in *Tables of Moments of Inertia*, by Frank C. Osborne, C. E. Eng. News Pub. Co., New York, 1889.

We shall now illustrate the methods of designing, by means of a series of selected examples, and shall take up first the subject of *Tension Members.*

A. TENSION MEMBERS.

The lower chords of ordinary bridges, the main and counter diagonals of trusses, and the diagonals in the upper and lower horizontal wind bracing, and the vertical sway bracing, are all ordinarily in tension, and never take compression.

Sometimes one or more cross ties rest directly upon the lower chord, between the panel points, in which case the bay in question acts as a beam, and at the same time has a tensile stress. This case is rare, as ordinarily the cross ties rest upon stringers, which in turn rest upon floor beams at every panel point. There is thus usually no transverse stress upon a lower bay, but a stress of tension only. Let us now apply both the "old" and "new methods" to the designing of tension members. In every case the strains in the piece considered are supposed to have been found by statical calculation, according to the principles of Part I. of this work.

VALUES OF β FOR TENSION MEMBERS.—For wrought iron, the value of the allowable working stress β for tension is for the "*old method:*"

	Lbs. per sq. in.
On lateral bracing, ...	$\beta = 15{,}000$
On bottom chords and main diagonals, ...	10,000
On counters and long vertical rods, ...	8 to 9,000

For the "*new method,*" we have for wrought iron, for links or rods of doubled rolled iron, page 329.

$$\beta = 7500\left(1 + \frac{\text{min. } B}{\text{max. } B}\right),$$

where min. B is the dead load tension and max. B is the greatest tension which ever acts upon the piece.

EXAMPLE I. *One of the lower bays of a bridge truss is subjected to a tension of 132,200 lbs. due to the dead load, which is, therefore, the least tension which ever acts upon the bay in question. The live load strain is 115,600 lbs. The total tension is, therefore, 247,800 lbs. What should be the area of cross section?*

By the "old method," we have for the area required

$$F = \frac{\text{max. } B}{\beta} = \frac{247800}{10000} = 24.78 \text{ square inches.}$$

If the bay is an end bay, we should probably distribute this area among two chord bars. Each bar should then have an area of 12.39 square inches.

A BAR OF IRON ONE YARD LONG AND ONE SQUARE INCH IN CROSS SECTION WEIGHS TEN POUNDS.

The student should make a note of this once for all, as we shall hereafter apply it without comment.

Each bar in question then will weigh $\frac{123.9}{3} = 41.3$ lbs. per foot, whatever the shape of cross section.

If each bar is to be a Union Iron Mills' channel, we see, by reference to *Carnegie*, page 98, that only two sizes rolled will suit us, *viz.*, a 15-inch or a 12-inch channel. If we decide on a 15-inch channel, then we see from the Table, its thickness of web would be 0.556 inch, and its width of flange 3.556 inches.

If we decide upon a 12-inch channel, its thickness of web would be 0.75 inch and width of flange 3 inches

The sizes can be specified and will be rolled and furnished of these dimensions.

If we decide upon a flat rectangular bar, as we probably would if the bay were not an end bay, we see from *Carnegie*, page 184, that the nearest bar rolled will be 11 inches by $1\frac{1}{8}$ inches, and from page 178, that such a bar weighs 41.25 lbs. per foot.

The reason why we should probably use a channel bar for an end bay and a flat bar for other bays, is that the strains in the horizontal wind bracing, when the bridge is empty, might cause a slight compression in the lower end bay. If so

a channel bar would better resist such stress, and would admit of being laced to its fellow so as to prevent lateral deflection.

By the "*new method*" our results would be somewhat different. For β we should have

$$\beta = 7500\left(1 + \frac{132200}{247800}\right) = 11500 \text{ lbs.},$$

and the necessary area would be

$$F = \frac{247800}{11500} = 21.55 \text{ sq. inches.}$$

Each bar should then have an area of 10.73 sq. inches, and would weigh $\frac{107.3}{3} = 35.76$ lbs. per foot. We see from *Carnegie*, page 98, that a 12-inch channel is the only one which will suit our purpose. The thickness of web of such a channel is 0.614 inch, and width of flange 2.874 inches.

We would call the special attention of the student here, to the fact that *min. B* does *not* stand for the least of the two stresses due to dead and live load. It stands for the steady stress which comes on the bay, which is evidently the *dead load* stress. Also *max. B* is *not* the greatest of the two stresses, but is the total stress which comes on the bay, or the *sum* of the dead and live stresses. The above error is likely to be committed without special warning. Thus, in our present case, min. B is the steady stress of 132200 lbs., *not* 115600 lbs , which is numerically less, and max. B is the total stress, 247800 lbs., and *not* the greater of the other two.

We see also that we obtain by the "new method" in this case a lighter bar than by the old method. This is as it should be. The "old method" only takes account of the action of a repeated stress by its specification of a different β for different members. But we see from our formula that β should vary with the ratio of $\frac{\text{min. } B}{\text{max. } B}$. If the dead load were equal to the maximum stress, or, in other words, if there were no live load at all, we should have the case of a steady stress, and, of course, could take β the largest allowable. This largest allowable stress is, we see from the formula, 15000 lbs., which corresponds with the largest allowable stress by the old method, for lateral bracing, which is seldom called into action. On the other hand, if there were no dead load stress at all, we should have by our formula, $\beta = 7500$ lbs., which agrees well with the value of β according to the old method, for counters, which are strained by every passing load, but not by dead loads. Between these limits, then, of 7500 and 15000, our values of β will range, according to the ratio of $\frac{\text{min. } B}{\text{max. } B}$, and our single formula replaces all the specifications of the old method. The value of 10000 lbs. for β, prescribed by the old method for chords, corresponds to a ratio of $\frac{\text{min. } B}{\text{max } B} = \frac{1}{3}$. For bridge construction, this is a good average value, but as this value is really not a constant, the new method gives the most rational means of taking it into account.

COMBINED TENSION AND FLEXURE.—The lower bay may have a weight resting upon it, due to a cross tie between the panel points. In this case it acts as a beam, and at the same time is in tension. This case has already been discussed on page 277.

EXAMPLE 2.—*If the bay in the preceding example is 20 feet long, and besides the longitudinal tension already given, has a weight of 2 tons at the centre, what should be the area?*

For this case we have, page 321,

$$F = \frac{Mv}{\beta r^2} + \frac{S}{\beta}.$$

By the old method, $\beta = 10000$ lbs., by the new method, we have just found for this case, $\beta = 11500$ lbs.

We cannot tell what value to take for r, however, unless we assume the depth of bar required. Guided by the preceding example, we should choose a 15 inch channel, because a 15 inch is the largest channel we can have by Carnegie's Table, and the area in the present case must be greater than in the previous case. If, then, we can have two bars at all, we shall need 15 inch channels. We may have to use more than two bars. From *Carnegie*, page 98, we see that the value for r varies for 15 inch channels between 5.48 and 5.12 inches. Let us assume 5.3 for r therefore. We have, then,

$$M = 2000 \times 10 \times 12 = 240000 \text{ inch lbs.}, \quad v = 7.5 \text{ inches}, \quad S = 247800 \text{ lbs.}$$

Hence, by the old method,

$$F = \frac{240000 \times 7.5}{10000 \times 28.09} + \frac{247800}{10000} = 6.40 + 24.78 = 31.18 \text{ sq. inches.}$$

By the new method,

$$F = \frac{240000 \times 7.5}{11500 \times 28.09} + \frac{247800}{11500} = 5.57 + 21.55 = 27.12 \text{ sq. inches.}$$

In the first case, if we have two bars, the area of each bar will be 15.59 sq. inches, and its weight $\frac{155.9}{3} = 51.96$ lbs. per ft. From *Carnegie*, page 98, this calls for thickness of web 0.77 inch, and width of flange 3.77 inches. This corresponds to a value of r of 5.37 inches, which is sufficiently close to our assumed value of 5.3 inches, not to necessitate another calculation.

In the second case, we have for the area of each bar 13.51 square inches, and $\frac{135.1}{3} = 45.03$ lbs. per ft. From *Carnegie*, page 98, this calls for thickness of web of 0.63 inch, and width of flange of 3.63 inches. The corresponding value of r is 5.39, which agrees sufficiently well with our assumed value of 5.3 inches.

If we wish a flat bar instead of a channel, we may assume the depth of bar at say 12 inches. Then,

$$r^2 = \frac{I}{A} = \frac{bd^3}{12bd} = \frac{d^2}{12} = 12.$$

Hence, by the old method,

$$F = \frac{240000 \times 6}{10000 \times 12} + \frac{247800}{10000} = 12 + 24.78 = 36.78,$$

and by the new method,

$$F = \frac{240000 \times 6}{11500 \times 12} + \frac{247800}{11500} = 10.40 + 21.55 = 31.95.$$

In the first case, if we have two bars, each will have an area of 18.39 sq. inches, and will weigh $\frac{183.9}{3} = 61.3$ lbs. per ft. From *Carnegie*, page 178, the nearest size is $11\frac{3}{4}$ inches by $1\frac{9}{16}$ inches.

In the second case, each bar has an area of 15.93 sq. inches, and will weigh $\frac{159.3}{3} = 53.1$ lbs. per ft. This calls for a bar $11\frac{1}{4}$ inches by $1\frac{3}{8}$ inches.

An increase in the assumed depth would, of course, diminish the material required, but as $12\frac{3}{4}$ inches is the limit in depth of the table, we have taken nearly the greatest depth procurable.

INITIAL TENSION.—Many of the tension members are made adjustable by means of turn-buckles or sleeve nuts. The screwing up of these may bring a strain upon the member independently of the stress which comes upon it from the loading. To allow for this we may add 1 ton for a rod 1″ in diameter, and $\frac{1}{4}$ of a ton for each increase of $\frac{1}{8}$″ in the diameter. That is *for round bars*,

$$\textit{Initial tension in tons} = 2d - 1,$$

where d is the diameter in inches. Flat bars are to have the same allowance as round rods of equal sectional area; or *for flat bars*,

$$\textit{Initial tension in tons} = 2.25\sqrt{A} - 1$$

where A is the area of cross section in sq. inches.

EXAMPLE.—*The maximum tension in a flat bar is 90000 lbs., and the working stress is found to be 10000 lbs. per sq. inch. If the bar is adjustable, what should be the area?*

The area, without allowance for initial tension, is $\frac{90000}{10000} = 9$ sq. inches. This area would give us 5.75 tons initial tension. Let us take 6.25 tons, or 12500 lbs., for the initial tension. Then the maximum tension would be 102500 lbs., and area required would be $\frac{102500}{10000} = 10.25$ sq. inches. This area substituted, gives us 6.2 tons initial tension, which is near enough to the initial tension assumed. The area required is then 10.25 square inches, instead of 9 square inches, called for by the loading alone.

COMPRESSION IN END LOWER BAYS.—The lower bays are all in tension by reason of the live and dead load. But the wind blowing upon one side bends the truss laterally, and acts as a horizontal load. The strains due to wind must then be resisted by the horizontal bracing, and it may happen that in one or more of the end bays there will be a compression due to wind, greater than the tension due to dead load, or even to dead and live loads combined. In such case the difference between wind compression and dead load tension, or dead and live load tension, will come as a strain of compression upon the lower end bays. The end bays should be able to take such excess, and must be treated for it as long struts in compression. It is for this reason that in the preceding examples we have taken channel bars in pairs when the bay was supposed to be an end one. Such bars can be joined to one another by lattice bars riveted to the flanges, and thus made to act together as a strut with a much greater least radius of gyration than either would have acting separately. Thus made, they will require no extra material to resist the slight compression which they may be called upon to sustain, as the area called for by the tensile strain will be ample, provided the radius of gyration is thus secured sufficiently large. As in general the chord bars go in pairs at the end, they may be spaced apart a distance always greater than their depth, and therefore, when latticed to each other, their least radius of gyration will be when the neutral axis is perpendicular to the web at centre. If not so latticed, we should have to take the radius of gyration for the axis coincident with centre line of web, which would of course be very small, and extra material might be required. As flat bars cannot easily be thus latticed, we see the propriety of making the end bays of channel bars, when compression due to the wind is to be feared.

CHOICE OF DEPTH OF LOWER CHORD BARS.—The maximum stress at any bay will determine the area required. According to the depth assumed for chord bars, the number required in each panel will vary. As the bars should go in pairs, and increase in number towards the centre, without increasing much in depth, a little preliminary figuring and judgment is required in order to assume, in any given case, such a depth as will allow of the requisite number of bars at each panel, without causing the depth to vary too greatly, or necessitating undue thickness. For this purpose, we may first find the area required in the end panel and centre panel. Then we may choose such a depth for the end panel bars as shall give the required area for a medium thickness, and at the same time will give, for about the same depth and thickness, the required number of bars at the centre. The bars in a panel need not have the same thickness necessarily.

EXAMPLE.—*The lower centre bay in a bridge truss has a tension of 526000 lbs. due to dead load and 427000 lbs. due to live load. In the first end bay, in which flat bars are used, we have 191600 lbs. due to dead load and 166400 due to live load. To choose a good depth for chord bars.*

We shall use the "new method" in our calculation. To apply the old method, we simply take $\beta = 10000$ lbs.

By the new method then, we have for centre bay,

$$\beta = 7500 \left(1 + \frac{526000}{953000} \right) = 11638 \text{ lbs.},$$

and for the end bay

$$\beta = 7500 \left(1 + \frac{191600}{358000} \right) = 11514 \text{ lbs.}$$

The area required in the centre bay is then $\frac{953300}{11638}$ = about 82 square inches, and in the end bay $\frac{358000}{11514}$ = about 31 square inches.

If we are to have four bars at the end, each bar will have an area of 7.75 square inches. A number of sizes may be chosen which will give this area. For such a heavy bar, we should not have less than 1 inch thickness. From *Carnegie*, page 182, we see that $7\frac{1}{4}$ inches by $1\frac{1}{8}$ inches will be ample for the end panel. If we take the same depth for the

centre panel and $1\frac{1}{8}$ inches thickness, we should have to have 4 bars $7\frac{1}{2}''$ by $1\frac{7}{8}''$ and 4 more $7\frac{1}{2}''$ by $1\frac{5}{8}''$, or 8 bars altogether in the centre panel. If this is not judged to be too many, we may then take $7\frac{1}{2}''$ for the depth. For a long truss, such as we have supposed, it would not be too many We could not take the depth much greater than $7\frac{1}{2}''$ without getting too small a thickness for the end bars, nor much less than $7\frac{1}{2}''$ without getting too many bars in the centre panel. For constructive reasons, it is preferable to have all the eye-bars of the same depth and as near as may be of the same thickness, and to increase the number as required. A depth of $7\frac{1}{2}''$ will then be satisfactory.

COUNTERS.—The main diagonals, lower chord, and vertical suspenders are generally made of forged eye-bars. All that is necessary for these is that the design of the head shall be such that upon being tested to destruction, the break shall occur in the bar, not in the head.

For the counter rods, square bars are preferably used. These have square loop eyes around the pin, and turn buckles for adjusting. If round bars are used with square loop eyes and turn buckles, they are more expensive.

Round bars are sometimes used for counters without turn buckles, but with loop swivels on the ends.

DETAILS OF LOWER CHORD.—The cross section is generally increased, as shown in Fig. 205, Plate 8, by increasing the number of eye-bars, and it is rarely that the dimensions are increased without increasing the number, or still more rarely that number and dimensions are both increased. A uniform size, as near as may be, at least in depth, is less expensive. This principle holds for all duplicated parts generally. In Fig. 205, Plate 8, we have shown the arrangement of eye-bars and ties at two panel points. The ties are distinguished by having their partly visible upturned ends shaded.

In Fig. 206, Plate 8, we have given an isometric drawing showing the details of bottom chords, etc. This figure, as well as Figs. 220, Plate 10, and 221 and 222, Plates 11 and 12, were kindly furnished by Kellogg & Maurice, from bridges built by them during 1881, for the C., C., C. & I. R. R. It will be seen in Fig. 206 that provision is made for an auxiliary timber stringer to support ends of ties in case of derailment. It is generally customary to use a light iron stringer for this purpose, or else to space the main stringers in such a manner as to accomplish the same purpose. There are many other points about Fig. 206 which will repay study, such as the connection of lower wind braces, the construction of posts and details at bottom of posts, etc. In Figs. 203, 204, and in Figs. 207, 208, and 209, we give still other illustrations of bottom chord and connections. Figs. 204, 208, and 209 are examples of old construction which would not, and, in fact, could not be built at the present day. Their value is simply historical. *No cast iron is allowed* in a bridge at the present time for *any purpose* except for bed plates and for the machinery of draw spans. Figs. 205, 206, 220, and 221 represent modern American practice. The other figures are specimens of riveted work. Foreign bridges consist largely of riveted

work. Though eye-bars and pins are sometimes used, it is rare, comparatively. This, and the necessary details, constitute the chief distinction between foreign and American practice. American engineers use eye-bars and pins almost exclusively for the bottom chords of bridges of any ordinary span.

The figures thus far given require but little explanation. The student can acquire, by a careful study of them, a good knowledge of the system of forming lower chord and connections. For other illustrations he can consult the illustrated albums of our various bridge companies, and better still, should seize every opportunity to study existing structures on the spot.

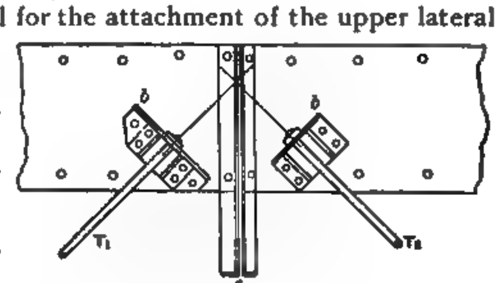
In Figs. 210 to 219, Plates 9 and 10, we have given illustrations of bottom chord riveted work, mostly foreign examples. We shall treat of riveted work hereafter in detail, and much information will be found in the specifications at the end of this work. Although riveting is but little used in the main trusses in American bridges, still it is of great importance, and a study of its application, as set forth in our illustrations, will be profitable.

In Fig. 210 we see how the area of the bottom chord may be increased by adding plates one over the other, as also the connection of the braces. Fig. 211 shows how the depth of bottom chord may be increased, as well as the use of an auxiliary plate to give greater area for riveting. Fig. 212 shows the introduction of a post, and Fig. 213 the same with auxiliary plate, when the depth of chord is not sufficient to attach the braces directly to it. Figs. 214, 215, 216, 217, 218, 219 give different styles of end connections.

Fig. 220 shows the details for inclined end posts or "batter braces," according to a drawing furnished by Kellogg & Maurice.

Plate 11*a* gives an isometric view of a double intersection Pratt Truss R. R. bridge, taken by permission from "A System of Railroad Bridges for Japan," by Prof. J. A. L. Waddell (Memoirs of the Tokiô Daigaku, No. 11). The names of the various members are written upon the drawing, and by inspection of the drawing and of actual bridges the student should familiarize himself with the name, duty, and connection of every member.

An excellent detail for the attachment of the upper lateral rods to the top chord is given by Professor Burr, *Stresses in Bridge and Roof Trusses*, Wiley & Sons, New York, 1886, and shown in the accompanying figure. At *b* a piece of angle iron 6″ by 4″, with the 6″ leg lying on the chord, carries two pieces of angle iron 3″ by 3″, with their edges parallel to the axis of T_1. One end of each of the latter angles rests squarely against the vertical 4″ leg of the 6″ by 4″ angle. The tie T_1 passes through the 4″ leg of the heavy angle, between the 3″ angles, and is adjusted by nut at the end.



The arrangement is effective, cheap, and the axes of the ties can be made to meet at the neutral axis of the chord. The student can compare this detail with that on Fig. 221.

PLATE 8.

Fig.205

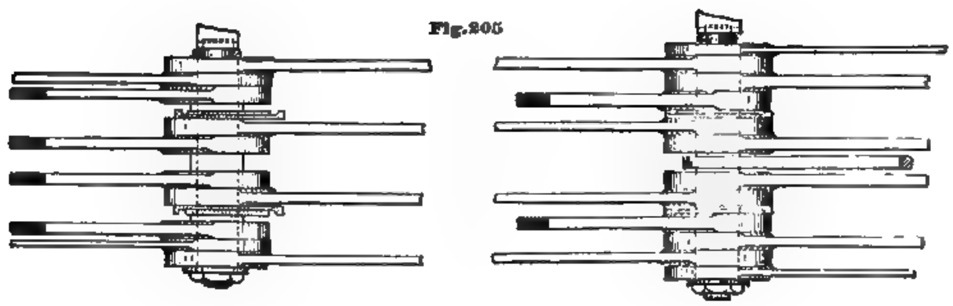


PLATE 9.

Fig. 213

Fig. 214

Fig. 215

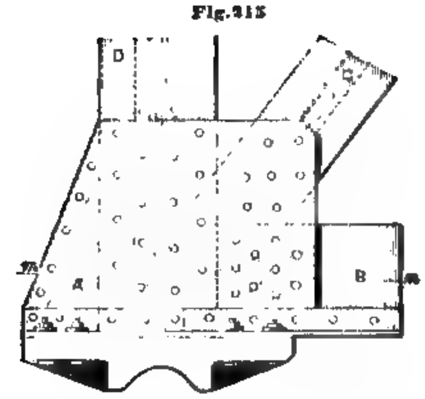


PLATE 10.

Fig. 218

Fig. 219

Fig. 220

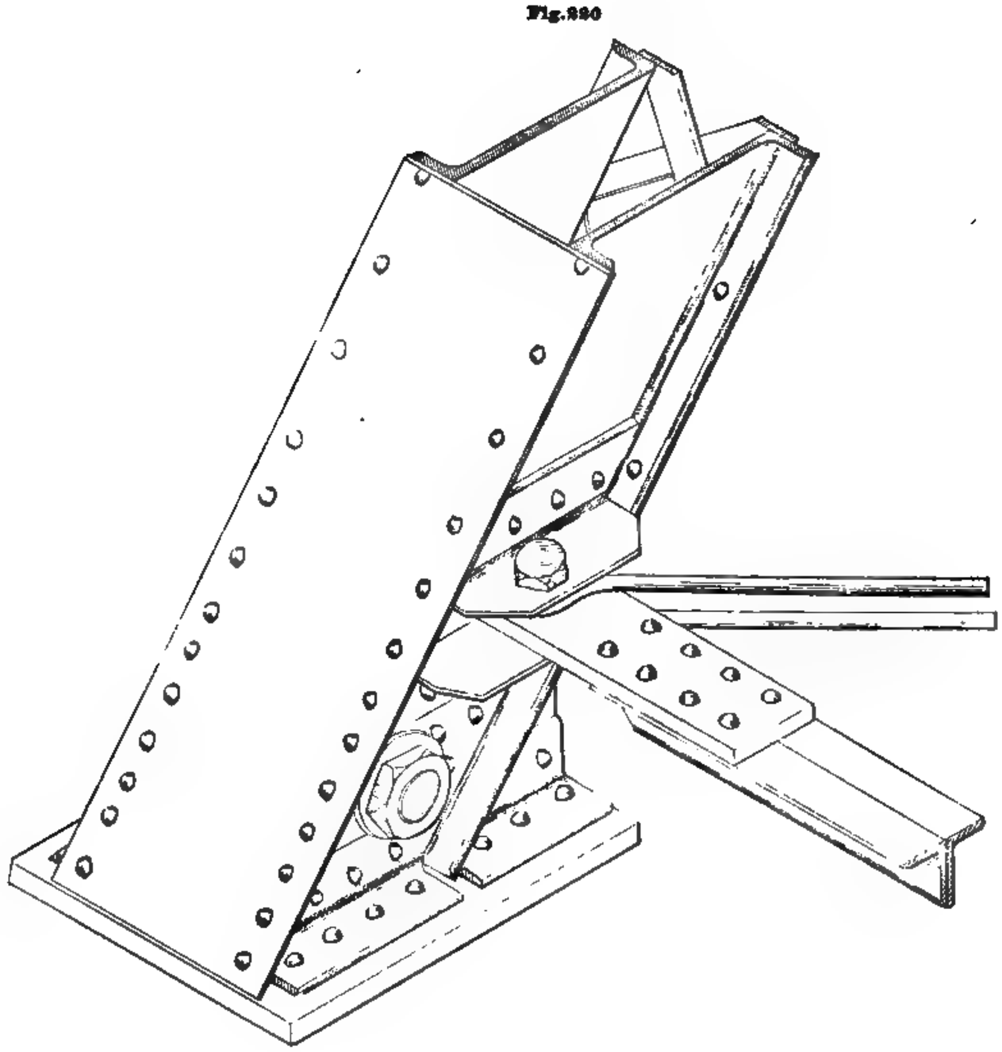
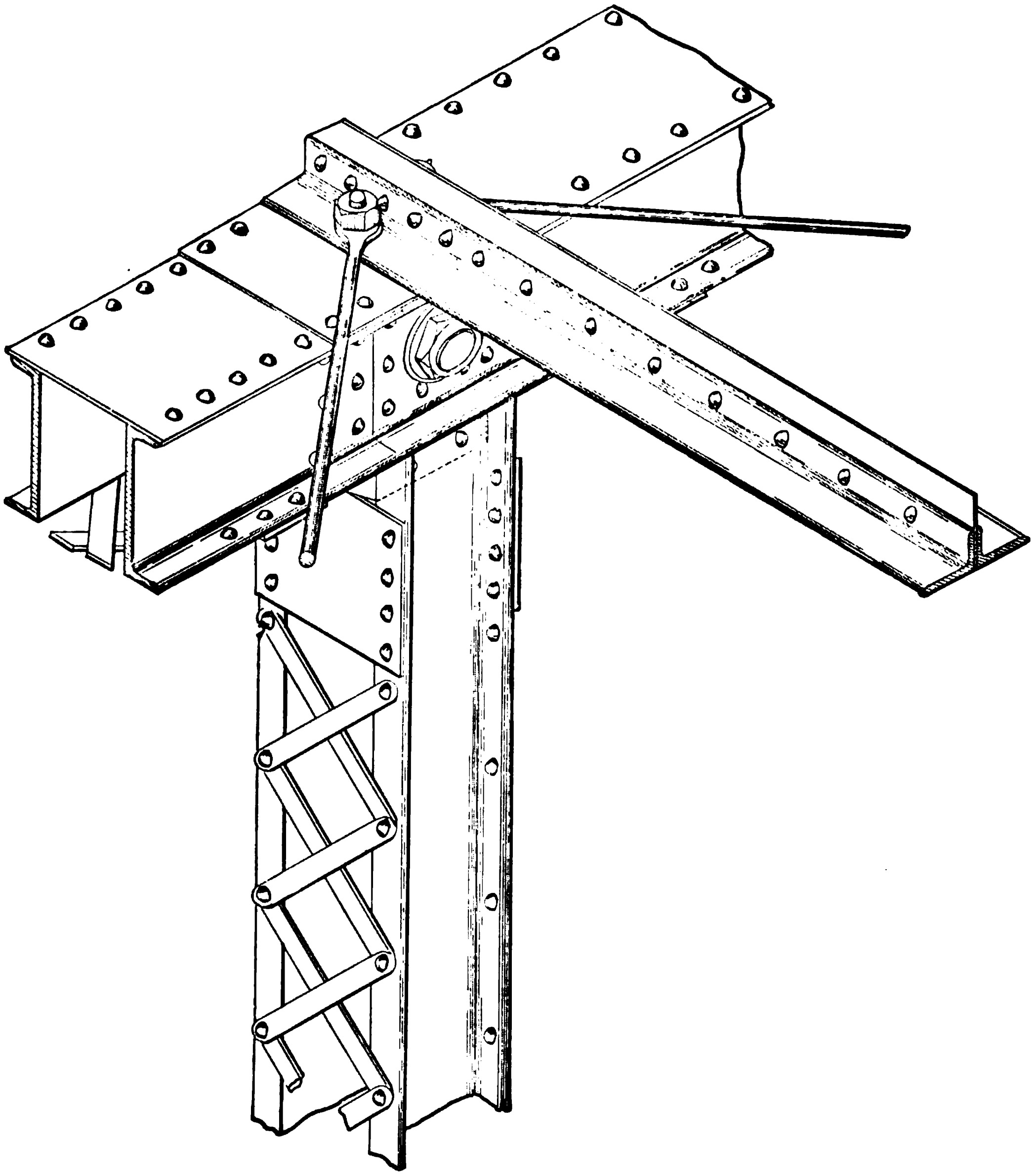


PLATE II.

Fig. 221



CHAPTER III.

CROSS-SECTIONING, DETERMINATION OF DIMENSIONS.

B. COMPRESSION MEMBERS.

WE have represented in Fig. 221, Plate 11, the ordinary method of forming compression members. It will be seen that both post and chord are composed of channels. The post is composed of two channels, united by lattice or lacing bars. When there is a single system of bars, the channels are said to be "*laced*." When there is a double system, as in Fig. 221, they are said to be "*latticed*." The upper chord is also composed of two channels, latticed or laced on the under side, and with a top plate. This constitutes the "common chord section." Sometimes, for short compression members, instead of lattice or lacing bars, we may have a rectangular strip or plate, riveted on at intervals, these plates or strips answering the same purpose as the lattice or lacing bars, viz., to unite the channels, and make them act together to resist lateral flexure. Without such lateral connections each channel would evidently bend more easily, and the resisting power of the combination is much greater than the sum of each channel acting separately. The strut for the over-head horizontal wind bracing is also shown in Fig. 221, composed of two angle irons riveted to a central plate. We also see the method of connection of the various pieces at the panel point or pin. The post channels are shaved off at the end, and the web strengthened at top and bottom by plates called "*reinforcing plates*." The pin goes through these plates and the web of the post channels, as well as through the web of the chord channels. The object of the reinforcing plates is not only to strengthen the ends of the posts, but also to give a sufficient bearing upon the pin. Each post has also, upon each side, at top and bottom, plates just between where the lacing or latticing ends and the pin, called "*stay plates*."

The channels composing the struts, whether posts or chords, are generally spaced farther apart than the depth of channel, so that the least radius of gyration, when the channels are laced or latticed, is with reference to the axis perpendicular to the web, and *not* coincident with the web of the channels.

RADIUS OF GYRATION.—We see from Chapter I., page 321, that in order to find the strength of a long strut we need to know r^2, or the square of the radius of gyration. The radius of gyration of two post channels is, in general, the same as the radius of gyration of a single channel, when the axis is at right angles to the web.

We have in general,

$$r^2 = \frac{I}{A},$$

where r is the radius of gyration, I is the moment of inertia of the cross section with reference to the required axis, and A is the area of the cross section.

For two post channels, the area is twice that for one, and the moment of inertia is also twice that for one, for axis at right angles to the web. So that the radius for the two, if they are connected by lattice or lacing bars, and spaced farther apart than their depth, is the same as for one.

But for the chord cross section, composed of two channels and a top plate, we must take into account the moment of inertia of this plate, with reference to the axis perpendicular to the web of the channels.

The moment of inertia of a rectangular cross section *with reference to the axis through its own centre of gravity* parallel to its breadth is, $\frac{1}{12}bd^3$, where b is the breadth and d is its depth.

The moment of inertia of any cross section with reference to an eccentric axis outside of it, is equal to the moment of inertia with reference to a parallel axis passing through the centre of gravity, *plus* the area into the square of the distance between the two axes. Or,

$$I' = I + AD^2,$$

where I' is the moment of inertia with reference to the eccentric axis, I is the moment of inertia with reference to the parallel axis through the centre of gravity, A is the area of cross section, and D is the distance between the two axes. Carnegie's Tables give us the moment of inertia of channel cross sections with reference to axes through the centre of gravity, perpendicular to the web, and an application of the preceding principle will enable us to find the moment of inertia and the radius of gyration of any compound cross section with reference to any given axis.

EXAMPLE.—*Suppose a top chord is composed of two 6 inch 10 lb. channels, spaced say 7 inches apart, back to back, with a top plate $\frac{1}{4}$ inch thick.*

Since the channels are spaced farther apart than their depth, flexure, if any will be in the direction of their depth, and we must find the moment of inertia and radius of gyration for an axis perpendicular to the web, passing through the centre of the channel cross section.

From *Carnegie*, page 100, we see that the width of flange is 2.04 inches. Hence the breadth of plate is $7 + 4.08 = 11.08$ inches.

The moment of inertia of the plate with reference to the required axis is then

$$I' = I + AD^2 = \frac{bd^3}{12} + bd \times (3\tfrac{1}{8})^2 = \frac{11.08 \times (\frac{1}{4})^3}{12} + 11.08 \times \tfrac{1}{4} \times (3\tfrac{1}{8})^2,$$

or, $$I' = 27.45.$$

The moment of inertia of the two channels is, from *Carnegie*, page 64, $16.9 \times 2 = 33.8$. Hence total moment of inertia is 61.25, and since the total area is $6 + 2.77 = 8.77$, we have $r^2 = \frac{61.25}{8.74} = 6.97$ inches, or $r = 2.64$ inches.

In this way we find r^2, or the square of the radius of gyration, for any cross section. Then from our formulæ, Chapter I., page 320, or from the Table, page 333, we can find the load which the strut will bear.

VALUE OF β FOR COMPRESSION MEMBERS.—For wrought iron, the value of the allowable working stress β, for compression, is, for the "*old method*," given by the formulæ on page 320 or at the top of the Tables at the end of Chap. I., when we take the proper factor of safety, as given at the head of every Table. The use of these Tables will greatly abridge the labor of calculation. Table I. applies generally to any form of cross section except hollow round, but, as we have just seen it requires some little calculation to find r for compound cross sections, it will ordinarily be more convenient to make use of the other tables, which only require d, or the least depth, to be known. We shall, therefore, in general, only apply Table I. in those cases where r can be taken at once from *Carnegie's* Tables, and in other cases may make use of one of the other tables of Chap. I.

The "straight-line" formulæ, page 330, can be at once applied without Table.

By the "*new method*,"

$$\beta = \frac{6500}{1 + c\frac{l^2}{r^2}}\left[1 + \frac{\text{min. } B}{\text{max. } B}\right],$$

where the value of $\dfrac{1}{1 + c\dfrac{l^2}{r^2}}$ in any case may be found from Table I., by dividing the crippling strength *in pounds*, as found from the Table, by 40000. When we use one of the other Tables, we have

$$\beta = \frac{6500}{1 + c\dfrac{l^2}{d^2}}\left[1 + \frac{\text{min. } B}{\text{max. } B}\right],$$

where the value of $\dfrac{1}{1 + c\dfrac{l^2}{d^2}}$ may be found by dividing the crippling strength *in pounds*, as found from the Table, by the ultimate strength taken for the case, as indicated by the formulæ given at the head of each Table.

EXAMPLE.—*A post in a bridge truss is subjected to a compression of 46900 lbs. due to the dead load, and 64100 lbs. due to the live load. The post is 30 feet long. What should be its area of cross section?*

Let us suppose that the post is composed of two channels latticed or laced. Then we should use Table I., Chapter I., page 333. We cannot enter the Table until we first know r, and therefore the value of $\frac{l}{r}$, and we cannot tell r until we first assume some size for the channels. Here judgment and experience will aid in making a suitable choice. Whatever choice we make we can soon test, however, and make another if not suitable.

Let us take two 10 inch channels between 20 and 35 lbs. per foot, and space these channels at least 10 inches apart, so that r must be taken for an axis at right angles to the web of the channels.

From *Carnegie*, page 98, we see that r varies between 3.85 and 3.47. Let us assume r then at 3.6. Then

$$\frac{l}{r} = \frac{360}{3.6} = 100.$$

Suppose the post to be pinned at both ends.

Then, by the "*old method*," we have from Table I., the factor of safety, $4 + \frac{l}{20d} = 4 + \frac{360}{200} = 5.8$, and the crippling strength = 12.855 tons, or 25710 lbs. per sq. inch. The allowable working stress is then $\frac{25710}{5.8} = 4433$ lbs. per sq. in. $= \beta$. The area required is then $\frac{111000}{4433}$ = about 25 sq. inches. Each channel will weigh therefore $\frac{250}{3 \times 2} =$ 41.66 lbs. per foot. We see from Carnegie that there is no 10 inch channel rolled as heavy as this, but that the size required will evidently come between 12 inch 30 lb. and 12 inch 50 lb. Taking for this size $r = 4.2$ inches, we have $\frac{l}{r} = \frac{360}{4.2} = 85.7$, and factor of safety $= 4 + \frac{360}{20 \times 12} = 5.5$. From Table I., therefore, we have the crippling strength = 14.180 tons = 28360 lbs. per sq. inch, and the allowable working stress is $\frac{28360}{5.5} = 5156$ lbs. per sq. in. $= \beta$. The area required is then $\frac{111000}{5156} = 21.52$ sq. inches.

This will give for each channel a weight of $\frac{215.2}{3 \times 2} = 35.86$ lbs. per foot. This comes well within the limits for 12 inch channels. The 12 inch channels required will then weigh 35.86 lbs. per ft. each, the thickness of web will be 0.616 inch, and width of flange 2.876 inches. The corresponding value of r is 4 31 inches, which is near enough to our assumed value.

By the "*new method*," we have from Table I., for $\dfrac{1}{1 + c\dfrac{l^2}{r^2}}$, for the 12 inch channels, $\frac{28360}{40000} = 0.709$; hence

$$\beta = 0.709 \times 6500\left[1 + \frac{46900}{111000}\right] = 6545 \text{ lbs. per sq. inch.}$$

The area required is therefore $\frac{111000}{6555}$ = about 17 sq. inches. This gives for each channel $\frac{170}{3 \times 2} = 28.33$ lbs. per ft. We see, from *Carnegie*, that this calls for 12-inch channels, 28.3 lbs. per ft., 0.44 inch thickness of web, and 3.13 inches width of flange. The corresponding value of r is 4.47 inches; our assumed value of 4.2 inches is near enough not to require recalculation, and is on the side of safety.

If we use the "*straight-line*" formula, with Cooper's values, page 332, we have, taking $r = 4.3$, for the live load $\beta = 7000 - 40\frac{360}{4.3} = 3651$, and for the dead load $\beta = 14000 - 80\frac{360}{4.3} = 7302$ lbs. The area required is therefore $\frac{46900}{7302} + \frac{64100}{3651} = 23.97$ sq. inches. This will give for each channel an area of $\frac{239.7}{3 \times 2} = 39.95$ sq. inches.

SPACING OF THE LATTICE OR LACING BARS.—The object of the lacing or lattice bars is to join the two channels composing the post or chord, and thus cause them to act together. Evidently, the principle which applies here is that the bars should be attached at intervals so close that there shall be no danger of failure of the channels between the points of attachment. In other words, the length of a single channel between the points of attachment of the bars, shall be as strong at least, considered as a short post, as the whole post or chord itself.

If then l is the distance in inches between the points of attachment of the bars, and r is the least radius of gyration of the channel cross section in inches, and L is the length of the whole post or chord in inches, and R its least radius of gyration in inches, we have

$$\frac{l}{r} = \frac{L}{R}, \text{ or } l = \frac{Lr}{R}.$$

The distance between the ends of bars cannot then be greater than the value of l thus determined.

Practice has made this distance much less, viz., never more than $0.6l$. Also, in order to avoid having the bars make too small an angle with the flanges, which would impair their action, lacing bars are not allowed to make an angle of more than 60° with each other, or less than 60° with the flanges. If then, the value of $0.6l$ comes out less than d or equal to d, where d is the distance between the channels in inches, we can use lacing bars with a distance of d between the points of attachment. If $0.6l$ is greater than d, we must use lattice bars. In case lattice bars are used, the ratio $\frac{l}{d}$ must not exceed $\frac{4}{3}$. The value of the $0.6l$ simply determines then whether lacing or lattice bars shall be used. This point settled, we take d for the distance between points of attachment for lacing and $\frac{4}{3}d$ for lattice bars.

LEAST RADIUS OF GYRATION FOR SINGLE CHANNELS.—The application of the preceding requires us to know the radius of gyration r for channels for axis parallel to the web, through the centre of gravity. This value of r is not given in Carnegie for the different sizes on page 64. We therefore give here these values of r, for the sizes in Carnegie's Pocket Book:

No. of shape	25		26	27		28		29	30	
Designation	15" Light.	15" Heavy.	12"	12" Light.	12" Heavy.	12" Light.	12" Heavy.	10"	10" Light.	10" Heavy.
Radius of gyration, axis parallel to web. r in inches	0.93	0.90	0.85	0.84	0.82	0.74	0.75	0.67	0.68	0.66

No. of shape	31		32	33		34		35		36		37	
Designation	10" Light.	10" Heavy.	9"	9" Light.	9" Heavy.	8" Light.	8" Heavy.	8" Light.	8" Heavy.	7" Light.	7" Heavy.	7" Light.	7" Heavy.
Radius of gyration. axis parallel to web. r in inches	0.72	0.71	0.69	0.68	0.68	0.56	0.55	0.65	0.66	0.56	0.55	0.64	0.65

No. of shape...	38		39		40		41		42		43		44	
Designation....	6″ Light.	6″ Heavy.	6″ Light.	6″ Heavy.	5″ Light	5″ Heavy.	5″ Light.	5″ Heavy.	4″ Light.	4″ Heavy.	4″ Light.	4″ Heavy.	3″ Light.	3″ Heavy.
Radius of gyration, axis parallel to web. r in inches..	0.51	0.50	0.58	0.58	0.47	0.46	0.55	0.52	0.46	0.46	0.50	0.51	0.45	0.46

EXAMPLE.—*Suppose the post channels, as determined in the last example, are 12 inch 28.3 lb. channels, spaced 15 inches apart, back to back, what should be the distance between the ends of bars, and shall we use lacing or lattice bars?*

Here we have from Carnegie, page 98, $R = 4.47$, and since $L = 360$ inches and $r = 0.84$ from our Table, we have

$$l = \frac{360 \times 0.84}{4.37} = 68 \text{ inches.}$$

Hence $0.6l = 41$ inches. This is greater than $d = 15$ inches, so we should use lattice bars, and space $\frac{4}{3} \times 15 = 20$ inches apart.

If, however, the post were only 10 feet long, instead of 30 feet, we should have

$$l = \frac{120 \times 0.84}{4.47} = 23 \text{ inches,}$$

and $0.6l = 13.8$ inches. As this is less than $d = 15$ inches, we could use lacing bars, and space 15 inches apart.

SIZE OF STAY PLATES.—Every compression member, composed of channels united by lacing or lattice bars, should have "stay plates" at the ends, as shown in Fig. 221, Plate 11, page 348. Lacing or lattice bars should never be used without such plates at the ends. No general principles can be laid down for determining the size of such plates.

In accordance with practice, we may be guided by the following rules:

Thickness of Stay Plates.—

For all depths of channel less than 8″................ $t = \frac{1}{4}$ inch.
From 8 to 10″ inclusive............................ $t = \frac{5}{16}$ "
Above 10″ .... $t = \frac{3}{8}$ "

Length of Stay Plates.—Let D = depth of channel in inches, d = distance between inner faces of the channels in inches, l = length of stay plate in inches. Then, for latticing or double riveted lacing,

$$l = 0.5D + \frac{d}{D} + 1.5;$$

for single riveted lacing,

$$l = D + \frac{2d}{D} + 2.$$

EXAMPLE.—*Thus in the preceding example, for two 12 inch channels, 15 inch spacing, what should be the size of stay plates for lattice bracing?*

We have, according to the above rules, the thickness of stay plate, $t = \frac{3}{8}$ inch, and for the length of plate,

$$l = 6 + \frac{15}{12} + 1.5 = 8.75 \text{ inches.}$$

SIZE OF LACING OR LATTICE BARS.—We can give no general principles for determining the sizes of the lacing or lattice bars, but the following rules are in accord with established practice:

Thickness of Lattice or Lacing Bars.—The same rule as for the thickness of stay plate holds good, viz.: For all depths of channel less than 8 inches, $t = \frac{1}{4}$ inch. From 8 inches to 10 inches inclusive, $t = \frac{5}{16}$ inch. Above 10 inches, $t = \frac{3}{8}$ inch.

Width of Lattice or Lacing Bars.—Let D = the depth of channel, d = the distance between the inner faces of the channels, and w = the width of the bar, all in inches. Then, *for lattice bars,*

$$w = \frac{9}{88} D + \frac{d}{4D} + \frac{37}{44};$$

for lacing bars,

$$w = \frac{17}{88} D + \frac{d}{2D} + \frac{31}{88}.$$

The ends of lattice and lacing bars are made semicircular, the centre being taken a little outside of the outer edge of the rivet hole.

EXAMPLE.—*For two 12 inch channels, 15 inch spacing, what should be the size of lattice bars adopted?*
According to our rules, we have for the thickness of bars, $t = \frac{3}{8}$ inch, the same as for the stay plates.
For the width of bars we have:

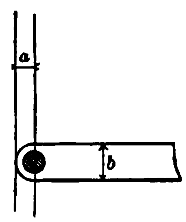
$$w = \frac{9}{88} \times 12 + \frac{15}{4 \times 12} + \frac{37}{44} = \text{about } 2\tfrac{3}{8} \text{ inches.}$$

The dimensions and weight of lattice bars may be figured from the following table adopted by the Phœnix Bridge Company:

LATTICE BARS FOR POSTS AND CHORDS.

THE DIMENSIONS OF SINGLE LATTICE BARS SHALL GENERALLY BE AS FOLLOWS:

		wt. ℔ ft.
For 6″ Rolled or Built Channels	$1\frac{3}{4} \times \frac{5}{16}$	1.82
" 7 " " "	$1\frac{3}{4} \times \frac{5}{16}$	1.82
" 8 " " "	$1\frac{3}{4} \times \frac{5}{16}$	1.82
" 9 " " "	$2 \times \frac{3}{8}$	2.50
" 10 " " "	$2 \times \frac{3}{8}$	2.50
" 11 " " "	$2 \times \frac{3}{8}$	2.50
" 12 " " "	$2\frac{1}{4} \times \frac{3}{8}$	2.81
" 13 " " "	$2\frac{1}{2} \times \frac{3}{8}$	3.13
" 14 " " "	$2\frac{1}{2} \times \frac{3}{8}$	3.13
" 15 " " "	$3 \times \frac{3}{8}$	3.75
" 16 " " "	$3 \times \frac{3}{8}$	3.75
" 18 " " "	$3 \times \frac{3}{8}$	3.75
" 21 " " "	$3 \times \frac{7}{16}$	4.38
" 24 " " "	$3 \times \frac{7}{16}$	4.38
" 27 " " "	$3 \times \frac{1}{2}$	5.00
" 30 " " "	$4 \times \frac{7}{16}$	5.83



$$a = \frac{b}{2} + \frac{1''}{4}$$

	b	a
for	$1\frac{1}{2}''$	$1''$
	$1\frac{3}{4}$	$1\frac{1}{8}$
	2	$1\frac{1}{4}$
	$2\frac{1}{2}$	$1\frac{1}{2}$

LENGTHS OF LATTICE BARS FOR ORDINARY CHORDS AND POSTS.

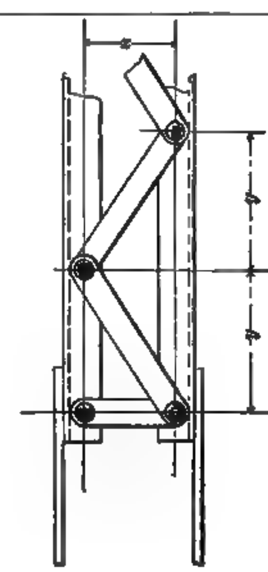
DISTANCE "x"	DISTANCE "y"	DISTANCE c to c	DISTANCE "x"	DISTANCE "y"	DISTANCE c to c
"	"	"	"	"	"
$4\frac{1}{2}$	6	$7\frac{1}{2}$	$14\frac{1}{2}$	$8\frac{1}{4}$	$16\frac{11}{16}$
5	6	$7\frac{13}{16}$	15	$8\frac{1}{2}$	$17\frac{1}{4}$
$5\frac{1}{2}$	$6\frac{1}{4}$	$8\frac{1}{4}$	$15\frac{1}{2}$	9	$17\frac{13}{16}$
6	$6\frac{1}{4}$	$8\frac{23}{32}$	16	$9\frac{1}{4}$	$18\frac{1}{2}$
$6\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{3}{16}$	$16\frac{1}{2}$	$9\frac{1}{2}$	$19\frac{1}{32}$
7	$6\frac{1}{2}$	$9\frac{9}{16}$	17	$9\frac{3}{4}$	$19\frac{17}{32}$
$7\frac{1}{2}$	$6\frac{1}{2}$	$9\frac{15}{16}$	$17\frac{1}{2}$	10	$20\frac{3}{32}$
8	7	$10\frac{3}{8}$	18	$10\frac{1}{4}$	$20\frac{11}{16}$
$8\frac{1}{2}$	7	11	$18\frac{1}{2}$	$10\frac{1}{2}$	$21\frac{9}{32}$
9	7	$11\frac{21}{32}$	19	11	$21\frac{13}{16}$
$9\frac{1}{2}$	7	$11\frac{15}{16}$	$19\frac{1}{2}$	$11\frac{1}{4}$	$22\frac{1}{2}$
10	7	$12\frac{7}{32}$	20	$11\frac{1}{2}$	$23\frac{1}{16}$
$10\frac{1}{2}$	7	$12\frac{5}{8}$	$20\frac{1}{2}$	$11\frac{3}{4}$	$23\frac{5}{8}$
11	$7\frac{1}{2}$	$13\frac{5}{16}$	21	12	$24\frac{5}{16}$
$11\frac{1}{2}$	$7\frac{1}{2}$	$13\frac{31}{32}$	$21\frac{1}{2}$	$12\frac{1}{2}$	$24\frac{7}{8}$
12	$7\frac{1}{2}$	$14\frac{5}{32}$	22	$12\frac{3}{4}$	$25\frac{7}{16}$
$12\frac{1}{2}$	$7\frac{1}{2}$	$14\frac{9}{16}$	$22\frac{1}{2}$	13	26
13	$7\frac{1}{2}$	15	23	$13\frac{1}{4}$	$26\frac{9}{16}$
$13\frac{1}{2}$	$7\frac{1}{2}$	$15\frac{7}{16}$	$23\frac{1}{2}$	$13\frac{1}{2}$	$27\frac{1}{8}$
14	8	$16\frac{1}{8}$	24	$13\frac{3}{4}$	$27\frac{25}{32}$

$$t = \frac{P}{7000w} + \frac{l}{27}$$

P = total compression carried by 1 Jaw.

LENGTHS OF LATTICE BARS FOR SMALL POSTS.

DISTANCE "x"	DISTANCE "y"	DISTANCE c to c	DISTANCE "x"	DISTANCE "y"	DISTANCE c to c
"	"	"	"	"	"
$4\frac{1}{2}$	$7\frac{3}{4}$	$8\frac{31}{32}$	8	$13\frac{1}{4}$	$15\frac{11}{16}$
5	$8\frac{1}{2}$	$9\frac{7}{8}$	$8\frac{1}{2}$	$14\frac{3}{4}$	$17\frac{1}{32}$
$5\frac{1}{2}$	$9\frac{1}{4}$	$10\frac{31}{32}$	9	$15\frac{3}{4}$	$18\frac{1}{8}$
6	$10\frac{1}{8}$	$12\frac{3}{32}$	$9\frac{1}{2}$	$16\frac{3}{4}$	$18\frac{23}{32}$
$6\frac{1}{2}$	$11\frac{1}{4}$	13	10	$17\frac{1}{4}$	$19\frac{13}{16}$
7	12	$13\frac{7}{8}$	$10\frac{1}{2}$	$18\frac{1}{4}$	$21\frac{1}{32}$
$7\frac{1}{2}$	13	15	11	19	$21\frac{13}{16}$



UPPER CHORDS.—The common chord section consists of two channels, latticed or laced on the under side, with a top or "cover plate." The same principles apply to this case as those already applied to posts. The size and spacing of lattice or lacing bars is the same, and the same rules hold good for stay plates.

For the common chord section, we may make use of Table IV., Chapter I., which gives the strength when the depth is known. We are thus saved the necessity of finding the radius of gyration, as described on page 350.

EXAMPLE.—*The upper chord of a bridge truss is 25 feet long, and is subjected to a strain of 292700 lbs. due to the dead load, and 253100 lbs. due to the live load. What should be the area of cross section?*

We suppose each chord member to be in the condition of a strut *fixed at both ends*, owing to the action of the splicing plates, etc. We must also assume the depth of chord. Here judgment and experience must aid us to make a good choice the first time, though any choice can be tested and its suitability determined.

Suppose we take the depth in this case at 15 inches, that being the greatest depth of channel rolled.

Then we have $\frac{l}{d} = \frac{300}{15} = 20$, and from Table IV., Chapter I., page 336, we have for flat ends, the crippling strength $= 36024$ lbs. The factor of safety is $4 + \frac{l}{20d} = 4 + \frac{300}{300} = 5$. Hence, by the "old method," the safe working stress is $\beta = \frac{36024}{5} = 7205$ lbs. per square inch. The total area required is then, $\frac{545800}{7205} = 75.75$ square inches.

If the channels are spaced 20 inches apart, back to back, and the cover plate is ⅜ inch thick, then, since by reference to *Carnegie*, page 98, we see that the width of flange will not be far from 3.6 inches for a 15 inch channel, the width of cover plate will not be far from $20 + 7.2 = 27.2$ inches, and its area will be about $27.2 \times \frac{3}{8} = 10.2$ square inches.

This will leave for the required area of the two channels $75.75 - 10.2 = 65.55$ square inches, or for each channel 32.77 square inches.

From *Carnegie* we see that this is far heavier than the heaviest single channel rolled. We must therefore build up our chord section in this case, by means of plates and angle irons. In this we may be guided by the principle of not having any plate less than ¼ inch or greater than ½ inch, or at most ⅝ inch in thickness.

We shall also find it advantageous to have a greater depth, and thus save material.

Let us take, therefore, the depth at 20 inches, then $\frac{l}{d} = \frac{300}{20} = 15$, and from Table IV. we have the crippling strength 37066 lbs. The factor of safety is 4.75, and hence $\beta = \frac{37066}{4.75} = 7803$ lbs. per square inch. The area required now is therefore $\frac{545800}{7803} =$ about 70 square inches.

If the spacing is as before, 20 inches, and our flanges 4 inches, we have width of top plate 28 inches, and area $= 28 \times \frac{3}{8} = 10.6$ square inches. This leaves for the built channels $70 - 10.5 = 59.5$ square inches, or for each channel, 29.75 square inches.

Let us take for the flanges, equal leg angle irons, 4'' by 4'', by ⅝ inch. The area of each is from *Carnegie*, page 107, $\frac{15.8 \times 3}{10} = 4.74$ square inches. For two, we have 9.48 square inches. This leaves for the web about 20 square inches. For a depth of 20 inches and thickness of ⅝'', the web would be 12.5 square inches. We have then 7.5 square inches remaining. We may rivet a flat plate to the bottom angle and make it 4'' by ½''. This would give 2 square inches more area, and leave 5.5 remaining. If now we add a side plate 12 inches by 7/16'', it will make up the area remaining and just fit in between the legs of the angles.

The chord then may be built up as follows: 1 top plate, 28'' × ⅜'', 2 web plates 20'' × ⅝'', 2 side plates 12'' × 7/16'', 4 angles 4'' × 4'' × ⅝'', 2 flats 4'' × ½''.

By the "new method," we have from Table IV., Chapter I., page 336, for $\frac{l}{d} = 15$, and flat ends, $\frac{1}{1 + c\frac{l^2}{d^2}} = \frac{37066}{38500} = 0.9627$, and hence $\beta = \frac{6500}{1 + c\frac{l^2}{d^2}}\left[1 + \frac{\text{min. } B}{\text{max. } B}\right] = 0.9627 \times 6500\left(1 + \frac{292700}{545800}\right) = 9613$ lbs. per square inch. The area required by the new method is therefore $\frac{545800}{9613} = 56.77$ sq. inches, instead of 70 sq. inches by the old method.

Using the same top plate, we have $56.77 - 10.5 = 46.27$, or 23.19 square inches for each channel. Taking angles 4'' × 4'' × ⅝'' for the flanges, we have $23.14 - 9.48 = 13.66$ square inches remaining. A web plate 20'' × ⅝'' will about cover this.

The chord then will consist of 1 top plate 28'' × ⅜'', 2 web plates 20'' × ⅝'', 4 angles 4'' × 4'' × ⅝''.

This section may now be tested by calculating the radius of gyration according to the principles of page 350, and using Table I., page 333.

The lattice work or lacing bars on the bottom are to be then spaced and dimensioned according to the rules on page 354.

Our example is for a very long bridge, and therefore very heavy chords. Ordinarily the size of channels required will fall within the limits of Carnegie's Table. At present prices of labor and material it is, however, cheaper to build up the chords by plates and angles than to roll heavy channels, and top chords are therefore usually built up.

If we use the straight-line formula, with Cooper's values, page 332, we have, taking $r = 7.5$, for the live load.

$\beta = 8000 - 30\frac{l}{r} = 6800$, and for the dead load, $\beta = 16000 - 60\frac{l}{r} = 13600$ lbs. The area required is therefore $\frac{292700}{13600} + \frac{253100}{6800} = 58.7$ sq. inches.

In using built-up chords the designer will find it indispensable to have on hand *Tables of Moments of Inertia*, by Frank Osborne, C. E. Eng. News Pub. Co., New York.

WIDTH OF UPPER CHORD AND TOP PLATE, AND THICKNESS OF TOP PLATE.—The width of top plate is determined by the conditions of the case. It must be wide enough to admit the posts and the main and counter ties.

The *least allowable width*, independently of these considerations, must be at least greater than the depth. The least allowable width of the top plate must be then equal to the distance between the channels, *plus* twice the width of the flange.

This least allowable width of the top plate may be taken at

$$w = \frac{7}{6}D + 1,$$

where D is the depth of channel, and w the width of plate in inches.

The least allowable *thickness* of top plate may be taken at $\frac{1}{4}''$ for depths of channel less than 8″. From 9 to 10 inches inclusive, $\frac{5}{16}''$. From 12 to 18 inches inclusive, $\frac{3}{8}''$. Above 20 inches, $\frac{7}{16}$ to $\frac{1}{2}''$. These thicknesses correspond to the least allowable width, as already given. Should the actual width exceed the least allowable by 50 per cent., we may add $\frac{1}{16}''$ to the thickness. If it exceeds by 75 per cent., we may add $\frac{1}{8}''$ to the thickness, as determined by the above rules.

DEPTH OF CHORD.—A little preliminary calculation will usually be necessary to fix upon a suitable depth for the top chord. As the depth ought to be constant from end to end of the bridge, and as the strain is much greater in the middle than at the ends of the truss, we must choose such a depth of channel as will allow of the necessary variation in thickness to meet the strain in centre and end panels.

If we find the area required in the end panel, then the depth which will give the least average area and allow for the area of centre panel and of end panel, will be the best depth to use.

EXAMPLE.—*Suppose the end upper panel is subjected to a strain of 47000 lbs. due to the dead load, and 46000 lbs. due to the live load, and the centre panel to a strain of 70000 lbs. due to dead load, and 54000 lbs. due to live load, what should be the depth of upper chord, if the panel length is 20 feet?*

Let us try 9 inch channels. The ratio $\frac{l}{d} = \frac{240}{9} = 26\frac{2}{3}$, and from Table IV., Chapter I., page 336, we have for flat ends, crippling strength = 17.153 tons = 34306 lbs. The factor of safety is 5.33, hence the safe working stress is $\beta = \frac{34306}{5.33} = 6470$ lbs. per sq. inch. The area required in the end panel, by the "*old method*," is then $\frac{93000}{6470}$ = about 14 square inches.

The minimum width of top plate, according to the rule just given, is, for 9″ channel 11 inches, and its thickness $\frac{5}{16}''$. Its area is then $11 \times \frac{5}{16} = 3.44$ square inches. This leaves $14 - 3.44 = 10.56$ for the channels, or 5.28 sq. inches for each channel. From *Carnegie*, page 99, we see that 9-inch channels will answer.

Let us see whether 9″ channels will give us enough area at centre. Here we have $\frac{124000}{6470} = 19.16$ sq. inches. Deducting 3.44 for the top plate, we have 7.86 sq. inches for each channel. This falls well within the limit of weight for 9 inch channels, and such a depth then will answer.

But we may perhaps choose a better depth. Let us try 10 inch channels. We have then $\frac{l}{d} = \frac{240}{10} = 24$, and from Table IV., $\beta = \frac{35034}{5.2} = 6737$ lbs. per sq. inch. This calls for an area in the end panel of $\frac{93000}{6737} = 13.8$ sq. in. The

area of top plate is $13 \times \frac{5}{16} = 4$ sq. inches. Deducting this, we have 4.9 for area of each channel. The lightest 10-inch channel is 5.25 sq. inches. If we use 10-inch channels we shall have a little excess in the end panel.

The *average area* for the 9 inch channels, is $\frac{5.4 + 7.86}{2} = 6.63$ square inches; for the 10 inch channels, it is $\frac{5.25 + 7.2}{2} = 6.27$ sq. inches. The 10 inch channels are then preferable.

Twelve inch channels will be found in like manner to call for 3.64 sq. inches at end, and 5.79 at the centre. No 12" channels are rolled as light as this. The lightest 12 inch channel, *of one weight only*, has 6 sq. inches cross section. We might therefore use this throughout the upper chord. It would give too great area throughout, but the average area would be only 6 sq. inches, a little less than for either 9 or 10" channels. There is also practical advantage in having all the chords of a size, as it makes all the splice plates and top cover plates of a size also, and secures economy in price, ease of erection, and uniformity of details.

By the "*new method*," we should proceed precisely as above, only the value of β would be determined from

$$\beta = \frac{6500}{1 + c\frac{l^2}{d^2}}\left(1 + \frac{\text{min. } B}{\text{max. } B}\right),$$

where $\frac{1}{1 + c\frac{l^2}{d^2}}$ can be found from Table IV., by dividing the crippling strength in lbs., as given by the Table, by 38500 for flat ends. By the straight-line formula we should also proceed precisely as above, only the value of β would be $\beta = 8000 - 30\frac{l}{r}$ for live load. and $\beta = 16000 - 60\frac{l}{r}$ for dead load, page 332.

COMPRESSION AND FLEXURE COMBINED.—The top chord of a deck bridge may have a load upon it due to a cross tie, between the panel points. It then acts as a beam as well as a strut.

For this case we have, page 328,

$$F = \frac{Mv}{\beta r^2} + \frac{S}{\beta},$$

where β is taken according to the "old" or "new" method, or straight-line formula for struts.

EXAMPLE.—*Suppose an upper panel to be subjected to compression of 30000 lbs. due to dead load, and 60000 lbs. due to live load, and to have a weight of 1 ton acting at the middle. If the bay is 15 feet long, what should be the area?*

Let us try 8 inch channels. The ratio $\frac{l}{d} = \frac{180}{8} = 22\frac{1}{2}$. For common chord section, flat ends, we have from Table IV., the crippling strength = 35482 lbs., and factor of safety = 5.11. By the "old method," $\beta = \frac{35482}{5.11} = 6943$ lbs. per sq. inch.

By the "new method" we have $\frac{1}{1 + c\frac{l^2}{d_1}} = \frac{35482}{38500} = 0.919$, and hence $\beta = 0.919 \times 6500\left[1 + \frac{30000}{90000}\right] = 7965$ lbs. per sq. inch.

In the present case $M = 1000 \times 7.5 \times 12 = 90000$ inch lbs., $v = 4$ inches. $r =$ not far from 3 inches, according to *Carnegie*, page 99.

Hence by "old method,"

$$F = \frac{90000 \times 4}{6943 \times 9} + \frac{90000}{6943} = 5.36 + 13 = 18.7 \text{ sq. ins.}$$

By the "new method,"

$$F = \frac{90000 \times 4}{7965 \times 9} + \frac{90000}{7965} = 4.7 + 11.3 = 16.3 \text{ sq. ins.}$$

The least allowable width of top plate is 8 inches, and thickness $\frac{1}{4}$". The area of top plate is then 2 sq. inches.

This leaves 16.7 sq. inches, or 8.35 sq. inches for each channel by the old method, and 14.3 sq. inches or 7.15 sq. inches for each channel by the new method. From *Carnegie*, page 99, we see that these channels can be rolled.

In the first case, then, we have two 8" channels, 27.32 lbs. per foot, 0.732 inches thickness of web, and 2.732 inches width of flange.

In the second case, we have two 8" channels, 23.3 lbs. per foot. 0.58 in. thickness of web, and 2.62 in. width of flange.

If we use the straight-line formula, with Cooper's values, page 332, we have $\beta = 6200$ lbs. for live load, and $\beta = 12400$ lbs. for dead load. Hence $F = \frac{30000 \times 4}{12400 \times 9} + \frac{30000}{12400} + \frac{60000 \times 4}{6200 \times 9} + \frac{60000}{6200} = 17.47$ sq. inches.

JAW PLATES.—When the flanges at the pin ends of compression members are cut away for the purpose of close packing, the webs of the channels remaining must be strengthened by "pin plates" or "jaw plates." These must give sufficient bearing on the pin. They must also have sufficient area as posts.

Their thickness as posts is determined by the formula

$$t = \frac{P}{7000w} + \frac{b}{27},$$

where P is the compression carried by one jaw in lbs., w = width of the jaw, b = length in inches from the centre of pin hole to the first rivet beyond the point at which the full section of the post begins, t = thickness in inches.

We give, in Figs. 221 and 222, Plates 11 and 12, details of upper chords and connections. The drawings explain themselves. These represent modern American practice.

In Fig. 221, Plate 11, we have the ordinary style of posts, formed of channels latticed or laced. Fig. 222, Plate 12, shows also the inclined-end posts or "batter braces," formed, like the chords, of latticed or laced channels, with top plate. It is designed precisely like the top chord. Figs. 233 and 234, Plate 14, show methods of riveting.

In Figs. 235 and 236 we have given details for light highway bridges. Such details are only allowable in light structures, and good modern practice would avoid bending the ties, as shown in Fig. 235. Fig. 237 shows the details for the ordinary Howe Truss.

Fig. 244, Plate 15, gives details of cross and wind bracing for a deck bridge. So, also, Figs. 245 and 246. These do not at present represent modern practice. Together with Figs. 223, 224, 225, 227, they may be considered as altogether superseded by later details, and possess only historical value. *No castings are permitted in modern bridges, for any purpose*, except for bed plates and for the machinery of draw spans. The ties would now be bolted to flanges, instead of passing through bearing blocks. These remarks apply to all of Plate 16 also. For best modern construction, the student should observe carefully, well-executed examples in the field, and sketch details. The recent editions of the illustrated albums of our best bridge companies will give much information. Also the reports of Geo. S. Morrison, C. E., upon the Bismarck Bridge, the Plattsmouth Bridge, and the Omaha Bridge. The illustrated albums of the various bridge companies are easily obtained upon application, for a small price, and many of them are excellently illustrated.

To attempt to give such details in a work of this character is to run the risk of becoming antiquated in a few years, as evidenced by Plates 13, 14, 15, and 16 of this work: most of the figures therein being already ancient history. It is necessary, therefore, to caution the student against taking them as examples of modern practice, and to refer him to the only source which is reliable, viz., the present practice of the best designers.

A careful and intelligent comparison of this, as he finds it, with the practice of a few years ago, as given in the figures referred to, may, however, be a valuable exercise, and not devoid of suggestion in the way of future improvement. A comparison of differences in practice and a consideration of the reasons for such differences is a most instructive exercise. These old examples of past practice have therefore been retained, as it is believed that they will still serve a useful end if properly used. The improvement of details is the constant aim of the designer, and the student should render himself familiar with the best practice attainable, and be on the alert to note new forms and improved methods.

PLATE 12.

Fig. 222

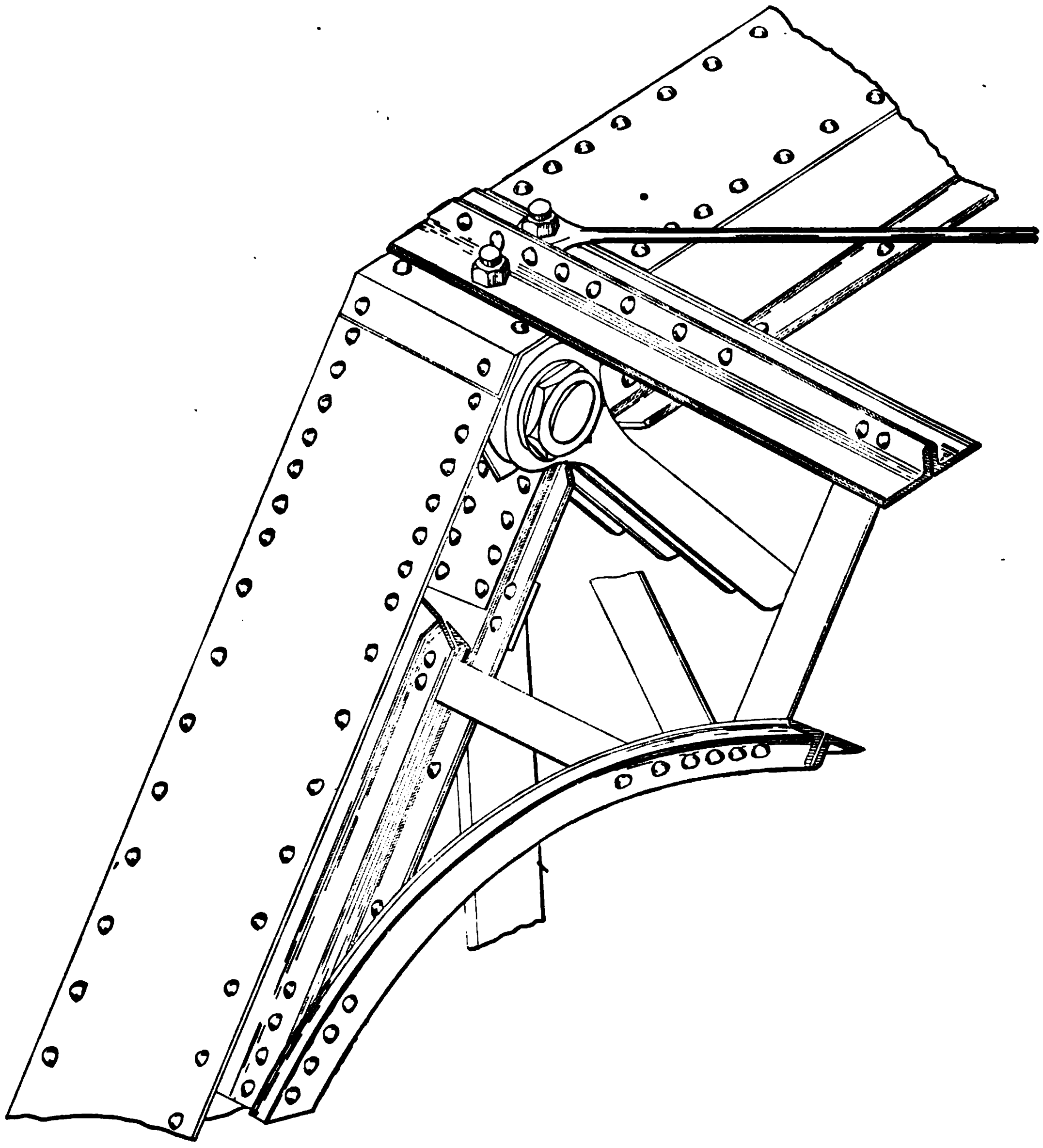


PLATE 13.

Fig.223

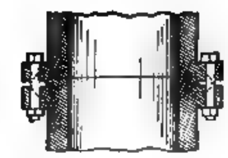


Fig.225

Fig.224

Fig.226

Fig.228

Fig. 227

Fig. 229

Fig.230

PLATE 14.

Fig. 235

Fig. 236

Fig. 237

Fig. 232

Fig. 233

Fig. 234

PLATE 15.

Fig. 244

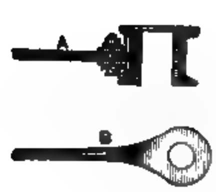


PLATE 16.

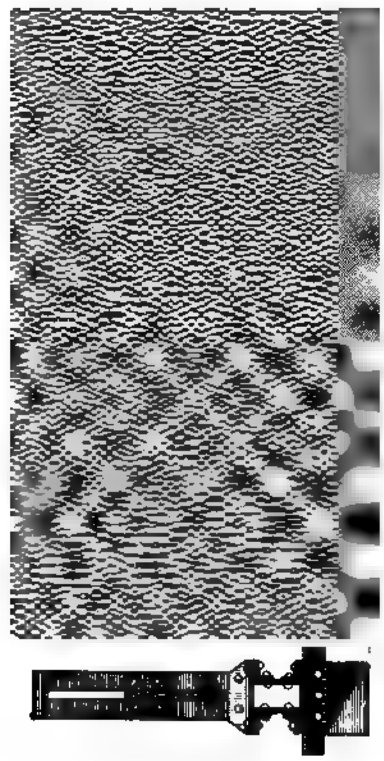
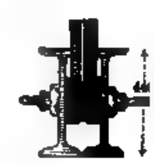


Fig.248



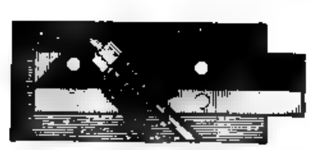


Fig.250

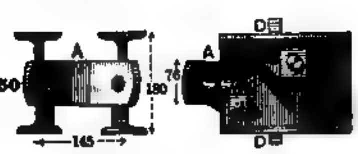


Fig.252

PLATE 17.

PLATE 18.

Fig. 255

CHAPTER IV.

PINS AND EYE BARS.

THE use of " pin connections " and " screw-end connections," is the characteristic of American bridge practice. Rivets are only used for such minor details as splice, cover, and re-enforcing plates, in the flanges of plate cross-girders and stringers, and their connections, and for stiffeners. The main connections of posts with chords, are by means of pins, whereas in England and on the Continent all the connections are usually riveted.

The price of labor, extent to which machine processes are employed, etc., are the main reasons which justify such diversity of practice. As regards the theoretical advantage of the two systems, the pin connection seems undoubtedly the best, and in this country has shown itself also the best practically.

Among the evident disadvantages of rivet connections, as compared with pin connections, we may mention the impossibility of getting the strains to act along the axis of the members. An indefinite amount of twisting is thus caused at each joint, which is entirely absent in pin joints. The practice of distributing rivets assumes that each takes its equal share. But in reality the first rivets must take a greater amount, and only the elasticity of the plates brings the others into play. With many rivets it is thus questionable whether some of them act at all, and it is impossible to determine to what extent. Rivet holes, even when laid out with the greatest care, cannot be made to always coincide, and the holes are then forced to match by the use of the " drift-pin," which distorts the holes and injures and mutilates the material. Imperfect workmanship cannot be avoided, even by the most careful supervision, and a rivet hole imperfectly filled presents the same appearance as a perfect one. Rivet heads will snap off under the contraction of the rivet when cooled, and often when carelessly driven by hand will shrink away from the hole without filling it at all. To guard completely against all these sources of imperfection would seem hopeless. To guard sufficiently requires that the greatest care and every precaution be taken to eliminate them. Thus, in Europe the holes are drilled in the plates while clamped in position, and machine riveting employed.

Such precautions mean increase of expense, and in this country cannot be employed in competition with pin connections. Hence, in this country at least, such precautions are not taken, and, whatever may be the case abroad, here our riveted bridges are inferior to the pin connected.

For this reason, our American practice would seem the best adapted to the circumstances, which uses pins for all the important main connections, and only employs rivets for those details, such as splice and cover plates, etc., whose office is simply to keep the members in line, or for such connections as the flanges of plate girders, where rivets are unavoidable. In such cases, as far as practicable, the riveting should be done in shop and not in the field. Field riveting is sure to be poor, and open to all the objections named.

There are no objections to the pin joint from a theoretical standpoint, and the only practical ones urged are the difficulty of securing a tight fit, and consequent expense, and the fact that the rupture of a single joint destroys the structure. The practical objections are practically answered by machine-made bearings and connections of the nicest fit, and by existing structures both economical and safe, which have given American engineers the reputation of being among the best bridge-builders.*

THEORY OF PINS AND EYE-BARS.—THICKNESS OF RE-ENFORCING PLATES.—The bearing resistance of the pin should equal the greatest pressure upon it due to any plate through which it passes.

If d is the diameter of pin in inches, $t =$ the thickness of any plate through which it passes in inches, then dt is the bearing area in square inches. Let C be the working compressive stress per square inch, then dtC is the bearing resistance of the pin. This should equal the stress transmitted through the plate, or

$$dtC = \text{stress}.$$

We may take C at 6.25 tons. The stress transmitted is always known. If the stress is *one ton*, the requisite bearing area is

$$dt = \frac{1}{6.25}, \text{ and hence we have}$$

$$\textit{lineal bearing on pin, in inches per ton of stress} = \frac{1}{6.25d}, \quad \ldots \quad (1)$$

From equation (1), having given the diameter, we can find the corresponding lineal bearing or thickness of plate, for every ton of stress to be transmitted. We have only to multiply this by the number of tons stress in any case, to find the requisite thickness of plate in any case. This equation is therefore to be applied in finding the thickness of re-enforcing plates.

EXAMPLE.—*The stress transmitted through a 12-inch post channel is 55500 lbs. The thickness of web is $\frac{6}{10}$ths of an inch, and diameter of pin is 3 inches. What thickness of re-enforcing plate is required?*

The thickness for each ton is $\frac{1}{6.25d} = \frac{1}{6.25 \times 3} = 0.0533$ inches. For $\frac{55500}{2000} = 27.75$ tons, we should have then a thickness of $0.0533 \times 27.75 = 1.48$ inches. As the channel web is 0.6 inch, this leaves $1.48 - 0.6 = 0.88''$ for the thickness of re-enforcing plate. Two plates, $\frac{7}{16}''$ thick upon each side of channel web, will then give the required thickness.

The thickness for each ton of stress, for different diameters, has been found from the formula (1), and is given in the Table, page 377, which follows.

LEAST DIAMETER OF PIN.—If t is the thickness of eye-bar, and w its depth, then tw is the area of cross section of eye-bar. If β is the working tensile stress for which the bar has been dimensioned, then $tw\beta$ is the stress transmitted from the bar to the pin.

* "The typical American railroad bridge is a skeleton structure, pin-connected at all the principal articulations. Its essential characteristics, in addition to being connected by pins," are stated by Cooper as follows: "*First*—So formed as to reduce all ambiguity of strains to a minimum. *Second*—Concentration of parts. *Third*—Facility of manufacture. *Fourth*—Perfection of lengths and fitting of all the members, so as to reduce to a minimum all riveting or mechanical work in the field. *Fifth*—Readiness with which the individual members can be assembled during erection."—*Trans. Am. Soc. C. E.*, July, 1889.

Now if d is the diameter of pin, and if the thickness of head is equal to the thickness of bar, t, we have td for the bearing of pin, and tdC for its bearing resistance.

We must have then for the smallest admissible value of d,

$$tdC = tw\beta, \qquad \text{or} \qquad d = \frac{\beta}{C}w.$$

The ratio of the tensile working stress β to the compressive working stress C, or $\frac{\beta}{C}$, may be taken at $\frac{3}{4}$. We have then for the *least diameter of pin admissible*,

$$d = \tfrac{3}{4}w. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The diameter of pin may need to be much greater than this, but it cannot be less, *unless the thickness of head of eye-bar is made greater than the thickness of bar itself.*

When this is the case, if t_1 is the thickness of bar and t the thickness of head, we have

$$tdC = t_1w\beta, \qquad \text{or} \qquad d = \tfrac{3}{4}\frac{t_1}{t}w,$$

for the least diameter of pin, and $\qquad t = \dfrac{3wt_1}{4d},$

for thickness of head when diameter is given.

It is seldom desirable and often impossible to use this smallest value $d = \frac{3}{4}w$ for lower chord bars. It is well simply to note it as a limit below which we cannot go without increasing the thickness of heads. For diagonals, counters, and hip verticals, the head must usually be thicker than the bar.

EXAMPLE 1.—*If the depth of eye-bar is 10 inches, what is the least diameter of pin which can be used without thickening the head of eye-bar?* *Ans.* $d = 7\frac{1}{2}$ *inches.*

EXAMPLE 2.—A hip vertical bar is 8" by $\frac{7}{8}$". If the diameter of pin passing through it at the upper end is $4\frac{5}{8}$", what should be the thickness of the head?

The least diameter allowable without thickening the head is $\frac{3}{4}w = \frac{3}{4}\,8 = 6''$. As the pin in this case is less than this, the head must be thicker than the bar. The thickness of head is $t = \dfrac{3wt_1}{4d} = \dfrac{3 \times 8 \times \frac{7}{8}}{4 \times 4\frac{5}{8}} = \dfrac{21}{18\frac{1}{2}} = 1\frac{5}{37}''$.

EXAMPLE 3.—*A main tie-bar is 5" by $1\frac{9}{16}$". If the diameter of pin is $3\frac{5}{8}$", what should be the thickness of head at that end?*

Here $\frac{3}{4}w = \frac{3}{4} \times 5 = 3\frac{3}{4}$, therefore the head must be thicker than bar. We have for thickness of head $t = \dfrac{3wt_1}{4d} = \dfrac{3 \times 5 \times 1\frac{9}{16}}{4 \times 3\frac{5}{8}} = 1.616''$.

EXAMPLE 4.—A counter rod is 1" diameter. What should be the thickness of head, if the pin is $3\frac{3}{4}$"?

We must replace here t_1w in the formula by $\dfrac{\pi w^2}{4}$, where w is the diameter of the rod. We have then

$$w = \frac{3\pi w^2}{16 \times 3\frac{3}{4}} = 0.157''.$$

If the head, then, is a loop of same diameter as rod, it will afford ample bearing.

SIZE OF PIN.—The pin should be treated as a beam which fails by flexure. The size as thus determined is greater than the diameter required for safe bearing or shearing resistance.

From the theory of flexure, page 246, we have

$$M = \frac{RI}{v} = \frac{RI}{r},$$

where r is the radius of pin, and R is the working stress in the outer fibre, and $I =$ the moment of inertia $= \frac{\pi r^4}{4}$. Hence the moment of resistance of the pin is

$$M = \frac{\pi R d^3}{32}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From this formula we may calculate the bending moment or moment of resistance of the pin for different diameters. The usual value for R is 15000 lbs. for iron and 20000 lbs. for steel. We have given the value of M for these two cases in the Table, page 377.

Now to find the requisite diameter in any case, we must find the maximum resultant moment M of all the forces acting upon the pin. From the Table, we can then pick out a diameter which gives a bending moment or moment of resistance equal to this maximum resultant moment.

It remains therefore to show how to find the maximum resultant moment M of all the forces acting upon the pin.

We have represented in Figs. 202 (*c*) and 202 (*d*), a pin joint in elevation and plan. See also Plate 8, Fig. 205.

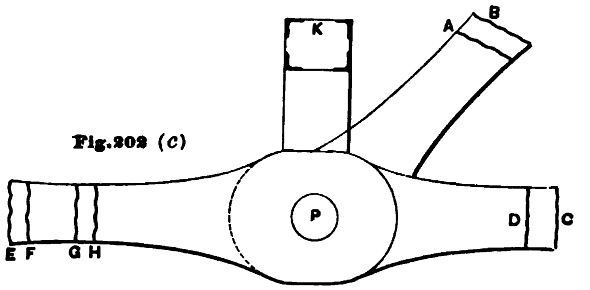


Fig. 202 (*c*)

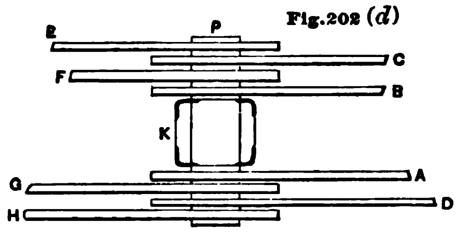


Fig. 202 (*d*)

K is the post, in this case composed of plate and angle irons, which takes compression only. A and B are the two main ties. The counter, if any, would be attached to centre of pin, but as the greatest stress on the pin will be for a full load, there will be no strain for this loading in the counter, and ***hence it may always be omitted in finding the size of pin at any joint in either upper or lower chord.*** The main ties, A and B, come next to the post upon each side. The chord bars, G, H, come next with the bar D between. The same arrangement holds on the other side of the middle.

The main ties are the only inclined members. All the others are either horizontal or vertical. The sum of the horizontal forces upon one side of the pin must be equal to the sum of all the horizontal forces upon the other side of the pin, including, of course, in this sum the horizontal component of the strain in the ties. The vertical component of the strain in the ties is equal to the post compression. These components may then be easily found. Thus the entire horizontal stress upon one side of the pin (the left in Fig. 202) is equal to the area of all the bars on that side $\times$ by the working stress β for which the bars were dimensioned. The entire stress upon the other side must be the same. The ***horizontal component of the tie stress***, may then be found by subtracting the sum of the stresses in the chord bars upon the main tie side, from the sum of the stresses in the chord bars on the other side. Thus in Fig. 202 (*d*), we multiply the area of D by β and subtract from the area of $H \times \beta +$ the area of $G \times \beta$. The result is the horizontal component of the stress in A. The ***vertical component*** is equal to the half post compression ***calculated for full loading.***

In general, for any pin, we must resolve the stress in every member through which that pin passes, *as found for full loading*, into its vertical and horizontal components. *The stress in each member is considered as acting along the centre line or axis*, and hence the point of application of each vertical and horizontal component is *at the centre of the bearing of the corresponding member.*

Let M_H be the maximum moment of all the horizontal stresses, and M_V the maximum moment of all the vertical stresses. Then the resultant moment is

$$M = \sqrt{M_H^2 + M_V^2},$$

and the size of pin required may be found from the Table, page 377, by taking that size whose bending moment is equal to M, or from the formula

$$d = \sqrt[3]{\frac{32M}{\pi R}}.$$

It remains to find M_H and M_V, or the maximum moment of all the horizontal forces and the maximum moment of all the vertical forces.

Let the horizontal forces or chord bar stresses acting upon the pin *on one side of the centre* be P_1, P_2, P_3, P_4, etc., the *odd* indices P_1, P_3, etc., acting in one direction, and the *even* indices P_2, P_4, etc., acting in the other direction. Let l_1 be the distance between centres of bearing of P_1 and P_2, l_2 the distance from P_2 to P_3, etc. Now the maximum moment will be at the point of application of some one of the forces. It is therefore easily found by trial.

Thus the moment at P_2 is P_1l_1. Add to this $(P_1 - P_2)l_2$, and we have the moment at P_3. Add again $(P_1 - P_2 + P_3)l_3$, and we have the moment at P_4, and so on. The greatest of all these is the moment required. Since all the forces upon one side, P_1, P_3, P_5, etc., are equal to all upon the other, P_2, P_4, P_6, etc., they will reduce to a couple at each end of the pin, and hence the moment at any point beyond the last force, that is, between the two inside horizontal forces (A and B in Fig. 202) is constant. We have only then to find the greatest moment by trial as above. To find M_V, we have simply to find the half post compression for full loading, and multiply by the distance between the centres of bearing of tie and post.

CHORD PACKING.—By means of washers, any two members may be separated and kept at any distance, so that the ties and posts may be in vertical planes. It is evident that a skilful packing of the bottom chord may diminish the value of M_H, and hence the size of pin. We should, in general, so arrange the packing that the points of application of the resultant on each side may as nearly as possible coincide. As the chord bars usually go in pairs of equal size, this is not difficult to arrange.

Thus, if we have two chord bars, P_1 and P_3, on one side, and one bar P_2 between them on the other side, with a tie P_4 beyond P_3, the distances being l_1, l_2, l_3; then the distance of one resultant from P_1 is $\frac{P_3}{R}(l_2 + l_1)$, and of the other, $\frac{P_4}{R}(l_3 + l_2) + l_1$. Equating the two and putting $R = P_1 + P_3$, we have, when the resultants coincide,

$$l_2 = \frac{P_1l_1 + P_4l_3}{P_3 - P_4}.$$

In such a case we should pack the first two bars snug, and the other bar and tie as close as possible, thus making l_1 and l_3 as small as circumstances allow. The distance l_2 can then be found.

If $P_1 = P_3 = 10$ and $P_2 = 15$, then $P_4 = 5$, and if $l_1 = 2$ and $l_3 = 3$, we have

$$l_4 = \frac{20 + 15}{10} = 3.5.$$

See Plate 24 at end of Work for this detail.

SIZE OF PIN AT CENTRE OF LOWER CHORD.—By an intelligent application of the preceding principles, we can find the size of pin at any joint. The application, however, admits of modification in special cases.

The largest pin in the lower chord will be at the centre for an even number of panels, or at the first joint right or left of the centre for an even number of panels. The chord bars are fully strained by a full load at every panel point. But for such a loading the post compression at the centre is very small, and as the tie can always be packed quite close to the post, the moment M_V can be disregarded.

We have, therefore, for the centre joint in the bottom chord, simply

$$M = M_H.$$

For ordinary spans all the pins in both upper and lower chord, except the pin at the hip, are made of the same size. In general, then, two calculations of size for *hip* and *centre of the lower chord* are sufficient. If we wish, however, to find the size of all the pins we may calculate the size of pin at end, at first joint or hip vertical, and at second joint and centre, and interpolate between these last for intermediate joints of the lower chord. For the upper chord, we may calculate the pin at the hip, at the first joint, and at the centre, and interpolate between the last two for intermediate joints of the upper chord. This is, as we have said, unnecessary in practice. The pins being so important, an excess of strength is desirable, and hence only two sizes are usually used, one for the *hip*, and the other for *all the other* joints, top and bottom. We shall, however, in what follows, illustrate the method of calculation very fully for any pin.

PRACTICAL SIZES FOR PINS.—PRACTICAL HINTS.—As we see by *Carnegie*, page 50, pins are furnished only in sizes differing either by $\frac{1}{4}$ or $\frac{1}{8}$ inch, and there are no intermediate sizes. All sizes are therefore an even number of 16ths.

When the size of a pin is calculated, we should always order it at least $\frac{1}{16}$ inch larger, in order that it may be turned down to exactly fit the hole.

We must therefore add $\frac{1}{16}''$ to the calculated size, and if this gives an even number of 16ths, it can usually be ordered. If not, we must order it $\frac{1}{16}$ larger still.

Thus, if the size of a pin is found by calculation to be $4\frac{3}{8}''$, it should be at least $4\frac{7}{16}''$, but from *Carnegie* we see that $4\frac{7}{16}$ is not rolled. We must therefore order $4\frac{1}{2}''$, and turn it down to fit the hole.

If the calculated size is $3\frac{1}{2}''$, it should be at least $3\frac{9}{16}$. But we see from *Carnegie* that only $3\frac{1}{2}$ and $3\frac{3}{4}$ are rolled. We must therefore order $3\frac{3}{4}''$, and turn down.

In general, pins in practice are between $4''$ and $6''$, and as these sizes are furnished at intervals of $\frac{1}{8}$ inch, we have, in all practical cases, between 4 and 6 inches, simply to add $\frac{1}{16}''$ to the calculated size, and if this gives an even number of 16ths, it can be ordered; if an odd number of 16ths, increase by $\frac{1}{16}$ inch.

The following Table gives about the sizes of pins used in practice. The pins should not be smaller than given in this Table, but if necessary can be made larger.

It will be noticed that the sizes given are all odd sixteenths. This is to allow turning off of $\frac{1}{16}$ as noticed above.

MINIMUM SIZES OF PINS.

Span in ft.	End Pins.	Intermediate Pins.	Span in ft.	End Pins.	Intermediate Pins.
75 to 85	$3\frac{7}{16}''$	$2\frac{15}{16}''$	140 to 155	$5\frac{1}{16}''$	$4\frac{9}{16}''$
85 " 95	$3\frac{11}{16}$	$3\frac{3}{16}$	155 " 175	$5\frac{5}{16}$	$4\frac{13}{16}$
95 " 105	$4\frac{1}{16}$	$3\frac{7}{16}$	175 " 200	$5\frac{9}{16}$	$4\frac{13}{16}$
105 " 115	$4\frac{5}{16}$	$3\frac{11}{16}$	200 " 225	$5\frac{13}{16}$	$4\frac{13}{16}$
115 " 125	$4\frac{9}{16}$	$4\frac{1}{16}$	225 " 250	$6\frac{1}{16}$	$5\frac{1}{16}$
125 " 140	$4\frac{13}{16}$	$4\frac{5}{16}$	250 " 300	$6\frac{9}{16}$	$5\frac{5}{16}$

DOUBLE TRACK.

Span in ft.	End Pins.	Intermediate Pins.
150′	$6\frac{9}{16}''$	$5\frac{5}{16}''$
140	$6\frac{1}{16}$	$5\frac{1}{16}$
130	$5\frac{13}{16}$	$4\frac{13}{16}$

Pin holes are bored about $\frac{1}{40}$ of an inch (0.025) larger than the pin. The size of pin taken from this Table must be tested at every joint, as illustrated by the following examples. We should not use less size than given by the Table, but may use larger. If, however, we find a larger pin required, we may very often reduce the required size, by re-arranging the chord bars, or by increasing their number. Only two sizes are used for end and intermediate pins.

Small pins should be tested for shear as well as bending, but in general, if the pin is safe against bending and bearing, it will be safe against shearing.

In finding the maximum bending moment, the counters may always be omitted, as they are not strained by a full load. The horizontal component of the strain in the tie bars at any joint is the difference in the strains of the chord bars on each side, and the vertical component is the post strain.

Chord bars are frequently not allowed to be placed next to each other in the same direction, owing to the difficulty of painting between them. They are, therefore, in couples, one in one direction and the next in the other direction. The lighter bar is always placed outside, so as to make the first moment, which is often the greatest, as small as possible.

All vacant spaces between pins should be filled by cast or wrought iron fillers.

In packing chords, and figuring the length of pin, it is customary to allow $\frac{1}{16}$ of an inch clearance for every thickness of metal on pin.

A cast-iron filler is never used for a space less than $\frac{15}{16}$ of an inch, after all clearances are allowed for. Over $\frac{15}{16}$ of an inch, we may use cast-iron fillers, and allow $\frac{3}{16}''$ for clearance, on account of the roughness of the casting.

If we use wrought-iron fillers, it is usual to allow $\frac{1}{16}''$ for clearance.

The ends of the pin are smaller in diameter than the pin, have a thread cut on them, and a hexagonal nut is screwed on, as shown in Fig. 268, Plate 20. The nut overlaps the shoulder of the pin $\frac{1}{4}''$ on each end, making $\frac{1}{2}''$ to be added to the figured length of pin, including all clearances, in order to find the length from shoulder to shoulder.

The following examples show how to test for size of pin at various joints.

EXAMPLE 1.—*In the centre panel of a bridge truss, we have 4 chord bars 7" by 1½", two at one end of pin and two at the other, with post between. In the next panel we have also 4 bars, 7" by 1", two on one side and two on the other side of post. We have also, on each side of centre of pin, a tie 1" thick. The tie is packed close to the inner re-enforcing plate of post, making the clearance between it and the next chord bar 1½". The chords are all packed snug, the lightest one on the outside, then a heavy and light one alternately. If the working stress $\beta = 10000$ lbs. per square inch, what size pin is required?*

The area of each chord bar on one side is $7 \times 1\frac{1}{2} = 10.5$ sq. inches, and on the other side $7 \times 1 = 7$ sq. inches. We have, then, $P_1 = P_3 = 7 \times 10000 = 70000$ lbs., and $P_2 = P_4 = 10.5 \times 10000 = 105000$ lbs. The horizontal component of the tie stress is $2 \times 105000 - 2 \times 70000 = 70000 = P_5$. The distances apart are $l_1 = l_2 = l_3 = \frac{1}{2}(1\frac{1}{2} + 1) = 1\frac{1}{4}"$ and $l_4 = \frac{1}{2}(1\frac{1}{2} + 1) + 1\frac{1}{2} = 2\frac{3}{4}"$.

Then the moment at P_2 is $P_1 l_1 = 70000 \times 1\frac{1}{4} = 87500$ inch lbs.

at P_3 we have $87500 + (P_1 - P_2)l_2 = 87500 - 43750 = 43750$ inch lbs.,

at P_4 we have $43750 + (P_1 - P_2 + P_3)l_3 = 43750 + 43750 = 87500$ inch lbs.,

at P_5 we have $87500 + (P_1 - P_2 + P_3 - P_4)l_4 = 105000$ inch lbs.

The maximum moment then is at P_5 and equal to

$$M = M_H = 105000 \text{ inch lbs.}$$

From the Table, page 377, we see that this will require an iron pin of about $4\frac{1}{4}"$ diameter.

But for a bar 7" deep, we have already seen that if the diameter of pin is less than $\frac{3}{4}w = 5\frac{1}{4}"$ in this case, the head must be thickened for safe bearing. If, then, we use this diameter of $4\frac{1}{4}"$, we have (page 379) for the thickness of head,

$$t = \frac{3wt_1}{4d} = \frac{3 \times 7 \times 1}{17} = 1\frac{4}{17}", \text{ and}$$

$$t = \frac{3 \times 7 \times 1\frac{1}{2}}{17} = 1\frac{7}{8}",$$

for the thickness of heads of eye-bars. These thicknesses would increase the moment and make a new determination of the size of pin necessary.

If we take the diameter at $5\frac{1}{4}"$, the heads need not be thicker than the bars. *We should always make this test for bearing.* The pin can be ordered $5\frac{3}{8}"$ commercial size.

EXAMPLE 2.—*Suppose the same arrangement as in the preceding example, but the bars to be 5" by 1⅜" and 5" by 1¼". The tie is $\frac{15}{16}"$ thick, and centre distance of its bearing from bearing of adjacent chord 1½". The outside bar is then 1¼", the next 1⅜", the next 1¼", the next 1⅜", and finally, with a clearance of 1½", comes the tie. The chord bars are packed snug. If the working stress $\beta = 10000$ lbs., what size pin is required?*

The area of each bar on one side is $5 \times 1\frac{3}{8} = 6\frac{7}{8}$ sq. inches, and on the other side $5 \times 1\frac{1}{4} = 6\frac{1}{4}$ sq. inches. Putting the lightest outside and alternating, we have $P_1 = P_3 = 6\frac{1}{4} \times 10000 = 625000$, and

$$P_2 = P_4 = 6\frac{7}{8} \times 10000 = 68750 \text{ lbs.} \quad P_5 = 2 \times 68750 - 2 \times 62500 = 12500 \text{ lbs.}$$

The distances are $l_1 = l_2 = l_3 = \frac{1}{2}(1\frac{1}{4} + 1\frac{3}{8}) = 1\frac{5}{16}$ and $l_4 = \frac{1}{2}(1\frac{3}{8} + \frac{15}{16}) + 1\frac{1}{2} = 2\frac{21}{32}"$.

The moment at P_2 is $P_1 l_1 = 62500 \times 1\frac{5}{16} = 82031$ inch lbs.

At P_3 we have $82031 + (P_1 - P_2)l_2 = 73828$ inch lbs.

At P_4 we have $73828 + (P_1 - P_2 + P_3)l_3 = 147656$ inch lbs.

At P_5 we have $147656 + (P_1 - P_2 + P_3 - P_4)l_4 = 114453$ lbs.

The maximum moment is then at P_4, and is equal to 147656. From the Table, page 377, this calls for a pin $4\frac{3}{4}"$ diameter. The least allowable diameter is $\frac{3}{4}w = 3\frac{3}{4}"$. The heads of bars do not require, therefore, to be thickened, and $4\frac{3}{4}"$ diameter may be taken. This gives $4\frac{7}{8}"$ commercial size.

SIZE OF PIN AT SECOND LOWER JOINT FROM END.—At this joint we must take into account M_V or the moment of the vertical forces. We have then

$$M = \sqrt{M_H^2 + M_V^2}.$$

EXAMPLE.—*Suppose we have 4 chord bars 4" by $1\frac{5}{16}"$ on one side, and on the other 2 chord bars 4" by $1\frac{7}{16}"$. The ties are $1\frac{5}{16}"$ thick. The tie is packed close to the post channel, the thickness of which, including the re-enforcing plate, is ⅞". The bars are packed snug. The vertical compression in the half post is 40000 lbs. for full loading. What is the size of pin required, taking the working stress β at 10000 lbs.*

We have here at each end of pin, 2 chord bars on one side, and one bar between them on the other. Then

$P_1 = P_3 = 4 \times 1\frac{3}{16} \times 10000 = 47500$, and $P_2 = 4 \times 1\frac{7}{16} \times 10000 = 57500$. The horizontal component of the tie stress is $P_4 = 2 \times 47500 - 57500 = 37500$ lbs.

The distances are $l_1 = l_2 = \frac{1}{2}(1\frac{3}{16} + 1\frac{7}{16}) = 1\frac{5}{16}''$, $l_3 = \frac{1}{2}(1\frac{3}{16} + 1\frac{9}{16}) + \frac{7}{8} = 2\frac{1}{4}$.

We have, then, at P_2 the moment $P_1 l_1 = 47500 \times 1\frac{5}{16} = 62344$ inch lbs.

At P_3 we have $62344 + (P_1 - P_2)l_2 = 49219$ inch lbs.

At P_4 we have $49219 + (P_1 - P_2 + P_3)l_3 = 133594$ inch lbs.

The maximum horizontal moment then is $M_H = 133594$ inch lbs. $= 66.797$ inch tons.

The vertical compression in post is 40000 lbs. Its lever arm is $\frac{1}{2}(1\frac{9}{16} + \frac{7}{8}) = 1\frac{7}{32}$. Hence $M_V = 40000 \times 1\frac{7}{32} = 48750$ inch lbs. $= 24.375$ inch tons.

The resultant moment is

$$M = \sqrt{M_H^2 + M_V^2} \times \sqrt{(66.8)^2 + (24.4)^2} = 71.11 \text{ inch tons} = 142220 \text{ inch lbs.}$$

This calls for a pin $4\frac{5}{8}''$ diameter, or $4\frac{3}{4}''$ commercial size.

The least diameter allowable is $\frac{3}{4}w = 3''$. Hence the bearing is abundant.

SIZE OF PIN AT FIRST LOWER JOINT FROM END AND AT END.—At this joint there are no ties or counters. We have the pin passing through the chord bars and hip vertical only. The chord bars on each side are equal and equal in number. The horizontal moment, then, is simply the stress on either side of pin at one end of pin, multiplied by the thickness of a chord bar.

If the cross-girder is riveted to the hip vertical above the pin, there is no vertical moment. If it is hung on floor-beam hangers, the vertical moment is the load supported by a hanger × by the distance from centre of bearing of a hanger to centre of bearing of hip vertical.

EXAMPLE 1.—*Suppose we have 2 bars in the first two panels, 4 by $1\frac{7}{16}''$, or 4 bars in all, one pair at one end of pin and one pair at the other. If the cross-girder is riveted to the hip vertical, and the working stress $\beta = 10000$ lbs., what size of pin is required?*

The stress on one side of pin at one end, in one direction is $P_1 = 4 \times 1\frac{7}{16} \times 10000 = 57500$ lbs., and the stress on the other side at one end, in the other direction, is $P_2 = 57500$ lbs. The distance $l_1 = 1\frac{7}{16}''$. Hence the horizontal moment is $M_H = 57500 \times 1\frac{7}{16} = 82656$ inch lbs. From the Table, page 377, this calls for a pin $3\frac{7}{8}''$. The least allowable for bearing is $\frac{3}{4}w = \frac{3}{4} \times 4 = 3''$. Hence $3\frac{7}{8}''$ can be used.

This diameter may be used also for the end pin.

EXAMPLE 2.—*Suppose at the same joint the load sustained by a beam hanger to be 32000 lbs., and the distance from centre of bearing of a beam hanger to centre of hip vertical $1\frac{1}{2}''$.*

Then

$$M_V = 32000 \times 1\frac{1}{2} = 48000 \text{ inch lbs.} = 24 \text{ inch tons.}$$

We have already found $M_H = 82656$ inch lbs. $= 41.328$ inch tons.

Hence, $$M = \sqrt{M_H^2 + M_V^2} = \sqrt{(41.33)^2 + (24)^2} = 47.79 \text{ inch tons} = 95580 \text{ inch lbs.}$$

This calls, from Table page 377, for a 4″ pin, or $4\frac{1}{8}''$ commercial size. The least allowable pin is $\frac{3}{4}w = 3''$, hence we need not increase thickness of head.

For the end pin, we have the diameter given in the preceding example.

SIZE OF PIN AT ANY INTERMEDIATE TOP CHORD JOINT.—Any pin in the top chord is acted upon simply by the full stress of the main ties through which it passes. The horizontal component of the tie stress gives the chord stress, and the vertical component the post stress. The main ties are packed close to the post end on the inside of post, and the counters, if any, between the main ties.

The horizontal moment, therefore, is the horizontal component of the stress in one main tie, multiplied by the distance between the tie and *chord bearings*.

The vertical moment is the vertical component of the stress in one main tie, multiplied by the distance between the tie and *post bearings*.

EXAMPLE.—*Suppose we have two main ties 5″ by $1\frac{1}{8}$″, the distance from centre of tie bearing to centre of chord bearing being $2\frac{5}{16}$″, and the distance from centre of tie bearing to centre of post bearing being $1\frac{1}{4}$″. If the working stress $\beta = 10000$ lbs., and the angle of tie with vertical 33° 11′, what size of pin is required?*

We have sin 33° 11′ = 0.547, and cos 33° 11′ = 0.837. The horizontal component of the tie stress is then $5 \times 1\frac{1}{8} \times 10000 \times 0.547 = 30769$ lbs., and the vertical component is $5 \times 1\frac{1}{8} \times 10000 \times 0.837 = 47081$ lbs.

Hence the horizontal moment is $M_H = 30769 \times 2\frac{5}{16} = 71153$ inch lbs. = 35.576 inch tons, and the vertical moment is $M_V = 47081 \times 1\frac{1}{4} = 58851$ inch lbs. = 29.425 inch tons.

The maximum moment then is

$$M = \sqrt{M_H^2 + M_V^2} = \sqrt{(35.6)^2 + (29.4)^2} = 46.17 \text{ inch tons} = 92340 \text{ inch lbs.}$$

From the Table, page 377, this calls for a pin 4″ diameter, or $4\frac{1}{8}$″ commercial size.

SIZE OF PIN AT HIP JOINT.—At the hip joint we have no post, but simply one, or at most two, hip verticals. The hip vertical is at the centre of pin, if there is but one, or packed as close to tie as possible on each side, if there are two. In either case, the pressure upon the chord bearing, due to the stress in the hip verticals, is *one half* of the full panel load for the truss. We have also the vertical component of the stress in a main tie, or if two ties meet at the hip on each end of pin, as is the case for a *double system*, then the sum of the vertical components of each. The vertical moment is then found as for a beam supported at the ends, with given vertical forces at given points. The horizontal moment is as before the horizontal component of the tie stress multiplied by the distance between the chord and tie bearing.

EXAMPLE.—*Suppose at the hip we have two main ties, 5″ by $1\frac{9}{16}$″, and one hip vertical at the centre. Let the load supported by the hip vertical be 60000 lbs., the distance between chord and tie bearing be $1\frac{1}{2}$″, and between tie and hip vertical bearing 3″. If the working stress β is 10000 lbs. and the angle of ties with vertical 33° 11′, what size of pin is required?*

One half of the hip vertical stress acts upon the chord bearing, or 30000 lbs. We have sin 33° 11′ = 0.547 and cos 33° 11′ = 0.837. The vertical component of the tie stress is $5 \times 1\frac{9}{16} \times 10000 \times 0.837 = 44204$ lbs. The total pressure on chord bearing is 44204 + 30000 = 74204 lbs. The moment at centre of hip vertical is

$$M_V = 74204 \times 4\frac{1}{2} - 44204 \times 3 = 20136 \text{ inch lbs.} = 100.653 \text{ inch tons.}$$

The horizontal component of the tie stress is

$$5 \times 1\frac{9}{16} \times 10000 \times 0.547 = 42734 \text{ lbs.}$$

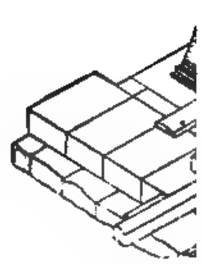
The horizontal moment is $M_H = 42734 \times 1\frac{1}{2} = 64101$ inch lbs. = 32 inch tons.

The maximum moment is

$$M = \sqrt{M_H^2 + M_V^2} = \sqrt{100^2 + 32^2} = 105 \text{ inch tons} = 210000 \text{ inch lbs.}$$

From the Table, this calls for a pin $5\frac{1}{4}$″ diameter, or $5\frac{3}{8}$″ commercial size.

TABLE FOR PINS.—We give below the Table referred to repeatedly in the preceding examples. The first column gives size of pin. The second the proper bearing for each size for one ton stress. This will enable us to find thickness of re-enforcing plates. The third and fourth give the bending moment for iron and steel pins.



PIN TABLE I.

LINEAL BEARING PER TON AND MAXIMUM BENDING MOMENT FOR PINS, FOR FIBRE STRAIN OF 15000 LBS. IRON, AND 20000 LBS. STEEL.

Lineal bearing in inches per ton $= \frac{1}{6.25d}$. Max. bending moment $= \frac{\pi R d^3}{32}$.

Least allowable diameter without thickening the head $= \frac{3}{4}w$.

Diameter of pin in inches.	Lineal bearing on pin in inches per ton.	Moment for $R = 15000$ for iron.	Moment for $R = 20000$ for steel.	Diameter of pin in inches.	Lineal bearing on pin in inches per ton.	Moment for $R = 15000$ for iron.	Moment for $R = 20000$ for steel.
1	0.16	1470	1960	4	0.04	94200	125700
1⅛	0.142	2100	2800	4⅛	0.038	103400	137800
1¼	0.128	2880	3830	4¼	0.038	113000	150700
1⅜	0.116	3830	5100	4⅜	0.037	123000	164400
1½	0.106	4970	6630	4½	0.035	134200	178900
1⅝	0.098	6320	8430	4⅝	0.034	145700	194300
1¾	0.091	7890	10500	4¾	0.034	157800	210400
1⅞	0.085	9710	12900	4⅞	0.033	170600	227500
2	0.08	11800	15700	5	0.032	184100	245400
2⅛	0.075	14100	18800	5⅛	0.031	198200	264300
2¼	0.071	16800	22400	5¼	0 03	213100	284100
2⅜	0.067	19700	26300	5⅜	0.03	228700	304900
2½	0.064	23000	30700	5½	0.029	245000	326700
2⅝	0.061	26600	35500	5⅝	0.028	262100	349500
2¾	0.058	30600	40800	5¾	0.028	280000	373300
2⅞	0.056	35000	46700	5⅞	0.027	298600	398200
3	0.053	39800	53000	6	0.026	318100	424100
3⅛	0.051	44900	59900	6⅛	0.026	338400	451200
3¼	0.049	50600	67400	6¼	0.025	359500	479400
3⅜	0.047	56600	75500	6⅜	0 025	381500	508700
3½	0.046	63100	84200	6½	0.025	404400	539200
3⅝	0.044	70100	93500	6⅝	0.024	428200	570900
3¾	0.042	77700	103500	6¾	0.023	452900	603900
3⅞	0.041	85700	114200	6⅞	0.023	478500	638000

EYE-BAR HEADS.—An eye-bar should be so proportioned that it will break first in the body rather than in the eye or head. Many experiments have been made to determine the proper relative dimensions of head and bar.

The following simple formulæ agree well with these experiments: Let D be the diameter of the head, d the diameter of pin, and w the depth of bar, then for *thickened heads*, or for

$$d < \tfrac{3}{4}w,$$

$D = d + 1.5w$, and *thickness of head*, $t = \frac{3wt_1}{4d}$, where t_1 is the thickness of bar.

For heads the same thickness as bar, or for

$$d > \tfrac{3}{4}w,$$

$$D = 1.25d + 1.29w, \text{ and } \textit{thickness of head}, \ t = t_1.$$

As to the shape of head, it is often made circular. Prof. Burr, in his *Stresses in Bridge and Roof Trusses*, gives a method of laying down an eye-bar head, as determined

by an extensive series of experiments, which is stated to have stood the test of long American experience.

As modified by the formulæ just given, it may be given as follows: Make DK equal to the value of D as found from the formulæ just given, and draw the semi-circle $DRBK$. Take the distance $QB = 0.87w$, that is, lay off $BC = 087w + \frac{d}{2}$ and draw the pin. Make $CA' = CA$. The curve GF is drawn with the centre A' and radius AD. GH is any curve with long radius, joining GF gradually with the body of the bar. HG should be gradual in order that there may be a large amount of metal in the vicinity of CG, for there the metal is subjected to flexure as well as direct tension. FD is a straight line parallel to the axis of the bar.

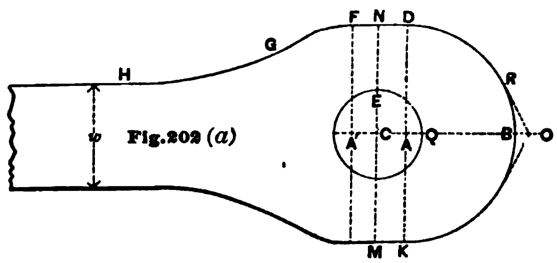


Fig. 202 (a)

We give in the following Table the diameter of the eye D, for different values of depth of bar and size of pin, according to the preceding formulæ; also the length of bar necessary to make an eye. This will be found useful in estimating the weight of iron in an eye-bar. These values we have taken approximately from Tables kindly furnished us by Jos. M. Wilson, C. E. In the column for diameter of eye, for each value of depth of bar, the value of D enclosed by lines is that for which the thickness of head is just equal to the thickness of bar, or $d = \frac{3}{4}w$. For all diameters less than this, the head must be thicker than the bar. Thus for depth of bar $w = 5''$, for all diameters of pin less than $3\frac{3}{4}$, the head must be thicker than the bar. For greater diameters than $3\frac{3}{4}$, the head and bar have the same thickness.

Different companies have different dies for heads, and it is only necessary that the designer shall know the form and size of head he has to expect. These are given from Pin Table II., or some similar Table furnished by the company. Specifications require only that upon being tested to destruction, the bar shall break in its body rather than in its head, and leave the form and size unspecified.

PIN TABLE II.

When $d < \frac{3}{4}w$, $D = d + 1.5w$. *When* $d > \frac{3}{4}w$, $D = 1.25d + 1.29w$.

$t = \dfrac{3wt_1}{4d}$. *d = diameter of pin in inches.* *D = diameter of eye in inches.* $t = t_1$.

t = thickness of head. *t_1 = thickness of bar.* *w = depth of bar in inches.*

Diameter of pin *d* in inches.	$w = 3''$.		$w = 4''$.		$w = 5''$.		$w = 6''$.		$w = 7''$.		$w = 8''$.		$w = 9''$.		$w = 10''$.	
	D.	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.	*D.*	Length of bar equal to one eye.
2	6.5	1′ 4″	8.00	0′ 11″												
$2\frac{1}{4}$	6.68	1′ 5″	8.25	1′ 0″												
$2\frac{1}{2}$	7.00	1′ 7″	8.50	1′ 1″												
$2\frac{3}{4}$	7.31	1′ 8″	8.75	1′ 2″												
3	7.62	1′ 10″	9.00	1′ 3″	10.5	1′ 3″	12.00	1′ 4″								
$3\frac{1}{4}$	7.93	2′	9.23	1′ 4″	10.75	1′ 4″	12.25	1′ 5″								
$3\frac{1}{2}$	8.24	2′ 2″	9.54	1′ 5″	11.00	1′ 5″	12.50	1′ 6″								
$3\frac{3}{4}$	8.56	2′ 5″	9.86	1′ 6″	11.25	1′ 6″	12.75	1′ 7″								
4	8.87	2′ 7″	10.16	1′ 7″	11.45	1′ 7″	13.00	1′ 8″	14.50	1′ 8″	16.00	1′ 8″				
$4\frac{1}{4}$	9.18	2′ 9″	10.47	1′ 8″	11 76	1′ 8″	13.25	1′ 9″	14.75	1′ 9″	16.25	1′ 9″				
$4\frac{1}{2}$	9.50	2′ 11″	10.78	1′ 9″	12.07	1′ 9″	13.50	1′ 9″	15.00	1′ 10″	16.50	1′ 10″				
$4\frac{3}{4}$	9.80	3′ 2″	11.10	1′ 10″	12.39	1′ 10″	13.68	1′ 10″	15.25	1′ 11″	16.75	1′ 11″				
5	10.12	3′ 4″	11.41	1′ 11″	12.70	1′ 11″	14.00	1′ 11″	15.50	2′ 0″	17.00	2′ 0″	18.50	2′ 0″	20.00	2′ 0″
$5\frac{1}{4}$	10.43	3′ 6″	11.72	2′ 1″	13.01	2′ 0″	14.30	2′ 0″	15.75	2′ 1″	17.25	2′ 1″	18.75	2′ 1″	20.25	2′ 1′
$5\frac{1}{2}$	10.74	3′ 8″	12.03	2′ 2″	13.32	2′ 3″	14.61	2′ 0″	15.90	2′ 2″	17.50	2′ 2″	19.00	2′ 2″	20.50	2′ 2″
$5\frac{3}{4}$	....	....	....	....	13.64	2′ 4″	14.93	2′ 1″	16.22	2′ 3″	17.75	2′ 3″	19.25	2′ 3″	20.75	2′ 3″
6	....	....	....	....	13.95	2′ 5″	15.24	2′ 2″	16.53	2′ 4″	18.00	2′ 4″	19.50	2′ 4″	21.00	2′ 4″
$6\frac{1}{4}$	....	....	....	....	14.26	2′ 6″	15.55	2′ 3″	16.84	2′ 5″	18.13	2′ 5″	19.75	2′ 5″	21.25	2′ 5″
$6\frac{1}{2}$	....	....	....	....	14.57	2′ 7″	15.86	2′ 4″	17.15	2′ 6″	18.44	2′ 6″	20.00	2′ 6″	21.50	2′ 6″
$6\frac{3}{4}$	....	....	....	....	....	....	....	....	17.47	2′ 7″	18.76	2′ 7″	20.25	2′ 7″	21.75	2′ 7″
7	....	....	....	....	....	....	....	....	17.78	2′ 8″	19.07	2′ 8″	20.36	2′ 8″	22.00	2′ 8″
$7\frac{1}{4}$	....	....	....	....	....	....	....	....	18.01	2′ 9″	19.38	2′ 9″	20.67	2′ 9″	22.25	2′ 9″
$7\frac{1}{2}$	....	....	....	....	....	....	....	....	18.40	2′ 10″	19.70	2′ 10″	20.98	2′ 10″	22.50	2′ 10″
$7\frac{3}{4}$	....	....	....	....	....	....	....	....	....	....	....	....	21.30	2′ 11″	22.58	2′ 11″
8	....	....	....	....	....	....	....	....	....	....	....	....	21.61	2′ 11″	22.90	3′ 0″
$8\frac{1}{4}$	....	....	....	....	....	....	....	....	....	....	....	....	21.92	3′ 0″	23.22	3′ 1″
$8\frac{1}{2}$	....	....	....	....	....	....	....	....	....	....	....	....	22.23	3′ 1″	23.50	3′ 2″

In the following Table we have given for different values of *d*, or for different sizes of pin, the weight of pin per inch in length, the corresponding diameter of screw at end, the size of hexagonal nut, and weight of one nut.

PIN TABLE III.

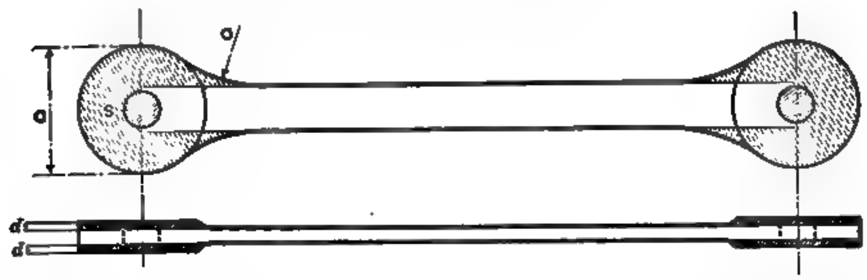
Diameter of pin.	Weight of pin per inch of length.	Diameter of screw.	Diameter of Hexagonal Nut.		Weight of one Nut.	Diameter of pin.	Weight of pin per inch of length.	Diameter of screw	Diameter of Hexagonal Nut.		Weight of one Nut.
			Least.	Greatest.					Least.	Greatest.	
2	0.87	1½	2⅜	2.74	1.20	5½	6.60	5	7⅝	8.81	12.32
2¼	1.10	1½	2⅜	2.74	1.20	5¾	7.21	5	7⅝	8.81	12.32
2½	1.36	2	3⅛	3.61	2.07	6	7.85	5½	8⅜	9.67	14.87
2¾	1.65	2	3⅛	3.61	2.07	6¼	8.52	5½	8⅜	9.67	14.87
3	1.96	2½	3⅞	4.47	3.18	6½	9.22	6	9⅛	10.54	17.65
3¼	2.30	2½	3⅞	4.47	3.18	6¾	9.94	6	9⅛	10.54	17.65
3½	2.67	3	4⅝	5.34	4.53	7	10.69	6½	9⅞	11.40	20.67
3¾	3.07	3	4⅝	5 34	4.53	7¼	11.47	6½	9⅞	11.40	20.67
4	3.49	3½	5⅜	6.21	6.13	7½	12.27	7	10⅝	12.27	23.96
4¼	3 94	3½	5⅜	6.21	6.13	7¾	13.10	7	10⅝	12.27	23.96
4½	4.42	4	6⅛	7.07	7.95	8	13.96	7½	11⅜	13.14	27.45
4¾	4.92	4	6⅛	7.07	7.95	8¼	14.85	7½	11⅜	13.14	27.45
5	5.45	4½	6⅞	7.94	10.02	8½	15.76	8	12⅛	14 0	31.19
5¼	6.01	4½	6⅞	7.94	10.02						

From these Tables we can find the weight of chord bars including heads, and also the weight of pins and nuts, and the proper sizes for every size of pin.

The rule upon which Table 3 is based is that the *least diameter of hexagonal nut or side of square nut in rough* = $1\frac{1}{2}$ *diameter of screw* + $\frac{1}{8}''$. *The greatest diameter of hexagonal nut in rough* = $1\frac{5}{32}$ *times the least diameter. The greatest diameter of square nut in rough* = 1.414 *times the side. Height of nut = diameter of screw. For finished sizes subtract* $\frac{1}{16}$.″ These rules are the standards of the *Franklin Institute*, recommended Dec., 1864. Tables for size and weight of nuts will be found in *Carnegie's Pocket Book.*

We give here a table for figuring the weight of eye-bars.

TABLE FOR FIGURING WEIGHT OF EYE-BARS.



SIZE OF BAR.	[illegible] OF PIN.	SIZE OF HEAD.	HEAD THICKER THAN BAR.	NO. OF DIE.	CUBIC INCHES FOR $\frac{1}{16}''$ THICKNESS OF SURFACE S.	CUBIC INCHES OF THICKENED EYE $2p$.	WEIGHT OF PIN $\frac{1}{16}''$ LONG.
"	"	"	"		cu. ins.	cu. ins.	lbs.
2 × ½	2 1/16	4 × ¾	¼	206	0 581	3.143	0.0580
2 × ¾	2 3/16	4½ × 1	¼	207	0.768	4.000	0.0653
2 × ⅞	2 7/16	5 × 1⅛	¼	204	0.985	5.000	0.0810
2 × 1	2 9/16	5½ × 1¼	¼	205	1.256	6.000	0 0895
2½ × ½	2 1/16	4½ × 11/16	3/16	203	0.701	[illegible].970	0.0580
2½ × ¾	2 11/16	5½ × 1⅛	⅜	156	1.137	8.610	0.0985
2½ × ⅝	2 13/16	6 × ⅞	¼	77	1.421	7 070	0.1177
2½ × ¾	3 3/16	6¼ × 1	¼	160	1.552	7.670	0.1611
3 × ¾	2 13/16	6 × 1	¼	172	1.286	7.070	0 0985
3 × ¾	2 15/16	7 × 1	¼	1	1.880	9 621	0.1177
3 × 15/16	3 7/16	7¼ × 1 3/16	¼	153	2.065	10.321	0.1611
3 × 1⅛	3 13/16	7½ × 1⅜	¼	152	2.222	11.045	0.1942
3 × 1	4	8½ × 1¼	¼	6	3.009	7.100	0.2182
3 × 1	4 3/16	7¾ × 1¼	¼	169	2.409	11.793	0.2391
3 × 1⅛	4 3/16	8¼ × 1½	⅜	144	2.682	19.443	0.2391
3 × 1	5 3/16	8¾ × 1⅜	⅜	137	3.060	21.909	0.3669
3 × 1	5¼	10¼ × 1¼	¼	5	4.789	10.824	0.3758
3½ × ⅞	2 13/16	7 × 1¼	⅜	155	1.773	14.432	0 0985
3½ × ¾	3 3/16	7¼ × 1⅛	⅜	176	2.076	16.566	0.1385
3½ × 1 3/16	3 7/16	8 × 1 7/16	¼	154	2.435	12.566	0 1611
3½ × ⅞	3 13/16	8¼ × 1¼	⅜	175	2.618	20.046	0.2114
3½ × 1	4	9¼ × 1¼	¼	4	3.406	8.400	0.2182
3½ × ¾	4 3/16	8½ × 1⅛	⅜	157	2.781	7.093	0.2685
3½ × 1 3/16	4 5/16	9 × 1 9/16	⅜	8	3.197	23.856	0 2841
3½ × 1	5¼	11 × 1⅜	⅜	3	5.135	11.879	0 3758
4 × 1	3	7¾ × 1⅜	⅜	159	1.815	15.768	0.1227
4 × 1	3 1/16	7¾ × 1½	½	177	2 089	17.691	0 1279
4 × 1¼	3 7/16	8¾ × 1⅝	⅜	150	2.629	21.279	0.1611

SIZE OF BAR.	SIZE OF HEAD.	HEAD THICKER THAN BAR.	NO. OF DIE.	CUBIC INCHES FOR $\frac{1}{16}''$ THICKNESS OF SURFACE S.	CUBIC INCHES OF THICKENED EYE $2p$.	WEIGHT OF PIN $\frac{1}{16}''$ LONG.
"	"	"		cu. ins.	cu. ins.	lbs.
4 × 1¼	¾ × 1⅝	⅜	171	2 860	22.549	0.2114
4 × 1	¾ × 1⅜	⅜	167	2.860	22.549	0 2391
4 × 1	¼ × 1⅜	⅜	158	3.273	25.200	9 2685
4 × 1	½ × 1⅜	⅜	168	3.479	26.580	0.3324
4 × 1 1/16	× 1 7/16	⅜	97	3.940	29 451	0 3669
4 × 1 1/16	¾ × 1 7/16	⅜	2	4.642	34 035	0 4219
4 × 1	½ × 1⅜	⅜	7	6.610	46.017	0.6443
4½ × 1½	× 1⅞	⅜	149	2.946	23.856	0.1611
4½ × ¾	½ × 1⅛	⅜	170	3.329	26.580	0 2114
4½ × 1¼	× 1⅝	⅜	151	3.778	29.451	0.2996
4½ × 1 5/16	¾ × 1 9/16	¼	9	4.253	25.970	0.3854
4½ × 1¼	¾ × 1⅝	⅜	[illegible]	5 261	32.472	0.4031
5 × 2	¾ × 2½	½	194	3.176	35.441	0.1942
5 × 1	× 1½	½	162	3.588	39 270	0 2391
5 × 2	× 2½	½	161	3.588	39.270	0.2391
5 × 1	½ × 1½	½	164	4 068	43.259	0 2996
5 × 2	¾ × 2½	½	163	4 068	43 259	0 2996
5 × 1¾	× 1¾	½	91	4.549	47.517	0 3669
5 × 1⅝	¾ × 2⅛	½	166	5.090	51.935	0.4411
5 × 2	¾ × 2½	½	165	5.090	51.935	0 4411
5 × 1¾	× 2¼	½	93	5.606	56.548	0 5210
5 × 1¾	½ × 2¼	½	71	6.195	61.359	0.6099
6 × 1¾	× 2¼	½	178	4 147	59.396	0 2841
6 × 2	× 2½	½	173	5 240	70.686	0.3324
6 × 2½	× 3	½	174	5 240	70.686	0 3324
6 × 1¾	× 2¼	½	68	6 304	82 959	0 5432
6 × 1¾	× 2¼	½	179	7 510	96 211	0.6563
6 × 2⅜	× 2 13/16	7/16	10	8 888	77 313	0 6328

EXAMPLE.—4″ × 1″ bar — 3″ pin — 7¾″ × 1⅜″ head — 20′ — 0″ *c.* of pins.

Thickness of bar = 1″ = $\frac{16}{16}$″. In table find $S = 1.815$; $1.815 \times 16 = 29.040$ cu ins. for 1 eye of same thickness as bar.
in table find, $2p = 15.768$ for additional thickness of 1 eye.
44.808 cub. ins.

Section of bar $= 4'' \times 1'' = 4$ sq. ins.; $\frac{44.808}{4} = 11''.2$ to be added to dist. *c.* of pins to make 1 eye.

Total length of 4″ × 1″ bar needed = 20′ + (2 × 11″.2) = say 21′ — 10¼″ = 21′.875.
Weight of 4″ × 1″ bar per foot (*Carnegie Handbook*) = 13.33 lbs.; 21.875 × 13.33 = 291.6 lbs.

Weight of 3″ pin $\frac{1}{16}$″ long given in table = 0.1227; $2 \times 1\frac{3}{8}'' = 2\frac{3}{4}'' = \frac{44''}{16}$

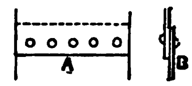
0.1227 × 44 = 5.4 = weight of cylinder bored out for both pins.
Weight of finished bar = 291.6 — 5.4 = 286.2 lbs.

CHAPTER V.

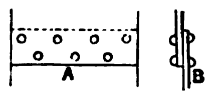
RIVETING.

In transmitting stress by rivets, it is customary to disregard the friction between the parts joined, as too uncertain an element to be relied upon to any extent. The rivets, then, must be proportioned for the entire stress transmitted.

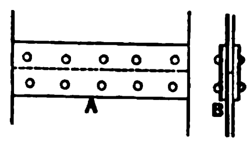
KINDS OF RIVETED JOINTS.—We may distinguish the following joints: 1st. *Simple "lap" joint, single riveted.* The Figure shows this joint, front and side view. The two plates to be joined are simply overlapped, by an amount equal to the "*lap*," and united by a single line of rivets. The distance from centre to centre of rivet, parallel to the joint, is called the "*pitch*."



2d. "*Lap*" *joint, double riveted.*—This joint is similar to the preceding, except two lines of rivets are used. In both cases, the rivets are in *single shear*.

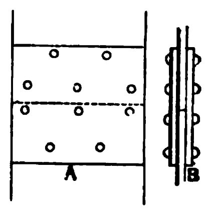


3d. "*Butt*" *joint, single riveted, two cover plates.*—Here the two plates are set end to end, making a "*butt joint*," and a pair of "*cover plates*" are placed on the back and front, and riveted through by a single line of rivets on each side of the joint. The plates in such a joint are not suffered to touch, and the entire stress, whether tensile or compressive, is transmitted through the rivets. The thickness of the cover plates should not be less than half the thickness of the plates joined, and when this rule would give a less thickness than the least allowable, *viz.*, $\frac{1}{4}$ inch, they should have this latter thickness. Owing to deterioration of the metal by the action of the weather, no plate is used less than $\frac{1}{4}$ inch in thickness, and this, therefore, makes a limit for the thickness of the cover plates.



4th. "*Butt*" *joint, one cover plate, single riveted.*—This is the same as the preceding, except that only one cover plate is used, of the same thickness as the plates themselves.

5th. *Double riveted "butt" joint, two cover plates.*—This joint is the same as case 3, except that we have two lines of rivets on each side of the joint. The thickness of the cover plates is determined by the same considerations as in case 3. In all cases where more than one row of rivets is used, the rivets are "staggered," or so spaced that those in one row come midway between those in the next, as shown in the Figure.



6th. "*Butt*" *joint, one cover plate, double riveted.*—This is the same as the preceding case, except that there is only one cover plate, the thickness of which is equal to that of the plates themselves.

7th. CHAIN RIVETING.—When we have more than two rows of rivets on each side of the joint, the system is called "*chain*" riveting. Such a disposition becomes necessary

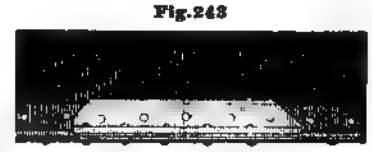
when the requisite number of rivets is so great that they cannot be placed in one or two rows without weakening the plates. We give in Figs. 238, 239, and 240, different forms

Fig. 238 Fig. 239 Fig. 240

of cover plate with chain riveting, and in Figs. 241, 242, 243, different methods of connections of chords by plates and angle irons.

Fig. 241 Fig. 242

Fig. 243



THEORY OF RIVETING.—A rivet may fail by shearing across, or by being crushed. The plate may fail by rupture between the rivets, or by tearing out of the rivets at the end. The rivets should be so proportioned and spaced that the strength for any case may be equal, and the plates weakened as little as possible.

Let b = the breadth of the joint in inches. This is usually a known quantity in any case. Let t = the thickness of the plates to be united, in inches, also a known quantity in any given case. Let d = the diameter of the rivet in inches, m = the number of rivets in a row, parallel to the joint, and n the number of rivets. Then $\frac{b}{m}$ will equal the "*pitch*" c, or the distance from centre to centre of rivet parallel to the joint, the distance of the end rivets from the edge being half of the pitch.

If W is the total stress to be transmitted by the joint, and T the unit stress, or allowable stress per square inch, of the material in tension, then, since the effective area of plate in a line through a row of rivet holes parallel to the joint, is $(b - md)t$, we have

Tearing area, ... $(b - md)t = \frac{W}{T}$ . . . . (1)

If C is the crushing stress per square inch, then, since the bearing area of a rivet is dt, we have

Bearing area, ... $ndt = \frac{W}{C}$ . . . . (2)

If S is the shearing stress per square inch, then, since the shearing area of a rivet is $0.7854d^2$, we have

Shearing area:

$$\left.\begin{array}{ll} \text{Single shear, or one cover plate, } \ldots\ldots & 0.7854nd^2 = \frac{W}{S} \\ \text{Double shear, or two cover plates, } \ldots\ldots & 1.5708nd^2 = \frac{W}{S} \end{array}\right\} \quad \ldots\ldots\ldots \quad (3)$$

Here we have three equations, and, in general, three quantities to be determined, *viz.*, m, n, and d.

We have, then, for the diameter in inches, for single shear,

$$d = \frac{tC}{0.7854S},$$

where t is the thickness of plate in inches, and C and S are the maximum allowable crushing and shearing stresses in lbs. per square inch.

For double shear, we substitute in place of 0.7854, 2×0.7854, for three-fold shear, 3×0.7854, and so on.

For the number of rivets we have

$$n = \frac{0.7854SW}{C^2t^2},$$

where W is the total stress transmitted by the joint, in lbs.

For the number of rivets in a row,

$$m = \frac{0.7854S}{tC}\left(b - \frac{W}{tT}\right),$$

where b = breadth of joint in inches, and T is the allowable tensile stress in lbs. per square inch.

It is customary to take $T = 10000$ lbs., $S = 7500$ lbs., $C = 12500$ lbs. Hence, for single shear,

$$d = 2.12t, \qquad n = \frac{W}{26500t^2}, \qquad m = \frac{1}{2.12t}\left(b - \frac{W}{10000t}\right).$$

PRACTICAL VALUES OF d.—SIZE OF RIVETS.—These are theoretical values, based upon the principle of equal strength, without restriction as to the diameter of the rivet. Practically, owing to risk of fracture and injury to the material, the diameter of the punch must be somewhat larger than the thickness of the plate.

Hence, we have the practical rule:

The diameter of rivet hole must not be less than the thickness of the thickest plate through which it passes.

As the least allowable thickness of plate is $\frac{1}{4}$ inch, this gives a practical lower limit of $\frac{3}{8}$ths of an inch for the rivet hole.

Rivets, however, as small as this are very rarely used. Diameters of $\frac{3}{4}$ to $\frac{7}{8}$ inch are of most frequent occurrence in girder work.

For all cross girders, stringers, and main compression members made of built sections $\frac{7}{8}$ *inch rivets is the size generally used.*

In other cases, we may be guided by the rule

$$d = 1\tfrac{1}{4}t + \tfrac{3}{16},$$

where the result is greater than $\frac{3}{4}''$, where d is the diameter of the rivet hole, and t the thickness of the plate in inches.

The rivet *hole* is punched $\frac{1}{16}''$ larger than the rivet, to allow for the increase in size of the hot rivet. As the hole must then be reamed out, the hole is to be assumed *as* $\frac{1}{8}''$ *larger*

than the rivet, in finding net section of tension members. The diameter of the hole is to be taken, rather than that of the cold rivet, which is always smaller, but when riveted fills the hole completely. The strength is therefore governed by the size of hole, and this, therefore, is our value of d.

NUMBER OF RIVETS.—Guided by these considerations and rules, we may select in any case a suitable size of rivet. This done, we may easily determine the requisite number.

A rivet is considered as failing in one of two ways—either by shearing across, or by crushing. In any case, then, the diameter being fixed, we must use such a number of rivets as shall give security against these two methods of failure. In general, if we determine the number required to resist crushing, it will be found ample to resist shear. It is, however, a work of little labor to determine the number of rivets required to resist either kind of stress, and to use the greatest of these two numbers. The bearing area of a rivet is the projection of the hole upon the diameter, or is equal to the diameter of the rivet, multiplied by the thickness of the plate. If both these dimensions are taken in inches, we obtain the bearing area in square inches.

The maximum allowable *bearing pressure* per square inch varies in practice from 15000 lbs. to 12000 lbs. In girder work 12500 lbs. seems sanctioned by the best practice.

Thus, for a $\frac{1}{2}$ inch rivet and $\frac{1}{4}$ inch plate, the bearing area is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ square inch, and taking 12500 lbs. per square inch allowable pressure, we have, for the safe resistance of the rivet to crushing, 1562· lbs. If, now, the total stress to be transmitted is, say, 18750 lbs., we should require $\frac{18750}{1562} = 12$ rivets.

The allowable *shear* is taken at 7500 lbs. per square inch for single shear. Thus, in our example, the area of the rivet is $\frac{\pi d^2}{4} = \frac{3.1416 \times 1}{4 \times 4} = 0.1963$ square inches, and hence its resistance to shear would be $0.1963 \times 7500 = 1472$ lbs. If the stress transmitted is 18750 lbs., we should require, then, $\frac{18750}{1472} = 13$ rivets. In this case, then, we see at once that about 13 rivets would be required, and this number would give ample security against crushing, which only requires 12 rivets. If, however, we had two plates of $\frac{1}{4}$ inch each, on each side of a central plate of $\frac{1}{2}$ inch, the rivets would be in double shear. The stress transmitted by each outer plate would be only one half of the whole stress upon the centre plate, and we should require for shear only 7 rivets, while for bearing we should still require 12. The number in this case would then be determined by the crushing strength.

In the following Table we have given the safe shearing and bearing resistance for rivets of different sizes, and for different thicknesses of plate, calculated as in the preceding example. Having chosen, then, the size of rivet, according to the rule already given, an inspection of the Table will give at once the number required in any given case, to resist either shear or crushing. The greatest of these two is to be taken. As most practical cases are in double shear, the greatest number will usually be determined by the crushing resistance.

We must then test the rivets in at least two ways, for shear and for bearing. In some cases it may be necessary also to test for bending, as in the case of pins. This is not usually done with rivets, however, as it is assumed that the head would be sheared off before the maximum bending would occur. A case where a rivet might fail by bending is in the attachment of a stringer to a floor beam, or a floor beam to a post, when there are filling plates. The filling plate increases the leverage, and may cause bending. For this reason this construction is to be avoided if possible.

Upon *field rivets the allowable stress is usually reduced* by $\frac{1}{3}$d, or we take $\frac{1}{3}$d more than

would be given by our Table. This is to allow for the imperfection of hand work. Of course, no rivet is ever to be used in direct tension, as the heads would be torn off.

RIVET TABLE I.

Shearing and Bearing Resistance of Rivets.

Diameter of Rivet in inches. Fraction.	Decimal.	Area of Rivet in square inches.	Single Shear at 7500 lbs. per square inch.	Bearing Resistance in lbs. for different thicknesses of plate at 12500 lbs. per square inch (= diameter × thickness of plate × 12500). $\frac{1}{4}''$	$\frac{5}{16}''$	$\frac{3}{8}''$	$\frac{7}{16}''$	$\frac{1}{2}''$	$\frac{9}{16}''$	$\frac{5}{8}''$	$\frac{11}{16}''$	$\frac{3}{4}''$	$\frac{13}{16}''$	$\frac{7}{8}''$
$\frac{3}{8}$	0.375	0.1104	828	1170	1465	1760								
$\frac{7}{16}$	0.4375	0.1503	1130	1370	1710	2050	2390							
$\frac{1}{2}$	0.5	0.1963	1470	1560	1950	2340	2730	3125						
$\frac{9}{16}$	0.5625	0.2485	1860	1760	2200	2640	3080	3520	3955					
$\frac{5}{8}$	0 625	0.3068	2300	1950	2440	2930	3420	3900	4390	4880				
$\frac{11}{16}$	0.6875	0.3712	2780	2150	2680	3220	3760	4290	4830	5370	5908			
$\frac{3}{4}$	0.75	0.4418	3310	2340	2930	3520	4100	4690	5270	5860	6440	7030		
$\frac{13}{16}$	0.8125	0.5185	3890	2540	3170	3800	4440	5080	5710	6350	6980	7620	8250	
$\frac{7}{8}$	0.875	0.6013	4510	2730	3420	4100	4780	5470	6150	6840	7520	8200	8890	9570
$\frac{15}{16}$	0.0375	0.6903	5180	2930	3660	4390	5130	5860	6590	7320	8050	8790	9520	10250
1	1	0.7854	5890	3125	3900	4690	5470	6250	7030	7810	8590	9370	10160	10940
$1\frac{1}{16}$	1.0625	0.8866	6650	3320	4150	4980	5810	6640	7470	8300	9130	9960	10790	11620
$1\frac{1}{8}$	1.125	0.9940	7460	3520	4390	5270	6150	7030	7910	8790	9667	10550	11420	12300
$1\frac{3}{16}$	1 1875	1.1075	8310	3710	4640	5570	6490	7420	8350	9280	10200	11130	12060	12990

Example.—*Required to unite two $\frac{1}{2}$ inch plates by a butt joint with two cover plates. The stress transmitted at the joint being 20000 lbs., what size of rivet and how many rivets are necessary?*

By our rule, we have for the diameter of rivet,

$$d = 1\tfrac{1}{4}\,t + \tfrac{3}{16} = \tfrac{5}{4} \times \tfrac{1}{2} + \tfrac{3}{16} = \tfrac{13}{16} \text{ inch.}$$

The stress in each cover plate is 10000 lbs. From our Table, we have for the resistance to shear of a $\frac{13}{16}$ inch rivet 3890 lbs. The shear will require then $\frac{10000}{3890}$ = about 3 rivets. The rivets in a butt joint with two plates are always in double shear.

From the Table we also have the bearing resistance of a $\frac{13}{16}$ inch rivet in a $\frac{1}{2}$ inch plate 5080 lbs. We shall require then for bearing $\frac{20000}{5080}$, about 4 rivets. This then is the number to be used.

Rivet Spacing, Pitch.—We thus know how to determine the size of the rivets and the required number. It remains to properly space the rivets, so that the plate shall be as strong as the rivets.

For this purpose we may take the shearing strength as equal to the tensile strength. The area then of a rivet cross section should be equal to the area of plate between the rivets. If c is the pitch or distance from centre to centre of rivets, and d the diameter of rivet, and t the thickness of plate, all in inches, and A the area of cross section of rivet in square inches, we have then

$$(c - d)\,t = A, \qquad \text{or} \qquad c = \frac{A}{t} + d,$$

for single shear. For double shear we put $2A$ in place of A, and so on.

Example.—Thus, in the preceding example, the diameter being $\frac{13}{16}$ inch, $t = \frac{1}{2}$ inch, and the rivets in double shear, we have from our Table, $A = 0.5185$, and hence the pitch in inches is

$$C = \frac{2 \times 0.5185}{\frac{1}{2}} + \tfrac{13}{16} = 2.887 \text{ inches.}$$

This rule, however, is subject to practical restrictions. *Rivets are not allowed to be placed nearer than* 3 *diameters, centre to centre.* If this distance is less than 3 inches, as it usually is, *we should take* 3 *inches for the pitch.*

If the rivets were spaced nearer than 3 diameters pitch, the holes would be liable to tear out, and there is danger of injury by punching.

Rivets should not have a pitch of *more than* 6 *inches in any case*, or when the plate is in compression, 16 *times the thickness of the thinnest outside plate.*

This is to guard against buckling of the plate between rivets.

With these restrictions, we may apply the preceding formula for the pitch c. In the preceding example, therefore, we are limited practically by $3 \times \frac{13}{16} = 2.44$ inches. But if this is less than 3 inches, we should take 3 inches for the pitch, or distance from centre to centre of rivets.

If the joint is in tension, the outside limit is 6 inches. If in compression, and the cover plates are $\frac{1}{4}$ inch, the outer limit would be 4 inches. Between 3 and 6 inches, or 3 and 4 inches, then, we should space our rivets in this case.

DISTANCE FROM END AND EDGE.—The distance between the end or edge of any plate and the centre of the rivet hole, or between rows, is fixed by practice at *never less than* $1\frac{1}{4}$ *inches*, except for bars less than $2\frac{1}{2}$ inches wide, and, whenever practicable, it should be at least 2 diameters for rivets over $\frac{5}{8}''$.

Since, now, we can find the diameter of rivet, the number of rivets, the pitch, and distance from end and edge, and between rows, we can space the rivets properly in any case where the breadth of plate is known, and determine the proper size of the cover plates.

EXAMPLE.—Let us take the same example as before, viz., butt joint with two cover plates each $\frac{1}{4}$ inch and a centre plate $\frac{1}{2}$ inch. The transmitted stress 20000 lbs.

We have already found the size of rivets $\frac{13}{16}''$, the number required 4, and the spacing or pitch 3 inches. Suppose the width of plate is 8 inches.

We should have for distance from each edge at least $1\frac{1}{4}$ inches. This leaves 5.5 inches, and for 3 rivets in a row we would have a pitch of 2.75 inches. We should have to have another row of two rivets, staggered with the first row, which would give 5 rivets in all, or one more than is strictly required. Taking 3 inches for distance from end and joint and between rows, we should have 9 inches for the half length of plate, or 18 inches for whole length.

It would be better, however, to make the pitch 3 inches, and use two rows of two rivets each. This would give same length of plate, 4 rivets, and distance from each edge of 2.5 inches.

JOINTS IN COMPRESSION.—The size and number of rivets are determined for joints in compression precisely as for joints in tension, because the joints are usually not considered as in contact, and hence the rivets must transmit the stress. The thickness and length of cover plates must also be the same as in tension joints. In general, compression joints are identical in proportions with tension joints, and have the same amount of shearing and bearing area. We may, if desirable, however, space the rivets somewhat more closely at right angles to the stress or across the plate. As the metal punched out does not affect the strength of a compression joint as it does that of a tension joint, the minimum pitch is determined by the nearest distance that holes can be punched without risk of cracking or injury to the metal. The pitch, for such reasons, should never be *less than two diameters*, or one diameter from edge to edge of holes, and, in any case, never less than $1\frac{1}{4}$ inches.

COMPRESSION CHORDS.—An exception to the preceding rule, that compression joints are not to be considered in contact, is found in the case of the main compression chords of a bridge. The joints in this chord being carefully planed, are considered in close contact, and hence the faces of the abutting joints are relied upon to transmit the strain. The splice plates at the side and on top of cover plate serve, therefore, merely to resist the dis-

placing action of the live load, jolts, jars, etc., and to hold the chords in line. The rivets for such plates are not calculated.

But at the hip, although the joint there is also carefully planed, it is not relied on, and the splice plates must therefore transmit the strains at the joint, and the rivets calculated accordingly.

SIZE OF RIVETS FOR STAY PLATES, RE-ENFORCING PLATES, LATTICE AND LACING BARS.—The preceding principles will enable us to find the size of rivets, number of rivets, pitch, distance from edge and side, distance between rows, number of rows, and length of cover or splice plate, for all tension or compression joints which occur in girder work. The same rules hold good for spacing, for the stay plates and re-enforcing plates at the ends of posts, as also for the lattice or lacing bars connecting the post channels. The *size* and corresponding number of rivets required however, for these details are best determined from the following rule, which conforms to established practice. For all post channels under 6″, the diameter of rivet employed to be not less than $\frac{1}{2}''$ or more than $\frac{5}{8}''$ or for $D < 6''$, $d \overset{=}{>} \frac{1}{2}$ and $< \frac{5}{8}$.

For channels over 6″, *up to* 12″ *inclusive*, we have

$$d = \frac{1}{16} D + \frac{1}{8}.$$

For 12″ channels we have

$$d = \frac{13}{16} \text{ to } \frac{15''}{16}.$$

TOP CHORD RIVETING.—Rivets are required for the splice plates and cover plates of the top chord, and top plate of chord and batter braces; for the stay plates and re-enforcing plates of the posts, or lateral and portal struts, which like the posts are formed of channels, laced or latticed; for the top and bottom flanges of plate floor girders and stringers; for lattice or lacing bars; and sometimes for the connection of floor girders and stringers with the posts and each other. The preceding rules and principles will enable us to properly treat any given case, and we shall proceed to illustrate their application by examples such as arise in practice.

The top chord is made up, as already described, of channels with a top plate. The joint in every panel does not come at the panel point, but a little to one side, towards the nearer end of the span. By this arrangement, the pin hole goes through the solid web, and is not bored partly through each abutting end, except at the hip, where this is unavoidable.

At each joint of the top chord we have two splice plates, besides a splice plate on top, which covers the abutting ends of the two chord plates. The rivets in these are not calculated, as they simply hold the chords in place.

At the hip, we have a plate on the outside of each chord channel, and one on the inside of each batter brace channel, the pin passing through all. The plates on the outside of the chord channels abut against plates riveted to the outside of the batter brace channels, and the plates on the inside of the batter brace channels abut against plates riveted to the inside of the chord channels, all abutting joints being planed to a true fit. Thus when the pin is driven, the joint is rigid. There is also a cover plate at the hip on top. A reference to Plate 12, Fig. 222, and also to Plate 26 at end, will explain these details. For these plates the rivets must be calculated.

EXAMPLE.—*The upper chord at the hip is composed of two 12-inch channels, each 35 lbs. per foot, area 10.5 sq. inches, and a cover plate 15" × ¼". The splice plates are $\frac{9}{16}$" thick, being the same thickness as the web of the channels. The allowable stress per sq. inch, which was used in dimensioning the chord, is $\beta = 6700$ lbs. What should be the dimension of the splice plate, and the size, number, and spacing of the rivets?*

Our rule for size of rivet, $d = 1\frac{1}{4}t + \frac{3}{16}$, gives, in this case, $d = \frac{5}{4} \times \frac{9}{16} + \frac{3}{16} = \frac{57}{64}$ or $\frac{7}{8}$". The area of the chord is $21 + 3.75 = 24.75$. Half of this is 12.375 sq. inches. Multiply this by $\beta = 6700$, and we have the strain to be transmitted by one plate, $12.375 \times 6700 = 82912.5$ lbs. From our Rivet Table, page 386, the shearing strength of a $\frac{7}{8}$" rivet is 4510 lbs., and, since the rivets are in double shear, we require for shear $\frac{41456}{4510}$ = about 9 rivets. For bearing, we have, from the Table for $\frac{9}{16}$" plate, bearing strength 6150, and we require $\frac{82912}{6150}$ = about 13 rivets.

We should space these rivets 3" pitch, and 1½" from end and edge. If now we know the size of the pin, we can sketch the plate. This detail is shown on Plate 26, at end of this work.

RIVETS IN TOP CHORD AND BATTER BRACE COVER PLATES.—The size of rivet may be chosen by our rule, $d = \frac{1}{4}t + \frac{3}{16}$, provided this gives a greater diameter than ¾", otherwise we take $d = \frac{3}{4}$".

It is usually customary to space the rivets 3" pitch for a distance on each side of joint equal to about 1½ times the width of top cover plate, and 6" pitch in the centre, unless this distance is greater than 16 times the thickness of the thinnest plate, in which case the centre rivets are spaced about 4½" pitch.

RIVETS IN LATTICE AND LACING BARS AND RE-ENFORCING PLATES.—The rule for size of bars has been already given, page 354, and for size of rivets for bars, page 388.

The same rule holds for re-enforcing plates. The *size* of rivets is thus easily determined in any case. The rules for spacing are the same as in all the preceding cases. The *number* required may now be readily determined.

The object of the re-enforcing plates, or extension plates, at the ends of post channels, is to give sufficient bearing area upon the pin. The proper thickness, therefore, can only be determined when the size of pin is known, as well as the thickness of channel. For practical reasons, the thickness of plate cannot be less than ¼" in any case. When this thickness is known the area can be found, because the width of plate is the same as that of the channel. The area of channel multiplied by the working stress β, used in dimensioning the channel, will give the stress on the channel. This stress must be divided between the end channel area and plate area (or plates, if there are two to a channel) in proportion to their respective areas. We thus find the stress transmitted by a plate. Thus the stress transmitted by a plate is equal to $\frac{\text{stress on channel} \times \text{area of plate}}{\text{total area of channel and plate (or plates)}}$.

The size of rivet being then fixed as above, the number of rivets can be found by using Rivet Table I., page 386, just as in the preceding examples. The rivets may then be spaced as in the preceding examples. Making allowance for the size of pin, we may then determine the length of plate. See Plate 25, at end of work, for this detail.

EXAMPLE.—*The stress in a 9 inch 30 lbs. post channel is 6000 lbs. per square inch. The flanges are shaved off for a certain distance at each end and two re-enforcing plates of ¼" thickness are put on. Find the transmitted stress in the plates and the size and number of rivets.*

The area of the channel is 9 square inches. The stress in the channel is $9 \times 6000 = 54000$ lbs. The thickness of web of channel is, from *Carnegie*, page 99, 0.7 inch The area of channel end cross section, after flanges are shaved off, is $0.7 \times 9 = 6.3$ sq. ins. Now the proper thickness of re-enforcing plates can only by determined when the size of pin is known, so that sufficient bearing area may be furnished The thickness cannot, however, be less than ¼", and the total area should be at least as great as the full cross section of the channel. If we take two re-enforcing plates each ¼", the total area will be $4.5 + 6.3 = 10.8$ sq. ins. The latter condition is therefore fulfilled by ¼" plates. We will suppose that the bearing area is also sufficient. Then the transmitted stress is $\frac{54000 \times 4.5}{10.8} = 22500$ lbs., or 11250 lbs. for each plate.

The size of rivet, by our rule, page 388, is $\frac{1}{16}D + \frac{1}{8} = \frac{9}{16} + \frac{1}{8} = \frac{11}{16}''$. The bearing resistance for this size of rivet and $\frac{1}{4}''$ plate is, from Table I., page 386, 2150 lbs. The number of rivets required therefore, is $\frac{11250}{2150}$ = about 5. The plate can now be drawn, the pin hole located and the rivets spaced properly.

RIVETS IN TRACK STRINGERS AND FLOOR BEAMS.—The *size* of rivets for track stringers and floor beams may be taken without discussion at $\frac{3}{4}''$ or $\frac{5}{8}''$ for light beams. This size is to be used for flanges, for all connections of floor beams with posts, of track stringers with floor beams, and for all stiffeners, unless the rule

$$d = 1\tfrac{1}{4}t + \tfrac{3}{16}$$

gives a greater value than $\frac{3}{4}$, when this greater value may be taken. The construction of floor beams and track stringers and their connections is shown on Plate 8, Fig. 206.

The spacing of rivets need not be calculated, as that will be determined by the number required. The pitch should not exceed 6 inches, nor be less than 3 diameters. At the ends it should be least, say about 3″ for a distance of 18 or 24″, but never less than 3 inches.

We need therefore to calculate only the *number*, which will determine the spacing in accordance with these rules.

To find the number of rivets necessary to connect a track stringer with a floor girder, or a floor girder with the post, also the number of rivets to connect the flanges with the web.

Half the total load W, carried by the girder, acts at each end, or $\frac{W}{2}$. Half of this is taken by each connecting angle. $\frac{W}{4}$ is then the stress transmitted by each angle. $\frac{W}{2}$ divided by the *bearing resistance* of a rivet, taken from Rivet Table I., page 386, will give then the number of rivets to resist the *bearing pressure*; and $\frac{W}{4}$ divided by the *shearing resistance* of a rivet, will give the number required to resist shear. The greatest of these two numbers is to be taken.

For the flanges the number of rivets must be calculated for the resultant stress. If H is the horizontal stress in the flanges, and V the vertical stress at any point, the resultant stress is $\sqrt{H^2 + V^2}$. It is customary to divide the girder into a number of lengths, say 4 or 6 or 8, and find the horizontal stress at each point of division. Then the horizontal stress at the first point from the end, together with the load on the first division, gives the resultant stress at the first point of division. The *difference* between the horizontal stress at the second and first points, together with the load on the second division, gives the resultant stress at the second point from the end. The difference between the horizontal stress at the third and second, together with the load on the third, gives the resultant at the third point from the end, and so on.

If W is the total load uniformly distributed, and l the length and d the effective depth of girder in feet,* the moment at any point distant x feet from the end, is $\frac{W}{2}x - \frac{W}{l}\frac{x^2}{2} = \frac{Wx}{2}\left(1 - \frac{x}{l}\right)$. The horizontal flange stress at any point is then

$$\frac{Wx}{2d}\left(1 - \frac{x}{l}\right).$$

* The distance between centres of gravity of the flange areas is the effective depth, and should be used in figuring all strains. Usually the effective depth of an ordinary girder is about two inches less than the depth over all.

EXAMPLE.—*A railway bridge track stringer is 17 feet long and 27 inches deep. The total load is equivalent to a distributed load of 55000 lbs. The thickness of the web is ½ inch, and of the flange angles $\frac{9}{16}$ of an inch. Find the size, number and spacing of the rivets.*

The size of rivets is $d = 1\frac{1}{4}t + \frac{3}{16} = \frac{5}{4} \times \frac{9}{16} + \frac{3}{16} = \frac{7}{8}''$. The bearing resistance for this size and ½ inch plate is, from Table I., 2730 lbs. The horizontal flange stresses at 2.5, 5 and 8.5 feet from the end, are given by $\frac{55000x}{4.5}\left(1 - \frac{x}{17}\right)$, where for x we put 2.5, 5 and 8.5. We have then 26062 lbs., 43137 lbs., and 51944 lbs. Subtracting each from the one following, we have 26062 lbs., 17075 lbs., 8807 lbs., for the horizontal stresses to be taken by the rivets in the different lengths.

The load on the first division of 2.5 feet is 8090 lbs., on the second division of 2.5 feet is 8090 lbs., on the third division of 3.5 feet is 11320 lbs. The resultant stress then for the first division is $\sqrt{13^2 + 4^2} = 13.6$ tons $= 27200$ lbs. In the next division it is $\sqrt{(8.5)^2 + 4^2} = 9.4$ tons $= 18800$ lbs. In the next division it is $\sqrt{(4.4)^2 + (5.66)^2} = 7.17$ tons $= 14340$ lbs. We require for bearing then, in the first 2.5 feet, $\frac{27000}{2730}$ or 10 rivets; in the next 2.5 feet, $\frac{18800}{2730}$ or 8 rivets; in the next 3.5 feet, $\frac{14340}{2730}$ or 6 rivets. We have then a pitch of about 3 inches for the first 2.5 feet, and if we take a pitch of 4 inches for the next 2.5 feet, and 5 inches for the 3.5 feet to the centre, we shall have more rivets than are needed.

Floor beams are treated in the same way. If W is the total load and a is the distance in feet from the ends to the point of attachment of the stringers, and d is the effective depth in feet,* then the flange stress at the distance a is $\frac{Wa}{2d}$. The vertical load between the end and point a, is $\frac{W}{2}$. The resultant stress is then $\sqrt{H^2 + V^2} = \sqrt{\left(\frac{Wa}{2d}\right)^2 + \left(\frac{W}{2}\right)^2}$. This stress divided by the bearing value of a rivet will give the number of rivets in the length a, or, since for two stringers *for single track*, the resultant stress beyond a is zero, the number of rivets *in the whole half length*.

See page 430 for an example.

As the load on the stringers comes through ties spaced about every 2 feet, the greatest wheel load may be taken as uniformly distributed. For the total load on stringers see page 425.

As the stringers are usually attached to the floor beams at the quarter points, the total load on the floor beam gives the same moment at centre as if it were uniformly distributed.

If the preceding remarks are carefully read and understood, and the examples worked over and checked, the student will find no difficulty in designing any riveted work.

RIVET HEADS—LENGTH FOR HEAD.—For button head rivets, if it is desired to know the size of head it may be found as follows:

$$\text{Height of head} = \tfrac{6}{10}d.$$
$$\text{Radius of head} = \tfrac{3}{4}d + \tfrac{1}{16}''.$$

For different sizes, these rules give the following dimensions:

Diameter of rivet	=	½″	⅝″	¾″	⅞″	1″.
Height of head	=	0.3	0.375	0.45	0.525	0.6.
Radius of head	=	$\frac{7}{16}$	$\frac{17}{32}$	$\frac{5}{8}$	$\frac{23}{32}$	$\frac{13}{16}$.

For countersunk heads, the greatest diameter is the same as for button heads. The angle of countersink = 30°.

Rivets are furnished with one head. The other head is made when the rivet is put in. The length of rivet should therefore be longer than the "*grip*" or thickness of metal through which it passes, by enough to make the head. This excess of length may be

* The distance between centres of gravity of the flange areas is the effective depth, and should be used in figuring all strains. Usually the effective depth of an ordinary girder is about two inches less than the depth over all.

determined by the rule—excess of length to make head $= d + \frac{3}{16}\left(\frac{\text{grip}}{d}\right)$. The diameter of rivet should be $\frac{1}{16}''$ less than that of the hole, so as to allow it to be inserted when hot. We give in the following Table the extra length in inches necessary to make one button head, for different diameters and length of grip.

DIAMETER OF RIVET IN INCHES.	EXTRA LENGTH IN INCHES FOR ONE BUTTON HEAD FOR DIFFERENT LENGTHS OF GRIP.			
	$\frac{3}{4}''$ and below.	$1\frac{1}{4}''$ to $\frac{3}{4}''$.	$2\frac{1}{4}''$ to $1\frac{1}{4}''$.	Above $2\frac{1}{4}''$.
	"	"	"	"
$\frac{1}{2}$	$\frac{7}{8}$	1	1	1
$\frac{5}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$
$\frac{3}{4}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$
	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$

To make one countersunk head, add $\frac{1}{2}''$ to the grip, or thickness of metal passed through.

Rivets have round or "button" heads, flat heads, and countersunk. If there is almost enough clearance for a button head, but not quite, a flat-head rivet, with a head $\frac{3}{8}''$ thick, may be used in preference to a countersunk rivet. Sometimes, however, either on account of clearance, or by reason of plates being in contact, it is impossible to avoid a countersunk rivet. These rivets are not nearly as capable of resisting strain as a button head, and should be counted upon very little, if at all, in figuring the number of rivets required at a joint.

PIN PLATES ON COMPRESSION MEMBERS.—For members in compression, there need only be an inch or two of metal between the edge of the pin hole and the end of the plate, as there is no strain on the metal on that side of the pin.

Sometimes, indeed, the end of a compression member may simply rest on the pin. This, however, is to be avoided, as a sudden jar might cause displacement. It is for this reason that an inch or two of metal is left beyond the pin hole.

To find the number of rivets, we assume each plate to take from the web its share of the strain, and figure the number as in the example, page 389.

In this example, the linear bearing on pin is $0.25 + 0.7 + 0.25 = 1.2$ inches. The strain is 54000 lbs., or 27 tons. If the diameter of pin were 4″, we see, from our Pin Table, page 377, that a linear bearing of 0.04 inch per ton is required. We therefore require $27 \times 0.04 = 1.08$ inches. As we have 1.2 inches, the bearing is sufficient. If the pin were $3\frac{1}{2}''$, the bearing would be insufficient, and the pin plates would have to be thicker.

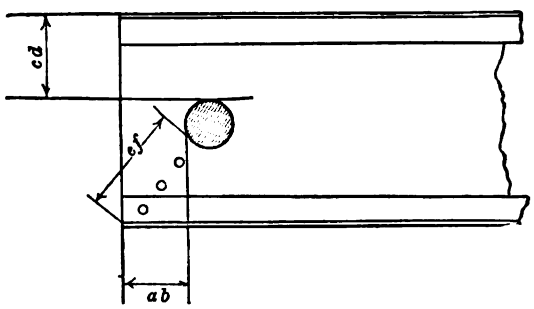
The number of rivets found for this example was 5. Knowing size of pin, we can distribute the rivets by making a sketch to scale of the end of channel, with pin hole in position.

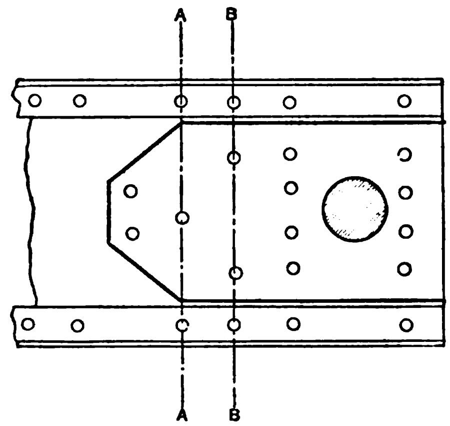
A compression joint is easier to arrange than a tension joint, because, in the latter, rivet holes weaken the section, while, in the compression joint, the section is not impaired by the rivet holes.

PIN PLATE ON TENSION MEMBERS.—In a tension joint, strain comes on the metal behind the pin, and we should have 50 per cent. more metal in the section *ab* than along *cd*. We should also guard against reducing the strength in any direction *ef*, by putting too many rivets in a line. We should also avoid putting rivets opposite the pin hole in either

direction, that is, directly above, below, or behind. They should be put to one side, above and below.

The number of rivets in the lines *AA*, *BB*, at the end of the pin plate, should be





reduced gradually, in order to take out as little section as possible along *AA*, as here we do not have the full section of the pin plate.

In tension pin plates we should always put some of the rivets on the side of the pin hole next to the end.

EXAMPLE.—*Let the section of a tension member be made up of two web plates 15" × ½", total area 15 sq. inches, and four angles 3" × 3", 7 lbs. per ft., area 8.4 sq. inches, latticed on both sides.*

The total area is 23.4 sq. inches, or 11.7 sq. inches for one plate and two angles. The angles are ⅜" thick.

We assume ⅞" rivets, and in calculating net section, it is customary to assume the hole 1" diameter, for a ⅞" rivet.

The lattice rivets will take out one hole 1 × ⅜", or 0.38 sq. inch. Only one hole is taken out, because the lattice rivets are, of course, staggered, and only one hole can come in a line at right angles to the length of member.

The rivets attaching the angles to the web plate will take out two holes 1 × ⅞, or 1.75 sq. inches.

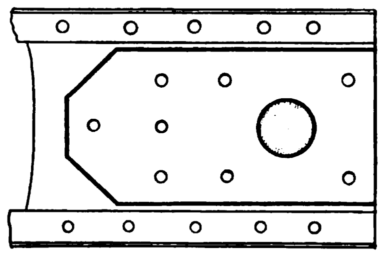
As the pin plate is 9 inches wide, we can have, at most, 3 rivet holes in a line. These take out $3 \times 1 \times \frac{1}{2} = 1.5$ sq. inches. The total section taken out is then 3.63 sq. inches, and the net section available for tension is $11.7 - 3.63 = 8.07$ sq. inches.

At 8000 lbs. per square inch, this gives a strain in one jaw of 64000 lbs., or 32 tons. If the pin hole is 5⅞", we have from Pin Table, page 377, for the necessary linear bearing $0.027 \times 32 = 0.864$ inch. As the web plate is only ½ inch, we must have a pin plate 0.364 inch thick, or ⅜ inch. The pin plate is 9 inches wide, its area is then 3.375 sq. inches, and at 8000 lbs per square inch, the pin plate takes 27000 lbs. strain.

The metal taken out by pin hole is $5\frac{7}{8} \times \frac{1}{2} = 3$ sq inches, and net area at pin is $11.7 - 3 = 8.7$ sq. inches. As this is larger than the net area over rivets, the thickness of the pin plate already found for bearing is sufficient. If it were not larger, we might have to increase thickness of pin plate to get sufficient section at pin.

The bearing value of a ⅞" rivet in a ⅜" plate is, from our Table, page 386, 4100 lbs. The value for shear is 4510. The number of rivets required in pin plate is then $\frac{27000}{4100} = 7$ rivets. We can add one more if required for spacing.

The pin plate can then be arranged as shown in the accompanying figure. Pitch of rivets, distance between rows, and from ends = 3 inches, and least distance from edge 1.5 inches.



In tension pin plates we should always put some of the rivets on the side of the pin hole next to the end.

In the preceding example, suppose that in order to pack close, the flanges of the angles are cut away for some distance from the end, so that the flanges of the angles are only one inch wide.

In any pin plate which we may use, we should run at least two rows of rivets back of the point where the angles are first cut away, and still further back if necessary to get in the required number of rivets.

For gross section, where we have full value of angles, we have, as before, 11.7 sq. inches, and for net section 8.07 sq. inches.

But over the pin, where the angles are cut away, the gross section is 10.2 sq. inches, and taking out 3 for the pin hole, we have 7.2 sq. inches net section over the pin. As this is less than 8.07, we require a pin plate in order to have

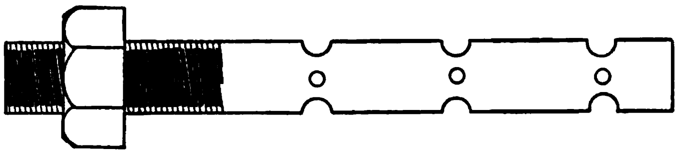
necessary net section over pin. For bearing, we have already found thickness of pin plate $\frac{3}{8}''$. The cross-section of pin plate is $9 \times \frac{3}{8} = 3.375$ sq. inches. Taking out pin $5\frac{7}{8} \times \frac{3}{8} = 2.2$ sq. inches, we have net section of pin plate over pin. 1.175 sq. inches, which, added to the net section 7.2 of the web and angles, gives 8.3 sq. inches. As this is somewhat greater than 8.07, the net section for full value of angles, no additional thickness of pin plate beyond that required for bearing is required.

BOLTS.—Bolts should be figured for shear and bearing, and if necessary for bending, just like pins and rivets.

A "joint" bolt is simply a rough bolt. A "skinned" bolt has the roughness taken off. A "turned" bolt is turned perfectly smooth.

The diameter of the bolt and the diameter of the thread are always the same, and, by the United States Standard, the nut is of the same thickness as the bolt. In ordering the bolt, the length of thread required must be specified.

A "swedged" bolt is a bolt which has indentations on its surface, as shown in the accompanying figure. They are used for foundation bolts, and the indentations allow the melted sulphur which is poured into the hole to obtain a better grip on the bolt.



Unless ordered to the contrary, pins, bolts, and nuts always come with a right-hand thread, as shown in figure.

CHAPTER VI.

WIND BRACING—MISCELLANEOUS DETAILS.

WIND FORCE.—The wind bracing should be proportioned upon precisely the same principles as the main truss members. The only difference is in the loading assumed.

The train surface is taken at 10 square feet for every foot in length, and the wind pressure, when the train is on, at 30 lbs. per square foot, or 300 lbs. per foot of length, *plus* 30 lbs. per square foot of exposed surface of truss. The 300 lbs. per lineal foot due to the train surface is treated as a moving load, and the pressure on the exposed surface of the trusses as a fixed load. When the bridge is empty we take 50 lbs. per square foot of exposed surface as the loading, and the greatest strain by either loading is used in determining the sectional area of the bracing.

The *exposed surface of truss* is estimated by the following rule: *Add to the surface, as shown on the drawing for upper chords and posts, one and a half times the surface of the ties and twice the surface of the lower chord.**

As soon, therefore, as the design has progressed far enough for us to determine the exposed surface of truss by this rule, we multiply this exposed surface in square feet by 30 and divide by the span in feet. To the result, we add 300 lbs., and we get the load per foot for which the wind bracing is to be calculated, when the train is on the bridge.

Again, we multiply this exposed surface in square feet by 50, and divide by the span in feet, and we obtain the load per foot for bridge empty. The greatest strains due to either loading are to be taken.

In preliminary estimates we may take the exposed surface for *both trusses* at 10 square feet per linear foot. At 30 lbs. per square foot this gives 300 lbs. per linear foot of truss, or 75 lbs. for each upper chord and 75 lbs. for each lower chord. This gives 450 lbs. per linear foot for top lateral bracing in deck bridges or bottom lateral bracing in through bridges, of which 300 lbs. is moving load. On the other chords we have 150 lbs. per lineal foot, or 75 lbs. for each chord, fixed load.

WIND BRACING.—The wind bracing consists of horizontal bracing under the floor; of horizontal bracing between the top chords in a through bridge, or the bottom chords in a deck bridge; and vertical sway bracing at every panel point of a through bridge when the truss is deep enough, and at every panel point of a deck bridge of any depth.

In through bridges the clear headway or vertical distance between the upper surface of the rails and the lowest part of the over-head bracing should be at least 12.5 feet. From 12.5 to 24 feet, we use horizontal over-head bracing only. Above 24 feet, vertical sway bracing is to be used also. Of course, *all* deck bridges have both upper and lower horizontal bracing, and vertical sway bracing also.

Of the wind load per foot found according to the preceding rule, $\frac{2}{3}$ds may be taken as acting at the panel points at the floor, and $\frac{1}{3}$d at the panel points of the other chord. Upon these assumptions we may find the strains in upper and lower horizontal wind bracing. The ties and posts of the vertical sway bracing may be taken the same as those of the horizontal bracing at the centre, without special calculation.

The reasons which have led to the adoption of 30 lbs. per square foot with train, or 50 lbs. without, although wind pressures have been registered as high as 90 lbs. per square foot, are that such extreme pressures are limited to narrow belts, less than the length of ordinary spans, so that the entire span is not subjected to this pressure, and also that no

* This is because the ties are in pairs and the lower chords consist of several bars.

train would venture across a bridge during such a tornado; so that the allowance of 30 lbs. per square foot, *with train*, may give higher strains even than 90 lbs. without, or even if not, still the strains on bracing so proportioned, due to the extreme pressure, will be within the limits of elasticity.

The working stresses used in proportioning the wind bracing are 15,000 lbs. per square inch in the ties, and rivets, pins, and struts the same as for the main trusses. The method of proportioning ties and struts is precisely the same as for the main trusses.

DETAILS.—When the cross girders are riveted to the posts, as in Fig. 206, Plate 8, the ties for the lower horizontal bracing may be pinned to the lower flange of the cross girder, as shown in Fig. 206, and no struts are necessary, the cross girder answering the purpose of a strut.

When the cross girder is slung below the chord by beam hangers from the pin, it is often customary to pin the ties to the upper flange in the same manner. This is evidently not good construction, and it is better, though of course somewhat more expensive, to insert struts above the cross girders. These struts may be of timber, shod with iron, riveted to the posts, or may be of latticed channels, with stay plates, riveted by angle irons to the posts.

The upper horizontal bracing for medium spans may be, as in Fig. 222, Plate 12, composed of two angle irons with the ties pinned to the flanges, the pin extending through the top chord plate.

When vertical sway bracing is required, the upper struts may be made like the posts, of two channels, with webs horizontal, latticed, with stay plates. The horizontal ties may be attached to a vertical pin through the horizontal webs of the channels, and the vertical sway braces to horizontal pins passing through the extension plates of the struts and the *ends of the chord pins*. The lower or intermediate struts may also be channels, latticed, the channel webs being in a vertical plane, riveted by angles to the post, and the vertical sway braces attached by pins passing through the vertical webs of the channels.

INCREASE OF CHORD SECTION DUE TO WIND.—When the train covers the span and the wind acts, the bridge is bent sideways and the lower chord on the side away from the wind has its maximum tension increased by the tension due to the wind. The chord on each side should be able to sustain the total maximum.

Again, the compression in the windward chord at end, when the bridge is empty, due to the wind, may exceed the tension due to dead load, in which case the chord may buckle unless made to resist compression. It is well, therefore, in the last two panels, to strap the inner lower chord bars to each other, so that they may act as a strut to resist compression. (See page 342.)

UPPER AND LOWER LATERAL WIND BRACING.—The upper and lower lateral wind bracing is calculated just as for a Pratt Truss. The upper lateral system in a deck bridge and the lower lateral system in a through bridge, or pony bridge, are calculated for a dead load of 30 lbs. per square foot of exposed surface of *both trusses*, and a live load of 300 lbs. per linear foot, or a dead load of 50 lbs. per square foot of exposed surface of *both trusses*, and the greatest strains in either case taken.

The exposed surface of truss may be found by the preceding rule, or may be assumed without calculation at 10 square feet per linear foot for *both trusses*. This is 5 square feet per linear foot for one truss, or $2\frac{1}{2}$ square feet per linear foot for each chord. At 30 lbs. per square foot, this is 75 *lbs. per linear foot for each chord*, and at 50 lbs. per square foot, it is 125 *lbs. per linear foot for each chord*.

When the panel length is known we can then easily find the panel load at each apex.

Although the train partially shelters one truss, it will be observed that this is disregarded, and each truss is considered as fully exposed, even when train is on.

For the lower lateral system in a deck bridge or the upper lateral system in a through bridge, we have simply to calculate for a dead load of 30 lbs. per square foot of exposed surface of both trusses, or 75 *lbs. per linear foot for each chord.*

The strains in the wind braces thus obtained are to be increased for *initial tension*, page 341, since each is furnished with a turn buckle.

CENTRIFUGAL FORCE.—If the track upon the bridge is curved, the strains in the lateral system under the train are increased by the centrifugal force.

The train load must first be reduced to an equivalent uniform load per foot; that is, a load per foot, which, spread over the whole span, will give the same moment at the centre of span as the maximum moment at centre due to the train. This is easily found.

Thus, if M is the maximum moment at the centre due to the train, and w is the equivalent load per foot, and $l =$ the span, we have

$$\frac{wl^2}{8} = M, \text{ or } w = \frac{8M}{l^2}.$$

Now, if p is the panel length, the panel load W at each panel point, is

$$W = wp = \frac{8Mp}{l^2}.$$

The centrifugal force at each panel point is then

$$C = \frac{Wv^2}{gr};$$

where v is the velocity in feet per second, $r =$ radius of curve, $g = 32\frac{1}{6}$.

We give in the following Table the values of $\frac{v^2}{gr}$ for a 1° curve, and different velocities. For any other degree multiply the tabular values by the degree of the curve.

TABLE FOR CENTRIFUGAL FORCE $C = \frac{Wv^2}{gr}$.

Values of $\frac{v^2}{gr}$ given for a 1° curve.

For any other degree, multiply by degree of curve.

v in miles per hour.	v in feet per sec.	$\frac{v^2}{gr}$ for 1° curve.	v in miles per hour.	v in feet per sec.	$\frac{v^2}{gr}$ for 1° curve.
10	$14\frac{2}{3}$	0.00117	40	$58\frac{2}{3}$	0 01866
15	22	0.00262	45	66	0 02361
20	$29\frac{1}{3}$	0.00467	50	$73\frac{1}{3}$	0.02915
25	$36\frac{2}{3}$	0.00729	55	$80\frac{2}{3}$	0.03527
30	44	0.01049	60	88	0.04196
35	$51\frac{1}{3}$	0.01428			

The "degree of a curve" is the angle subtended at the centre by a chord of 100 *feet.*

We can thus find the centrifugal force at each panel point for a given train, degree of curve, and assumed maximum velocity, and find the strains in the lateral system for this loading. These strains are to be added to the wind strains already found, and *initial tension* added, page 341.

EXAMPLE.—*Through bridge, span c to c 153 feet, no. of panels* = *9, panel length 17 feet, width c* to *c* $16\frac{1}{2}$ feet. *Find the strains in upper and lower lateral bracing.*

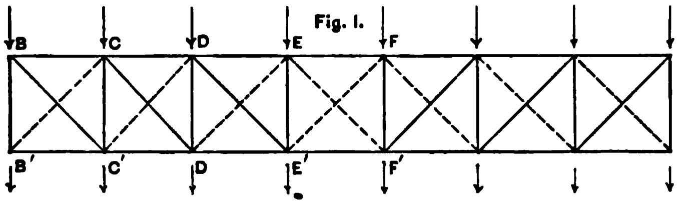


Fig. I.

We have in this case the panel load for the upper lateral system, $75 \times 17 = 1275$ lbs. This load acts at each apex of the windward and leeward chords. In the upper system there will be seven panels, as shown by the Fig I.

We have sec $\theta = 1.447$, tan $\theta = 1.046$, and hence, in the upper lateral system,

$EE' = + 1275$ lbs. $\qquad DE' = - 2 \times 1275 \times 1.447 = - 3690$ lbs.

$DD' = + 3 \times 1275 = + 3825$ lbs. $\qquad CD' = - 4 \times 1275 \times 1.447 = - 7380$ "

$CC' = + 5 \times 1275 = + 6375$ " $\qquad BC' = - 6 \times 1275 \times 1.447 = - 11070$ "

$BB' = + 7 \times 1275 = + 8925$ " $\qquad BC = + 3 \times 2550 \times 1.046 = + 7000$ "

$CD = + 5 \times 2550 \times 1.046 = + 13340$ lbs. $\qquad DE = FF = + 6 \times 2550 \times 1.046 = + 16000$ lbs.

The chord *BCDE* is in compression under the action of the train. The compression due to the train is increased by that due to the wind, as given above.

The strains in the braces *BC'*, *CD'*, etc., must be increased for initial tension, page 341.

When the wind blows from the other side we shall have the same strains in *B'C'*, *C'D'*, etc., as in *BC* and *CD*, and the other system of braces will act.

The braces *EF'*, *E'F*, are not strained theoretically.

They are inserted, however, for appearance, of same size as *ED'*, *E'D*.

Since the strains in the top lateral system are all so small, we would in practice make the ties all of the same size, and the struts all of the same size, as this would cost less than to have different sizes of details and connections.

We may, if desired, find the strains for the actual exposed surface at 30 lbs. per square foot, but the preceding is the customary method.

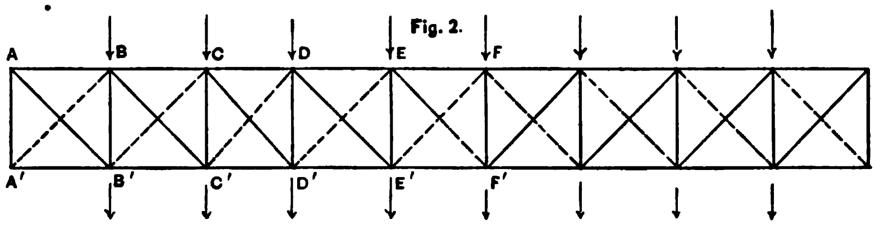


Fig. 2.

For the lower lateral system, Fig. 2, we have nine panels, and the panel load for 30 lbs. per square foot will be, as before, 1275 lbs. at each apex right and left. This we treat as a dead or fixed load. The train wind load of 300 lbs. per linear foot gives a panel load of $17 \times 300 = 5100$ lbs. This we treat as a moving load.

We may, if desired, find the actual exposed surface, and take this at 30 lbs. per square foot, but the preceding is the customary method.

We have then

For the Chords.

$B'C' = - 7650 \times 4 \times 1.046 = - 32008$ lbs. $\qquad AB = + 32008$ lbs.

$C'D' = - 7650 \times 7 \times 1.046 = - 56013$ " $\qquad BC = + 56013$ "

$D'E' = - 7650 \times 9 \times 1.046 = - 72017$ " $\qquad CD = + 72017$ "

$E'F' = - 7650 \times 10 \times 1.046 = - 80019$ " $\qquad DE = + 80019$ "

For the Braces.

$$EF' = - \frac{10}{9} \times 5100 \times 1.447 = - 8200 \text{ lbs.} \qquad DE' = - \left(2550 + \frac{15}{9} \times 5100\right) 1.447 = - 15989 \text{ lbs.}$$

$$CD' = \left(2 \times 2550 + \frac{21}{9} \times 5100\right) 1.447 = - 24599 \text{ lbs.} \qquad BC' = - \left(3 \times 2550 + \frac{28}{9} \times 5100\right) 1.447 = - 34028 \text{ lbs.}$$

$$AB' = - (4 \times 2550 + 4 \times 5100) 1.447 = - 44280 \text{ lbs.}$$

For the Struts.

$AA' = +8 \times 1275 + 4 \times 5100 = +30600$ lbs. $BB' = +7 \times 1275 + \frac{28}{9} \times 5100 = +24791$ lbs.

$CC' = +5 \times 1275 + \frac{21}{9} \times 5100 = +18275$ " $DD' = +3 \times 1275 + \frac{15}{9} \times 5100 = +12325$ lbs.

$$EE' = 1275 + \frac{10}{9} \times 5100 = +6941 \text{ lbs.}$$

If we should take 50 lbs. per square foot as a fixed load, we have the panel load at each apex, right and left, 2125 lbs., and the strains would be all less than those already found. We therefore take the latter.

When the wind blows from the other side, the other system of braces will act, and $A'F'$ will be in compression and AF in tension.

The tensile strains in the lower chords due to the train should therefore be increased by the tension just found due to wind, and the chords designed for the combined result.

The compression in AB due to the fixed wind load of 50 lbs. per square foot, when bridge is empty, is $4 \times 4250 \times 1.046 = +17782$ lbs.

If this compression were greater than the tension due to the dead load of the bridge itself, the chord bars in AB would have to be stiffened to take the difference, or resultant compression.

The tie rods of the lower lateral system are usually fastened to the bottom flanges of the cross-girders, thereby relieving the tensile strains in those flanges. There need, therefore, be no bottom lateral struts at all.

If the track were on a curve, we should find the strains due to centrifugal force as directed in the preceding article, and add them to those already found.

VERTICAL SWAY BRACING.—In deck bridges, besides the upper and lower lateral wind bracing, there is always vertical sway bracing at each panel, at right angles to the axis of the bridge. In through bridges also, if the headway allows of it, we have vertical sway bracing.

In Fig. 3, let P be the pressure concentrated at the upper panel point of a through bridge, windward and leeward. It is, according to usual assumptions, 75 lbs. per lineal ft., and hence, if p is the panel length, $P = 75p$.

If it is desired to take the actual exposed surface, P is the pressure at 30 lbs. per sq. ft. upon the surface of one panel length of top chord, one-half the surface of the diagonal braces meeting at the top chord, and one-half of the distance BC on a post. It is customary in most ordinary cases to take P as $75p$ lbs.

Let P' be the pressure concentrated at one end of the intermediate strut CC'. It is the pressure at 30 lbs. per sq. ft. upon one-half of the post. If we take the post as one foot wide, and d = depth of truss c to c,

$$P' = 30\frac{d}{2} = 15d \text{ lbs.}$$

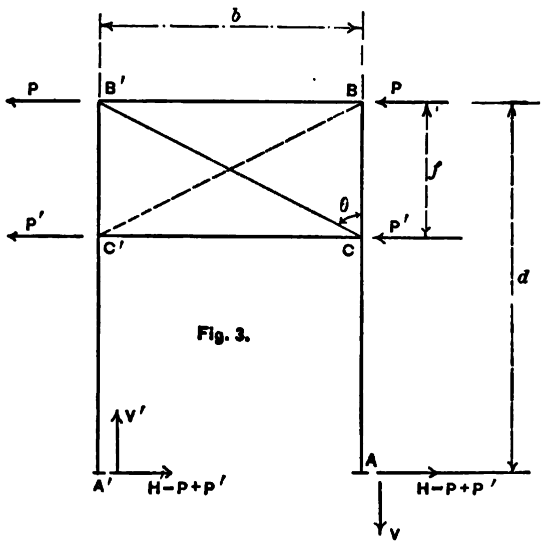


Fig. 3.

Let f = the distance BC, b = width of bridge c to c, θ = angle of vibration rods, CB' or $C'B$, with vertical.

The total pressure $2(P + P')$ is resisted by H and H', the horizontal forces at the foot of the posts, and V and V' acting vertically. V' is an increase of pressure on the lee side and acts up; V acts down on the windward side.

We have for equilibrium the three conditions,

$$2(P + P') + H + H' = 0. \qquad V + V' = 0,$$

and, taking moments about B',

$$(H + H')\,d + Vb + 2P'f = 0.$$

From the first and third of these equations, we have

$$V = \frac{2\,(P + P')\,d - 2P'f}{b} = -\,V', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which is independent of the values of H and H'.

Since, then, we have three equations only, and four unknown quantities, V, V', H and H', the latter are strictly indeterminate.

It is customary to assume

$$H = H' = -\,(P + P'), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and this assumption is probably as correct as any other that can be made.

For the strain in the vibration rod CB', we have strain in

$$CB' = -\,V \sec\theta = -\,\frac{2\,(P + P')\,d - 2P'f}{b}\sec\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the minus sign denoting tension.

To find the strain in the intermediate strut CC', consider it cut, and take moments about B', and we have strain in $CC' \times f + H'd + P'f = 0$; or, putting for H' its value,

$$\text{strain in } CC' = (P + P')\,\frac{d}{f} - P' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The maximum strain in BB' has already been found, since it is in the upper lateral system. Its strain in this case is not, therefore, needed, as it will be less than already found.

When the wind blows from the other side, we have the same compression in CC' and tension in $C'B$, instead of CB'.

The moment at C or C' on the post is, if the ends are free, $H\,(d - f)$, or $(P + P')$ $(d - f)$. But it will be more correct to consider the ends as fixed, since they are rigidly attached to the cross girders. We have, therefore, the moment only one-half as much as for free ends, or

$$\text{moment at } C = \frac{1}{2}\,(P + P')\,(d - f).$$

The post is composed of channels latticed together. If the distance between the channels c to c is m, we have the compression on one post channel due to the bending alone, $\frac{(P + P')\,(d - f)}{2m}$. The post also has a direct compression of V, or for one channel $\frac{V}{2}$. The total compression on one post channel is, therefore,

$$\text{compression on one post channel} = \frac{V}{2} + \frac{(P + P')\,(d - f)}{2m} \quad . \quad . \quad . \quad . \quad (5)$$

This is to be added to the compression due to train and weight of bridge where V is given by (1). This compression is to be added to that due to the train and the weight of the bridge itself.

Formulæ (3), (4), and (5), for the vibration rod, intermediate strut and post channels,

will hold equally for the inclined portal and batter braces, if for d we put the length of batter brace $= \sqrt{d^2 + p^2}$, for f the distance f_1 between upper and lower portal struts, for P' the pressure on one-half the batter brace $= P'_1$, and for P *one-fourth the sum of all the pressures concentrated at windward and leeward panel points of the upper lateral system* $= P_1$.

For the strain in the strut BB' *at the portal*, we have, then, considering this strut as part of the sway bracing only, by taking moments about C,

$$+ BB' \times f_1 - P_1 f_1 - (P_1 + P'_1)(\sqrt{d^2 + p^2} - f_1) = 0,$$

or

$$BB' = \frac{\sqrt{d^2 + p^2}}{f_1}(P_1 + P'_1) - P'_1.$$

But this only gives the strain in BB' as part of the sway bracing. It is also part of the top lateral system, and as such has the compression $P_1 - P_e$, where P_e is the pressure concentrated at the leeward hip. Hence, *for the portal*

$$\text{strain in } BB' = \frac{\sqrt{d^2 + p^2}}{f_1}(P_1 + P'_1) - P'_1 + P_1 - P_e, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where p is the panel length, d is the depth of truss, and f_1, P_1, P'_1, and P_e, have the values given above.

For the portal, (3), (4), and (5) become, therefore,

$$\text{strain in } CB' = -\frac{2(P_1 + P_1')\sqrt{d^2 + p^2} - 2P_1' f_1}{b} \sec \theta_1; \quad . \quad . \quad . \quad . \quad (7)$$

where θ_1 is the angle made by CB' with the batter brace,

$$\text{strain in } CC' = (P_1 + P_1')\frac{\sqrt{d^2 + p^2}}{f_1} - P_1'; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\text{compression on one batter-brace channel} = \frac{V}{2} + \frac{(P_1 + P_1')(\sqrt{d^2 + p^2} - f_1)}{2m}; \quad . \quad . \quad (9)$$

$$\text{where } V \text{ is given by } V = \frac{2(P_1 + P_1')\sqrt{d^2 + p^2} - 2P_1' f_1}{b}. \quad . \quad . \quad . \quad . \quad (10)$$

DECK BRIDGE.—SWAY BRACING.—For a *deck bridge*, we have simply to make $f = d$, $f_1 = \sqrt{d^2 + p^2}$, and as now CC' is part of the lower lateral system, and BB' of the top, we only have to find at any intermediate panel

$$\text{strain in } CB' = -\frac{2Pd}{b}\sec\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\text{Compression on post channel} = \frac{V}{2} = \frac{2Pd}{b}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

And at the end,

$$\text{strain in } BB' = 2P_1 - P_e. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

KNEE BRACES.—When, in a through bridge, there is not headway enough for sway bracing, as in Fig. 3, stiffness is obtained by the use of knee braces or brackets, as in Fig. 4.

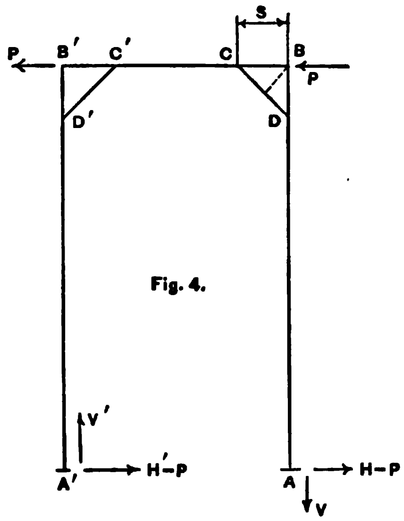


Fig. 4.

In this case, taking moments about A', we have,

$$Vb = 2Pd, \text{ or } V = \frac{2Pd}{b}, \quad . \quad . \quad . \quad . \quad (14)$$

and the compression on a post channel is $\frac{V}{2}$. This is to be added to the compression due to train and weight of bridge.

The strain in CD is found by taking moments about B.

The lever arm is $s \cos \theta$, where s is the distance CB, and θ the angle of CD with vertical.

Hence, $CD \times s \cos \theta = Pd$, or,

$$\text{strain in } CD = -\frac{Pd}{s \cos \theta}, \quad . \quad . \quad . \quad (15)$$

where the minus sign denotes tension.

There is a moment at C and C', which is equal to

$$V(b - s) - Pd = \frac{Pd}{b}(b - 2s). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

If m' is the distance c to c between the two channels of which the upper lateral strut is composed, then we have,

$$\text{compression on each channel of } BB' = \frac{Pd}{bm'}(b - 2s). \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Twice this is to be added to the compression on BB' as part of the upper lateral system.

At the portal we have to put, for d, $\sqrt{d^2 + p^2}$, and for P, P_1 as before, and θ_1, and (14), (15), and (16) become

$$V = \frac{2P_1\sqrt{d^2 + p^2}}{b}, \qquad CD = -\frac{P_1\sqrt{p^2 + d^2}}{s \cos \theta_1}, \qquad \frac{P_1\sqrt{d^2 + p^2}(b - 2s)}{b}. \quad . \quad . \quad (18)$$

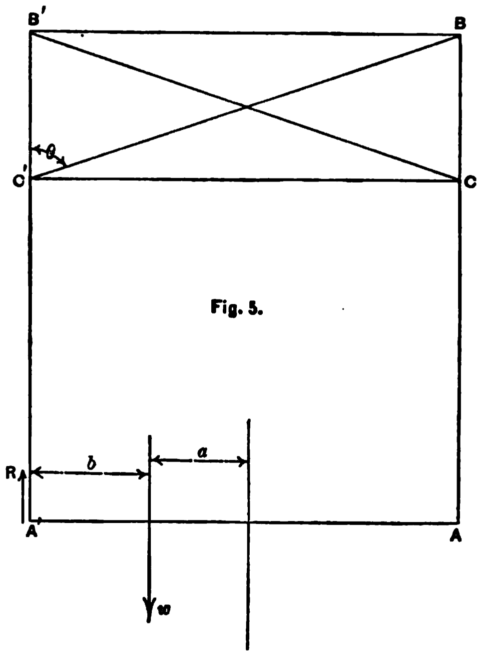
The compression on BB' at the portal as part of the upper lateral system is $2P_1 - P_e$, and adding to this the compression due to the moment at C, we have at the portal,

$$\text{compression in } BB' = \frac{2P_1\sqrt{d^2 + p^2}(b - 2s)}{bm'} + 2P_1 - P_e. \quad . \quad . \quad . \quad . \quad (19)$$

If there are no knee braces, but the strut BB' is a flanged beam, rigidly fastened at the ends B and B' to the posts, the post compression is, as in the previous case, $V = \frac{2Pd}{b}$.

There is a moment at the end of the strut equal to $Vb - Pd$ or Pd. If the effective depth of the beam BB' is d', the compression in each flange is $\frac{Pd}{d'}$, or $\frac{2Pd}{d'}$ for both flanges due to this moment. This compression must be added to that in BB' as part of the upper lateral system.

DOUBLE TRACK.—The preceding holds good for wind bracing for single track. For double track we have not only the strains, as already found, but also strains due to transference of the load when one track only is covered by the train, the other being empty.



Thus, in Fig. 5, let W be the weight on a cross-girder, for one track loaded. Then the reaction R is given by

$$R \times 2\,(a + b) = W\,(2a + b), \text{ or}$$

$$R = W\frac{2a + b}{2\,(a + b)}.$$

If the sway bracing were perfectly rigid, however, both trusses would have to deflect together, and each truss would carry half the load.

The weight G, transferred by the sway bracing, would then be,

$$G = R - \frac{W}{2} = \frac{Wa}{2(a + b)}.$$

The strain in the vibration rod due to this action is,

$$\text{strain in } C'B = -\frac{Wa \sec\theta}{2(a + b)}. \quad \ldots \ldots \ldots \quad (20)$$

For $C'C$, taking moments about B, we have

$$C'C \times f = G \times 2(a + b), \text{ or } C'C = \frac{Wa}{f}. \quad \ldots \ldots \ldots \quad (21)$$

The strain in BB' is zero.

The moment at C is Wa.

If m is the distance between post channels, the compression on one channel is $\dfrac{Wa}{m}$.

$$\text{The compression due to bending is} = \frac{2\,Wa}{m}. \quad \ldots \ldots \ldots \quad (22)$$

These strains must be *added* to the wind strains as already found.

For deck bridge, make $f = d$ in (21); (20) and (22) remain unchanged.

This action does not take place at the portal.

When knee braces are used, eq. (20) gives the strain in $D'C'$, Fig. 4.

$$\text{For the moment at } C', \text{ we have } \frac{Wa}{2(a + b)}\,[2(a + b) - s].$$

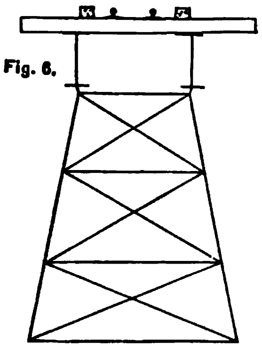
If d' is the depth of strut BB', Fig. 4, this gives a compression on the strut BB' of

$$\frac{Wa}{(a + b)\,d'}\,[2\,(a + b) - s].$$

These strains must be added to wind strains.

STRAINS IN BRACED PIERS AND TRESTLE BENTS.—A trestle "bent" is simply a pair of columns connected transversely by bracing, as shown in Fig. 6. A trestle tower consists of two bents, or four columns, united by longitudinal and transverse bracing. It is customary to unite only every other span in this manner. The usual transverse batter given to the "bent" column is 6 vertical to 1 horizontal.

Fig. 6.



In Fig. 7 we show a side view of the towers. Every other two bents are united by longitudinal bracing.

Each tower must have sufficient base, longitudinally, to be stable when standing alone without other support than its anchorage. That is, no dependence is to be placed on the girder connection between two towers at top, but the entire tower should be capable of standing alone, with the maximum wind-force on either side transverse to axis of bridge. Tower spans for high trestles are usually about 30 feet, the intermediate spans 60 feet.

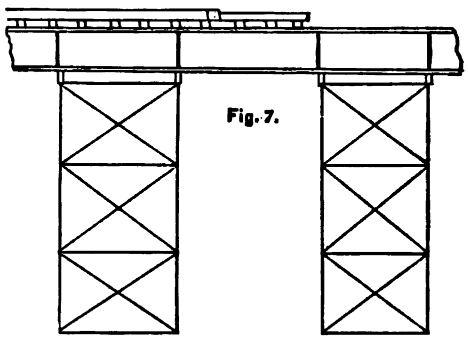


Fig. 7.

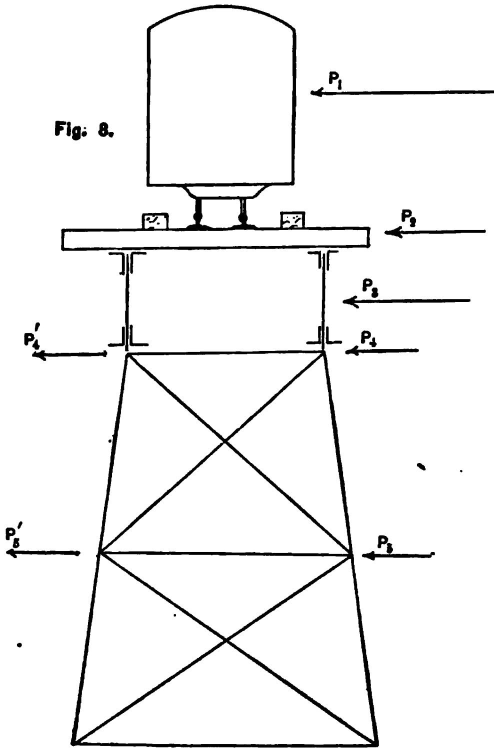
The longitudinal bracing of each tower must be capable of resisting the greatest tractive force of the engines, or any force induced by suddenly stopping, upon any part of the trestle, the assumed maximum trains.

If W is the maximum weight due to train on a bent, and ϕ is the coefficient of friction, usually taken at $\frac{1}{8}$th, then ϕW is the tractive force acting longitudinally at the top of the tower, for which the longitudinal bracing must be figured.

OUTER FORCES.

WIND STRAINS IN A BENT.—We have first the wind force on train P_1, Fig. 8. This is taken at 300 lbs. per linear foot of span, and the span *on each side* of the bent is covered by train. We may take P_1 as acting 9 feet above the base of the rail on the windward side only. P_2 is the wind on the ties and guard-rails, considered as acting at the foot of the rail, and may be taken at 30 lbs per square foot of exposed surface. The exposed surface of ties and guard-rails may be taken at 1 square foot per linear foot; so that the wind force may be taken at 30 lbs. per linear foot on ties and guard-rails. P_2 will then be 30 × the half span on each side of the bent, and acts on the windward side only.

Fig. 8.



We have, next, the wind force P_3 on the truss. This also is taken at 30 lbs. per square foot of exposed surface. If the girder is a plate girder, it acts on the windward side only. If a framed truss, it acts upon both windward and leeward sides. In the first case P_3 is $\frac{1}{2}$ (30 × area of girder on one side of bent) + $\frac{1}{2}$

(30 × area of girder on other side of bent). In the second case P_3 is $\frac{1}{2}$(30 × area of truss on one side of bent) + $\frac{1}{2}$ (30 × area of truss on other side of bent), and it acts at *both* windward and leeward sides. For a framed girder we may take the area of a truss of 4 square feet per linear foot, and hence we have 120 lbs. per linear foot for wind on each truss.

The wind on the towers is taken at 125 lbs. per vertical linear foot for the whole side, or one-half of this for one column; and it acts on both windward and leeward sides of bent. Hence, we have for P_4, $\frac{125}{2} \times ab$, and P'_4 the same. For P_5, $\frac{125}{2} \times bc$, and P'_5 the same.

We have also, at the top of the bent at a and a', the quarter weight of the superstructure with train for span on each side, and that part of the weight of tower itself, which is concentrated at a and a'. We denote these forces by W_4, W'_4.

Also at P_5 and P'_5, we have that part of the weight of the tower itself which is concentrated at these points, W_5, W'_5.

These constitute all the outer forces, and in any given case they are easily estimated.

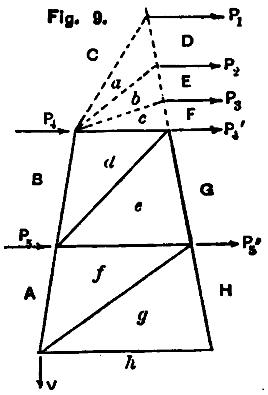
STRAINS IN A BENT.—The simplest and easiest method of finding the strains due to the wind is by diagram.

We first estimate the wind forces P_1, P_2, P_3, P_4, P'_4, P_5, P'_5, etc., as directed in the preceding article.

Then draw the bent carefully to scale, as shown in Fig. 9 (a), which represents the bent with wind from the left.

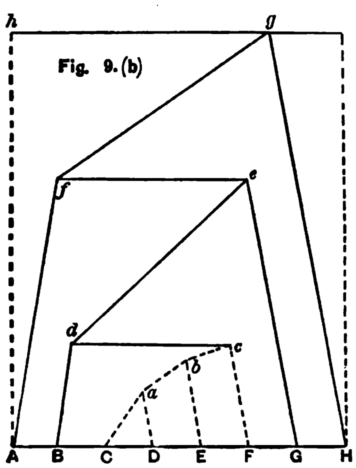
We have simply to prolong the leeward column as indicated, till it meets P_1, or CD, according to our notation, acting at 9 feet above the rails, and draw the imaginary tie Ca. In the same way, prolong P_2, or DE, and P_3 or EF, to intersection with the leeward column, P_2 acting at foot of rails, and P_3 at the centre of the girder, and draw the ties ab, bc.

Fig. 9.



We can now diagram the strains due to these forces, as shown in Fig. 9 (b). (This method of diagram is explained in Chapter I. of Part I.) The wind forces on train P_1, and on ties and guard-rails P_2, will always be above the top strut. The wind force on the girder P_3 is above the top strut when the girder is on top of the bent. But in the case of a deck span, it may be below the top strut. In any case, prolong it to intersection with leeward column, and draw bc, either above or below.

Fig. 9.(b)



If the girder is framed, P_3 is to be taken as the *sum* of the pressures on both windward and leeward trusses. For a plate girder, it is, of course, only the pressure upon the exposed side.

The vertical forces W_4, W'_4, include the weight of girder, weight of track, dead weight of structure, and weight of train; W_5 and W'_5 are simply the apex loads due to weight of structure.

The calculation of strains due to the vertical forces is very simple. If θ is the angle which a column makes with the vertical, the strain in the top strut, cd, is $W_4 \tan \theta$, and in Bd or Ge, $W_4 \sec \theta$. In the next strut, ef, we have $(W_4 + W_5) \tan \theta$, and Af or Hg $= (W_4 + W_5) \sec \theta$, and so on.

All these strains are compression. There are no strains due to vertical loading in the inclined braces.

When the train is off, W_4 and W'_4 will be diminished by the weight of train, and the wind strains will be diminished by the strains due to P_1. There should be no tension in the windward columns, Af and Bd, under any circumstances. We can easily calculate the strains in Af and Bd for train off.

One of the last pieces in our diagram should always be checked by calculation.

EXAMPLE.—*A bent is 20′ $4\frac{3}{4}$″ at bottom, and 9′ $2\frac{1}{2}$″ at top, c to c. The height from base to top strut is 45 feet. From top strut to next strut, 18′ $3\frac{1}{8}$″, and from there to bottom strut, 26′ $8\frac{3}{4}$″. The span on each side of the bent is 30 feet. The plate girder on top is 3 feet deep, and its bottom is 6 inches above the top strut. The foot of rail is 4 feet above the top strut. Find the strains.*

Let us first estimate the outer forces.

We have for P_1, the wind force on train, 300 lbs. per linear foot for both spans covered, or $\frac{300 \times 30}{2} + \frac{300 \times 30}{2} = 9000$ lbs.

This acts at 9 feet above foot of rail, or 13 feet above top strut.

For P_2, the wind force on ties and guard-rails, we have 30 lbs. per linear foot, or 90 lbs. acting at foot of rail, or 4 feet above the top strut.

For P_3, the wind force upon the exposed surface of the girder, we have 30 lbs. per square foot, or $\frac{90 \times 30}{2} + \frac{90 \times 30}{2} = 2700$ lbs., acting at $1.5 + 0.5 = 2$ feet above the top strut.

For P_4, the wind force on bent, we have $\frac{125}{2} \times 9 = 560$ lbs., and P'_4 the same.

For P_5, we have $\frac{125}{2}(9 + 13.5) = 1400$ lbs., and P'_5 the same.

An estimate of the weight of the structure gives $W_5 = W'_5 = 2000$ lbs. For W_4, we have for weight of structure 500 lbs. at each cap; taking the track at 400 lbs. per linear foot, we have 6000 lbs. at each cap; the weight of the girder is estimated at 4850 lbs. at each cap; the train, taking the loading of our diagram, page 215, is 68715 lbs. at each cap.

For the train on, $W_4 = W'_4 = 80000$ lbs., $W_5 = W'_5 = 2000$ lbs. We have then, for the strains due to vertical loading, *train on*, since $\tan\theta = 0.124$, $\sec\theta = 1.007$,

$Bd = Ge = +80000 \times 1.007 = +80560$ lbs. $\qquad Af = Hg = +82000 \times 1.007 = +82574$ lbs.

$cd = +80000 \times 0.124 = +9920$ lbs. $\qquad ef = +82000 \times 0.124 = +10168$ lbs.

$V = +82000$ lbs. $\qquad de = fg = 0$.

For the *train off*, $W_4 = W'_4 = 11285$, $W_5 = W'_5 = 2000$ lbs., and we have

$Bd = +11285 \times 1.007 = +11364$ lbs. $\qquad Af = +13285 \times 1.007 = +13378$ lbs.

$V = +13285$ lbs.

V is the pressure on the anchorage of the windward side.

For the strains due to P_1 *alone*, in Af and Bd, we have lever arm for $Bd = 9.2\cos\theta = 9.13$, lever arm for $Af = 15.7\cos\theta = 15.6$ feet, and

$$Af \times 15.6 = -9000 \times 31.3, \text{ or } Af = -18057 \text{ lbs.}$$
$$Bd \times 9.13 = -9000 \times 13, \text{ or } Bd = -12814 \text{ ''}$$
$$V \times 20.36 = -9000 \times 58, \text{ or } V = -25638 \text{ ''}$$

where V is the tension on anchorage of windward side.

Making now our diagram, we have, for wind strains, *train on*,

$cd = +9700$ lbs.	$ef = +8300$ lbs.	$gh = +11600$ lbs.	$de = -16250$ lbs.
$fg = -15000$ lbs.	$Bd = -14000$ lbs.	$Af = -27750$ lbs.	$Ge = +27850$ lbs.
$Hg = +40650$ lbs.	$V = -40340$ lbs.		

A convenient scale for the diagram, which has been adopted in finding these results, is ten feet to an inch, for the bent, and 4000 lbs. to an inch for the forces.

We can check the value of V as follows:

$$V \times 20.36 = -9000 \times 58 - 900 \times 49 - 2700 \times 47 - 2(560 \times 45) - 2(1400 \times 26.7), \text{ or } V = -40184 \text{ lbs.}$$

We have now for the maximum strains,

$cd = +9700 + 9920 = 19620$ lbs. $ef = +8300 + 10168 = +18468$ lbs.

$gh = +11600$ lbs. $de = -16250$ lbs. $fg = -15000$ lbs.

$Ge = +27850 + 80560 = +108410$ lbs. $Hg = +40650 + 82574 = +123224$ lbs.

$V = +82000 - 40340 = +41660$ lbs.

These strains are obtained by combining the strains due to wind, *train on*, with vertical loading, *train on*. If the wind were from the other side, we would have same strains in Bd and Af as in Ge and Hg, already found, the struts cd, ef, and gh would be the same, and the other ties would act. We do not care to put down the strains for Bd and Af, for wind from left, because they are less than Ge and Hg, already found. But we should see if they are in tension or not. They will be

$Bd = +80560 - 14000 = +66560$ lbs. $Af = +82574 - 27750 = +54824$ lbs.

Both are therefore in compression, but maximum compression is when wind is on other side.

Also, on windward side,

$$V = +82000 - 40340 = +41660 \text{ lbs.},$$

or there is no tension on anchorage.

Let us now see whether the bent with the *train off* has no tension in the columns.

Subtract from the diagram strains the strains for P_1 alone, which we have calculated, and we have wind strains when *train is off*. Combine these with vertical load strains when train is off, and see if Af and Bd are still in compression.

We have

$$Bd = +11364 - (14000 - 12814) = +10178 \text{ lbs.}$$
$$Af = +13378 - (27750 - 18057) = +3685 \text{ lbs.}$$
$$V = +13285 - (40340 - 25638) = -1417 \text{ lbs.}$$

We see that there is no tension in the columns of the windward side, under any circumstances, but there may be a small tension of 1417 lbs. on the anchorage.

The *longitudinal bracing* in this case would be figured for a force of $68715 \times \varphi$, where $\varphi = \frac{1}{5}$, or 13743 lbs., acting at the top of the tower.

It causes strains in the longitudinal ties, which can be easily found by calculation, by multiplying by the sec of the angle α, which the ties make with the horizontal. Thus, in the present case, $\sec \alpha = 1.171$ for the first tie, and $\sec \alpha = 1.338$ for the next tie.

The strains in these ties are then $-13743 \times 1.171 = -16090$ lbs., and $-13743 \times 1.338 = -18388$ lbs.

PONY TRUSSES.—As pony trusses do not admit of over-head bracing, we have only horizontal bracing under the floor. If the floor beams are riveted to the posts, these latter may be continued below the floor beams, and horizontal and vertical sway bracing introduced. The floor beams may also be prolonged and stays or knee braces introduced to support the truss sideways.

WEIGHT OF WIND BRACING.—For preliminary estimates of weight the following formulæ will be found useful:

FOR PONY TRUSSES.—*Depth below* 12.5 *feet*,

$$\textit{weight per foot lineal of wind bracing} = 3.6\,N + \frac{540}{p}.$$

FOR THROUGH TRUSSES WITHOUT VERTICAL SWAY BRACING.—*Depth between* 12 5 *and* 24 *feet*,

$$\textit{weight per foot lineal of wind bracing} = 6.4\,N + \frac{672}{p}.$$

FOR THROUGH TRUSSES WITH VERTICAL SWAY BRACING (*depth above* 24 *feet*), OR DECK BRIDGES,

$$\textit{weight per foot lineal of wind bracing} = \frac{6Nl}{170} + \frac{1136}{p},$$

where l = length in feet, N = number of panels, p = panel length in feet.

All these formulæ are for single track, and give the total weight *for both trusses.* For double track multiply by $\frac{b}{15}$, where b = width or breadth of bridge in feet.

FRICTION ROLLERS.—The specifications of the New York, Penn. & Ohio R. R. require all bridges over 70 feet span to have at one end a nest of turned friction rollers of wrought iron, running between planed surfaces.

"The rollers shall not be less than 2 inches in diameter, and shall be so proportioned that the pressure per lineal inch of rollers shall not exceed the product of the square root of the diameter of the roller in inches multiplied by 500," or permissible pressure,

$$p = 500\sqrt{d},$$

where p is the permissible pressure per lineal inch, and d is the diameter in inches.

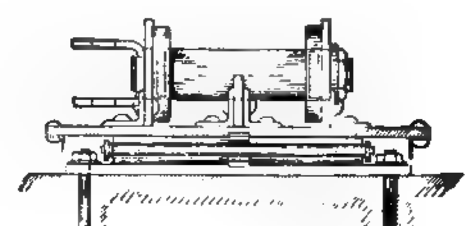


Fig. 253

We give, in Figs. 253, the construction of rollers, roller nut, and cover plate for rollers.

A more rational formula has been deduced by Professor Burr (*Stresses in Bridge and Roof Trusses*),

$$p = \frac{4}{3}R\sqrt{\frac{4w^3}{E}},$$

where w = the greatest allowable pressure on a roller, or 12000 lbs. per sq. inch for wrought iron. R = radius of roller in inches. E = coefficient of elasticity = 28000000 for wrought iron.

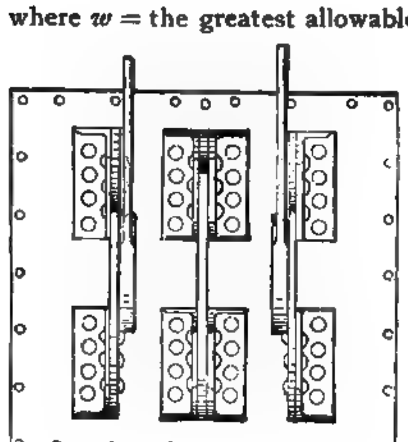


Fig. 253

For $R = 1''$, this gives $p = 662$ lbs. per lineal inch, while the first equation gives $p = 707$ lbs.

We give, in Plates 19, 20, and 21, illustrations of various details which have been referred to in the foregoing pages.

EQUIVALENT LENGTH OF RODS FOR UPSET ENDS, NUTS, SLEEVE NUTS, AND TURN BUCKLES.—We have already given, in Pin Table II. of the preceding chapter, the equivalent length of chord bar required to make the head. For main diagonals and hip verticals it will be sufficient in general to add 3 feet for eyes, and for adjustable rods, such as counters and wind ties, 5 feet for turn buckles. If greater accuracy is required for the latter, we may ascertain what length of

rod is needed at each end for the connection, and how much for adjusting nuts and upset ends, by the following Table:

($\frac{1}{4}''$—$1''$) 1 upset end and 1 nut = $1\frac{1}{7}$ feet of rod.
($1\frac{1}{16}''$—$1\frac{1}{2}''$) " " " " = $1\frac{3}{8}$ " " "
($1\frac{9}{16}''$—$2''$) " " " " = $1\frac{3}{5}$ " " "
($2\frac{1}{8}''$—$2\frac{1}{2}''$) " " " " = $1\frac{7}{10}$ " " "
($\frac{1}{2}''$—$1\frac{1}{4}''$) 2 upset ends and 1 nut = $2\frac{3}{8}$ " " "
($1\frac{5}{16}''$—$2\frac{1}{2}''$) " " 1 nut, 1 turn buckle, 3 ft. of rod.

These equivalent lengths do not include the lengths of the upset ends themselves; they represent simply the extra length to be added to the bar to allow for the weight of the nuts, sleeve nut, or turn buckle, and the extra iron for the enlarged ends, which are generally about 8 inches long.

CAMBER.—In practice the upper and lower chords of bridges are not perfectly horizontal, but are curved upward by such an amount that even when fully loaded they do not quite reach the horizontal.

This upward deflection is called the "*camber*."

The two chords form, thus, concentric arcs, and since the unit strain is constant, these arcs are circular.

In order to produce the camber the upper chord is made longer than the lower.

The finding of the actual lengths of the members "*c* to *c*," or centre to centre of pin holes, is one of the most important points of the design.

Let u' be the allowable working stress per square inch in the upper chord for combined dead and live loads, and u for the lower chord.

Some specifications call for a different unit stress for dead and live loads.

Let L = the live load strain in any member in lbs., and D = the dead load strain in the same member, and let β = the allowed unit stress for the dead load, and β' for the live load. Then, if U is the combined unit stress for *any* member, for both dead and live loads, we have

$$\frac{D+L}{U} = \frac{D}{\beta} + \frac{L}{\beta'}, \text{ or } U = \frac{D+L}{\frac{D}{\beta} + \frac{L}{\beta'}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I.)}$$

when, as is most often the case, $\beta = \beta'$, we have $u' = \beta$.

From (1), by introducing the values of L and D, and β, β', in any case for the *upper* chord, we can find u', and introducing the values of L and D, β and β' for the *lower* chord, we can find u.

Let s = the length of span, and E = the coefficient of elasticity. Then the compression of the upper chord, under the combined dead and live loads, if the truss deflected from a horizontal, would be $\frac{u's}{E}$ (page 233), and its new length would be $s - \frac{u's}{E}$. In the same way the new length of the lower chord, after deflection from the horizontal, would be $s + \frac{us}{E}$.

If we camber the truss upward, in order to just counteract the deflection, we should make the *upper* chord $s + \frac{u's}{E}$, and the *lower* chord $s - \frac{us}{E}$.

Let d = the depth of truss c to c, and r the radius of the lower chord. Then $r + d$ is the radius of the upper chord, and we have

$$r + d : r :: s + \frac{u's}{E} : s - \frac{us}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence,

$$r = \frac{Ed - du}{u + u'},$$

or, since u is small compared to E,

$$r = \frac{Ed}{u + u'}. \qquad (2)$$

Let $\triangle$ be the camber. Then, from the right-angled triangle formed by the half span, the radius of the lower chord at end, and the vertical through centre of span, we have

$$(r - \triangle)^2 + \frac{s^2}{4} = r^2, \text{ or } \triangle = r - \sqrt{r^2 - \frac{s^2}{4}}. \qquad (3)$$

We have, also, if $\frac{I}{2}$ is the increase of the upper chord over the lower in the half span,

$$\frac{I}{2} : d :: \frac{s}{2} : r - \triangle,$$

or, since $\triangle$ is very small compared to r, we can neglect it, and hence, $I = \frac{ds}{r}$.

The increase of length of the upper over the lower chord *per unit of length*, that is, in inches for every inch of length, or in feet for every foot of length, is then $i = \frac{d}{r}$, or, substituting the value of r from (2),

$$i = \frac{d}{r} = \frac{u + u'}{E}.$$

To allow for the additional deflection due to the web, let us increase this amount by one-third. We then have

$$i = \frac{4(u + u')}{3E}, \qquad (\text{II.})$$

where u is the unit stress for the lower chord, and u' for the upper chord, for combined dead and live loads, to be found from (I.).

A convenient practical rule for this increase of length of the top chord, per unit of length, which is given in Cooper's specifications, may be deduced by taking $u = 10000$ lbs. and $u' = 8000$ lbs., and $E = 24000000$ lbs. Then, from (II.),

$$i = \frac{1}{1000}.$$

For a length of 10 feet this becomes $\frac{1}{100}$, or about $\frac{1}{8}$th of an inch. Hence the rule, "*$\frac{1}{8}$th of an inch for every* 10 *feet of panel.*"

In figuring the lengths of the various members, we give them such lengths that, under the action of the *dead load only*, any lower panel shall have the length p from c to c, any post the length d from c to c, and the remaining camber *shall still be that given by* (*II.*) *for dead and live loads combined.* This insures that, when the bridge is fully loaded, the truss shall still be above the horizontal at the centre, by an amount about equal to the dead load deflection.

The unit stress due to dead load only is from (I.) $\dfrac{D}{\frac{D}{\beta} + \frac{L}{\beta'}}$. Hence, the extension or

compression of any member per unit of length, for dead load only, is

$$e = \frac{D}{E\left[\frac{D}{\beta} + \frac{L}{\beta'}\right]}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(III.)}$$

when $\beta = \beta'$ this becomes

$$e = \frac{D\beta}{E(D+L)}.$$

We must allow for this extension or compression due to the dead load, in figuring the lengths, so that, when the dead load only acts, the lower chord panel may be p, the posts d, and the upper chord panel $p + ip$, c to c where i is found from (II.).

Also, since the pin hole is always bored one-fortieth of an inch (0.025″) larger than the pin, we must allow for this clearance.

Equations (I.), (II.), and (III.) completely solve our problem. We recapitulate them here for convenience of reference.

Unit stress for combined dead and live loads in any member.

$$U = \frac{D+L}{\frac{D}{\beta} + \frac{L}{\beta'}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(I.)}$$

where D is the dead load, L the live load strain in the member, and β and β' the unit stresses for dead and live loads. When $\beta = \beta'$, this becomes $U = \beta$.

Increase of length of upper chord per unit of length.

$$i = \frac{4(u + u')}{3E}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(II.)}$$

where u is the unit stress for lower chord, and u' for upper chord, found from (I.).

Extension or compression per unit of length, of any member, due to the dead load only.

$$e = \frac{D}{E\left[\frac{D}{\beta} + \frac{L}{\beta'}\right]}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(III.)}$$

When $\beta = \beta'$, this becomes $e = \dfrac{D\beta}{E(D+L)}$.

ACTUAL LENGTH OF LOWER CHORD BARS.—Since the lower chord bars are in tension, we must make them a little short, so that, allowing for pin clearance and dead load extension, they will pull out to the length p. We have therefore

$$\textit{actual length of lower chord bars c to c} = p - ep - 0.025, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (a)$$

when p is the panel length c to c in inches and e is found from (III.).

The length is figured only to the nearest $\frac{1}{32}$d of an inch, as that is the least shop measurement.

ACTUAL LENGTH OF POST.—The post is in compression, and we therefore make it longer than d, to allow for dead load compression and pin clearance. We have therefore

$$\textit{actual length of post c to c} = d + ed + 0.025, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (b)$$

where d is the depth c to c in inches, and e is found from (III.).

The length is figured to the nearest $\frac{1}{32}$d of an inch.

ACTUAL LENGTH OF UPPER CHORD PANELS.—Since the chords are in close contact, there is no allowance for pin clearance. We must make it a little long to allow for dead load compression, and also increase it by the amount ip. We have then

$$\textit{actual length of upper chord panel} = p + ip + ep, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

where p is the panel length c to c in inches, e is found from (III.), and i from (I.) and (II.).

The length is figured to the nearest $\frac{1}{32}$d of an inch.

ACTUAL LENGTH OF INCLINED TIES.—The length of the tie is to be

$$l = \sqrt{d^2 + \left(p + \frac{ip}{2}\right)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (d)$$

where d is the depth c to c in inches, p is the panel length c to c in inches, and $p + \frac{ip}{2}$ is the mean of the upper and lower chord panel lengths. We find i from (I.) and (II.). We have then, allowing for dead load extension and pin clearance,

$$\textit{actual length of ties c to c} = l - el - 0.025, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

where l is found from (d) and e from (III.).

The length is figured to the nearest $\frac{1}{32}$d of an inch.

In a *draw* span each arm may be considered as one span in giving the camber, but whole amount of lengthening of the upper chord must be taken out of the upper chord at centre, or the ends will sink below their original positions.

EXAMPLE.—*Let the span c to c be 200 feet, depth c to c 25 feet, panel length c to c 20 feet. In a given panel we have the following strains and unit stresses:*

	L	D	β'	β
LOWER CHORDS,	240000 lbs.	120000 lbs.	8000 lbs.	16000 lbs.
UPPER CHORDS,	180000 "	90000 "	7000 "	14000 "
TIES,	100000 "	40000 "	8000 "	16000 "
POSTS,	87000 "	35000 "	4000 "	8000 "

Let $E = 24000000$ lbs. *Find the required lengths.*

For the lower chords, we have from (III.) $e = \frac{1}{7500}$, and from (a)

$$\textit{length of chord bars c to c} = 240 - \frac{240}{7500} - 0.025 = 239.943'' = 19 \text{ ft. } 11\tfrac{15}{16} \text{ in.}$$

For the posts, we have from (III.) $e = \frac{1}{17914}$, and from (b)

$$\textit{length of posts c to c} = 300 + \frac{300}{17914} + 0.025 = 300.042'' = 25 \text{ ft. } 0\tfrac{1}{32} \text{ in.}$$

For the upper chords, we have from (III.) $e = \frac{1}{8571}$.

From (I.) we have

$$u = 96000, \ u' = 8400, \text{ and from (II.) } i = \frac{1}{1000}. \quad \text{We have then from } (c)$$

$$\textit{length of upper chord panel} = 240 + \frac{240}{1000} + \frac{240}{8571} = 240.268'' = 20 \text{ ft. } 0\tfrac{9}{32} \text{ in.}$$

For the ties, we have from (III.) $e = \frac{1}{9000}$, and from (I.) and (II.),

$$i = \frac{1}{1000}. \quad \text{Then } p + \frac{ip}{2} = 240.12'', \ l = \sqrt{300^2 + 240.12^2} = 384.262'',$$

and from (d)

$$\textit{length of ties c to c} = 384.262 - \frac{384.26}{9000} - 0.025 = 384.195'' = 32 \text{ ft. } 0\tfrac{3}{16} \text{ in.}$$

EXAMPLE.—*Let the span c to c be 250 feet, depth c to c 45 feet, panel length c to c 25 feet. In a given panel we have the following strains and unit stresses:*

	L	D	$\beta = \beta'$
LOWER CHORDS,	300000 lbs.	150000 lbs.	10000 lbs.
UPPER CHORDS,	340000 "	170000 "	8000 "
POSTS,	100000 "	35000 "	8000 "
TIES,	114000 "	40000 "	10000 "

Let E = 26000000 lbs. *Find the actual lengths.*

For the lower chords we have from (III.) $e = \frac{\beta}{3E} = \frac{1}{7800}$, and from ($a$),

$$\textit{actual length of chord bars c to c} = 300 - \frac{300}{7800} - 0.025 = 299.94'' = 24 \text{ ft. } 11\tfrac{15}{16} \text{ in.}$$

For the posts we have from (III.) $e = \frac{1}{12536}$, and from (b),

$$\textit{length of posts c to c} = 540 + \frac{540}{12536} + 0.025 = 540.068'' = 45 \text{ ft. } 0\tfrac{1}{16} \text{ in.}$$

For the upper chords, we have from (III.) $e = \frac{1}{7800}$.

From (I.) we have

$$u = 10000,\ u' = 8000, \text{ and from (II.) } i = \frac{24}{26000}.$$

From (c).

$$\textit{length of upper chord panel} = 300 + \frac{300 \times 24}{26000} + \frac{300}{7800} = 300.315'' = 25 \text{ ft. } 0\tfrac{5}{16} \text{ in.}$$

For the ties, we have from (III.) $e = \frac{20\beta}{77E} = \frac{1}{10010}$.

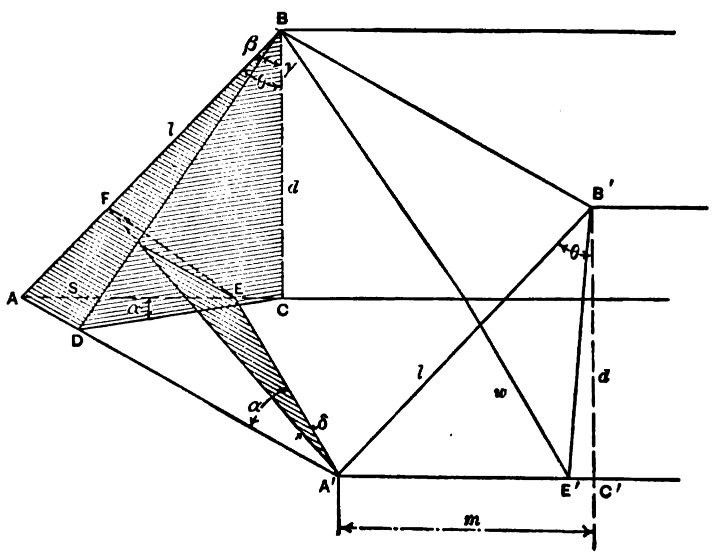
From (I.) and (II.),

$$i = \frac{24}{26000},\ p + \frac{ip}{2} = 300.138,\ l = \sqrt{540^2 + 300.138^2} = 617.804.$$

From (d)

$$\textit{length of ties c to c} = 617.804 - \frac{617.804}{10010} - 0.025 = 617.718'' = 51 \text{ ft. } 5\tfrac{23}{32} \text{ in.}$$

BEVEL ANGLES FOR SKEW PORTALS.—The angles required for laying out a skew portal are the angles $ABD = \beta$, or the amount by which the angle ABB', between the inclined end post and the portal strut, differs from 90°; the angle $DBC = \gamma$, or the angle between the plane of the portal and a vertical plane through BB'; the angle $FA'E = \delta$, or the amount by which the angle between the plane of the portal and the plane of the truss differs from 90°.



In the figure, the line BD lies in the plane of the portal, and is perpendicular to AA'. Therefore, $90 + \beta$ gives the angles ABB', and $AA'B'$ and $90 - \beta$ gives the angles $BB'A'$ and BAD, all in the plane of the portal. The line DC is

in the plane of the bottom chords and is perpendicular to AA'. Therefore the angle $ACD = \alpha$ is the skew angle, or is equal to $AA'E$, $A'E$ being in the plane of the bottom chords and perpendicular to them. The line BC is vertical and in the plane of the truss, so that the angle $DBC = \gamma$ is the angle between the plane of the portal and a vertical plane through the portal strut BB'. Through $A'E$ we pass a plane perpendicular to AB, the inclined end post. Then the angle $FEA' = 90°$, and the angle $FA'E = \delta =$ the amount by which the angle between the plane of the portal and the plane of the truss differs from 90°.

Let the depth of truss, $BC = B'C' = d$; the width of truss $A'E = w$; the horizontal projection of inclined end posts $AC = A'C' = m$; the length of inclined end posts $= l$; the angle between inclined end posts and vertical, $ABC = A'B'C' = \theta = FEA$; the skew angle $AA'E = \alpha = ACD$; the skew $AE = s$; the length of portal strut $AA' = BB' = \varepsilon$.

Then

$$\left.\begin{array}{lll} \sin\theta = \dfrac{m}{l} = \dfrac{m}{\sqrt{m^2+d^2}}, & \cos\theta = \dfrac{d}{l} = \dfrac{d}{\sqrt{m^2+d^2}}, & \tan\theta = \dfrac{m}{d} \\ \sin\alpha = \dfrac{s}{\varepsilon} = \dfrac{s}{\sqrt{s^2+w^2}}, & \cos\alpha = \dfrac{w}{\varepsilon} = \dfrac{w}{\sqrt{s^2+w^2}}, & \tan\alpha = \dfrac{s}{d} \end{array}\right\} \quad . \; . \; . \quad (1)$$

We have also, $l \sin\beta = AD = l \sin\theta \sin\alpha = AC \sin\alpha$.

Hence,

$$\sin\beta = \sin\theta \sin\alpha; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

$$d \tan\gamma = DC = AC \cos\alpha = d \tan\theta \cos\alpha.$$

Hence,

$$\tan\gamma = \tan\theta \cos\alpha; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (b)$$

$$\varepsilon \sin\alpha = AE, \qquad AE \cos\theta = FE, \qquad \varepsilon \cos\alpha = A'E, \qquad A'E \tan\delta = FE.$$

Hence,

$$\varepsilon \sin\alpha \cos\theta = \varepsilon \cos\alpha \tan\delta,$$

or

$$\tan\delta = \tan\alpha \cos\theta. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (c)$$

Equations (*a*), (*b*), and (*c*) give the required angles β, γ, δ. $90 + \beta$ and $90 - \beta$, give the angles between inclined end posts and portal strut, in the plane of the portal. $90 + \theta$ gives the angle between the top chords and inclined end posts in the plane of the truss. γ gives the angle between the plane of the portal and a vertical plane through portal strut, and gives the bevel for bending plates to connect with top chords. $90 - \delta$ gives the angle between plane of the portal and plane of the truss.

If we substitute in (*a*), (*b*), and (*c*) the values of (1), we have also,

$$\sin\beta = \frac{ms}{\sqrt{m^2+d^2}\,\sqrt{s^2+w^2}}; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a')$$

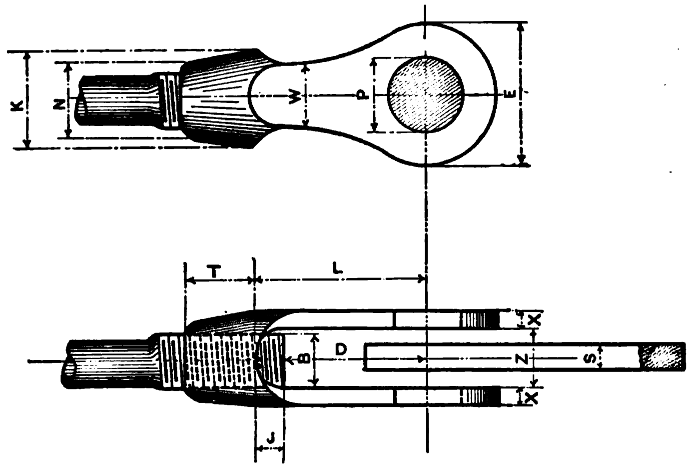
$$\tan\gamma = \frac{mw}{d\sqrt{s^2+w^2}}; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (b')$$

$$\tan\delta = \frac{s}{\sqrt{m^2+d^2}}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (c')$$

From which the required angles can be found in terms of s, w, and m and d.

STANDARD CLEVISES.—For attaching lateral rods, clevises are often used, as illustrated. We give, in the following Table, the dimensions and weight as furnished by the Phœnix Bridge Company.

STANDARD CLEVISES.



Tension (Rod), 12000 lbs. per sq. inch. Bearing, 15000 lbs. per sq. inch. Bending, 15000 lbs. per sq. inch.

Side of Square Bar.	Diameter of Round Bar.	*B* Diameter of Upset.	*W* Width of Strap.	*X* Thickness of Strap.	*P* Diameter of Pin.	*E* Diameter of Eye.	*Z* Width of Fork.	*S* Thickness of Plate.	*L* Length of Fork.	*y* Projection.	*D* End of Rod to C. of Pin.	*T* Length of Nut.	Nuts. *N* & *K*	Stock.	Number.	Estimated Weight in Lbs.	Shipping Weight.
"	"	"	"	"	"	"	"	"	"	"	"	"	"	"		LBS.	
		⅞	1½ ×	5/16	1	2¼	1	.35	5	½	4½	1⅛		1¾ sq. × 8	1	2¼	
		1	1½ ×	5/16	1⅛	2½	1⅛	.38	5	⅝	4⅜	1½	1½ & 1⅞	1¾ " × 8	2	3	
	⅞	1⅛	1½ ×	⅜	1¼	3	1¼	.384	5	¾	4¼	1½	1½ & 1⅞	1⅞ " × 8	3	3½	
	1	1¼	2 ×	⅜	1⅜	4 & 3¾	1⅜	.45	8	⅞	7⅛	2	2 & 2½	1⅞ " × 10	4	7¼	
	1⅛	1⅜	2 ×	⅜	1¾	4½	1½	.46	8	⅞	7⅛	2	2 & 2½	2 " × 12	5	8	
1	1¼	1½	2½ ×	⅜	2	5 & 4½	1⅝	.57	8	1	7	2	2 & 2½	2 " × 12	6	9	
1⅛		1⅝	2½ ×	⅜	2	5 & 4½	1¾	.57	8	1	7	2	2 & 2½	2 " × 12	7	9¼	
	1⅜	1¾	2½ ×	½	2	5 & 4½	1⅞	.64	8	1	7	2½	2½ & 3	2⅜ " × 15	8	10½	
1¼	1½	1⅞	2½ ×	⅝	2	5 & 4½	2	.7	8	1⅛	6⅞	2½	2½ & 3	2⅜ " × 15	9	12	
1⅜	1⅝	2	2½ ×	⅝	2⅜	5	2⅛	.67	8	1⅛	6⅞	3	2¾ & 3½	2½ " × 16	10	16	
1½	1¾	2⅛	3 ×	⅝	2⅜	5–5½–6½	2¼	.84	8	1¼	6¾	3	2¾ & 3½	2½ " × 16	11	17	
	1⅞	2¼	3 ×	¾	2⅜	5–5½–6½	2⅜	.96	8	1¼	6¾	3¼	3 & 3⅞	2¾ " × 16	12	20	
1[illegible]	2	2⅜	3 ×	⅞	2⅞	6 & 6½	2½	.88	8	1⅜	6⅝	3¼	3 & 3⅞	2¾ " × 16	13	22	
1¾	2⅛	2½	3 ×	⅞	2⅞	6 & 6½	2⅝	1	8	1⅜	6⅝	3½	3¼ & 4	3 " × 16	14	24	
	2¼	2⅝	3½ ×	⅞	2⅞	6 & 6½	2¾	1.11	8	1½	6½	3½	3¼ & 4	3¼ " × 17	15	26	
1⅞	2⅜	2¾	4 ×	⅞	3	7¼	2⅞	1.18	10	1½	8½	3½	3½ & 4¼	3½ " × 18	16	37	
2	2½	2⅞	4 ×	15/16	3⅛	7¼	3	1.25	10	1⅝	8⅜	3½	3½ & 4¼	3½ " × 18	17	39	
	2⅝	3	4 ×	1	3¼	7¼	3⅛	1.33	10	1⅝	8⅜	3¾	3¾ & 4½	3½ " × 18	18	43	
2⅛	2¾	3⅛	4 ×	1⅛	3⅜	7¾	3¼	1.36	10	1⅝	8⅜	3¾	3¾ & 4½	3¾ " × 18	19	48	
2¼	2⅞	3¼	4 ×	1¼	3¾	7¾	3⅜	1.4	10	1⅝	8⅜	4	4 & 5	3¾ " × 18	20	57	
2⅜	3	3⅜	4 ×	1 5/16	3¾	7¾	3½	1.5	10	1⅝	8⅜	4	4 & 5	3¾ " × 18	21	60	

PLATE 19.

Elevation of pedestal and section of rollers and roller plate.

End view of pedestal and roller plate.

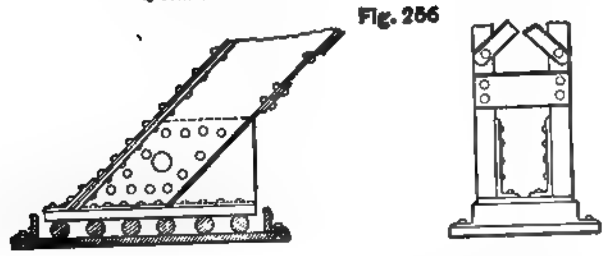
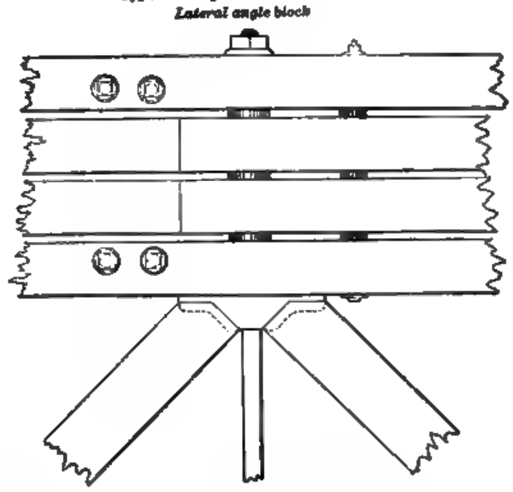


Fig. 256

Fig. 257

Upper chord pannel connection and Lateral angle block



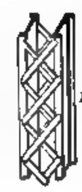


Fig. 258

Latticed post

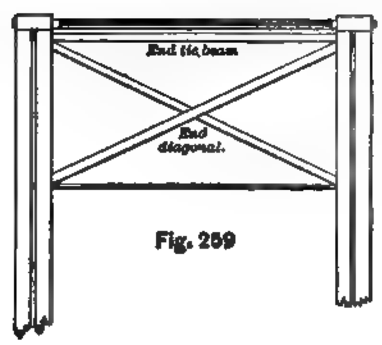


Fig. 259

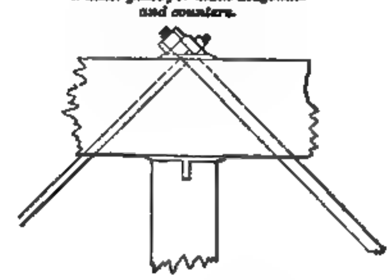
Packing washer.

Fig. 260

Washer.

Fig. 261

Washer plate for main diagonals and counters.



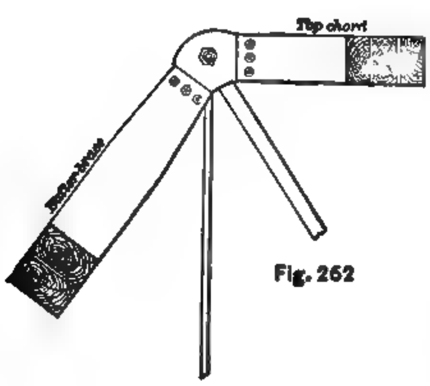


Fig. 262

Fig. 263 Lower lateral strut showing Jaw.

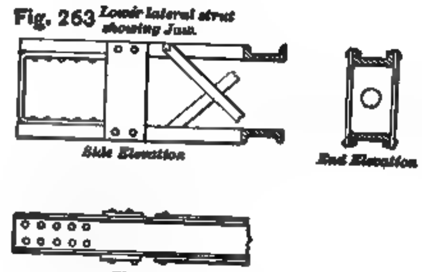
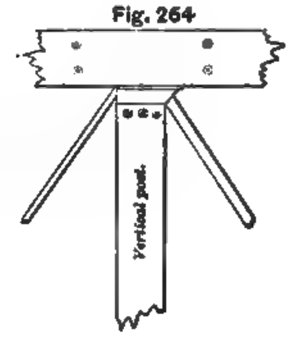


Fig. 264



Side brace connection to top chord

Fig. 266

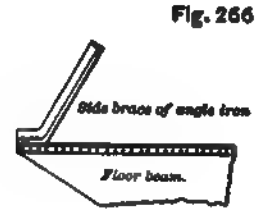


Fig. 267

Vibration rod with bent eyes. Elevation.

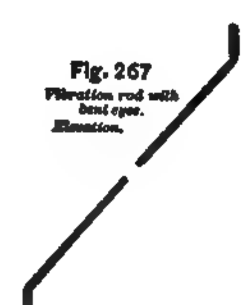


PLATE 20.

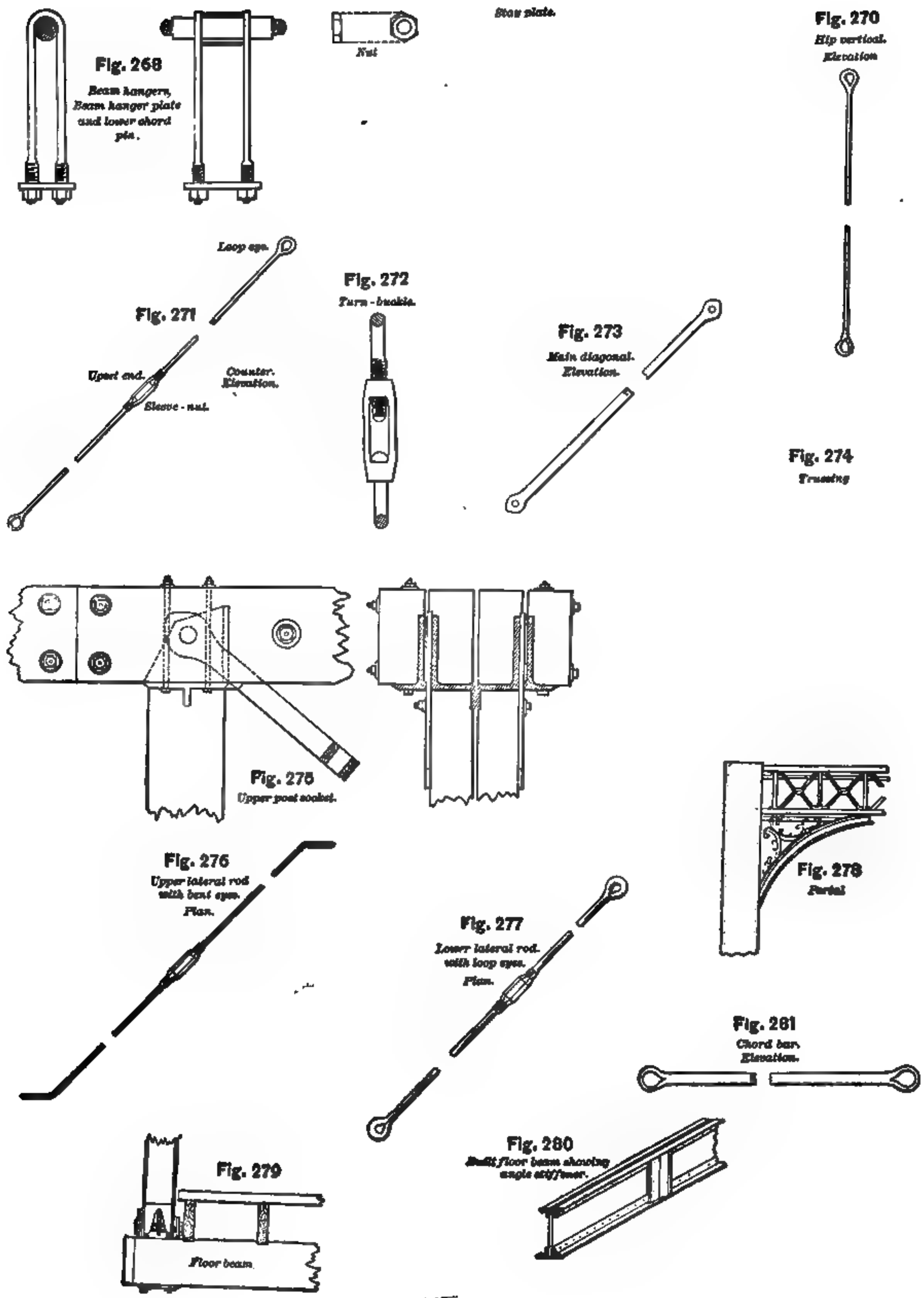
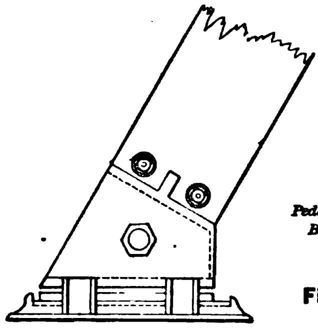


Fig. 268
Beam hangers, Beam hanger plate and lower chord pin.
Nut
Stay plate.
Fig. 270
Hip vertical.
Elevation
Fig. 271
Loop eye.
Upset end.
Sleeve-nut.
Counter.
Elevation.
Fig. 272
Turn-buckle.
Fig. 273
Main diagonal.
Elevation.
Fig. 274
Trussing
Fig. 275
Upper post socket.
Fig. 276
Upper lateral rod with bent eyes.
Plan.
Fig. 277
Lower lateral rod with loop eyes.
Plan.
Fig. 278
Portal
Fig. 281
Chord bar.
Elevation.
Fig. 280
Built floor beam showing angle stiffener.
Fig. 279
Floor beam

PLATE 21.



Pedestal and Bed plate

Fig. 282

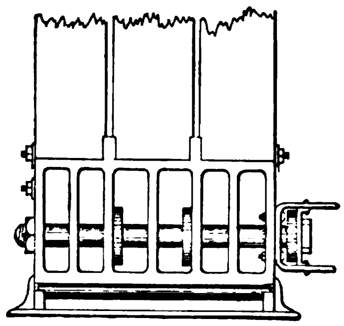
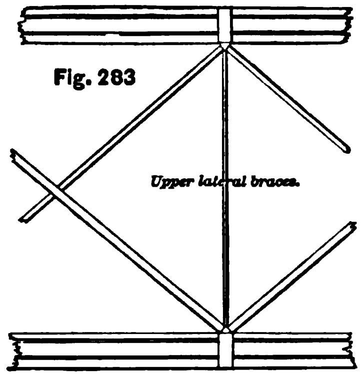
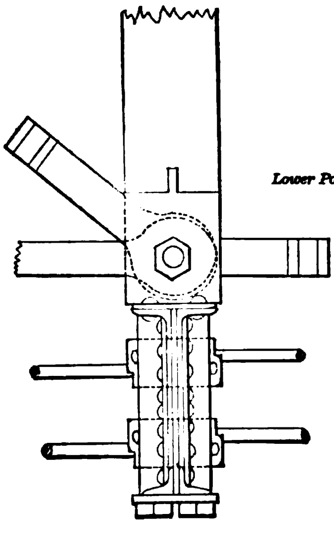


Fig. 283

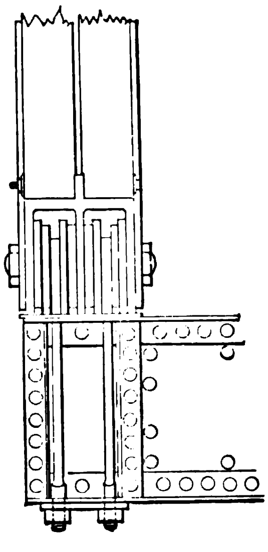


Upper lateral braces.



Lower Post Socket

Fig. 284



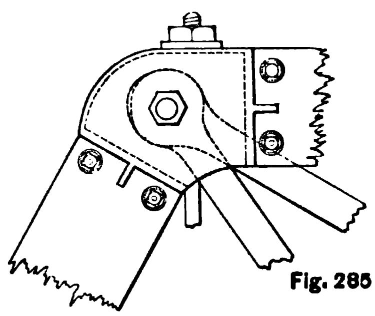


Fig. 285

Hip joint box

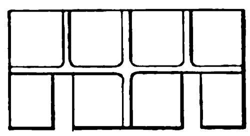
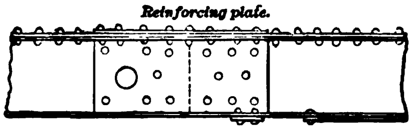
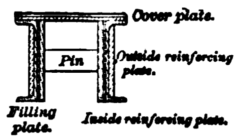


Fig. 286





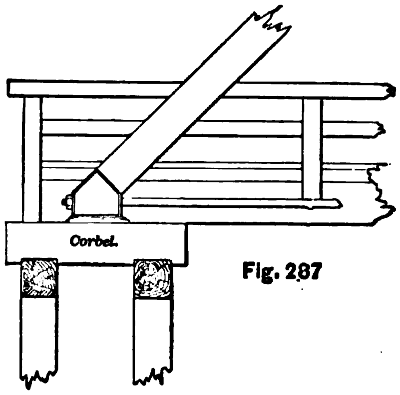


Fig. 287

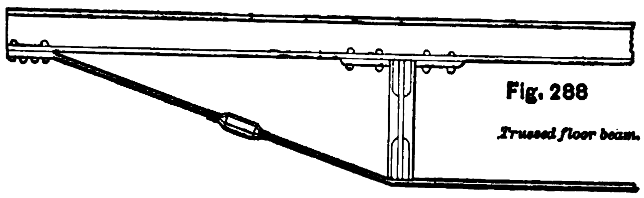


Fig. 288

Trussed floor beam.

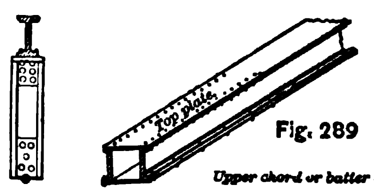


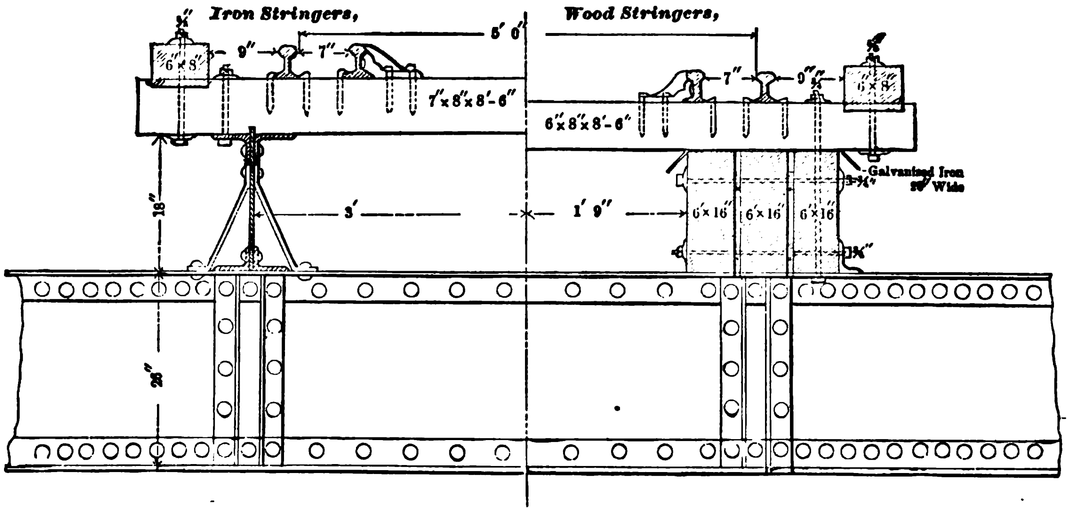
Fig. 289

Upper chord or batter braces

CHAPTER VII.

FLOOR SYSTEM—CROSS GIRDERS—STRINGERS—FLOOR.

FLOOR SYSTEM.—The arrangement of the floor system is shown in Fig. 206, Plate 8, and also by the following Fig.



The cross girder at every panel point is composed of double angle irons for the upper and lower flanges, of such a uniform section as will satisfy the stress at the middle due to the maximum loading. The web consists of a single plate, the thickness of which rarely exceeds $\frac{3}{8}''$, even for the heaviest cross girders, and is never less than $\frac{1}{4}''$ in the lightest. This web is riveted to the flanges above and below, and its edge is very nearly flush with the upper and lower surfaces of the angles. The cross girder is usually of uniform depth and square ends. Sometimes it is of uniform depth only in the central portion, and tapers off at the ends. More rarely still, the cross girder is a trussed frame, as shown in Fig. 228, Plate 21.

The cross girders are either slung from the pin at the panel point by beam hangers, as shown in Fig. 268, Plate 20, or they are riveted to the posts by angle irons, as shown in Fig. 206, Plate 8.

The stringers may be either of wood or of iron, as shown in the Fig. preceding. They either rest upon the cross girders, or are riveted to them as shown in Fig. 206, Plate 8.

Upon the stringers are laid the cross ties, which are usually of white oak, about 8 feet 6 inches long, and 7 inches inches deep by 8 inches wide, for single track, spaced about $16\frac{1}{2}$ inches from centre to centre, notched on to the stringers about $\frac{3}{4}''$, and bolted to the

stringer flanges. For double track, the ties are about 20 feet 6 inches long, 9 inches deep and 8 inches wide.

Pine guard rails or strips are bolted and notched to the ties, outside of and parallel to the rails, spliced at their ends.

The stringers are usually spaced about 6 feet apart, and for double track usually four stringers are used, so that the load per stringer is the same whether for double or single track.

Upon the ties the rails are laid and secured in the usual manner, and between the rails, a few planks for a foot walk are provided.

The entire weight of rails, spikes, chain, etc., and also, planking, cross ties and guard strips, is taken at 400 lbs. per ft. lineal, for single track, or 750 for double track.

For highway bridges this weight will be very different according to the style of roadway adopted and the locality and traffic, and must be estimated for the case in hand.

LIVE LOAD.—The live load adopted for railway bridges is that assumed as the basis of our diagram, page 89 or 215. This system of wheel loads is somewhat in excess of the heaviest locomotives now used, thus allowing for future increase, while it approximates closely to the actual loading. By means of the diagram, the strains may be found with more exactness than by any other method. The train load is small, but the tabular values can easily be increased to suit any given loading.

For *highway bridges*, the live load may be varied according to the situation, as given in the following Table.

TABLE OF LIVE LOADS FOR HIGHWAY BRIDGES.

Span in Feet.	City and Suburban Bridges liable to heavy traffic. Class A.	Bridges in Manufacturing Districts—Ballasted Roads. Class B.	Bridges in Country Districts—Unballasted Roads. Class C.
100 and under	100 lbs. per sq. ft.	90 lbs. per sq. ft.	70 lbs. per sq. ft.
100 to 200	80 " " "	60 " " "	60 " " "
200 to 300	70 " " "	50 " " "	50 " " "
300 to 400	60 " " "	50 " " "	45 " " "
400 and over	50 " " "	50 " " "	45 " " "

The stringers of highway bridges are usually of wood, and the floor beams of iron. The weight of these may be easily estimated, as detailed in what follows. The *flooring* varies too much for any general values to be given. For simple pine flooring, we may take 0.35 lb. for 12 cubic inches, and the flooring is usually 3 inches thick. The weight of railing posts, hand rails, hub rails, guard rails, etc., must be estimated according to the design.

WOOD STRINGERS—TOTAL LOAD, SIZE, WEIGHT.—For highway bridges the load W supported by a stringer will depend upon the weight of roadway and the live load assumed, according to the preceding Table.

For railway bridges, taking a system of wheel loads very similar to Class A of Cooper's *Specifications*, and taking floor, cross ties, rails, bolts, guard strips, etc., at 400 lbs. per lineal foot, we have the equivalent distributed live load for *one stringer*, as given in the following Table.* For double track and 4 stringers, we may take the same loading, because

* Increase the tabular values by about 18 per cent. for the system of loads assumed in our diagram, page 89 or 215. The values given are for a lighter system, and it is scarcely necessary to change them, as they now represent good average practice.

although the load is twice as much, there are twice as many stringers. If to the equivalent distributed live load, we add 200 lbs. per lineal foot for track, and also allow a percentage for impact, we have the total equivalent distributed external load W, not including the weight of the stringer itself, which in the case of wood may be disregarded. The allowance for impact is taken at 30 per cent. of the external load for all spans below 25 feet, and $40 - \frac{2}{5}l$ for spans above 25 feet. (l = span in feet.)

WOOD STRINGERS.*

Equivalent distributed live load and total distributed external load W, upon one stringer, for railway bridges. Allowance for shock 30 per cent. for spans below 25 feet, and $40 - \frac{2}{5}l$ for spans above 25 feet. Rails, ties, etc., 200 lbs. per ft. per stringer.

Length or panel length in feet.	Live load for one stringer in lbs.	Total external load W in lbs., including allowances for impact and flooring.	Length or panel length in feet.	Live load for one stringer in lbs.	Total external load W in lbs., including allowances for impact and flooring.
5	25000	33800	18	47222	66068
6	25000	34060	19	48685	68230
7	25000	34320	20	50000	70200
8	25000	34580	21	52380	73554
9	25000	34840	22	54545	76628
10	25000	35100	23	56521	79457
11	27272	38314	24	58333	82073
12	33333	43453	25	60000	84500
13	36538	50879	26	61537	86491
14	39286	54711	27	63518	89042
15	41666	58066	28	65355	91390
16	43750	61035	29	67068	93562
17	45588	63684	30	68832	95784

From this Table we can at once take for any given length of stringer, that is, for any given panel length, the corresponding equivalent *total external load W*, including the live load and weight of rails, ties, etc., at 200 lbs. per foot per stringer.

This load W being known, we may take at once from the following Table, the size of beam which will safely carry it.

The table gives the *safe load for one inch in breadth*, for different lengths and depths, on the condition that the deflection shall not exceed $\frac{1}{480}$th of the length, calculated from Trautwine's formula,

$$W = \frac{bd^3}{Bl^2},$$

where d = depth in inches, b = breadth in inches, l = length in feet, and $B = 0.00575$ for white oak, and 0.008 for white or yellow pine, hemlock, and red and black oak. The smaller values in the Table are for white or yellow pine, hemlock, red and black oak, and the larger values for white oak. For a concentrated load at the centre, one half of the tabular values may be taken.

In taking dimensions from this Table, it is well to bear in mind that beams over 14″ deep are not readily obtained, also that market sizes are usually even inches in depth and *always* even feet in length. Thus, beams 3″ × 8″, or 3″ × 10″, or 3″ × 12, are easily procured, while 3″ × 9″, 3″ × 11″, etc., are not.

* Increase these values by 18 per cent. for the system of loads assumed in our diagram, page 89 or 215.

WOOD STRINGERS.

SAFE DISTRIBUTED LOAD FOR ONE INCH BREADTH, FOR DIFFERENT LENGTHS AND DEPTHS, LARGER VALUES FOR WHITE OAK, SMALLER VALUES FOR WHITE OR YELLOW PINE, HEMLOCK, RED AND BLACK OAK. FOR CONCENTRATED LOAD HALF THESE VALUES TO BE TAKEN.

Length in ft.	5′	6′	7′	8′	10′	12′	14′	16′	18′	20′	22′	24′	26′
Depth in inches. 6″	1080 1502	750 1044	552 766	422 588	270 374	188 262	138 192	106 146	84 116				
7″	1716 2386	1192 1656	876 1218	670 932	428 596	298 414	218 304	168 232	132 184	108 148			
8″	2560 3562	1778 2474	1306 1818	1000 1390	640 890	444 618	326 454	250 348	198 274	160 222	132 184		
9″	3644 5072	2532 3522	1860 2588	1414 1982	912 1268	634 880	466 646	356 496	282 392	228 316	188 262	158 220	
10″	5000 6956	3472 4830	2552 3550	1954 2718	1250 1740	868 1208	638 888	488 680	386 536	312 434	258 358	218 302	186 258
12″	8640 12020	6000 8348	4408 6134	3376 4696	2160 3006	1500 2088	1102 1534	844 1174	666 928	540 752	446 622	376 522	320 444
14″	13720 19088	9528 13256	7000 9740	5360 6456	3430 4772	2382 3314	1750 2434	1340 1864	1058 1472	858 1192	708 986	596 828	508 706
16″	20480 28494	14222 19788	10448 14524	8000 11130	5120 7124	3556 4948	2612 3634	2000 2782	1580 2198	1280 1782	1058 1472	890 1234	758 1054
18″	29160 40570	20250 28174	14878 20698	11390 15848	7290 10142	5062 7044	3720 5174	2848 3962	2250 3130	1822 2536	1506 2096	1266 1762	1078 1500
20″		27778 38648	20408 28394	15626 21740	10000 13914	6944 9662	5102 7098	3906 7436	3086 4294	2500 3478	2066 2874	1736 2416	1480 2058
22″			27164 37792	20796 28934	13310 18518	9244 12860	6792 9448	5200 7234	4108 5716	3328 4630	2750 3826	2312 3214	1970 2738
24″				27000 37566	17280 24042	12000 16696	8816 12266	6750 9392	5334 7420	4320 6010	3570 4970	3000 4174	2556 2556

When the dimensions of stringer have been chosen, the weight may be found from the formula,

$$\text{weight} = bdl\gamma,$$

where b and d are the breadth and depth in inches, l the length in feet, and γ the weight of 12 cubic inches. We may take γ as equal to 0.35 lbs. for ordinary purposes, and hence

$$\text{weight of wood stringer} = 0.35bdl.$$

EXAMPLE.—*A white oak stringer in a railway bridge is 12 feet long. What dimensions should it have, and what is its weight?*

The distributed load W, is from the Table, 43453 lbs. From the last Table, we see that a beam 18″ deep and one inch in breadth will carry safely 7044 lbs. Our stringer then may be 18″ deep by 6″ wide. Other dimensions may be taken from the Table, as for instance 16″ deep and 9″ wide, etc. If the latter dimensions are adopted, the weight is

$$bdl\gamma = 9 \times 16 \times 12 \times 0.35 = 605 \text{ lbs.}$$

We can seldom take more than 16″ to 18″ depth and 6″ to 8″ width, as heavier timbers are costly. Where a single beam would be too large, several may be used side by side. Thus instead of one beam 16″ by 9″, we may have three each 16″ by 3″. Two beams 14″ by 6″ would be more easily procured and would be sufficient.

EXAMPLE.—*A white oak stringer in a railway bridge is 16 feet long. What dimensions should it have, and what is its weight?*

Here the distributed load is 61035 lbs. If we take the depth at 14 inches, the safe load is 1864 for one inch width. This would require a width of about 32 inches or 4 beams of 8 inches width. Such beams would be better replaced by iron stringers.

IRON PLATE STRINGERS, THICKNESS, DEPTH AND WEIGHT.—Iron stringers for railway bridges, whether single or double track, of less than 15 feet in length, may usually be made of rolled I beams. Above this length such beams are not heavy enough, and plate girders or built beams of plate and angle irons must be used.

The thickness of plate or web will usually be determined by the size and bearing of rivets. If the web is not thick enough, it will not be possible to have rivets enough in the flanges.

Let the total load, $W' + W$, including therefore the weight W' of the girder itself, be reduced to an equivalent uniformly distributed load, and represented by $W' + W$, and let l be the span in feet and d the depth in inches from outside to outside. Then, with sufficient accuracy for our purposes, the moment at the centre is $\frac{(W' + W)l}{8}$. If we divide this by the depth in feet or by $\frac{d}{12}$ where d is in inches, we get a fair estimate of the strain in one flange. The number of rivets to resist this would be $\frac{6l}{\text{pitch}}$, and the resistance of a single rivet is diameter $\times$ t $\times$ bearing resistance per square inch, where t is the thickness of web, and the diameter of rivet is in inches. We have then

$$t = \frac{(W' + W) \times \text{pitch}}{4 \times \text{diameter} \times \text{bearing resistance} \times d}.$$

Taking the bearing resistance per square inch at 12000 lbs. and the diameter of rivet at $\frac{7}{8}$ inch, and the minimum pitch at 3 inches, we have

$$t = \frac{W' + W}{14000d}.$$

The thickness of web must never be less than $\frac{1}{4}$ inch, the least allowable thickness of plate. It will rarely by the above formula be greater than $\frac{3}{8}$ inch.

For stringers, the flanges are usually of uniform cross section. Let the weight of the stringer itself be W'. Then if R is the mean stress per square inch in both flanges, and we disregard the web, the moment at the centre will be, accurately enough for our purposes,

$$\frac{(W + W') \times 12l}{8} = \text{area of one flange} \times R \times d.$$

The area of both flanges then will be

$$\frac{(W + W')\,12l}{4Rd}.$$

If the thickness of the web is t, its area will be dt. The total area is then about

$$\frac{(W + W')\,12l}{4Rd} + dt.$$

If we multiply this by $\frac{10}{3}$ we have the weight of one foot in length. The total weight is then about

$$\left[\frac{(W + W')\,12l}{4Rd} + dt\right]\frac{10}{3}l = W'.$$

As the thickness of web is rarely more than $\frac{3}{8}''$, if we take it $\frac{1}{2}''$, we make an allowance to cover connections, etc.; we have then

$$W' = \frac{12Wl^2 + 2Rld^2}{1.2Rd - 12l^2}. \qquad (1)$$

From equation (1) we can make a close estimate of the weight in pounds W' of any plate stringer of uniform depth, when the length in feet l, clear depth in inches d, mean working stress in lbs. per square inch R, and total equivalent load in lbs. W, are known.

Differentiating and putting the first differential equal to zero, we have the depth in inches corresponding to least weight

$$\text{least weight depth} = \frac{10l^2}{R} + \sqrt{\frac{6Wl}{R} + \left(\frac{10l^2}{R}\right)^2}. \qquad (2)$$

From equation (2), we can find the "least weight depth" in inches for an iron plate stringer, when the total equivalent external load W in lbs., length in feet l, and mean working stress R in lbs. per sq. inch, are known.

The least weight depth is not necessarily the depth for *least cost*, or best depth. Moreover, the depth is usually governed by considerations depending upon the design, so that formulæ for depth are of little practical value. If no such considerations apply, the best depth or least cost depth from centre to centre of rivet holes may be taken as not far from $\frac{8}{10}$ths of the least weight clear depth as given by equation (2). As this latter serves then as a basis of estimation, we have thought it well to give it in the Tables which follow. In view of the preceding remarks, the practical value of the equation (2) should not, and probably will not, be over estimated.

TOTAL EXTERNAL EQUIVALENT LOAD W FOR IRON PLATE RAILWAY STRINGERS.—The total external load W, for railway stringers, is composed of the equivalent distributed live load, the allowance for impact and the allowance for weight of rails, ties, etc., *viz.*, 200 lbs. per foot per stringer. It is usual to make allowance for impact by adding to the equivalent live load a certain percentage, depending upon the length of the stringer. We take here in addition, 30 per cent. of the equivalent live load for spans below 25 feet, and $40 - \frac{2}{5}l$ per cent. for spans above 25 feet, where l is the span in feet. The equivalent distributed live load is that found for a system of wheel loads very similar to Class A of Cooper's *Specifications*.

We give, in the following Table, the equivalent distributed live load, and the total

external load W, for different lengths of stringer, for the above allowance for impact and floor and live load. We also give the weight and least weight depth as found from equations (1) and (2), taking $R = 8000$ lbs. per square inch.

IRON PLATE STRINGERS OF UNIFORM DEPTH.*

Equivalent distributed live load and total external load W upon one stringer, for railway bridges. Also weight and least weight depth. Allowance for shock 30 per cent. of external load for all spans below 25 feet, and $40 - \frac{2}{5}l$ for all spans above 25 feet (l = span in feet). Rails, ties, etc., 200 lbs. per ft. per stringer. $R = 8000$ lbs. per square inch.

Length or panel length in feet.	Equivalent live load per stringer in lbs.	Total external load W in lbs., including allowance for impact and flooring at 200 lbs. per ft.	Weight in lbs. W'.	Least weight depth in inches.	Length or panel length in feet.	Equivalent live load per stringer in lbs.	Total external load W in lbs., including allowance for impact and flooring at 200 lbs. per ft.	Weight in lbs. W'.	Least weight depth in inches.
5	25000	33800	188	11.3	18	47222	66068	1816	30.3
6	25000	34060	248	12.4	19	48685	68230	2003	31.6
7	25000	34320	315	13.5	20	50000	70200	2197	33
8	25000	34580	386	14 5	21	52380	73554	2421	34.6
9	25000	34840	463	15.4	22	54545	76628	2652	36.2
10	25000	35100	545	16.4	23	56521	79457	2900	37.7
11	27272	38314	657	18	24	58333	82073	3133	39.2
12	33333	46453	825	20.6	25	60000	84500	3382	40.7
13	36538	50879	974	22.5	26	61537	86491	3633	42
14	39286	54711	1130	24.2	27	63518	89042	3904	43.4
15	41666	58066	1292	25.8	28	65355	91390	4180	44.8
16	43750	61035	1460	27.4	29	67068	93562	4463	46.2
17	45588	63684	1634	28.8	30	68832	95784	4756	47.5

Any depth may of course be taken in designing, which seems desirable. As the weight varies but little with a change of depth, the Table will in all cases give a good estimate of the weight. For the best depth, if no other considerations affect it, $\frac{8}{10}$ths of the least weight clear depth as given by the Table, will not be far from the best or least cost effective depth.

Flanges of Stringers.—From the preceding Table we can find at once the maximum load $W' + W$, sustained by a stringer, and its least weight clear depth. Since W' is small compared to W, the weight W' given in the Table is near enough for any depth which may be taken. The effective depth is the depth from centre to centre of rivet holes. It may be taken as $\frac{8}{10}$ths of the clear depth given in the Table, if no other considerations affect it. The loading assumed in our Table is intended to be large enough to cover future increase of traffic.

The maximum load $W' + W$ being thus known, the moment at the centre in inch pounds will be $\frac{(W' + W)l}{8}$, where l is the length in inches. If d is the effective depth in inches, the moment of resistance of the web is $\frac{RI}{v} = \frac{Rtd^2}{6}$, where R is the allowable stress in lbs. per square inch, and t is the thickness of the web in inches. The area of the upper flange at the centre is then

$$\frac{(W' + W)l}{8Rd} - \frac{td}{6},$$

where $(W' + W)$ is taken from the Table in lbs., d is $\frac{8}{10}$ths of the value for d in inches given in the Table, if no other considerations determine the depth, and l is the span in inches.

* Increase values for W by 18 per cent. for the system of loads of our diagram, page 89. The depths remain the same.

The nearest angle iron which will suit can then be taken from Carnegie's Pocket Book. The bottom flange should be calculated from net section or area, with rivet holes deducted. The rivets are usually taken at from $\frac{5}{8}$ to $\frac{7}{8}$ inches.

Web Plate of Stringers.—The web is composed of plate, not less than $\frac{1}{4}$ inch and rarely more than $\frac{3}{8}$ths inch. The upper limit may be found by the formula already given, $t = \dfrac{W' + W}{14000d}$. The shear at any point ought not to exceed 8000 lbs. per sq. inch. The shear is of course greatest at the ends, where it is equal to half the total load or $\dfrac{W' + W}{2}$. The web must also be prevented from buckling.

This condition is attained when the shear per square inch of cross section at any point does not exceed the

$$\textit{safe resistance to buckling per square inch} = \frac{10000}{1 + \dfrac{d^2}{3000t^2}},$$

where d and t are the depth and thickness of web in inches.

Stiffeners.—Ordinarily this formula gives a lower strain per square inch than 8000 lbs., so that when it is fulfilled, the web is safe against shearing also. When, however, the web is safe against shearing, at 8000 lbs. per square inch, but not safe against buckling, as tested by the preceding formula, instead of increasing the thickness of the whole web, "*stiffeners*" are used.

These stiffeners consist of vertical strips or angle irons, riveted to the web at intervals. The intervals between stiffeners, in girders over 3 feet in depth, should not exceed the depth of girder, with a maximum limit in any case of 5 feet. Under 3 feet depth, they may be spaced every 3 feet when needed. They should be calculated as columns by the formula, *safe resistance to buckling per square inch of cross section* $= \dfrac{10000}{1 + \dfrac{d^2}{3000t^2}} \leqq$ *the shear per square inch at the point where the stiffener is placed*, where $d =$ depth in inches, and $t =$ thickness of web and stiffener in inches.

Stiffeners should always be placed at the ends, wherever the web plate is spliced, and at any point where a concentrated load acts, as the point of attachment of the stringers to the cross girders. Splicing of the web sheets is unnecessary in stringers and cross girders, as sheets of the requisite depth and length can be supplied in one piece.

EXAMPLE.—*Required to design a railway track stringer 17 feet long.*

From our Table, the weight of such a stringer is about 1634 lbs. and the least weight clear depth about 29 inches. If no other considerations influence our choice of depth, we may then take about $\frac{8}{10} \times 29 = 23$ inches for the least cost or best effective depth. $W' + W$ is then $63684 + 1634 = 65318$, or about 65000 lbs.

Flanges.—If we take the web at $\frac{1}{4}$ inch, the area of the top flange is

$$A = \frac{65000 \times 17 \times 12}{8000 \times 8 \times 23} - \frac{23}{4 \times 6} = \text{about 8 square inches.}$$

This requires angles weighing $\dfrac{8 \times 10}{3 \times 2} = 13.3$ lbs. per ft. From Carnegie we see that angles $4\frac{1}{2} \times 3 \times \frac{9}{16}$ will answer. For the bottom flanges we must have 8 sq. inches net. Taking $\frac{5}{8}''$ rivets, the gross section should be $8 + 2 \times \frac{9}{16} \times \frac{5}{8} = 8.70$ sq. inches. This calls for angles weighing $\dfrac{8.7 \times 10}{3 \times 2} = 14.5$ lbs. per foot. From Carnegie we have angles $5 \times 3 \times \frac{9}{16}$ for the lower flanges.

The application of our rule for thickness of web gives $\frac{65000}{14000 \times 23} = 0.20$ inch.

Web.—Let us therefore take the web plate at $\frac{1}{4}''$ thick, and see if this is safe against shear. The shear at the end is $\frac{65000}{2} = 32500$ lbs ; at 8000 lbs. per sq. inch, this requires 4.06 square inches. But the actual cross section is $23 \times \frac{1}{4} = 5.75$ sq. inches. The thickness is then more than sufficient to resist the shear.

Stiffeners.—At the ends we always need stiffeners, unless the stringers are riveted at the ends to the web of the cross girders, when the rivet angles will answer the purpose. We must also have stiffeners every three feet if found necessary. The shear at the end is $\frac{32500}{5.75} = 5652$ lbs. per square inch. But as the web is $t = \frac{1}{4}''$ thick, its safe resistance to buckling is $\frac{10000}{1 + \frac{d^2}{3000t^2}} = \frac{10000}{1 + \frac{529 \times 16}{3000}} = 2617$ lbs. per sq. inch. As this is less than 5652, we need stiffeners at the end. If we take two filling plates $2'' \times \frac{9}{16}''$, giving an area of 2.25 sq. inches, and two angle irons $2 \times 2 \times \frac{1}{4}$, area 1.86 sq. inches, the total area, including the web, is $2.25 + 1.86 + 0.5 = 4.61$ sq. ins., and the total thickness is $\frac{9}{8} + \frac{1}{2} + \frac{1}{4} = \frac{15}{8}$ inches. The resistance to buckling is then $\frac{10000}{1 + \frac{529 \times 64}{3000 \times 225}} = 9524$ lbs. per square inch of cross section, or $9524 \times 4.61 = 43905$ lbs. As this is greater than the end shear of 32500 lbs., the stiffeners are ample.

At 3 feet from the end, the shear is $32500 - 3 \times \frac{65000}{17} = 21030$ lbs. or $\frac{21030}{5.75} = 3657$ lbs. per square inch. As this is greater than the safe resistance of the web to buckling, 2617 lbs., we need stiffeners here also. Let us take here simply two filling plates, $2 \times \frac{9}{16}$, area 2.25 sq. ins., or total area, including the web, 2.75 sq. ins., and total thickness $\frac{9}{8} + \frac{1}{4} = \frac{11}{8}$ ins. Then the resistance to buckling is $\frac{10000}{1 + \frac{529 \times 64}{3000 \times 121}} = 9150$ lbs. per square inch of cross section, or $9150 \times 2.75 = 25162$ lbs. As this is greater than the shear, 21030 lbs., it is sufficient.

At 6 feet from the end, the shear is $32500 - 6 \times \frac{65000}{17} = 9560$ lbs., or $\frac{9560}{5.75} = 1662$ lbs. per sq. inch. As this is less than the safe resistance of the web to buckling, 2617 lbs., no stiffeners are needed.

Rivets and Rivet Spacing.—The size of rivets may be found by the rule $d = 1\frac{1}{4}t + \frac{3}{16}$, except that if this rule in any case gives a less diameter than $\frac{3}{4}$ or $\frac{5}{8}$ at least, the latter diameter is to be taken.

We have already illustrated the method of determining the number of rivets quite fully, page 410. In the present case our rule gives $d = \frac{5}{4} \times \frac{9}{16} + \frac{3}{16} = \frac{7}{8}''$ rivets. The distributed load is 65000 lbs. The bearing resistance for $\frac{1}{4}''$ plate and $\frac{7}{8}''$ rivet is, from Rivet Table I., 2730 lbs. The horizontal stress at any distance x from end is $\frac{65000x}{3.83}\left(1 - \frac{x}{17}\right)$, see page 390. If we take $x = 2.5$, 5 and 8.5 feet, we have the horizontal stresses 14.35 tons, 23.75 tons, 28.6 tons. Subtracting each from the one following, we have 14.35 tons, 9.4 tons, 4.85 tons, for the horizontal stresses to be taken by the rivets in the different lengths. The load on the first length of 2.5 feet is $\frac{65000}{17} \times 2.5 = 4.78$ tons ; on the next 2.5 feet, 4.78 tons ; on the last 3.5 feet, 6.7 tons. The resultant stress for the first division of 2.5 feet is then $\sqrt{14.35^2 + 4.78^2} = 15.12$ tons or 30240 lbs. In the next division of 2.5 feet it is $\sqrt{9.4^2 + 4.78^2} = 10.54$ tons or 21080 lbs. In the last division of 3.5 feet it is $\sqrt{4.85^2 + 6.7^2} = 8.27$ tons or 16540 lbs. We require for bearing then $\frac{30240}{2730} = 11$ or 12 rivets ; in the next 2.5 feet, $\frac{21080}{2730} = 8$ rivets ; and in the last 3.5 feet $\frac{16540}{2730} = 6$ rivets. If we take a pitch of 2.5 inches in the first 2.5 feet, which is just 3 times the diameter, and therefore the least allowable, 4 inches in the next 2.5 feet, and 5 inches in the last 8.5 feet, we shall have rivets enough.

We should always arrange to have rather more than less rivets as calculated. We see also that if the depth is taken too small, the flange strains will be so great that it may be impossible to get in rivets enough without overcrowding.

FLOOR BEAMS OR CROSS GIRDERS.—EXTERIOR LOADING, THICKNESS, WEIGHT, DEPTH.—Equations (1) and (2) apply to plate cross girders also. The total external load W upon a cross girder consists of the greatest live load, the weight of the stringers, weight of rails, ties, etc., and the allowance for impact. We may take W therefore, for double track, at about twice what it is for single track.

Since the stringers are attached to the cross girders at or near the quarter points, the total load upon a cross girder may be taken as uniformly distributed, so far as the moment

at the centre is concerned. The thickness of web is never to be less than $\frac{1}{4}$ inch. The other limit is, as for stringers, already found

$$t = \frac{W' + W}{14000d},$$

where W is the equivalent distributed load.

The same remarks as to depth hold here as to stringers. The least weight clear depth given in the Tables which follow, multiplied by $\frac{8}{10}$ths, will give the best effective depth near enough, if no other considerations limit the depth.

We give, in the following Table, the live load on a cross girder for different panel lengths, based upon a load system very similar to Class A of Cooper's *Specifications.* Also the total external load W, including the live load, the weight of two stringers, the weight of rails, ties, etc., taken at 400 lbs. per ft. and the allowance for impact, taken at 30 per cent. of the load. The Table is for single track. For double track, double the tabular values may be taken.

Iron Plate Cross Girders of Uniform Depth.*

Live load and total external load W for single track. For double track take double these values. Rails, ties, etc., 400 lbs. per ft., allowance for impact 30 per cent.

Panel length in feet.	Live load in lbs.	Total external load W in lbs., including live load, weight of two stringers and floor, and allowance for impact.	Panel length in feet.	Live load in lbs.	Total external load W in lbs., including live load, weight of two stringers and floor, and allowance for impact.
5	25000	35590	18	78055	115560
6	33333	47100	19	81578	121140
7	39285	55530	20	84750	126290
8	43750	62040	21	87619	131400
9	47222	67270	22	90226	135630
10	50000	71620	23	93260	140740
11	54545	78340	24	96041	145480
12	58332	84220	25	98400	150040
13	62690	90790	26	100960	154220
14	66428	96575	27	103147	158280
15	69666	101725	28	105714	162860
16	72500	106370	29	108103	167220
17	75000	110590	30	110333	171400

The total external load W being known, we can easily find the least weight clear depth and the weight W' of the cross girder for any given length and loading from equations (1) and (2), page 424.

We give in the following Table, the least weight clear depth and the corresponding weight for cross girders 15 and 25 feet long, for single and double track respectively. Any depth may of course be taken in designing, which may be desired, and as the weight varies but little with a change of depth, the Table will in all cases give a good estimate of the weight, for the lengths assumed. About $\frac{8}{10}$ths of the depth given in the Tables will be the best effective depth, if no other considerations affect it.

* Increase these values by 18 per cent. for the system of loads given by our diagram, page 89 or 215.

IRON PLATE CROSS GIRDERS.*

WEIGHT AND ECONOMIC DEPTH FOR SINGLE TRACK, 15 FEET WIDE, AND DOUBLE TRACK, 25 FEET WIDE. RAILS, TIES, ETC., 400 LBS. PER FT. $R = 8000$ LBS. PER SQUARE INCH. ALLOWANCE FOR IMPACT 30 PER CENT.

Panel length in feet.	Single track 15 feet wide.		Double track 25 feet wide.		Panel length in feet.	Single track 15 feet wide.		Double track 25 feet wide.	
	Depth in inches.	Weight in lbs.	Depth in inches.	Weight in lbs.		Depth in inches.	Weight in lbs.	Depth in inches.	Weight in lbs.
5	20	1014	37	3110	18	36.3	1811	66.5	5551
6	23	1165	43	3567	19	37.2	1860	68.2	5682
7	25	1263	46	3868	20	38	1898	69.6	5743
8	27	1358	49	3985	21	38.7	1937	71	5915
9	28	1390	51	4270	22	39	1967	72	6008
10	29	1433	52.6	4384	23	40	2003	73.4	6060
11	30	1500	55	4582	24	40.7	2036	74.6	6161
12	31	1553	57	4748	25	41.4	2068	75.8	6257
13	32	1612	59	4886	26	42	2096	76.8	6345
14	33	1662	61	5080	27	42.4	2124	77.8	6427
15	34	1706	62.5	5212	28	43	2154	79	6564
16	35	1744	64	5328	29	43.6	2172	80	6664
17	35.5	1772	65	5400	30	44.2	2209	81	6746

The designing of the cross girder is the same as that of a stringer, as already illustrated, page 426.

EXAMPLE.—*A single track railway bridge has a width of 15 feet and panel length of 17 feet. What is the best depth and weight of the cross girders? Also, if the stringers are attached at 4 feet from the ends, required to design the girder.*

The best effective depth by the preceding Table is $\frac{8}{10} \times 35.5 = 28.5$ inches. The total external load is by our Table 110590 lbs. The weight by the preceding Table is about 1772 lbs.

We can now design the cross girder just as in the case of a stringer.

Flanges.—Thus the total load is $110590 + 1772 = 112362$ lbs. $= W' + W$. If the stringers are attached at say 4 feet, or 48 inches, from the ends, the moment at the centre is $\frac{W' + W}{2} \times 48 = 2696688$ inch lbs. If the web is $\frac{1}{4}$ inch, the moment of the web is $\frac{Rtd^2}{6}$. Subtracting this from the moment at the centre, we have the moment for the upper flange. Dividing by R and by d, we have the area

$$A = \frac{2696688}{8000 \times 28.5} - \frac{28.5}{4 \times 6} = 11 \text{ sq. inches.}$$

The upper angles should weigh then $\frac{11 \times 10}{2 \times 3} = 18.3$ lbs. per ft. each. From Carnegie, this calls for angles $5 \times 4 \times \frac{5}{8}$ for the upper flanges.

For the lower flange we must have 11 sq. inches net. Our rule $1\frac{1}{4}t + \frac{3}{16}$ gives $\frac{5}{4} \times \frac{5}{8} + \frac{3}{16} = \frac{15}{16}$ for the rivets. The gross area then, should be $11 + 2 \times \frac{5}{8} \times \frac{15}{16} = 12.17$ square inches. The bottom angles weigh then $\frac{12.17 \times 10}{2 \times 3} = 20.3$ lbs. per ft. From Carnegie this calls for angles about $6 \times 4 \times \frac{5}{8}$.

Web.—For the web we have the thickness by our rule $\frac{112362}{14000 \times 28.5} = 0.28$. If we take the web $\frac{5}{16}$ of an inch thick, then there will be more bearing than the rivets require. The area at end then is $\frac{5}{16} \times 28.5 = 8.9$ sq. ins. at 8000 lbs. per square inch, this gives 71200 lbs. safe resistance. The shear at end is $\frac{112362}{2} = 56181$ lbs. There is therefore ample resistance to shear.

* Increase the values for weight by 18 per cent. for the system of loads given by our diagram, page 89. The depth remains the same.

If the cross girder is riveted at the ends to the posts, no stiffeners will be needed at the ends, and if the stringers are riveted to the web of the cross girder, no stiffeners will be needed there. If, however, the girder is hung by beam hangers from the chord pin, and if the stringers are laid on top, stiffeners may be needed.

In such case, the safe resistance of web per square inch to buckling is

$$\frac{10000}{1+\frac{d^2}{3000t^2}} = \frac{10000}{1+\frac{28.5^2 \times 256}{3000 \times 25}} = 2652 \text{ lbs.}$$

The shear per square inch at the end is $\frac{56181}{8.9} = 6312$ lbs. We need then stiffeners at the end and also at the points where the stringers cross.

If we use for the end stiffeners, two filling plates $\frac{3}{8}''$ thick, the total thickness is $1\frac{5}{16}$ inches. The resistance to buckling is then $\frac{10000}{1+\frac{28.5^2 \times 256}{3000 \times 625}} = 9009$ lbs. per sq. inch. If the filling plates are 4 inches wide, the area, including the web, will be 6.25 square inches, and the safe resistance $9009 \times 6.25 = 56306$ lbs. As the shear at the end is 56181 lbs., these plates will be sufficient. The same stiffeners may be used under the stringers.

As to the rivets, the size already determined is $\frac{1}{1}\frac{3}{6}''$. The bearing resistance for this size and $\frac{5}{16}''$ plate is, from Rivet Table 1, 3660 lbs. The horizontal stress at the first stringer, which is 4 feet from the end, is $\frac{56181 \times 48}{28.5}$ = about 94620 lbs. The horizontal stress beyond this point at any point is the same. The vertical load is 56181 lbs. The resultant stress is then *for the half span*, $\sqrt{47.3^2 + 28^2} = 55$ tons = 110000 lbs. This requires $\frac{110000}{3660}$ = about 30 rivets. If we pitch the rivets then at 3 inches for the entire length, which is allowable, as this pitch is greater than 3 times the diameter, we shall have about 30 rivets in the half span.

BEAM HANGERS.—When the cross girder is not riveted to the post, it is hung from the pin by beam hangers, as represented in Fig. 268, Plate 20. The hangers go in pairs, and each one takes therefore $\frac{1}{4}$ and each leg $\frac{1}{8}$ of the total load $W + W'$, on a cross girder. Owing to impact, the unit strain is taken very low, about 5000 lbs. per square inch. The allowance for upset ends and nuts will be found on page 408.

EXAMPLE.—*In the preceding example, what should be the size of the beam hangers?*

The total load $W' + W$ has been found to be 112362 lbs. The tension on each leg is therefore 14045 lbs. At 5000 lbs. per square inch, this gives 2.809 sq. ins, or about $1\frac{7}{8}''$ diameter. The length of rod required to make a beam hanger, since the cross girder is 35.5 inches deep, will be about 74 inches. To this add 2 feet (page 408) for upset ends, and we have about 8 feet. The weight will be, from Carnegie, page 99, about 73 lbs. for each hanger.

PLATE GIRDER BRIDGES, LIVE LOAD.—Below 80 feet, plate girder bridges are usually preferred to pin connected trusses with open web.*

The designing of a plate girder is similar to that of a track stringer or cross girder, except that the flanges cannot usually be made of angle irons alone, but must be re-enforced by cover plates laid on top of the angles and riveted to them. The flange area can thus be adjusted to the stress at different points, by increasing the number of cover plates or their thickness from end to centre of girder.

Total External Load.—Our equations (1) and (2), will give us a good estimate of the weight of girder and least weight clear depth, provided the total external load W is known. About $\frac{8}{10}$ths of this depth may be taken as the least cost effective depth. The total external load is composed of the flooring, rails, ties, etc., which for single track railway may be taken at 400 lbs. per foot; of the weight of the track stringers and cross girders; of the wind bracing; and of the live load. We have just learned how to design the track

* Plate girders are practically limited to lengths which do not require more than two ordinary flat cars 33 feet long for transport, *i.e.*, 65 feet span. The length is more rarely extended to three car lengths, or about 100 feet maximum. They are riveted at the shops, and are preferable to lattice girders, being cheaper, costing less for maintenance, and having greater security; as faulty rivets produce less reduction of strength. They are also more free from corners and recesses, and are therefore cleaner and less exposed to oxidation.

stringers and cross girders and find their weight. The formulæ for weight of wind bracing, page 407, will give a good estimate. The equivalent distributed live load for any span can be found from our diagram, page 89. We give in the following Table the equivalent distributed live load thus found for a system very similar to Class A of Cooper's *Specifications*. The values given are for single track, and for all the girders. Thus, if *two* plate girders are used, one half the tabular values should be taken. For double track, double the tabular value gives the total live load, which is to be divided among the girders according to their number. The allowance for impact is 30 per cent. of the load for spans under 25 ft., and $40 - \frac{2}{5}l$ for greater spans, where l is the span in feet.

EQUIVALENT DISTRIBUTED LIVE LOAD FOR SINGLE TRACK PLATE GIRDER BRIDGES.*

Span in feet.	Live load.	Span in feet.	Live load.	Span in feet.	Live load.	Span in feet.	Live load.
10	50000	28	130700	46	192860	64	235520
11	54550	29	134140	47	195190	65	237280
12	66670	30	137670	48	198420	66	239130
13	73080	31	140310	49	200340	67	240850
14	78580	32	1431[illegible]0	50	203500	68	242500
15	83340	33	152700	51	205270	69	244750
16	87500	34	154950	52	207970	70	246770
17	91180	35	157000	53	209790	71	248530
18	94450	36	158900	54	212480	72	250470
19	97370	37	163000	55	213960	73	252380
20	100000	38	170000	56	217300	74	253940
21	104760	39	172800	57	218770	75	256070
22	109090	40	176280	58	222040	76	258470
23	113040	41	179190	59	224900	77	260760
24	116670	42	181890	60	227770	78	262770
25	120000	43	184580	61	229860	79	264490
26	123080	44	186940	62	231820	80	267120
27	127040	45	189300	63	233700		

Girder Spacing.—Single track plate girder deck bridges usually have the girders spaced 6′ 6″ from centre to centre, and double track deck bridges have usually 3 girders spaced 9′ 3″ apart, so that each girder will carry an equal share of the total load on both tracks.

Single track plate girder through bridges should be at least 15 feet from centre to centre of girders, and double track have usually 3 girders likewise 15 feet apart, in which case the outer girders carry one-half the total load on one track, and the middle girder the entire load of one track. If only two girders are used, they can be spaced 28 feet from centre to centre, each one carrying the entire load for one track.

The spacing of the main girders determines the length of panel for the wind bracing, which may be taken a little longer than the width, and so as to make an even division of the length. The number of panels being thus chosen, the *total* weight per ft. lineal of the wind bracing may be found from the formula of page 407, viz.:

$$\textit{total weight per ft. lineal of wind bracing} = 3.6\,N + \frac{540}{p},$$

where l = length in feet of span, N = number of panels, p = panel length in feet.

For double track, multiply by $\frac{b}{15}$, where b is the width in feet.

The length of cross girders will be also determined by the width, and the length of track stringers by the panel length just found. The cross girders and stringers may therefore be calculated as already illustrated. The flooring, rails, ties, etc., being then esti-

* Increase these values by 18 per cent. for the system of loads given by our diagram, page 89 or 215.

mated, and finally the live load taken from the preceding Table, we can find the total external load W.

Weight and Depth.—This load can then be divided among the girders according to their number and spacing. We can then find the weight and least weight clear depth of the girders, from the equations,

$$\text{weight} = \frac{12\,Wl^2 + 2\,Rld^2}{1.2\,Rd - 12l^2},$$

$$\text{depth} = \frac{10l^2}{R} + \sqrt{\frac{6Wl}{R} + \left(\frac{10l^2}{R}\right)^2},$$

where $W =$ the total external load per girder, $R =$ allowable unit stress $= 8000$ lbs., $l =$ length in feet, $d =$ depth in inches.

About $\frac{8}{10}$ths of the least weight clear depth may be taken as the least cost effective depth.

Floor System.—For single track deck plate girders the cross ties are notched to the girder flanges and secured to them through the guard strips by bolts. The rails are laid over the cross ties in the usual way. No stringers or floor beams are required. For the girder spacing already given, the ties are of sawed white oak, about 8′ 6″ long, 7″ deep and 8″ wide, spaced about 16″ from centre to centre. For double track, for the girder spacing as given, the cross ties may be about 20′ 6″, 9″ deep and 8″ wide, spaced 16″ between centres. The pine guard strips are 6″ by 8″, laid outside of and parallel to the rails, and spliced and bolted at joints.

For through plate girder bridges, iron floor beams and stringers should be used. The ties and rails are laid upon the stringers precisely as in the case of deck bridges. The iron track stringers are usually spaced 6′ 6″ between centres. They may rest upon the floor beams or be riveted between them. In the first case they must be strongly spliced at the joints, and continued over the piers, and rest at the pier ends upon bearing plates so as to allow of contraction and expansion. In the second case they may be supported by bracket angles riveted to the floor beams, and the stringer ends should be riveted to the web of floor beams by angle irons.

Web and Flanges.—The web for the heaviest bridge is rarely over $\frac{5}{8}$″ thick, and never less than $\frac{1}{4}$″. Within these limits it may be proportioned by our rule $\dfrac{W' + W}{14000d}$.

Crippling or buckling of the web is to be guarded against by the formula

$$\text{safe resistance per sq. inch} = \frac{10000}{1 + \dfrac{d^2}{3000t^2}},$$

where d is the depth and t the thickness in inches, and the stiffeners are to be calculated by the same formula, and spaced, for girders over three feet in depth, at distances apart not exceeding the depth, with a maximum limit of five feet. Under three feet depth, 3 feet apart.

The flange angles are of equal section throughout and re-enforced by cover plates as the stress increases towards the centre. These cover plates should project slightly beyond the outer edges of the horizontal legs of the angles, and are riveted to them by rivets of proper size and number.

Web and flange angles can nearly always be ordered in one length. If the web is ordered in sections, it can be so arranged that the splices come at the stiffeners. The flange angles at least should be always in one length.

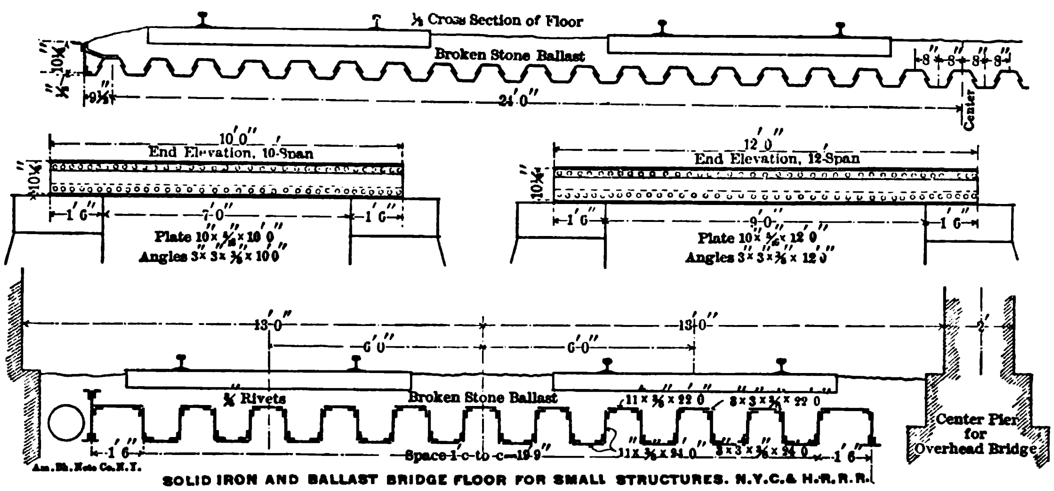
In through bridges knee braces may be introduced at every floor beam for lateral support to the girders. The wind bracing offers no special points of difference from ordinary bridges.

Rivets.—The size of rivet may be taken from our rule,

$$d = 1\tfrac{1}{4}t + \tfrac{3}{16},$$

where d is the diameter and t the greatest thickness of plate, in inches, provided that the result is not less than $\frac{3}{4}''$. The pitch should never exceed 6″, or to prevent buckling, 16 times the thinnest outside plate, nor be less than 3 diameters. The distance from edge to centre of rivet hole should not be less than $1\frac{1}{2}''$, and if practicable at least 2 diameters. When the flange plates are over 12 inches wide, or more than 3″ project beyond the angles, an extra line of rivets with a pitch of not over 9″ should be driven along the edges to draw the plates together and keep out water.

SOLID FLOOR PLATE GIRDER.—Instead of the style of floor consisting simply of cross girders, stringers, and ties, there is a decided tendency on the part of some of our leading railroads to "solid floors," the ballast and roadway being continued on the bridge itself. The accompanying illustration, taken from the *Engineering News* for November 16, 1889, shows the practice in this respect of the N. Y. C. & H. R. R.R., as given by George H. Thomson, C.E., Bridge Engineer of the Company.



SOLID IRON AND BALLAST BRIDGE FLOOR FOR SMALL STRUCTURES. N.Y.C.& H.R.R.R.

The top section shows a floor built of Pencoyd standard heavy trough sections, fastened by rivets. This form of floor is for small spans. The lower section shows half of a four track bridge, length, 24 feet, clear span, 21 feet, 2 feet depth of floor.

The two elevations given between the two sections show short spans of similar floors.

In the next illustration, we have the system for larger spans.

Mr. Thomson, in an abstract of a paper on "Railway Structural Economics," summarized in *Engineering News*, November 23, 1889, classifies floors under the following heads:

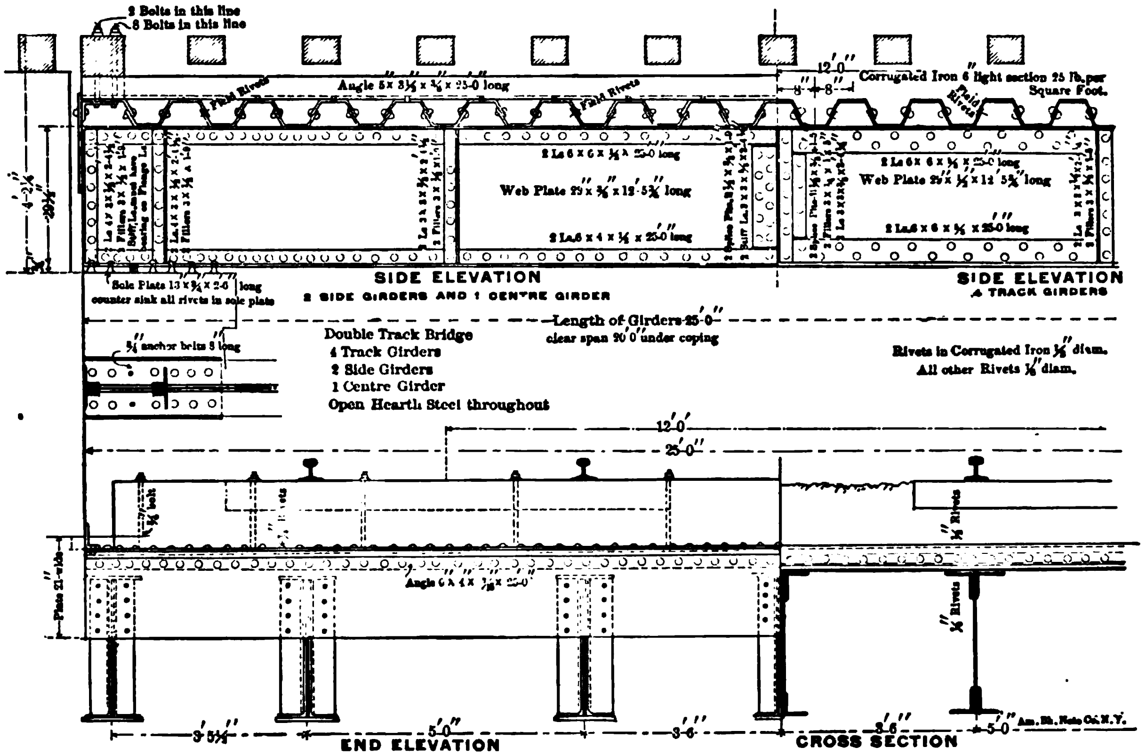
By a first-class floor is meant a solid floor of the type illustrated.

By a second-class floor is meant the usual system of cross girders and stringers.

By a third-class floor is meant wooden floor beams and cross ties.

The phenomena of "bunching" and "scooping," which occur with second and third, cannot obtain with first-class floors.

A derailed train is admirably provided for in the first class. The danger of fires is *nil.* The cost of masonry is less than for second and third class.



First class are often cheaper than the other classes, especially where the thickness of floor is limited.

There is no shock on entering and leaving the bridge, such as always occurs when the ballast is followed on the bridge by cross ties resting on the back walls.

Bed Plates and Rollers.—Under 50 feet one end may be left free to slide on planed surfaces. The pressure of bed plates on masonry should not exceed 250 lbs. per square inch.

Over 50 feet, nests of turned friction rollers should be used at one end as described on page 408.

Much valuable information upon plate girders may be found in a paper by M. J. Becker, Eng. P. C. & St. L. R. R., published in the Sixth Annual Report of the Ohio Soc. of Civil Engineers, from which many of the points above given are taken.

EXAMPLE.—*Required to design a single track through plate girder bridge, 63 feet long and 15 feet wide, centre to centre.*

In this case we may take the distance between the floor beams at $\frac{63}{4} = 15.75$ ft., and we thus have 4 panels and 5 floor beams. The stringer length will be 15.5 feet. The rails, cross ties, etc., we may take at 400 lbs. per foot, or 200 lbs. for each girder.

The total weight per foot for the wind bracing is then

$$3.6N + \frac{540}{p} = 48 \text{ or say } 50 \text{ lbs.}$$

For each girder then, we have 25 lbs. per ft. for wind bracing. The weight of stringer is given by our table at about 1292 lbs. As there are 8 stringers, the total weight is 10336 lbs., or 5168 lbs. for each girder.

The economic depth of floor beams is, from our Table for floor beams, $\frac{8}{10}$ths of $34 = 27$ inches, and weight about 1706 lbs.

Weight and Depth.—We have found, then, for each girder, the wind bracing 25 lbs. per foot, or $25 \times 63 = 1575$ lbs. per girder. The stringers give 5168 lbs. per girder. The floor beams give $\frac{1706 \times 5}{2} = 4265$ lbs. per girder. The rails, ties, floor, etc., give $200 \times 63 = 12600$ lbs. per girder. The live load from our Table is $\frac{233700}{2} = 116850$ lbs. per girder. The total is 140458 lbs. per girder. The allowance for impact is $40 - \frac{2}{5}l$ per cent., or 14.8 per cent., or 20787. The total external load for one girder then, including impact, is $W = 161245$ lbs.

Taking this value for W, we have the depth for our case,

$$d = \frac{10 \times 63^2}{8000} + \sqrt{\frac{6 \times 161245 \times 63}{8000} + \left(\frac{10 \times 63^2}{8000}\right)^2} = 92.5 \text{ inches or } 7.7 \text{ feet.}$$

Taking $\frac{8}{10}$ths of this, we have about 6 feet for the effective economic depth, from centre to centre of rivet holes The weight of one girder is then

$$W' = \text{weight} = \frac{12 \times 161245 \times 63^2 + 2 \times 8000 \times 63 \times 72^2}{1.2 \times 8000 \times 72 - 12 \times 63^2} = 20050 \text{ lbs.}$$

The total load per girder, including weight of girder, is then

$$W' + W = 161245 + 20050 = 181295 \text{ lbs.}$$

We may take the thickness of web at $\frac{3}{8}$ inch.

Flanges.—If we take the effective depth in calculation of the flanges, the web should be taken into account. If the clear depth, the web may be neglected. The moment of the strain in the web is $\frac{RI}{v} = \frac{Rtd^2}{6}$, where t is the thickness of the web in inches, and d is the depth in inches, and R is the allowable stress per square inch.

The moment in inch lbs. at any point distant x feet from the left end due to the loading is

$$\frac{(W + W')\,12x}{2}\left(1 - \frac{x}{l}\right),$$

where x is in feet and l in feet. If we subtract the moment due to the web, we have the moment to be resisted by the flanges. Dividing then by d in inches, we have the strain in the upper flange, taking the web into account,

$$\frac{12(W + W')\,x}{2d}\left(1 - \frac{x}{l}\right) - \frac{Rtd}{6},$$

or if x, l and d are all in feet, and t in inches,

$$\frac{(W + W')\,x}{2d}\left(1 - \frac{x}{l}\right) - 2\,Rtd.$$

The last term is omitted if the web is to be disregarded in the calculation.

In the present case, $W + W' = 181295$, $d = 6$, $R = 8000$, $t = \frac{3}{8}$, $l = 63$, hence we have for any distance of x feet from the left, the upper flange strain

$$15108x\left(1 - \frac{x}{63}\right) - 36000.$$

Let us take for the angle irons in the top flange, angles $6 \times 6 \times \frac{5}{8}$, area 14.52 sq. ins. This is nearly the largest size given by Carnegie. At 8000 lbs. per sq. inch, such angles will sustain a stress of $14.52 \times 8000 = 116160$ lbs.

Putting then $116160 = 15108x\left(1 - \frac{x}{63}\right) - 36000$, we find $x =$ about 13 feet for the point at which the angles need to be re-enforced by a top plate. For the sake of security and to allow for the net area of the lower flange, let us take $x = 10$ feet.

The first top plate then must be $63 - 20 = 43$ feet long. Let us take this plate 13 inches wide by $\frac{1}{2}$ inch thick. Its area then is 6.5 sq. ins. and the total area of flange is now $14.52 + 6.5 = 21.02$ sq. ins. At 8000 lbs. per sq. inch, this will give a resistance of $21.02 \times 8000 = 168160$ lbs. We have then $168160 = 15108x\left(1 - \frac{x}{63}\right) - 36000$,

or $x =$ about 20 feet, for the distance from the end at which the angles and top plate cease to be sufficient. Taking $x = 18$, we have the second top plate $63 - 36 = 27$ feet long.

Let us take this plate 13″ wide by $\frac{3}{8}$″ thick, area 4.87. The total area is then $21.02 + 4.87 = 25.89$, or at 8000 lbs. per square inch, $25.89 \times 8000 = 207120$ lbs. But the strain at the centre is $15108 \times \frac{63}{2}\left(1 - \frac{63}{2 \times 63}\right) - 36000 = 201951$, which is less than the flange resistance. No other plate is needed.

The size of rivets for the angles is by our rule $1\frac{1}{4}t + \frac{3}{16} = \frac{5}{4} \times \frac{5}{8} + \frac{3}{16} = \frac{31}{32}$, or about 1″ diameter, and for top plates $\frac{5}{4} \times \frac{3}{8} + \frac{3}{16} = \frac{21}{32}$″. The net area of lower flange is then, at 10 feet from end, $14.52 - \frac{10}{8} = 13.27$ sq. ins. At 8000 lbs. this gives 106160 lbs., while the strain at 10 feet from the end is $15108 \times 10\left(1 - \frac{10}{63}\right) - 36000 = 91100$ lbs. The net area of lower flange is therefore sufficient here.

At 18 feet from the end the net area is $21.02 - \frac{10}{8} - \frac{13}{16} = 18.96$, and resistance $18.96 \times 8000 = 151680$ lbs. The flange strain is $15108 \times 18\left(1 - \frac{18}{63}\right) - 36000 = 158245$ lbs. The net area is therefore a little small. If we take the second lower flange plate as commencing at 17 feet from the end, instead of 18, and therefore $63 - 34 = 29$ ft. long, it will be sufficient.

If the top plates are in two lengths, a splice plate at centre will be required. If in one length, no splice is necessary. The angles should be in one length always.

Web.—The shear at the end is $\frac{181295}{2} = 90647$ lbs. If we take the web $\frac{3}{8}$″ thick, its area is $72 \times \frac{3}{8} = 27$ sq. ins. At 8000 lbs. this gives a resistance of 216000, or much greater than the shear.

Stiffeners.—At the end the resistance of the web to buckling is

$$\frac{10000}{1 + \dfrac{d^2}{3000\,t^2}} = \frac{10000}{1 + \dfrac{66^2 \times 64}{3000 \times 9}} = 1052 \text{ lbs. per square inch.}$$

$$\text{The shear is } \frac{90647}{27} = 3357 \text{ lbs. per square inch.}$$

As this is more than the web will stand without buckling, we need stiffeners. These we must space at intervals of 5 feet, and calculate them for the shear at each point.

At the end, if we take two angles $4 \times 4 \times \frac{3}{8}$, area 5.7 sq. ins., and two filling plates $4 \times \frac{5}{8}$, area 5 sq. ins., the total area, including the web, is $5.7 + 5 + 1.5 = 12.25$ sq. ins, and the thickness is $t = \frac{3}{4} + \frac{5}{4} + \frac{3}{8} = 2\frac{3}{8}$″. The safe resistance per square inch of cross section is then

$$\frac{10000}{1 + \dfrac{60^2 \times 64}{3000 \times 19^2}} = 8264.$$

$$\text{The total resistance is then } 8264 \times 12.25 = 101234 \text{ lbs.}$$

As the shear at the end is 90647 lbs., the resistance of these stiffeners is sufficient.

For the other stiffeners, let $x =$ any distance from end, and u the static load, and w the live load per foot. Then the shear at any point distant x feet from the end is

$$\frac{ul}{2} - ux + \frac{w\,(l - x)^2}{2l}.$$

In the present case, the live load per foot is

$$w = \frac{116850 + 116850 \times 0.148}{63} = 2130 \text{ lbs.},$$

and the static load per foot is

$$u = \frac{181295}{63} - 2130 = 748 \text{ lbs.}$$

In the present case, therefore, the maximum shear at any point is

$$23562 - 748x + 16.9\,(63 - x)^2.$$

At 5 feet from the end, the maximum shear is then 76673 lbs. If we take 2 angles $3\frac{1}{2} \times 3 \times \frac{3}{8}$, area 4.62, and two filling plates $3 \times \frac{5}{8}$, area 3.75, the total area is $4.62 + 3.75 + 1.12 = 9.5$, and since the thickness is still $2\frac{3}{8}$″, the safe resistance is as before 8264 lbs. per sq. inch. The total safe resistance is then $8264 \times 9.5 = 78508$, or greater than the maximum shear.

At 10 feet from the end, the maximum shear is 63554 lbs. If we take here two angles $2 \times 2 \times \frac{3}{8}$, area 2.88, and two filling plates $3 \times \frac{5}{8}$, area 3.75, the total area is $2.88 + 3.75 + 1.12 = 7.75$; the thickness is as before, and the resistance $8254 \times 7.75 = 64046$, or greater than the maximum shear.

At 15 feet from the end, the maximum shear is 51279 lbs. If we take here two angles $2 \times 2 \times \frac{3}{8}$, area 2.88, and two filling plates $2.5 \times \frac{5}{8}$, area 3.125, the total area is $2.88 + 3.125 + 0.75 = 6.75$; the thickness is as before, and the resistance $8264 \times 6.75 = 55782$, or greater than the maximum shear.

At 20 feet from the end, the maximum shear is 39850 lbs. If we take here two filling plates $3.5 \times \frac{5}{8}$, area 4.375, the total area is 5.68, the thickness is $1\frac{5}{8}''$, the safe resistance $\dfrac{10000}{1 + \dfrac{60^2 \times 64}{3000 \times 13^2}} = 6900$, and the resistance is $6900 \times 5.68 = 39192$, or about the same as the shear.

At 25 feet from the end, the maximum shear is 29265 lbs. If we take here two filling plates $3 \times \frac{5}{8}$, area 3.75, the total area is 4.87, the thickness as before $1\frac{5}{8}''$, the resistance $6900 \times 4.87 = 33603$ lbs.

At the centre the maximum shear is 16770 lbs. Two filling plates $2 \times \frac{5}{8}$ will give a total area of 3.25 and a resistance of $3.25 \times 6900 = 22425$ lbs.

Rivets.—The size of rivets, as already found, is about $1''$ for the flange angles, and $\frac{7}{8}''$ for the flange plates. In the top plate, we may have a pitch of 4 inches. As the top plate is $13''$ wide, we can run an extra line of rivets along the edge with a pitch of 8 inches, to draw the plates together. The bearing resistance of a flange rivet for $\frac{3}{8}''$ web is, from Rivet Table I., 4690 lbs. The total load is 181295 lbs. We have found the horizontal stress at 13 feet from the end to be 116160 lbs. or 58 tons. At 20 feet, 168160 lbs. or 84 tons. At the centre, 201951 lbs. or 100 tons. We have then for the first 13 feet 58 tons, for the next 7 feet $84 - 58 = 26$ tons, for the next 11.5 feet $100 - 84 = 16$ tons, to be taken by the rivets. The load per ft. is 2878 lbs. or 18.7 tons on the first 13 feet, 10 tons on the next 7 feet, and 16.5 on the next 11.5 feet. The resultant stresses are

$$\sqrt{58^2 + 18.7^2} = 61 \text{ tons} = 122000 \text{ lbs.}, \quad \sqrt{26^2 + 10^2} = 28 \text{ tons} = 56000 \text{ lbs.}, \quad \sqrt{16^2 + 16.5^2} = 23 \text{ tons} = 46000 \text{ lbs.}$$

We have then for the number of rivets in the first 13 feet $\dfrac{122000}{4690} = 27$, in the next 7 feet $\dfrac{56000}{4690} = 13$, in the remaining 11.5 feet $\dfrac{46000}{4690} = 10$. If we space the rivets at 5 inches for the first 20 feet, and at 6 inches, which is the largest pitch allowable, for the rest of the way to centre, we shall have more than are called for.

CHAPTER VIII.

ROOF AND BRIDGE TRUSSES—DEAD WEIGHT—ECONOMIC DEPTH.

FOR highway trusses the allowable live load has been given on page 420. The weight of flooring, etc., will depend upon the circumstances of the case, and must be estimated in accordance with such circumstances. The weight of stringers and floor beams, and of lateral bracing, can be estimated as in Chap. VII., page 424, and Chap. VI., page 407. It remains to find the dead weight of the truss itself. When this is known, the maximum strains due to loading can be found, and the various members designed in accordance with the preceding rules and principles.

The same remarks hold good for railway bridges, except here the weight of flooring, rails, cross ties, etc., may be taken at once at 400 lbs. per foot for single track and 750 per foot for double track. The entire external load of a truss can then be easily found. It remains to estimate the dead weight of the truss itself.

For roof trusses, we have already taken the horizontal wind pressure at 50 lbs. per ft. and have given in Part I., page 65, a Table giving the normal pressure upon an inclined surface due to the wind force, and shown how to find the strains due to it. The only other loads to which a roof truss is subjected are the snow load and the weight of roof covering. The total external load being known, it remains to estimate the dead weight of the truss itself. The maximum strains may then be found and the various members proportioned.

ROOF TRUSSES—SNOW LOAD AND ROOF COVERING.—The snow load for roofs may be taken at 30 lbs. per square foot *as a maximum*. Locality should be considered of course in the design, as also pitch of roof.

The wind pressure can be found as in Part I., page 65, and the pressure at the end supports due to it can be found.

The weight of roof covering can be estimated from the following Table:

WEIGHT OF VARIOUS ROOF COVERINGS IN LBS. PER SQUARE FOOT.

Shingles, 16 inch	2	Cast iron plates ($\frac{3}{8}''$)	15
Shingles, long	3	Sheet iron ($\frac{1}{16}''$)	3
Thatch	6.5	Slates (ordinary)	5 to 9
Felt and asphalt	1	Slates (large)	9 to 11
Felt and gravel	8 to 10	Tiles (average)	12
Tin	0.7 to 1.25	Tiles (large)	7 to 20
Sheet lead	5 to 8	Tiles (with mortar)	25 to 30
Copper	0.8 to 1.25	Slates and iron laths	10
Zinc	1 to 2	Sheathing, pine, 1 inch thick	3
Iron, galvanized	1 to 3	Sheathing, chestnut or maple	4
Iron, corrugated	1 to 3.75	Sheathing, ash, hickory, pine, oak	5
Sheet iron and laths	5	Laths and plaster	9 to 10

To the weight of roof covering thus estimated, must be added the weight of the "*purlins*" or stringers, whether wood or iron, which are laid across from truss to truss at the apices, to support the roof and covering. In any case then, we may make a close estimate of the total external load, due to wind, snow, roof covering and purlins. Call this total external load W. It is now required to estimate the dead weight W' of the truss.

ROOF TRUSSES—DEAD WEIGHT.—Let the length of span in feet be l, and rise in feet be r, and the allowable stress per square inch be β.

Then the strain in the tie will be $\frac{(W + W')}{2} \tan \theta$, where θ is the angle of the rafter with the vertical. Since $\tan \theta = \frac{l}{2r}$, we have the tie strain $\frac{(W + W')l}{4r}$. The cross section of tie is then $\frac{(W + W')l}{4\beta r}$. Let γ be the weight of 12 cubic inches of material. Then the weight of the tie per foot is $\frac{\gamma(W + W')l}{4\beta r}$, and the weight of the entire tie is $\frac{\gamma(W + W')l^2}{4\beta r}$.

The rafter strain is

$$\frac{(W + W')}{2} \sec \theta = \frac{(W + W')}{2} \frac{\sqrt{\frac{l^2}{4} + r^2}}{r}.$$

Divide by β and we have the cross section. Multiply by γ and we have the weight per per ft. Multiply by $\sqrt{\frac{l^2}{4} + r^2}$ and we have the weight of one rafter. For two rafters then the weight is

$$\frac{\gamma(W + W') \left(\frac{l^2}{4} + r^2\right)}{\beta r}.$$

The total weight of both rafters and the tie, is then, disregarding the bracing, approximately the weight of the truss, or

$$W' = \frac{\gamma(W + W')}{\beta r} \left(\frac{l^2}{2} + r^2\right),$$

hence,

$$W' = \frac{W}{\frac{\beta r}{\gamma \left(\frac{l^2}{2} + r^2\right)} - 1}.$$

For iron, we have $\gamma = \frac{10}{3}$, $\beta = 10000$. For wood $\gamma = 0.35$, $\beta = 1200$. We have neglected the web in this formula, but for short spans its influence is small. On the other hand, we have treated the rafter as of constant cross section, which for long spans gives an excess and tends to balance the error in disregarding the web.

EXAMPLE.—*An iron roof truss, with corrugated iron covering, has a span of 100 feet and a rise of 20 feet. It is spaced 7 feet from the adjacent trusses on each side. Each rafter is divided into 4 equal bays, and the purlins are rolled iron beams. What is the dead weight?*

Here $l = 100$, $r = 20$, $\gamma = \frac{10}{3}$, $\beta = 10000$. It remains to estimate the total external load W.

The maximum snow load is $100 \times 7 \times 30 = 21000$ lbs. The angle of roof with horizon is $21° 48'$. From our

Table, Part I., page 65, the normal pressure per square ft. of wind is 24.77 lbs. The length of rafter is 53.85 ft. The exposed area is $53.85 \times 7 = 376.95$ sq. ft. The normal wind pressure is $376.95 \times 24.77 = 9326$ lbs. The vertical component of this pressure is $9326 \cos 21^\circ\ 48' = 9326 \times 0.9285 = 8660$ lbs.

The weight of roof covering is say 3 lbs. per sq. ft. The whole weight is $53.85 \times 7 \times 3 \times 2 = 2262$ lbs. One-seventh of this acts at each apex. So also for the snow load. For the wind load $\frac{1}{4}$ of $8660 = 2165$ lbs. acts at an apex. The total apex load is then

$$\frac{2262}{7} + \frac{21000}{7} + 2165 = 5488 \text{ lbs.}$$

This is the load on a purlin.

From Carnegie, page 51, we see that a 5 inch I beam, 10 lbs. per foot, will be required for the purlins. Each purlin weighs then 70 lbs. There are 8 purlins, and their weight is 560 lbs.

The total external load W is now

$$W = 21000 + 8660 + 2262 + 560 = 32482 \text{ lbs.}$$

The dead weight of the truss is therefore

$$W' = \frac{32482}{\dfrac{10000 \times 20}{\dfrac{10}{3}\left(\dfrac{100^2}{2} + 20^2\right)} - 1} = \frac{32482}{10.11} = 3212 \text{ lbs.}$$

Now that we know the dead weight of the truss itself, also the snow load, the weight of roof covering and of purlins, we can find the strains due to total static loading. Then, as detailed in Part I., page 65, we can find the wind strains, and can then make out the maximum strain for each member. The various members can then be proportioned in accordance with preceding principles.

Bridge Trusses—Dead Weight.—For highway bridges we must estimate the flooring and roadway according to the design and case in hand. No general estimate can be given. For railway bridges, we may take the rails, ties, planking, etc., at 400 lbs. per ft. for single track, and 800 lbs. per ft. for double track.

We can then find the weight of the stringers, as detailed in the preceding chapter, whether the stringers are of wood or iron. Next we can find the weight of the cross girders or floor beams.

We can then estimate the weight of the lateral system or wind bracing by the formulæ of page 407. If we denote by w_3 the weight per ft. *per truss* of the wind bracing, these formulæ may be written as follows:

For *single track*,

$$\text{for pony trusses—depth below 12.5 feet, } w_3 = 1.8\,N + \frac{270}{p};$$

for through trusses, without vertical sway bracing—depth between 12.5 and 24 feet,

$$w_3 = 3.2\,N + \frac{336}{p};$$

for through trusses, with vertical sway bracing, depth above 24 feet, or for deck bridges,

$$w_3 = \frac{3Nl}{170} + \frac{568}{p},$$

where l = span in feet, N = the number of panels, and p = panel length in feet.

For double track, multiply by $\frac{b}{15}$ where b = width in feet.

We represent the weight per foot *per truss* of the stringers, cross girders, and of the rails, ties, planking, etc., by w_2, and the weight per foot *per truss* of the uniformly distributed load, equivalent to the live load assumed, by w_1. This equivalent load can easily be found from our diagram, page 89, for any span. Let the weight per foot of one main

truss be w_4, and let w_0 be the weight per foot of lattice bars, pins, eye bar heads, splice and cover plates, rivets, etc. Then the total load per foot per truss, is $w_1 + w_0 + w_2 + w_3 + w_4$. Let the length of panel be p, then $(w_1 + w_0 + w_2 + w_3 + w_4)\,p$ will be the total panel load for one truss. Let N be the number of panels, $d =$ the depth in feet, and $l =$ the span in feet.

Let us consider first the Warren girder, Fig. 88, page 97. The reaction at end for full load is, according to our notation,

$$\frac{(w_1 + w_0 + w_2 + w_3 + w_4)(N - 1)\,p}{2}.$$

The strain in the 1st lower panel is the reaction multiplied by the half panel length $\frac{p}{2}$, and divided by the depth d. If this strain is divided by the stress per square inch for tension, R_t, we have the area in square inches. The area multiplied by the length of panel, p, will be the volume, and this multiplied by $\frac{10}{3}$ will give the weight. We have then for the weight of the first lower panel,

$$\frac{10\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2}{12R_t d}\,[N - 1].$$

In a similar way we can easily find the weight of each lower panel, and thus obtain the following:

Wt. of 1st lower panel,		$\frac{10\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2}{12R_t d}$	$[N - 1]$.
" 2d "	"	"	$[3\,(N - 1) - 2]$.
" 3d "	"	"	$[5\,(N - 1) - 8]$.
" 4th "	"	"	$[7\,(N - 1) - 18]$,

and so on.

Summing up by series, we have for the weight of N lower panels,

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,Np^3\,(N^2 - 1)}{18R_t d}.$$

Since $Np = l =$ the span, the weight *per ft.* per truss of the lower chord will be given by

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2\,(N^2 - 1)}{18R_t d}.$$

In a precisely similar manner, we find for the weight of the upper chord per ft. per truss,

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,p^2\,(N^2 - 1)}{18R_c d},$$

where R_c is the stress per square inch for compression.

For the braces, the $\sec\theta = \dfrac{\sqrt{\dfrac{p^2}{4} + d^2}}{d} = \dfrac{\sqrt{p^2 + 4d^2}}{2d}$.

We have then for full loading, the strain in the first tie $=$

$$\frac{p\,(w_1 + w_0 + w_2 + w_3 + w_4)\sqrt{4d^2 + p^2}}{4d}\,[N - 1].$$

We have, then, multiplying by the length $\frac{\sqrt{4d^2+p^2}}{2}$, dividing by R_t and multiplying by $\frac{10}{3}$:

$$\text{Weight of 1st tie, } \frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(4d^2 + p^2)\,p}{12R_t d}\,[N - 1].$$

"	" 2d "	"	$[(N-1) - 2]$.
"	" 3d "	"	$[(N-1) - 4]$.
"	" 4th "	"	$[(N-1) - 6]$,

and so on.

The weight of N ties is then $\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N^2 p}{24R_t d}$.

The weight per foot per truss of the ties is then

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N}{24R_t d},$$

and for the struts we have, in precisely similar manner,

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)\,(p^2 + 4d^2)\,N}{24R_c d},$$

where R_c is the stress per square inch for compression.

The whole weight per foot is therefore, exclusive of details, $w_4 =$

$$\frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)}{18d}\left[\frac{(N^2-1)\,p^2}{R_t} + \frac{(N^2-1)\,p^2}{R_c} + \frac{0.75\,N\,(p^2+4d^2)}{R_t} + \frac{0.75N(p^2+4d^2)}{R_c}\right].$$

For the sake of brevity let us put

$$w_4 = \frac{5\,(w_1 + w_0 + w_2 + w_3 + w_4)}{18d}\left[\frac{T}{R_t} + \frac{C}{R_c} + \frac{S}{R_s}\right],$$

where T refers to the lower chord and ties, C to the upper chords, and S to the struts; hence $T = (N^2 - 1)\,p^2 + 0.75N\,(p + 4d^2)$, $C = (N^2 - 1)\,p^2$, and $S = 0.75N\,(p^2 + 4d^2)$.

We have then,

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{\dfrac{3.6d}{\dfrac{T}{R_t} + \dfrac{C}{R_c} + \dfrac{S}{R_s}} - 1}.$$

Rankine's formula for long struts is, for the upper chords, $R_c = \dfrac{\mu}{1 + \dfrac{p^2}{250r_1^2}}$, and for the struts $R_s = \dfrac{\mu}{1 + \dfrac{p^2 + 4d^2}{4 \times 125\,r_2^2}}$, where $\mu = 8000$ and r_1, r_2 are the least radii of gyration of the cross section. We have then,

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{\dfrac{3.6\mu d}{\dfrac{\mu}{R_t}\,T + C + S + \dfrac{Cp^2}{250r_1^2} + \dfrac{S\,(p^2 + 4d^2)}{500r_2^2}} - 1}.$$

Now R_t is on the average about 9000 lbs., and $\mu = 8000$ lbs. We shall make but slight error in taking $\frac{\mu}{R_t} = 1$. For r_1^2 the simple expression $r_1^2 = \frac{(N-1)p^2}{100}$ gives very close values as compared with practice. For the struts we take $r_2^2 = \frac{N-1}{50}$ multiplied by the square of the length, or, in this case,

$$r_2^2 = \frac{(N-1)(p^2 + 4d^2)}{200}.$$

If, then, we put, for the sake of brevity,

$$T + C + S = p^2\left(2N^2 + \frac{3N}{2} - 2\right) + 6Nd^2 = \alpha p^2 + \beta d^2,$$

we have

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{\dfrac{3.6\mu d}{\alpha p^2 + \beta d^2 + \dfrac{\alpha p^2 + 4\beta d^2}{5(N-1)}} - 1}.$$

The form of this equation is entirely rational, and only the constants α and β and w_0 remain to be determined.

For w_0 we have the empiric formula $w_0 = \frac{Nd}{3} + A$, where $A = 0.875N(12 - N) + 6$.

We have finally, then, the following formula for the weight per foot of one truss:

Formula for the Dead Weight of one Main Truss.

Let w_1 = the weight per ft. per truss of the equivalent uniform load.
w_0 = the weight per ft. per truss of details.
w_2 = the weight per ft. per truss of the stringers, cross girders, and of the rails, ties, planking, etc.
w_3 = the weight per ft. per truss of the wind bracing.
w_4 = the weight per ft. of one main truss.
p = the panel length in feet.
d = the depth in feet.
N = the number of panels.
μ = the numerator of Gordon's formula = 8000 for iron. Then

$$w_4 = \frac{w_1 + w_0 + w_2 + w_3}{L - 1}; \qquad \text{(I.)}$$

$$\text{where } L = \frac{3.6\mu d}{\alpha p^2 + \beta d^2 + \dfrac{\alpha p^2 + 4\beta d^2}{5(N-1)}},$$

and

$$w_0 = \frac{Nd}{3} + A, \quad A = 0.875N(12 - N) + 6.$$

We have, then, *total weight of iron per foot* $= 2(w_2 - 200 + w_0 + w_3 + w_4)$; also for the values of α and β, we have

For Warren girder,

$$\alpha = (2N^2 + 1.5N - 2); \quad \beta = 6N.$$

In precisely the same way as for the Warren girder we may deduce for

Single intersection Pratt truss,

$$\alpha = (2N^2 + 3N - 2); \quad \beta = 6N - 12 + \frac{33}{N}.$$

Double intersection Whipple,

$$\alpha = 2N^2 + 6N - 20 + \frac{24}{N}; \quad \beta = 3N - 6 + \frac{48}{N}.$$

For Post truss,

$$\alpha = 2N^2 + 3.75N - 21.5 + \frac{30}{N}; \quad \beta = 3N - 6 + \frac{24}{N}.$$

For parabolic bow-string,

$$\alpha = 3N^2p; \quad \beta = 16Np.$$

For double parabolic bow-string,

$$\alpha = 3N^2p; \quad \beta = 24Np.$$

TABLES FOR FACILITATING CALCULATION OF DEAD LOAD.—For ready application of the formula I., for dead weight of truss, we recapitulate here the formulæ for weight per ft. per truss of wind bracing, w_3, and also give Tables for the value of the equivalent uniformly distributed live load per ft. per truss, or w_1, for the value of the weight of stringers, floor beams, flooring, rails, ties, etc., per ft. per truss, or w_2, and for the values of A for single and double intersection Pratt truss, Post truss, and Warren girder. This Table can easily be extended to the other systems for which the value of A is given, if desired.

FORMULÆ FOR w_3.—For *single track*, width 15 feet,

$$\text{for pony trusses, depth below 12.5 feet, } w_3 = 1.8N + \frac{270}{p};$$

for through trusses, without vertical sway bracing, depth between 12.5 and 24 feet,

$$w_3 = 3.2N + \frac{336}{p};$$

for through trusses, with vertical sway bracing, depth above 24 feet, or for deck bridges,

$$w_3 = \frac{3Nl}{170} + \frac{568}{p};$$

where l = span in feet, N = number of panels, p = panel length in feet. For any width, divide by 15 and multiply by the width.

TABLE I.

VALUES OF α AND β FOR DIFFERENT TRUSSES.

N	Single Intersection.		Double Intersection.		Warren.		Post.	
	α	β	α	β	α	β	α	β
2	12	16.5	12	24	9	12	9	12
3	25	17	24	19	20.5	18	17.75	11
4	42	20.25	42	18	36	24	33	12
5	63	23.6	64.8	18.6	55.5	30	53.25	17.8
6	88	29.5	92	20	79	36	76.33	16
7	117	34.714	123.48	21.857	106.5	42	107.036	18.45
8	150	40.125	159	24	138	48	139.875	21
9	187	45.666	198.666	26.333	173.5	54	177.584	23.67
10	228	51.3	242.4	28.8	213	60	219	26 4
11	273	57	290.18	31.36	266.5	66	264.477	29.15
12	322	62.666	342	34	304	72	314	32
13	375	68.538	397.846	36.69	355.5	78	367.557	34.8465
14	432	74.357	457.714	39.4285	411	84	425.143	37.715
15	493	80.2	521.6	42.2	470.5	90	486 75	40.6
16	558	86.0626	589.5	45	534	96	551.4375	43.5
17	627	91.941	661.412	47.8235	601.5	102	622.018	46.412
18	700	97.833	737.333	50.666	673	108	695.666	49.333
19	777	103.737	817.263	53.5263	748.5	114	773.329	52.263
20	858	109.65	901.4	56.4	828	120	855	55.2

TABLE II.—VALUES OF w_1.*

EQUIVALENT UNIFORM LOAD w_1, IN LBS. PER FOOT, PER TRUSS, FOR THE LOAD SYSTEM SIMILAR TO "CLASS A" OF COOPER'S SPECIFICATIONS.

Table gives values of w_1 for single track for one truss. For double track take double these values.

Span =	50	55	60	65	70	75	80	90	100	110	120	130 feet.
w_1 =	1848	1733	1718	1677	1618	1570	1546	1552	1560	1569	1579	1586 lbs.

Span =	140	150	160	170	180	190	200	210	220	230 feet and over.
w_1 =	1574	1562	1556	1546	1533	1516	1511	1506	1502	1500 lbs.

The table gives w_1 for one truss, single track, on the assumption of two trusses to the bridge.

TABLE III.—VALUES OF $w_0 = \dfrac{Nd}{3} + A$.

For the weight per foot per truss of details, $w_0 = \dfrac{Nd}{3} + A$, where $A = 0.875N(12 - N) + 6$ *we have the following values of* A.

N =	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A =	+ 34	+ 36.6	+ 37.5	+ 36.6	+ 34	+ 29.6	+ 23.5	+ 15.6	+ 6	− 5.37	− 18.5	− 33.4	− 50	− 68.4	− 88.5	− 110.4	− 134.

For the values of w_2, we have the following Table, based upon the Tables for weight of stringers and cross girders already given, the weight of rails, ties, etc., being taken at 400 lbs. per foot for single track, and 800 lbs. for double track.

* Increase these values by 18 per cent. for the system of loads given by our diagram, page 89 and 215.

TABLE IV.*

Panel length in feet.	Single track—15 feet wide.				Panel length in feet.	Double track—25 feet wide.			
	½ cross gird'r	1 stringer.	½ floor.	w_2		½ cross gird'r	2 stringers	Floor.	w_2
5	507	188	1000	339	5	1555	376	2000	786
6	582	248	1200	338	6	1783	496	2400	780
7	631	315	1400	335	7	1934	630	2800	766
8	679	386	1600	333	8	1992	772	3200	745
9	695	463	1800	327	9	2135	926	3600	740
10	716	545	2000	326	10	2192	1090	4000	728
11	750	657	2200	328	11	2291	1314	4400	727
12	776	825	2400	333	12	2374	1650	4800	735
13	806	974	2600	337	13	2443	1958	5200	738
14	831	1130	2800	340	14	2540	2260	5600	743
15	853	1292	3000	343	15	2606	2584	6000	746
16	872	1460	3200	345	16	2664	2920	6400	748
17	886	1634	3400	348	17	2700	3268	6800	751
18	906	1816	3600	351	18	2775	3632	7200	755
19	930	2003	3800	354	19	2841	4006	7600	760
20	949	2197	4000	357	20	2871	4394	8000	763
21	968	2420	4200	361	21	2957	4840	8400	771
22	983	2652	4400	365	22	3004	5304	8800	777
23	1001	2900	4600	369	23	3030	5800	9200	783
24	1018	3133	4800	373	24	3080	6266	9600	789
25	1034	3382	5000	376	25	3128	6764	10000	795
26	1048	3633	5200	380	26	3172	7266	10400	801
27	1062	3904	5400	384	27	3213	7808	10800	808
28	1077	4180	5600	387	28	3282	8360	11200	816
29	1086	4463	5800	391	29	3332	8926	11600	822
30	1104	4756	6000	395	30	3373	9512	12000	829

With the aid of these Tables and Formula I., we can readily and easily compute the weight of any R.R. bridge.

EXAMPLE I.—*Let us take a single track R. R. bridge, 150 feet span, 9 panels, double intersection, 27.8 feet deep.*

Since the depth is greater than 24 feet, we have the weight per foot per truss for wind bracing, from the formula, page 440, $w_3 = 57$ lbs. per foot per truss.

If there are 9 panels, each panel is 16⅔ feet long. We have then, from Table IV., $w_2 = 347$ lbs. per foot per truss.

From Table II., we have $w_1 = 1562$ lbs. per foot per truss. From Table III., $w_0 = 113$. Hence $w_1 + w_2 + w_3 + w_0 = 2079$ lbs. per foot per truss. From Table I., for 9 panels, double intersection, we have $\alpha = 198\frac{2}{3}$, $\beta = 26\frac{1}{3}$, and from Formula I., taking $\mu = 8000$, $w_4 = 327$ lbs. per foot per truss.

The weight of each main truss is then 327 lbs. per foot. For the total weight of iron in the structure we have for the trusses $327 \times 2 = 654$ lbs. per foot. For the details $113 \times 2 = 226$ lbs. per foot. For the floor and wind bracing we have $347 - 200 = 147$ lbs. per foot per truss, for the floor, and 57 lbs. per foot per truss for wind bracing, or $147 + 57 = 204$ lbs. *per truss*, or 408 lbs. per foot for the structure. We subtract 200 lbs. from 347, because the rails, ties, planking, etc., weigh 400 lbs. per foot for both trusses, and this portion is not part of the structure. The structure weighs then $408 + 654 + 226 = 1288$ lbs. per foot, or $1288 \times 150 = 193200$ lbs. The panel dead load is $\frac{193200}{2 \times 9} + 200 \times 16\frac{2}{3} = 14066$ lbs. per truss. The strains can now be found for this loading and for the live load assumed.

In the same way we can find the weight of truss and of structure for any other kind of truss, single or double track.

ECONOMIC DEPTH AND BEST NUMBER OF PANELS.—If our Formula (I.) is reliable, and gives even with tolerable accuracy the weight of truss, then, since it is rational in form, the least weight depth, or the depth which gives the least weight, can also be deter-

* Increase the values for cross girder and stringer by 18 per cent. for the system of loads given by our diagram, page 89. The floor remains the same.

mined. The least cost depth, or economic depth, ought to be somewhat less than this, usually by about ⅛th.*

Differentiating and putting the first differential equal to zero, we have

$$\frac{d}{l} = \frac{1}{N}\sqrt{\frac{\alpha\left[1 + \dfrac{1}{5\,(N-1)}\right]}{\beta\left[1 + \dfrac{4}{5\,(N-1)}\right] + \dfrac{1.2\,\mu N}{(w_1 + w_2 + w_3 + w_0) + A}}} \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

The values of α and β are taken from Table I. and of A from Table III. For standard specifications and the locomotive system adopted, we may take $w_1 + w_2 + w_3 + w_0 = 2000$ lbs. without noticeable error.

If we use this value of $w_1 + w_2 + w_3 + w_0$, we can make at once the following tabulation, which will enable us to find directly the best depth for any span, single or double intersection, or Warren.

TABLE V.

Least Weight Depth, $d = Cl$. Values of C Given in Table.

N	Warren. C	Single Intersection. C	Double Intersection. C
4	0.2207	0.2510	0.2592
5	0.1978	0.2258	0.2436
6	0.1805	0.2018	0.2272
7	0.1669	0.1846	0.2122
8	0.1559	0.1708	0.1992
9	0.1467	0.1596	0.1875
10	0.1389	0.1502	0.1773
11	0.1348	0.1420	0.1684
12	0.1263	0.1350	0.1605
13	0.1211	0.1290	0.1534
14	0.1164	0.1235	0.1470
15	0.1123	0.1196	0.1413
16	0.1084	0.1142	0.1360
17	0.1049	0.1102	0.1312
18	0.1016	0.1065	0.1267
19	0.0986	0.1031	0.1226
20	0.0958	0.0999	0.1187

For constructive reasons it is well to limit p to about 30 feet and d to 50 feet. Within these limits we can find best depth from Table V., and best number of panels N by trial. The total weight per foot of all the iron is $2\,(w_2 + w_3 + w_4 + w_0 - 200)$. That value of N which gives this a minimum is the best.

Thus, for span 104 feet, single intersection, we have for $N = 4$,

$N = 4$, $w_1 = 1564$, $d = 0.251$, $l = 26$ feet, $p = 26$ feet, $w_2 = 380$, $w_3 = 29$, $w_0 = 68$, $\alpha = 42$, $\beta = 20\frac{1}{4}$.

* Least weight does not necessarily mean least cost. The relative amount of the various kinds of iron, the cost of manufacturing the various shapes and members, whether riveted, rolled, or forged, facility of transportation and erection, all influence the cost. The influence of these factors varies from time to time, and the factors themselves may even vary at the same time at different manufactories. Constant employment in the preparation of competitive designs and alternate plans is necessary to enable a designer to choose best proportions. But even to such, the determination of proportions for least weight will be valuable, and to others most important as a guide to the judgment.

Therefore,

$w_1 + w_2 + w_3 + w_0 = 2041, \quad 3.6\mu d = 748800, \quad \alpha p^2 = 28392, \quad \beta d^2 = 13689, \quad L = 15.72.$

We have, then, from (I.), $w_4 = 138$, and total weight per foot of iron = 830 lbs.

For $N = 5$, we have,

$w_1 = 1564, \quad d = 23.5, \quad p = 20.8, \quad w_2 = 360, \quad w_3 = 36, \quad w_0 = 75, \quad \alpha = 63, \quad \beta = 23.6,$

$w_1 + w_2 + w_3 + w_0 = 2035, \quad 3.6\mu d = 676800, \quad \alpha p^2 = 27256, \quad \beta d^2 = 13033, \quad L = 15.26,$

and $w_4 = 142$; total weight per foot of iron = 826 lbs.

For $N = 6$,

$w_1 = 1564, \quad d = 21, \quad p = 17\frac{1}{3}, \quad w_2 = 349, \quad w_3 = 39, \quad w_0 = 79, \quad \alpha = 88, \quad \beta = 29.5,$

$w_1 + w_2 + w_3 + w_0 = 2031, \quad 3.6\mu d = 604800, \quad \alpha p^2 = 26365, \quad \beta d^2 = 13005, \quad L = 14.23,$

and $w_4 = 152$; total weight per foot of iron = 838 lbs.

We see at once that, as the number of panels diminishes, or the panel length increases, the truss grows lighter, but at the same time the floor grows heavier, as shown by Table IV. There is, then, a best number of panels, in this case *five*, for which the total weight is a minimum. The best panel length is, then, 20.8 feet.

The corresponding best depth is 23.5 feet. Formula I., however, shows that for the best number of panels *a considerable change in depth affects the weight of truss but little.* This may then be taken more or less than the value from Table V., without much effect on weight. In any case the best value of N is easily found by trial, and in case p is greater than 30 feet, it would be well to limit it to that value.

For span 150 feet, double intersection, we have, $N = 5$,
$w_1 = 1562, \quad d = 36, \quad p = 30, \quad w_2 = 395, \quad w_3 = 32, \quad w_0 = 96, \quad \alpha = 64.8, \quad \beta = 18.6,$
$w_4 = 198$; total weight of iron per foot = 1042 lbs.

$N = 6, \quad w_1 = 1562, \quad d = 34, \quad p = 25, \quad w_2 = 376, \quad w_3 = 38, \quad w_0 = 105, \quad \alpha = 92, \quad \beta = 20,$
$w_4 = 201$; total weight of iron per foot = 1040 lbs.

$N = 7, \quad w_1 = 1562, \quad d = 28, \quad p = 21.43, \quad w_2 = 363, \quad w_3 = 37, \quad w_0 = 102, \quad \alpha = 123.428,$
$\beta = 21.857, \quad w_4 = 221$; total weight of iron per foot = 1046 lbs.

We thus establish $N = 6$ as the best number of panels.

For span 320 feet, double intersection, we find that the total weight decreases as N decreases, until we have $p = 29$ for $N = 11$. As it is not advisable to have a longer panel than this, we may take $N = 11$ or more.

As we have repeatedly said in the proper connection, double intersection trusses are no longer built. The single advantage, that for large depths the long posts can be pinned at centre, is equally well obtained by some modification of the Baltimore Truss (page 58), such as the "sub-Pratt," illustrated on page 61, Fig. 60 (*a*).

LIMITING LENGTH OF GIRDER.—If we denote by L the limiting length of girder, or that length for which the girder will just support its own weight, we have

$$w_4 L = (w_1 + w_2 + w_3 + w_4)\, l, \text{ or } w_4 = \frac{w_1 + w_2 + w_3}{\frac{L}{l} - 1}.$$

This expression is precisely similar in form to Formula I.

We have, therefore, by reference to Formula I.,

$$L = \frac{3.6\,\mu d}{\alpha p^2 + \beta d^2 + \dfrac{\alpha p^2 + 4\beta d^2}{5\,(N-1)}},$$

and this equation will give for any case the limiting length. Thus, for a span of 104 feet, single intersection, $N = 6$, $p = 17\frac{1}{3}$, $d = 24$, if we take $w_1 + w_2 + w_3 + w_0 = 2031$, we have $L = 1480$ feet.

HIGHWAY BRIDGES—DEAD LOAD.—Our method is equally applicable to highway bridges. We have only to figure up the value of w_1, w_2 and w_3 for the case in hand.

EXAMPLE.—*A single intersection iron Pratt truss highway bridge, of "Class A," page 420, is 160 ft. long and 14 ft. wide, and has 8 panels. What should be the depth? Also if the flooring is 3 inch pine, what is the weight of each main truss, what is the total weight of iron, and what is the total static load? The cross girders are to be of iron, and the stringers of white pine.*

The 3 inch flooring will weigh $14 \times 12 \times 3 \times 0.35 = 176.4$ lbs. per ft. lineal, or 3528 lbs. per panel. From our Table, page 420, we have for the live load for "Class A," 80 lbs. per sq. ft. or 11200 lbs. per panel per truss. The total panel floor load is then $3528 + 22400 = 25928$ lbs. If this is carried by 6 joists or stringers, each one must carry $\frac{25928}{6} = 4320$ lbs. From our Table, page 422, we see that joists 6″ × 14″ will carry, if 20 ft. long, $858 \times 6 = 5148$ lbs., and will therefore be sufficiently strong. Each joist will weigh $0.35 \times 6 \times 14 \times 20 = 588$ lbs.

The load on each cross girder is $3528 + 588 \times 6 + 22400 = 29456$ lbs. The least weight depth of cross girder is by our formula, page 424,

$$\frac{10 \times 14^2}{8000} + \sqrt{\frac{6 \times 29456 \times 14}{8000} + \left(\frac{10 \times 142}{8000}\right)^2} = 17.6 \text{ inches.}$$

The weight of such a cross girder, is from the formula, page 424,

$$\frac{12 \times 29456 \times 14^2 + 2 \times 8000 \times 14 \times 17.6^2}{1.2 \times 8000 \times 17.6 - 12 \times 14^2} = 832 \text{ lbs.}$$

There are seven such cross girders, the total weight being $832 \times 7 = 5824$ lbs. or $\frac{5824}{160} = 36$ lbs. per ft., or 18 lbs. per ft. per truss. The wind bracing is, from page 407, $6.4 \times 8 + \frac{672}{20} = 84$ lbs. per ft., or 42 lbs. per ft. per truss $= w_3$. We have now for the live load per ft. per truss, $w_1 = \frac{11200}{20} = 560$ lbs. For the flooring we have $\frac{176.4}{2} = 88.2$, for the joists $\frac{588 \times 6}{20 \times 2} = 88.2$, for the cross girders 18, and hence $w_2 = 194.4$, and $w_1 + w_2 + w_3 = 791$ lbs.

The best depth from our Table V. is then about 27 ft., $w_0 = 106$, and the weight of one truss is, from our formula, page 443, 219 lbs. per ft.

Add to this 18 for the cross girders and 42 for the wind bracing, and 106 for details, and we have $385 \times 2 = 770$ lbs. per ft. of iron in the entire structure. The total static load is $w_2 + w_3 + w_4 + w_0 = 561$ lbs. per ft. per truss. The lumber weighs 342 lbs. per ft. for the entire structure.

We can now find the strains and design the structure. The total weight of iron will be 123200 lbs., and of lumber 54720 lbs.

RESULTS OF APPLICATION OF FORMULÆ.—We give here the tabulated results of our formulæ for dead weight, for single and double intersection railway bridges, on the assumption of two trusses, and in accordance with the specifications assumed in this chapter. For the system of loads assumed in our diagram, page 98, the weight should be increased 18 per cent. The best number of panels and best depth as determined by our formulæ are also given. A change of depth of a few feet will not materially affect the weights given. The total weight of iron given does not include shoe-plates, rollers, etc.

The general formulæ can be adapted to any practice and specifications, by proper Tables for w_1, w_2, and w_3.

DEAD WEIGHT OF IRON RAILWAY BRIDGES, ACCORDING TO OUR FORMULÆ, WITH ECONOMIC DIMENSIONS.

Table gives the weight per foot of iron, exclusive of shoe-plates, rollers, etc.
For single track add 400 *lbs. per foot for ties, rails, chairs, spikes, etc.*
For double track add 800 " " " " " " "

Span in ft., l	SINGLE INTERSECTION.				DOUBLE INTERSECTION.			
	Best Number of Panels, N	Best Depth in Ft., d	Single Track, 15 ft. Total Weight of Iron per ft.	Double Track, 25 ft. Total Weight of Iron per ft.	Best Number of Panels, N	Best Depth in Ft., d	Single Track, 15 ft. Total Weight of Iron per ft.	Double Track, 25 ft. Total Weight of Iron per ft.
60	4	15	628	1194	5	15	632	1186
70	4	18	658	1238	5	17	660	1234
80	5	18	695	1310	5	20	696	1288
90	6	18	740	1438	5	22	730	1342
100	6	20	822	1504	5	24	784	1436
110	6	23	872	1594	5	27	840	1532
120	6	24	924	1686	5	29	892	16'2
130	5	29	968	1770	5	26	947	1700
140	5	32	1020	1864	6	32	996	1780
150	5	34	1076	1956	6	34	1044	1866
160	6	32	1138	2056	6	36	1100	1938
170	6	34	1212	2188	7	36	1160	2048
180	6	36	1264	2278	7	38	1206	2130
190	7	35	1340	2398	7	40	1258	2216
200	7	37	1404	2512	7	42	1316	2318
210	7	40	1468	2614	7	46	1370	2398
220	9	35	1560	2776	8	44	1460	2546
230	8	39	1634	2916	8	46	1504	2618
240	8	41	1722	3046	8	48	1570	2726
250	9	40	1816	3220	9	47	1656	2870
260	9	41	1924	3406	9	49	1730	2982
270	9	43	1984	3506	9	50	1782	3086
280	10	42	2120	3770	10	50	1906	3260
290	10	46	2210	3884	11	49	2006	3432
300	10	45	2304	4056	12	48	2120	3636

EMPIRIC FORMULÆ FOR TOTAL WEIGHT.—As variations in depth do not greatly affect the weight of truss, it would seem possible to construct an empiric formula, which shall contain the span as the only variable, and give at once, with little calculation, and with sufficient accuracy, the entire weight of iron.

We have seen that the total weight of iron in lbs. per foot is given by

$$2\,(w_2 - 200 + w_3 + w_0 + w_4),\ \text{or putting for } w_4 \text{ its value } \frac{w_1 + w_2 + w_3 + w_0}{L - 1},$$

we have

$$2\left[\frac{(w_1 - 200 + w_3 + w_0)\,L + w_1 + 200}{L - 1}\right].$$

Now we find that $L = \dfrac{\text{constant}}{l}$ very nearly. We have, then, at once, for the form of empiric formula,

$$\text{total weight per foot} = \frac{a + bl}{c - l}.$$

This formula, we find, gives very excellent results when the proper values of a, b, and c are used, and these values will vary according to specifications and kind of bridge.

For the specifications of this chapter and live load similar to Class A of Cooper's *Specifications*, we have,

For SINGLE INTERSECTION,

single track, weight per foot of iron in lbs. $= \dfrac{276250 + 1890l}{666 - l}$;

double track, weight per foot of iron in lbs. $= \dfrac{536900 + 3294l}{676 - l}$.

For DOUBLE INTERSECTION,

single track, weight per foot of iron in lbs. $= \dfrac{271230 + 1630l}{654 - l}$;

double track, weight per foot of iron in lbs. $= \dfrac{566340 + 3010l}{704 - l}$.

For DECK PLATE GIRDERS, 8 feet wide, ties on top chord,

single track, weight per foot of iron in lbs. $= \dfrac{228612 + 7774l}{1110 - l}$.

For double track, about 70 per cent. greater.

For *Iron Highway Bridges*, of Class A (page 420),

$$\text{weight of iron in lbs. per foot} = \frac{7600 + 124l}{1100 - l}w;$$

weight per foot of lumber $= 120 + 12w$, where $w =$ width of roadway in feet.

For the live load of our diagram, page 89, add 18 per cent. to weight.

PRACTICAL FORMULÆ FOR WEIGHT OF TRUSS.—The dead load is made up of the weight of the track, which ranges from 300 to 500, usually taken at 400 lbs. per linear foot, the floor system, and the trusses and lateral system.

As the weight of the floor has no connection with the weight of the rest, it is in practice designed first, and its correct weight is then always known. There remains, therefore, only to estimate the weight of the trusses and lateral system.

For this purpose the following empiric formulæ are in general use:

For SINGLE TRACK PLATE GIRDER SPANS,

weight per foot of girders and lateral system $= 10l$.

For AVERAGE SINGLE TRACK PRATT TRUSS,

weight per foot of trusses and lateral system $= 5l$,

where $l =$ span in feet.

For lattice girder spans take the weight intermediate between plate girders and Pratt truss.

For AVERAGE SINGLE TRACK PIN-CONNECTED PIVOT SPANS,

weight per foot of trusses and laterals $= 6$ to $7l$,

where $l =$ length of one arm.

For double track double these values.

These formulæ are purely empiric, and the coefficients must be varied according to judgment, to suit different specifications and live loads.

EXAMPLE.—*Single track through Pratt truss $l = 153$ feet. To be designed for the live load of our diagram, page 89, and by Cooper's Specifications.*

From our Table, page 450, we have, for best proportions, that is, for $N = 5$ and d about 34 feet, the weight of iron per foot $= 1100$ lbs. for live load, similar to Class A of Cooper's *Specifications*. We also find, page 444, $w_1 = 57$, from Table IV., page 446, $w_2 = 348$, and from Table III., $w_0 = 108$. The floor and laterals weigh, therefore, $2(w_0 + w_1 + w_2 - 200) = 626$ lbs. per foot.

If we wish weight for the live load of our diagram, page 89, we add 18 per cent., and have weight = 1300 lbs. per foot for best dimensions. From our empiric formula, page 451, we have,

$$\frac{276250 + 1890 \times 153}{666 - 153} = 1100 \text{ lbs.},$$

agreeing perfectly with our Table, page 450.

If we use the formula, page 443, and take $N = 9$, $d = 26$, we have weight of iron per foot = 1186 instead of 1100. This shows the result of departing from the best dimensions. For the live load of our diagram, page 89, we add 18 per cent., and have weight of iron per foot = 1400 lbs.

From the practical formula, page 451, we have for weight of trusses and lateral system $5l = 765$ lbs. per foot. If the floor is found by actual design to weigh 340 lbs., we would have weight = 1105 lbs. per foot, agreeing with preceding results.

ECONOMICAL SPAN.—When there are a number of spans and piers, the question arises, what length of span, taking into account the cost of the piers, will be the best, that is, corresponds to the least cost.

We have seen, page 450, that the weight of iron per foot is given by $\frac{a + bl}{c - l}$, where l is the span in feet, and a, b, and c are constants for which we have already given the values. Let there be n spans of length l, and let $L = nl$ be the total length. Let C be the cost in cents per pound of the iron, including manufacture, freight, erection, etc. Then, $\frac{nC(al + bl^2)}{100(c - l)}$ will be the cost in dollars of all the spans, or, inserting $l = \frac{L}{n}$, $\frac{C(aLn + bL^2)}{100(cn - L)}$.

Now let the average cost of a pier be P, then the total cost will be

$$y = \frac{C}{100}\left[\frac{aLn + bL^2}{cn - L}\right] + P(n + 1).$$

Differentiating, and placing the first differential equal to zero, we have for minimum cost, after reduction,

$$P = \frac{Cl^2}{100}\left[\frac{cb + a}{(c - l)^2}\right] \quad \ldots \ldots \ldots \ldots \quad (1)$$

From this formula, when the estimated average cost of a pier is known, the economical span l is easily determined.

If we take, for instance, $C = 5$ cents, and as already given, page 451, $a = 276250$, $b = 1890$, $c = 666$, then for single track, single intersection,

$$P = 0.05l^2\left(\frac{1534990}{(666 - l)^2}\right).$$

Suppose that in any case the estimated average cost of piers is \$5000. Then we have, $4.91l = 666$, or $l = 135$ feet. If the distance to be spanned were from 500 to 600 feet we should then have four spans.

The formula (1) can be adapted to any cost C, and any form of span, by giving to a, b, and c proper values, as given, page 451.

It will be seen that the usually accepted rule, that the economical span is that which costs the same as one pier, is not strictly correct. For the same values of C, and a, b, and c, the cost of a span of 135 feet would be \$6755, or 1.35 times as much as the average pier.

In case of a long structure, where the erection of piers offers no special difficulty, and the cost of a pier can be accurately estimated, our formula may give valuable information as to the length of span which should be selected.

The Bismarck Bridge, for instance, consists of three spans, single track, double intersection, each 400 feet long. The cost of the spans was about eight cents per lb., the freight being very high. The cost of the piers was actually as follows:

1st pier	$54144
2d "	171123
3d "	155800
4th "	65372
Total	$446439

The cost of a span was $84,000. Total cost, $698,439 for piers and spans.

By comparison with the actual weight of a span, we find that we should take only $\frac{7}{10}$ of the weight given by our formula, page 443, for double intersection, the difference being due to the use of steel and different train load.

For the present case we have, then, $P = \$111,609$, and,

$$P = \frac{56l^2}{1000}\left(\frac{1337250}{(654 - l)^2}\right).$$

This gives for economical span $l = 360$ feet. As the distance to be spanned is 1200 feet we should have either three spans of 400 feet each, as actually built, or two spans of 350 and two of 250 feet, or three spans of 350 and one of 150 feet. The extra pier can be taken at $170,000, and the cost of these different suppositions easily estimated.

In the present case the actual choice of three spans of 400 feet is justified by the calculation.

The practical difficulty of estimating the average cost of pier may, in many cases, prevent our formula from being used. Where this difficulty does not exist, its use may be a guide in the selection of length of spans.

CHAPTER IX.

SPECIFICATIONS—LIST OF BRIDGE MEMBERS.

WHEN a bridge is to be built, either for a railway or a city, the work is generally advertised and let to some responsible company, bridge-builder, or contractor, who gives bonds for the satisfactory performance of the work within a certain specified time and for a certain specified price. In such case, it is the duty of the engineer of the city or railway to draw up a "specification," which shall give precisely and in sufficient detail the requirements as to construction and finish of the work. The contractor must execute the work in exact accordance with these specifications, and it is the duty of the engineer and his assistants to see that he does so.

The drawing up of a complete list of specifications, then, is a labor implying thorough knowledge on the part of the engineer who draws them up, not only of all the principles which enter into the construction, of the strength of the materials employed and the best way of utilizing them, of the processes of erection, etc., but also of those practical difficulties and sources of disagreement which often arise between the engineer and the contractor. A complete list of specifications is therefore an epitome of the science of bridge construction.

There are many such specifications in use, and the student can easily obtain them by application to the engineers of our leading railroads and bridge companies. They differ in many minor points of more or less importance. Indeed, in this respect no two are alike, embodying as they do the special experience and personal preferences of the authors. No exercise will be more profitable to the student than the careful and intelligent comparison of such points of difference.

In the preceding chapters we have covered one by one nearly all the points of construction, and an orderly *résumé* of these points would constitute the specifications of this work, and the practice here illustrated and endorsed. This practice varies as intimated, and other specifications would show points of difference as well as of agreement. The designer must be prepared to work to any given specification, and must follow it closely in his work.

We give here, by permission of the author, the *Specifications* of Theodore Cooper, C. E., which are deservedly well known and widely adopted. We shall, in future chapters, design a bridge entirely according to these specifications, referring to the preceding chapters upon construction, already given, for principles and illustration of methods.

The *Specifications* of Mr. Cooper are published by the Engineering News Publishing Company, Tribune Building, New York, and are easily obtainable.* We give them here for convenience of reference. The portion in ordinary type comprises the specifications *verbatim* as given by Mr. Cooper. We have given on each page, in fine print, in connection with each article, such explanatory remarks as seem desirable for the student. For many of these remarks we are indebted to Morgan Walcott, C. E., formerly with the Phœnix Bridge Company.

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* By the same author can be obtained, *General Specifications for Iron and Steel Highway Bridges and Viaducts.*

GENERAL SPECIFICATIONS

FOR

IRON AND STEEL RAILROAD BRIDGES AND VIADUCTS.

NEW AND REVISED EDITION, 1888.

BY THEODORE COOPER, CONSULTING ENGINEER.

ENGINEERING NEWS PUBLISHING COMPANY,

TRIBUNE BUILDING, NEW YORK.

BY PERMISSION OF THE AUTHOR.

GENERAL DESCRIPTION.

1. All parts of the structures shall be of wrought iron or steel, except ties and guard rails. Cast iron or steel may be used in the machinery of movable bridges, and in special cases for bed-plates.

2. The following kinds of girders shall preferably be employed: Bridges.

Spans, up to	16 feet	....	Rolled beams.
" 16 to	70 "	....	Riveted plate girders.
" 70 to	100 "	....	Riveted plate or lattice girders.
" over	100 "	....	Pin-connected trusses.

Generally "double track through" bridges will have but two trusses, to avoid spreading the tracks at bridges.

In calculating strains the length of span shall be understood to be the distance between centres of end-pins for trusses, and between centres of bearing-plates for all beams and girders. Length of Span.

3. The girders shall be spaced, with reference to the axis of the bridge, as required by local circumstances, and directed by the engineer of the railroad company. (§ 5.) Longitudinal floor girders shall in no case be less than three feet and three inches from centre line of tracks. (§ 6.) Spacing of Girders.

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1. The flooring, floor joists, ties, and guard rails are of wood. The machinery of movable bridges, of course, allows of the use of cast iron. But it is not allowed in any part of the structure proper. It is of no value in tension, and is not so good as wrought iron in compression. It is considered as unreliable by reason of brittleness and want of homogeneousness.

For bed-plates, a special case where it might be allowed is when a space occurs between the bottom of the pedestal and the masonry, of, say 3 inches. This must be filled up, and as wrought iron is not rolled so thick, it might be cheaper to use a cast plate rather than build up a wrought gridiron.

Again, if the span is on a grade, and the bed-plate has to be made with a slant or bevel, it is cheaper to cast it, as, if it were of wrought iron, it would have to be faced down.

2. The tracks are generally 13 feet apart, c. to c. on straight lines. If we had a "double track through bridge," with three trusses, one in centre, we should have to allow about 2 feet for width of centre truss, and 7 feet clearance from centre of each track, making 16 feet from c. to c. on the bridge. This would require the tracks to be spread, which the railroad company would wish to avoid.

The length c. to c. of girders is their "effective length," and should be distinguished from actual length, or "length over all."

3. To space the stringers nearer than 6′ 6″ makes the cross-girders heavier. The moment for a cross-girder is its reaction at end multiplied by the distance from end to the stringer. The less this distance the smaller the moment for the cross-girder.

On the other hand, the track is 4′ 8½″, and if the stringers are spaced much farther than this, there is large bending in the ties.

Head-room. 4. For all through bridges and overhead structures, there shall be a clear head-room of 20 feet above the base of the rails.

Clear width. 5. In all through bridges the clear width from the centre of the track to any part of the trusses shall not be less than seven (7) feet at a height exceeding one foot above the rails, where the tracks are straight, and an equivalent clearance shall be provided where the tracks are curved.

6. The standard distance, centre to centre of tracks on straight lines, will be thirteen (13) feet.

Trestle Towers. 7. Each trestle bent shall, as a general rule, be composed of two supporting columns, and the bents united in pairs to form towers; each tower thus formed of four columns shall be thoroughly braced in both directions. Transversely the columns shall have a uniform batter sufficient to nearly or quite prevent tension at the base under the greatest wind force specified, either during erection or after completion.

8. Each tower shall have sufficient base, longitudinally, to be stable when standing alone, without other support than its anchorage. (§§ 25, 26.)

Trestle Spans. 9. Tower spans for high trestles shall not be less than 30 feet; intermediate spans about 60 feet.

Form of Trusses. 10. Unless otherwise specified, the form of bridge trusses may be selected by the bidder; but, to secure uniformity in appearance, it is desired that all "through" trusses shall be built with inclined end-posts; for pin-connected trusses, preference will be given to those of single intersections.

11. All "deck" trusses shall have top chord bearings at abutments, which are retaining walls, unless otherwise ordered for particular structures.

Wooden Floor. 12. The wooden floors shall consist of transverse ties or floor timbers; their scantling will vary in accordance with the design of the supporting iron floor. (§ 15.) They shall be spaced with openings not exceeding six inches, and shall be secured to the supporting girders by $\frac{3}{4}$-inch bolts at distances not over six feet apart. For deck bridges the ties will extend the full width of the bridge, and for through bridges at least every other tie shall extend the full width of bridge for a footwalk.

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4. This clear head-room is only requisite at the centre of the bridge, for a space of about 6 feet for single track. The brackets or knee-braces reduce this clear depth at the sides.

5. The cover plate on the inclined end-post is usually the widest part, so that the distance c. to c. of trusses, on a straight line, is 14 feet in clear, plus the width of a cover plate.

"Equivalent clearance" means that, on a curve, the circle *tangent* to the sides of the cars must have the clearance specified. This requires that the circle through the corners of the cars shall have an equivalent clearance.

6. This is to agree with the railroad company.

7. A trestle bent consists of two columns, one on each side of track, each inclined or battered toward the axis in a vertical plane, and connected by transverse bracing. A tower consists of two trestle bents united by longitudinal bracing. Every other pair is thus united, making every other span an expansion span, with a fixed span between. The usual transverse batter is 6 vertical to 1 horizontal; often, however, 8 vertical to 1 horizontal.

8. That is, the fixed or tower spans must be stable when standing alone with the maximum wind force, and no dependence is placed on the girder connections at the cap.

9. This is to secure stability.

11. The train on the abutment increases the pressure on the retaining wall. But if the top chords bear on the wall, the weight of the truss and of the train assist the wall.

12. It is always necessary, especially in deck spans, to figure the sizes of ties required. If P is the weight of the heaviest single driver and a the distance from rail to end bearing of tie, then Pa is the moment, and (page 246) $Pa = \frac{CI}{v}$, where C is the allowed fibre strain and v is the distance from centre of gravity of cross section to outer fibre. For a rectangular cross section $I = \frac{bd^3}{12}$ and $v = \frac{d}{2}$, where b = breadth and d = depth in inches. Therefore $d = \sqrt{\frac{6Pa}{Cb}}$,

13. There shall be a guard timber (scantling not less than 6 x 8″) on each side of each track, with its inner face parallel to and not less than 3 feet 3 inches from centre of track. Guard timbers must be notched one inch over every floor timber, and be spliced over a floor timber with a half-and-half joint of four inches lap. Each guard timber shall be fastened to every third floor timber, and at each splice, with a three-quarter ($\frac{3}{4}$) inch bolt. **Guard Timbers.**

14. The guard and floor timbers must be continued over all piers and abutments.

15. The maximum strain allowed upon the extreme fibre of the best yellow pine or white oak floor timbers will be 800 pounds per square inch. The weight of a single engine wheel being assumed as distributed over three tie spaces, as per § 12. **Allowed Strain on Timber.**

16. The floor timbers, from centre to each end of span, must be notched down over longitudinal girders so as to reduce the camber in the track, as directed by the engineer.

17. All the floor timbers shall have a full and even bearing upon the stringers; no open joints or shims will be allowed.

18. On curves the outer rail must be elevated, as may be directed by the engineer.

19. In comparing different proposals, the relative cost to the railroad company of the required masonry or changes in existing work will be taken into consideration. **Proposals.**

20. Contractors, in submitting proposals, shall furnish complete strain sheets, general plans of the proposed structures, and such detail drawings as will clearly show the dimensions of all the parts, modes of construction, and the sectional areas.

21. Upon the acceptance of the proposal and the execution of contract, all working drawings required by the engineer must be furnished free of cost.

22. No work shall be commenced or materials ordered until the working drawings are approved by the engineer in writing; if such working drawings are detained more than one week for examination, the contractor will be allowed an equivalent extension of time. **Approval of Plans.**

23. All the structures shall be proportioned to carry the following loads:

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where d can be found for any assumed value of b. As the rails are stiff it is customary to assume the load as carried by three ties.

EXAMPLE.—Tie of white oak, weight of driver $P = 25,600$ lbs., $a = 1' \; 4''$. Take $C = 800$ lbs. per square inch. Then for one tie we have $\frac{1}{3} P = 8,533$ lbs. instead of P. If we take $b = 9$ inches, we have $d = 10$ inches.

15. We have assumed these values in the example to § 12.

16. The camber causes the centre of the truss to be higher than its ends. This notching down reduces the track on bridge to the desired grade.

18. This is the same as is done on all curves.

19. One proposal may require entirely new masonry, or great changes in the existing masonry. Another may utilize the existing masonry without material change, by taking dimensions for the truss to suit. If in such case the truss is more costly, it is but fair to consider the saving in masonry. Again, one plan may call for more costly masonry than another, even when there is none already existing.

20. In many cases a complete strain sheet is considered sufficient without detail drawings.

21. These drawings are required for use by the inspectors.

22. Many roads do not require the working drawings at all.

23. The dead load can be estimated as directed, page 450. A large number of bridges are designed for a loading about like Class A. The specifications of several roads call for much heavier loading than this. Whatever the system adopted, the maximum strains are found as illustrated, page 89, by the use of a diagram prepared for each system. The introduction of this method by diagram, and its invention, are due to Mr. Cooper, and also, independently, Mr. Robert Escobar, C. E., of the Union Bridge Company.

1st. The weight of iron in the structure. 2d. A floor weighing 400 pounds per linear foot of *track*, to consist of rails, ties, and guard timbers only.

Dead Load. These two items, taken together, shall constitute the "dead load."

Live Loads. Class Extra Heavy A. 3d. For Class Extra Heavy A.—A moving load for each *track*, supposed to be moving in either direction, and consisting of two "consolidation" engines coupled, followed by train weighing 3,000 pounds per running foot. This "live load" being concentrated upon points distributed as in Diagram No. 1. Or, 80,000 pounds equally distributed upon two pairs of drivers, seven feet centre to centre; or,

Class A. 3d. For Class A.—A moving load for each *track*, supposed to be moving in either direction, and consisting of two "consolidation" engines coupled, followed by train weighing 3,000 pounds per running foot. This "live load" being concentrated upon points distributed as in Diagram No. 2. Or, 80,000 pounds equally distributed upon two pairs of drivers, seven feet centre to centre; or,

Class, Extra Heavy, A.

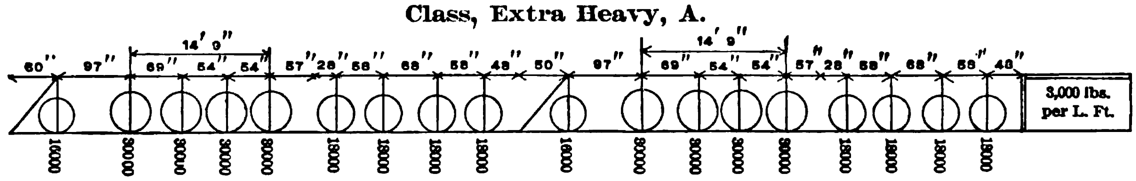

DIAGRAM No. 1.

Class A.

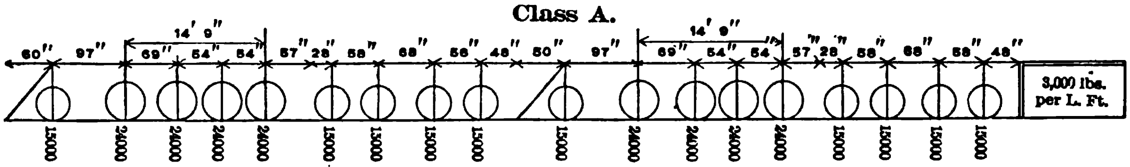

DIAGRAM No. 2.

Class B.

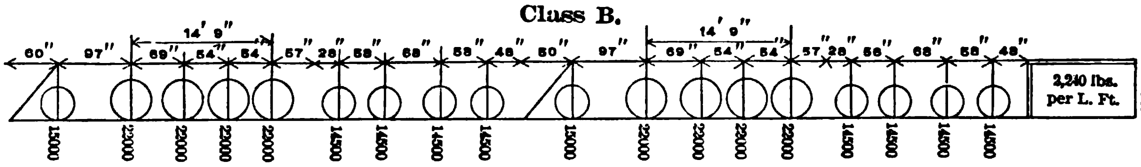

DIAGRAM No. 3.

Class C.

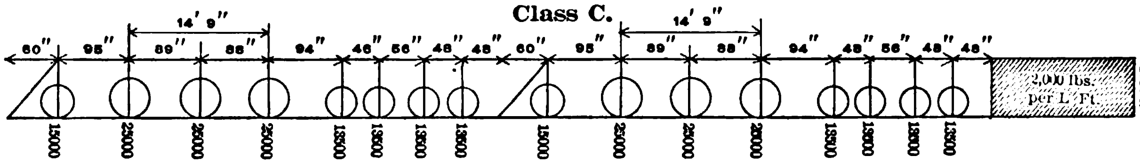

DIAGRAM No. 4.

Class B. 3d. For Class B.—A moving load for each *track*, supposed to be moving in either direction, and consisting of two "consolidation" engines coupled, followed by train weighing 2,240 pounds per running foot. This "live load" being concentrated upon points distributed as in Diagram No. 3. Or, 80,000 pounds, equally distributed upon two pairs of drivers, seven feet six inches centre to centre; or,

Class C. 3d. For Class C.—A moving load for each *track*, supposed to be moving in either direction, and consisting of two "mogul" engines coupled, followed by train weighing 2,000 pounds per running foot. This "live load" being

concentrated upon points distributed as in Diagram No. 4. Or, 80,000 pounds, equally distributed upon two pairs of drivers, eight feet centre to centre.

The maximum strains due to all positions of either of the above "live loads" of the required class, and of the "dead load," shall be taken to proportion all the parts of the structure.

24. To provide for wind strains and vibrations, the top lateral bracing in deck bridges, and the bottom lateral bracing in through bridges, shall be proportioned to resist a lateral force of 450 pounds for each foot of span; 300 pounds of this to be treated as a moving load. **Wind Bracing.**

The bottom lateral bracing in deck bridges, and the top lateral bracing in through bridges, shall be proportioned to resist a lateral force of 150 pounds for each foot of the span.

Preference will be given to lateral bracing in the floor system, which is capable of resisting both compression and tension.

25. In trestle towers the bracing and columns shall be proportioned to resist the following lateral pressures, in addition to the strains from dead and live loads:

1st. The trusses fully loaded, a lateral pressure, at the level of the tracks, of 650 pounds for each longitudinal linear foot of the structure; and a lateral pressure of 125 pounds for each vertical lineal foot of the trestle bents; or,

2d. The trusses unloaded, a lateral pressure, at the level of the tracks, of 600 pounds for each longitudinal lineal foot of the structure; and a lateral pressure of 225 pounds for each vertical lineal foot of the trestle bents.

26. Longitudinally, the bracing of the trestle towers and the attachments of the fixed ends of all trusses shall be capable of resisting the greatest tractive force of the engines, or any force induced by suddenly stopping, upon any part of the work, the assumed maximum trains; the coefficient of friction of the wheels upon the rails being assumed at 0.20. **Longitudinal Bracing.**

27. Variations in temperature, to the extent of 150 degrees, shall be provided for. **Temperature Strains.**

28. When the structures are on curves, the additional effects due to the centrifugal force of trains moving at high velocities shall be considered. **Centrifugal Force.**

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24. The exposed area of the train is about 10 square feet for every foot in length. At 30 lbs. per square foot this gives 300 lbs. for every foot of length, which should be treated as a moving load.

The truss would probably not have more than about 10 square feet of exposed surface for every foot in length; this also, at 30 lbs. per square foot, would give 300 lbs. per foot of length for wind pressure on the whole truss. Taking one-half of this on each chord, upper and lower, we have a fixed load of 150 lbs. per linear foot on each chord. We have thus, as specified, 150 lbs. fixed load, per linear foot, for the unloaded chord, and 450 lbs. per linear foot for the loaded chord, of which 300 is live, and 150 fixed. We have used these values in the example of page 398.

Stiff lateral bracing in the floor system is preferred because it better resists the effects of vibration and shock, and makes the structure more rigid.

25. Generally, the true wind forces are taken at their actual points of application, as we have done in the example, page 406.

26. The strains from traction should always be figured for viaducts. If W is the weight on a bent, and φ the coefficient of friction, the tractive force, F, acting longitudinally at the top of the bent, is $F = \varphi W$.

27. A bar of iron 1 foot long will lengthen about 0.000006 foot for a rise of temperature of 1 degree. For 150 degrees this gives 0.0009 foot per foot, or, for a bar 100 feet long, 0.09 foot, or about 1 inch. Hence the rule, "*one inch per one hundred feet.*" In designing the roller-bed plates, allowance should be made so that the checks for the rollers shall permit of the expansion and contraction of the truss.

28. The centrifugal force for curves has been given, page 397.

29. All parts shall be so designed that the strains coming upon them can be accurately calculated.

PROPORTION OF PARTS.

The following clauses are all intended to apply to wrought-iron construction.

Tensile Strains. 30. All parts of the structure shall be proportioned in tension by the following allowed unit strains:

	Pounds per square inch.
Floor beam hangers, and other similar members liable to sudden loading (bar iron with forged ends)	6,000
Floor beam hangers, and other similar members liable to sudden loading (plates or shapes), net section	5,000
Lateral bracing	15,000
Solid rolled beams, used as cross floor beams and stringers	8,000
Bottom flanges of riveted cross girders, net section	8,000
Bottom flanges of riveted longitudinal plate girders, *over* 20 feet long, net section	8,000
Bottom flanges of riveted longitudinal plate girders, *under* 20 feet long, net section	7,000

	For live loads.	For dead loads.
Bottom chords, main diagonals, counters, and long verticals (forged eye-bars)	8,000	16,000
Bottom chords, main diagonals, counters, and long verticals (plates or shapes), net section	7,500	15,000

For swing bridges and other movable structures, the dead-load unit strains, during motion, must not exceed three-fourths of the above allowed unit strains for dead load on stationary structures.

The areas obtained by dividing the live-load strains by the live-load unit strains will be added to the areas obtained by dividing the dead-load strains by the dead-load unit strains, to determine the required sectional area of any member.

31. Angles subject to direct tension must be connected by both legs, or the section of one leg only will be considered as effective.

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29. If it is impossible to avoid an indeterminate member, the member should be designed for the maximum strains which can occur, whichever way the strains go.

30. The floor beam hangers are liable to sudden loading and impact, and this is allowed for by taking a small unit strain. The lateral bracing is called in play only at long intervals, perhaps never to its full extent, and the strain is applied slowly. The unit strain is therefore taken large. For short stringers the effect of impact is greater than for long ones, and the unit strains are taken accordingly. For the main chords, the dead load forms quite a large percentage of the live load, and hence the unit stress is large for the dead load, and reduced for the live load.

By net section is meant the section after rivet-holes are deducted. The strains are reduced for swing bridges to allow for effects of motion.

31. Thus, if the diagonal ties of a Warren girder are angles, and are riveted to the chords by one leg only, the section of one leg only is to be considered as effective. To make both legs effective, the other leg must be also attached to the chords by means of connecting angles. It is, however, sometimes considered allowable, in the first case, to take the *gross* section of the leg, under the assumption that the metal taken out by rivet-holes is balanced by the metal in the other leg.

[32]. In members subject to tensile strains, full allowance shall be made for reduction of section by rivet-holes, screw threads, etc. **Net Section.**

33. Compression members shall be proportioned by the following allowed unit strains: **Compressive Strains.**

Chord segments,	$P = 8000 - 30\frac{l}{r}$ for live-load strains.
	$P = 16000 - 60\frac{l}{r}$ for dead-load strains.
All posts,	$P = 7000 - 40\frac{l}{r}$ for live-load strains.
	$P = 14000 - 80\frac{l}{r}$ for dead-load strains.
	$P = 10500 - 60\frac{l}{r}$ for wind strains.
Lateral struts,	$P = 9000 - 50\frac{l}{r}$ for assumed initial strain. (§ 34.)

P = the allowed compression for square inch of cross section.
L = the length of compression member, in inches.
r = the least radius of gyration of the section, in inches.

No compression member, however, shall have a length exceeding 45 times its least width.

For swing bridges and other movable structures, the dead-load unit strains during motion must not exceed $\frac{3}{4}$ of the above allowed unit strains for dead load on stationary structures.

34. The lateral struts shall be proportioned by the above formula to resist only the resultant due to an assumed initial strain of 10,000 pounds per square inch, upon all the rods attaching to them, assumed to be produced by adjusting the bridge or towers. (§ 41.) **Struts.**

35. In beams and plate girders the compression flanges shall be made of same *gross* section as the tension flanges. **Compression Flanges.**

36. Riveted longitudinal girders shall have, preferably, a depth not less than $\frac{1}{10}$ of the span. **Depth of girders.**

Rolled beams used as longitudinal girders shall have, preferably, a depth not less than $\frac{1}{12}$ of the span.

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32. The diameter of hole multiplied by thickness of plate, gives area to be taken out. In compression there is evidently no deduction to be made.

33. The chords are considered as having fixed ends, while the posts have pin ends.

These formulæ are the "straight-line formulæ," given page 332. For initial strain see page 341.

34. Other specifications design the struts for the wind strains with a slightly larger unit strain. The difference is practically nothing in the sizes.

35. This gives the compression flange a larger available section. But the compression flange resembles in a measure a post in compression, and some specifications therefore, introduce the ratio $\frac{l}{b}$, of length to breadth of flange. Without such refinement, Mr. Cooper practically reduces the unit stress for the top flange, by making it the same section as the bottom.

36. The greater the depth of girder the less material is required for the flanges, while theoretically the web requires no more material. But, practically, the web gets too thin at great depth, and has to be stiffened, or requires more material. There is then a limit to depth. Mr. Cooper fixes the least depth at $\frac{1}{10}$ of span. For depth of stringers see page 425.

Alternate Strains. 37. Members subject to alternate strains of tension and compression shall be proportioned to resist each kind of strain. Both of the strains shall, however, be considered as increased by an amount equal to $\frac{8}{10}$ of the least of the two strains, for determining the sectional areas by the above allowed unit strains. (§§ 30, 33.)

Effect of Wind on Chord Strains. 38. The strains in the chords and end-posts from the assumed wind forces need not be considered, except as follows:

1st. When the wind strains on any member exceed one-quarter of the maximum strains due to the dead and live loads upon the same member. The section shall then be increased until the total strain per square inch will not exceed by more than one-quarter the maximum fixed for dead and live loads only.

2d. When the wind strain alone, or in combination with a possible temperature strain, can neutralize or reverse the tension in any part of the lower chord.

Rivets, Bolts, and Pins. 39. The rivets and bolts connecting the parts of any member must be so spaced that the shearing strain per square inch shall not exceed 7,500 pounds, or $\frac{3}{4}$ of the allowed strain per square inch upon that member; nor the pressure upon the bearing surface per square inch of the projected semi-intrados (diameter × thickness of piece) of the rivet or bolt hole exceed 12,000 pounds, or one and a half times the allowed strain per square inch upon that member. In the case of field riveting the above limits of shearing strain and pressure shall be reduced one-third part. Rivets must not be used in direct tension.

40. Pins shall be so proportioned that the shearing strain shall not exceed 7,500 pounds per square inch; nor the crushing strain upon the projected area of the semi-intrados of any member (other than forged eye-bars, see article 79) connected to the pin be greater per square inch than 12,000 pounds, or one and a half times the allowed strain per square inch; nor the bending strain exceed 15,000 pounds per square inch when the centres of bearings of the strained members are taken as the points of application of the strains.

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37. If a member has tension of 130,000 lbs., and compression of 90,000 lbs., $\frac{8}{10}$ of the latter is 72,000 lbs. The increased strains are, therefore, tension 202,000 lbs., compression 162,000 lbs. If the allowable unit strain is 10,000 lbs. per square inch for tension, and 7,000 for compression, the member should have 20.2 square inches, net, for the tensile, or 23.14 square inches, gross, for the compressive strain, whichever comes out largest.

38. If the dead load strain on a chord is 60,000 lbs., the live load 140,000 lbs., and the wind strain 80,000 lbs., the total strain from live and dead is 200,000 lbs. One quarter of this is 50,000 lbs., which the wind strain exceeds.

Now if β' is the allowable unit strain for live load L, and β for dead load D, and u is the unit strain for dead and live loads combined, we have $\frac{L}{\beta'} + \frac{D}{\beta} = \frac{L + D}{u}$, or $u = \frac{L + D}{\frac{L}{\beta'} + \frac{D}{\beta}}$. If, in our present case, $\beta = 16,000$ lbs., $\beta' = 8,000$ lbs., then $u = 9,400$ lbs. Increasing this by $\frac{1}{4}$, we have 10,750 lbs. as the allowable unit strain for combined dead, live, and wind strains of 280,000 lbs. This calls for 26 square inches, while, if the wind were disregarded, only 21.2 square inches would be needed.

If the compressive wind strains in the lower chord are greater than the tensile due to dead load, it will be necessary to stiffen the lower chord to take the difference.

39. We must therefore test the rivets for both shear and bearing (page 385). The allowable stress on field rivets is reduced $\frac{1}{3}$ to allow for the imperfections of this kind of work. If rivets were used in direct tension, the heads would tear off. The allowance of 7,500 lbs. assumes a unit strain of 10,000 lbs. If, however, the unit stress is reduced from 10,000 to 8,000 lbs. the shearing strain on rivets should be reduced to $\frac{3}{4}$ 8,000 = 5,000 lbs.

40 Main pins are only figured for bending and bearing (page 377). If large enough for these they are also large enough for shear. Bolts and small pins should be figured for shear also.

41. In case any member is subjected to a bending strain from local loadings, such as distributed floors on deck bridges, in addition to the strain produced by its position as a member of the structure, it must be proportioned to resist the combined strains. Combined Strains.

If the fibre strain, resulting from the weight only, of any member exceeds ten per cent. of the allowed unit strain on such member, such excess must be considered in proportioning the areas.

42. Plate girders shall be proportioned upon the supposition that the bending or chord strains are resisted entirely by the upper and lower flanges, and that the shearing or web strains are resisted entirely by the web-plate; no part of the web-plate shall be estimated as flange area. Plate Girders.

The distance between centres of gravity of the flange areas will be considered as the effective depth of all girders.

43. The iron in the web-plates shall not be subjected to a shearing strain greater than 4,000 pounds per square inch; but no web-plates shall be less than three-eighths of an inch in thickness. Web Plates.

44. The webs of plate girders must be stiffened at intervals, about the depth of the girders, wherever the shearing strain per square inch exceeds the strain allowed by the following formula:

$$\text{allowed shearing strain} = \frac{12000}{1 + \dfrac{H^2}{3000}},$$

where H = ratio of depth of web to its thickness.

45. Rolled beams shall be proportioned (§§ 30, 35) by their moments of inertia. Rolled Beams.

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41. For combined flexure and direct strain see page 321. Let M = the maximum bending moment in the member. Let S = the direct strain, tension, or compression. Let β_1 = the allowable unit strain for direct strain, and β_2 for bending. Let a = the area of the member, and I = the moment of inertia of its cross section $= ar^2$, where r is the radius of gyration. Let $\beta = \beta_1 + \beta_2$. Then, from theory of flexure (page 246),

$$M = \frac{\beta_2 I}{v}, \text{ where } v \text{ is the distance from neutral axis to extreme fibre.}$$

Hence

$$\beta_2 = \frac{Mv}{I} = \frac{Mv}{ar^2}. \quad \text{But } \beta_1 = \frac{S}{a}. \quad \text{Therefore}$$

$$\beta_1 + \beta_2 = \beta = \frac{S}{a} + \frac{Mv}{ar^2}, \text{ or } a = \frac{1}{\beta}\left(S + \frac{Mv}{r^2}\right).$$

If the fibre strain due to weight of member were just 10 per cent. of the allowed unit strain, it would add $\frac{1}{10}$ of a square inch to the cross section. If it exceeds this, the specification requires it should be considered. This is rarely the case. The inclined end-posts are the most apt to exceed the limit. For very large bridges the limit may be exceeded.

42. This is contrary to some specifications, which allow $\frac{1}{8}$ of the web to aid each flange, or $\frac{1}{4}$ of the web in all available for flange section. The web undoubtedly does assist the flanges. The "effective depth" is to be used in figuring all strains.

43. If the shearing strain were greater than 4,000 lbs. per square inch, it would not give sufficient bearing for the rivets in the flanges. These rivets have a pitch of not less than 3 inches. With a thin web this pitch would have to be reduced, which is not allowable.

Web plates are not made less than $\frac{3}{8}$ inch thick, in order to resist the action of rust and to prevent the web from being unsteady, and to enable it to resist impact as well as to reduce the stiffening angles.

44. The practice of stiffening the webs of plate girders differs widely. Some specifications require many stiffeners. Some require them spaced closer at the ends, others at equal distances throughout the length.

45. For rolled beams we have $M = \frac{\beta_2 I}{v}$, where M is the maximum moment, I the moment of inertia of the cross

DETAILS OF CONSTRUCTION.

Details. 46. All the connections and details of the several parts of the structures shall be of such strength that, upon testing, ruptures shall occur in the body of the members rather than in any of their details or connections.

47. Preference will be had for such details as shall be most accessible for inspection, cleaning, and painting; no closed sections will be allowed.

Web Splices. 48. The webs of plate girders must be spliced at all joints by a plate on each side of the web.

Stiffeners. 49. All web plates must have stiffeners over bearing points and at points of local concentrated loadings.

Riveting. 50. The pitch of rivets in all classes of work shall never exceed 6 inches, or sixteen times the thinnest outside plate, nor be less than three diameters of the rivet.

51. The rivets used shall generally be $\frac{3}{4}$ and $\frac{7}{8}$ inch diameter.

52. The distance between the edge of any piece and the centre of a rivet-hole must never be less than $1\frac{1}{4}$ inches, except for bars less than $2\frac{1}{2}$ inches wide; when practicable, it shall be at least two diameters of the rivet.

53. In punching plate or other iron, the diameter of the die shall in no case exceed the diameter of the punch by more than $\frac{1}{16}$ of an inch, and all holes must be clean cuts, without torn or ragged edges.

54. All rivet-holes must be so accurately spaced and punched that when the several parts forming one member are assembled together, a rivet $\frac{1}{16}$ inch less in diameter than the hole can generally be entered, hot, into any hole, without reaming, or straining the iron by "drifts;" occasional variations must be corrected by reaming.

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section, β_2 the allowable fibre strain, v the distance from neutral axis to extreme fibre. If $a =$ the area of the cross section, then $I = ar^2$, where r is the radius of gyration, and $a = \frac{Mv}{\beta_2 r^2}$.

If $v = r$, as is the case for a pin, we have $a = \frac{M}{\beta_2 r}$, where r is the half depth. If d denote the depth, $a = \frac{2M}{\beta_2 d}$, or, total area of both flanges $= a\beta_2 = \frac{2M}{d}$, or, area of one flange $= \frac{1}{2} a\beta_2 = \frac{M}{d} = \frac{\text{Bending Moment}}{\text{Depth}}$.

46. This makes the main members limit the safety of the span.

47. This prevents us from packing two chord bars together in pairs. It may sometimes thus cause trouble in getting the pin-bending moments small enough. It also throws out Phœnix or other closed columns.

48. The sizes of these plates are usually as follows:

For $\frac{5}{16}$ inch web $\frac{1}{4}$ inch splice plate. For $\frac{7}{16}$ inch web $\frac{3}{8}$ inch splice plate.
" $\frac{3}{8}$ " " $\frac{5}{16}$ " " " " $\frac{1}{2}$ " " $\frac{7}{16}$ " " "

Two rows of rivets should be placed on each side of the joint, and no intermediate stiffener angle need necessarily be placed on these splice plates.

49. Some specifications do not require all the stiffeners to bear at tops and bottoms. There should always be, as the clause requires, a stiffener under a concentrated load, and this should at least bear at the top, so that the whole effect of the load may come on the stiffener and not on the web. There should also always be a stiffener at bearing points, and this should at least bear on the bottom.

50. A greater pitch than 6 inches in compression might allow the plate to "buckle." For this reason, if 16 times the thickness of the plate is less than 6 inches, that should be the limit. A less pitch than three diameters renders the holes liable to tear out, as well as injures the metal when punched.

51. For girders and main compression members, $\frac{7}{8}''$ is the size generally used.

52. This for the same reason as § 50.

53. If the clearance between the punch and the die is over $\frac{1}{16}''$, there is a tendency to draw and bunch the iron and make a ragged hole.

54. Reaming is expensive, and forcing holes into opposition by driving through a steel drifting-pin is injurious to the metal. On the shop drawings, the rivet-holes are always ordered to be punched $\frac{1}{16}''$ larger than the rivet.

55. The rivets when driven must completely fill the holes. The rivet-heads must be round, and of a uniform size for the same-sized rivets throughout the work. They must be full and neatly made, and be concentric to the rivet-hole, and thoroughly pinch the connected pieces together.

56. Wherever possible, all rivets must be machine driven. The machines must be capable of retaining the applied pressure after the upsetting is completed. No hand-driven rivets exceeding $\frac{7}{8}$ inch diameter will be allowed.

57. Field riveting must be reduced to a minimum or entirely avoided, where possible.

58. The effective diameter of a driven rivet will be assumed the same as its diameter before driving. In deducting the rivet-holes, to obtain net sections in tension members, the diameter of the rivet-hole will be assumed as $\frac{1}{8}$ inch larger than the undriven rivets.

59. When members are connected by bolts which transmit shearing **Bolts.** strains, the holes must be reamed parallel and the bolts turned to a driving fit.

60. The several pieces forming one built member must fit closely together, and when riveted shall be free from twists, bends, or open joints.

61. All joints in riveted tension members must be fully and symmetrically **Splices.** spliced.

62. In compression members, abutting joints with planed faces must be **Abutting Joints.** sufficiently spliced to maintain the parts accurately in contact against all tendencies to displacement.

63. In compression members, abutting joints with untooled faces must be fully spliced, as no reliance will be placed on such abutting joints. The abutting ends must, however, be dressed straight and true, so there will be no open joints.

64. All the angles, filling, and splice plates on the webs of girders and riveted members must fit at their ends to the flange angles, sufficiently close to be sealed when painted against admission of water; but need not be tool finished.

65. Web plates of all girders must be arranged so as not to project **Web Plates.** beyond the faces of the flange angles, nor on the top be more than $\frac{1}{16}$ inch below the face of these angles, at any point.

66. Wherever there is a tendency for water to collect, the spaces must be filled with a suitable waterproof material.

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55. Rivets which do not fill the hole when driven are called "loose rivets." The inspector should require them to be replaced. The rivets, by pinching the plates, develop friction which increases their value.

56. The rivet spacing should be so designed that all may be machine driven. It is sometimes impossible to avoid driving some by hand, owing to the locality. For instance, when two rivets are placed opposite in the two legs of a small angle. Field rivets are all driven by hand.

If the machine is not capable of retaining the applied pressure after the upsetting is completed, the plates will not be thoroughly pinched together.

In the case of a large rivet exceeding $\frac{7}{8}''$ it would be impossible to properly upset it by hand driving.

58. For a $\frac{7}{8}''$ rivet the hole would be punched $\frac{15}{16}''$. If this hole is reamed it may easily reach 1'', and the net section is assumed on this basis.

62. The faced joints are relied upon to transmit the strain, but there should be enough splice plates to prevent displacement from jars, etc. Tension joints must, of course, be fully spliced to take the entire strain.

63. This is a "full splice." Open joints admit rain, and are hard to paint or protect from rust.

65. This clearance allows cover plates to fit closely against the backs of the flange angles. If there is no cover plate, a clearance of more than $\frac{1}{16}''$ would collect and hold water, and would be difficult to protect.

Flange Plates. 67. In girders with flange plates, at least one-half of the flange section shall be angles or else the largest sized angles must be used.

68. In lattice girders, the web members must be double, and connect symmetrically to the web of the flanges.

69. The compression flanges of beams and girders shall be stayed against transverse crippling when their length is more than thirty times their width.

70. The unsupported width of plates subjected to compression shall not exceed thirty times its thickness; except cover plates of top chords and end-posts, which will be limited to forty times their thickness.

71. The flange plates of all girders must be limited in width so as not to extend beyond the outer lines of rivets connecting them with the angles, more than five inches, or more than eight times the thickness of the first plate. Where two or more plates are used on the flanges, they shall either be of equal thickness or shall decrease in thickness outward from the angles.

72. No iron shall be used less than $\frac{1}{4}$ inch thick, except for lining or filling vacant spaces.

Eye-Bars. 73. The heads of eye-bars shall be so proportioned and made, that the bars will preferably break in the body of the original bar rather than at any part of the head or neck. The form of the head and the mode of manufacture shall be subject to the approval of the engineer of the railroad company. The heads must be formed either by the process of upsetting and forging, or by the process of upsetting, piling, and forging. No welding will be allowed in the body of the bars, nor, in the process of piling, welding seams in any other direction than parallel to the sides of the original bars.

74. The bars must be free from flaws and of full thickness in the necks. They shall be perfectly straight before boring. The holes shall be in the centre of the head, and on the centre line of the bar.

75. The bars must be bored to lengths not varying from the calculated lengths more than $\frac{1}{64}$ of an inch for each 25 feet of total length.

76. Bars which are to be placed side by side in the structure shall be

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67. This corresponds pretty well with the rule that the flange angle shall be $\frac{1}{16}''$ thicker than the cover plate. The object of the clause is to get most of the metal as near as possible to the centre of gravity of the flange.

68. Single web members would have the rivets in single shear, and we could not use enough to develop full strength of the members.

69. If a girder has a top flange 12" wide, it would be necessary to brace it against transverse crippling if its length was over 30 feet. In a deck-plate girder this would be done by transverse bracing to the other girder. In a through plate girder knee-braces can be used at every ten or fourteen feet, depending upon the distance c. to c. of girders.

70. A cover-plate for a top chord $\frac{1}{2}''$ thick, should not have an unsupported width exceeding 20 inches. The unsupported width would be the distance between the lines of rivets in the flanges. This clause is to prevent the cover-plate from buckling transversely.

71. If the cover-plates extended over 5 inches beyond the outer line of rivets, there would be a tendency to buckle along their outer edges.

72. Iron, less than $\frac{1}{4}''$ thick, might, after a little exposure, become unfit to perform its duty.

73. Forging is the process of making out of a single piece of iron the shape required, by pressure at a high heat. Small forgings are made in the blacksmith shop; larger ones by large steam hammers and hydraulic pressure.

Welding is the process of uniting two pieces of iron by hammering or by compression while softened by heat. The degree of heat required for iron is just above whiteness, and is known as welding heat.

Piling is the process of piling up separate pieces of iron on the original bar, and then welding them to the bar, so as to get enough metal to make the head.

75. It is essential to have the bars of exactly the correct length, otherwise the camber would not be provided for, and the appearance of the structure would be injured.

76. When bars are side by side, it is still more necessary that their lengths should be the same, otherwise they are strained unequally.

bored at the same temperature and of such equal length that upon being piled on each other the pins shall pass through the holes at both ends without driving.

77. The lower chord shall be packed as narrow as possible.

78. The pins shall be turned straight and smooth, and shall fit the pin-holes within $\frac{1}{50}$ of an inch, for pins less than $4\frac{1}{2}$ inches diameter; for pins of a larger diameter the clearance may be $\frac{1}{32}$ inch. **Pins.**

79. The diameter of the pin shall not be less than two-thirds the largest dimension of any tension member attached to it. The several members attaching to the pin shall be so packed as to produce the least bending moment upon the pin, and all vacant spaces must be filled with wrought-iron filling rings.

80. All rods and hangers with screw ends shall be upset at the ends, so that the diameter at the bottom of the threads shall be $\frac{1}{16}$ inch larger than any part of the body of the bar. **Upset Ends.**

81. All threads must be of the United States standard, except at the ends of the pins.

82. Floor beam hangers shall be so placed that they can be readily examined at all times. When fitted with screw ends they shall be provided with check nuts. Preference will be given to hangers without screw ends. **Hangers.**

83. When bent loops are used they must fit perfectly around the pin throughout its semi-circumference.

84. All nuts on floor beam hangers and counter rods must have the bearing faces faced square to the axes of the screw ends.

85. Compression members shall be of wrought iron and of approved forms. **Compression Members.**

86. The pitch of rivets at the ends of compression members shall not exceed four diameters of the rivets for a length equal to twice the width of the member.

87. The open sides of all compression members shall be stayed by batten

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77. This is to reduce the bending on the pins and reduce their size.

78. It is customary to turn off from the rough pins, $\frac{1}{16}$ to $\frac{3}{16}$ of an inch, according to the size of the pin, in order to get a smooth and straight pin surface. If the pin-hole and pin were of exactly the same size, the erectors would be unable to drive the pin without injury to it.

79. Eye-bars cannot have pin plates riveted to them in order to get sufficient pin bearing. It is therefore necessary to have enough pin bearing without any pin plates. Too small a pin would not give sufficient bearing. The ratio we have deduced, page 369, is $\frac{2}{3}$. For a less diameter the head must be thicker than the bar, in order to get sufficient bearing. Vacant spaces on pins must always be filled with filling rings, to prevent displacement of the members on the pin. As the faces of castings are rough, large clearances have to be allowed for cast rings. Wrought filling rings are smooth, and require smaller clearance (see page 373). Therefore wrought iron filling rings are specified.

80. This is to make the screw ends at least as strong as the body of the bar. The process of "upsetting" consists in making the member larger at a particular point than it is elsewhere. This is done by forging.

81. This standard will be found on page 120 of *Carnegie*, edition of 1889.

82. Owing to their severe duty, importance, and position, the hangers should be specially examined, and such examination should be aided by making every part accessible. It is the duty of the inspector to see that all nuts are tight and not wearing loose. To prevent this, check nuts are used. Check nuts are regular nuts screwed on over the first or main nut. This second nut is found to prevent the first from working loose.

83. In order to secure a fit, after the loops are bent around the pin in the blacksmith's shop, they are taken to the machine which bores the pin-holes, and the semi-circumference of the loop bearing on the pin is bored true.

84. This is to insure getting all the bearing surface provided.

86. It is usually the custom to space the rivets 3" apart for two or three feet at the ends of compression members, and in the centre 6" apart, unless this distance is not greater than 16 times the thickness of the thinnest plate, in which case the rivets in centre would be spaced, say, $4\frac{1}{2}$" apart.

87. The battens or stay plates cannot be put, in general, directly at the ends, because of the inclined ties and

plates at the ends and diagonal lattice-work at intermediate points. The batten plates must be placed as near the ends as practicable, and shall have a length of $1\frac{1}{2}$ times the width of the member. The size and spacing of the lattice bars shall be duly proportioned to the size of the member. They must not be less than $2 \times \frac{1}{4}$ inches for posts 6 inches wide, nor $4 \times \frac{3}{8}$ inches for posts 15 inches wide. They shall be inclined at an angle not less than 60° to the axis of the member. The pitch of the latticing must not exceed the width of the channel plus nine inches.

88. Where necessary, pin-holes shall be re-enforced by plates, so that the allowed pressure on the pins shall not be exceeded. These re-enforcing plates must contain enough rivets to transfer their proportion of the bearing pressure, and at least one plate on each side shall extend not less than six inches beyond the edge of the batten plates. (§ 87.)

89. Where the ends of compression members are forked to connect to the pins, the aggregate compressive strength of these forked ends must equal the compressive strength of the body of the members; in order to insure this result the aggregate sectional area of the forked ends, at any point between the inside edge of the pin-hole and six inches beyond the edge of the batten plate, shall be about double that of the body of the member.

90. In compression chord sections the material must mostly be contentrated at the sides, in the angles and vertical webs. Not more than one plate, and this not exceeding $\frac{1}{2}$ inch in thickness, shall be used as a cover plate, except when necessary to resist bending strains. (§ 41.)

Top Chord Splices.

91. The sections of compression chords shall be connected at the abutting ends by splices sufficient to hold them truly in position.

92. The ends of all square-ended members shall be planed smooth, and exactly square to the centre line of strain.

93. All members must be free from twists or bends. Portions exposed to view shall be neatly finished.

94. Pin-holes shall be bored exactly perpendicular to a vertical plane passing through the centre line of each member, when placed in a position similar to that it is to occupy in the finished structure.

Abutting Joints.

95. Abutting joints in truss bridges shall be in exact contact throughout.

Lateral Bracing.

96. In no case shall any lateral or diagonal rod have a less area than $\frac{3}{4}$ of a square inch.

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counters which would interfere. Practice varies somewhat as to size, and stay plates about as long as wide are common. For main members the angle of lattice bars should not be less than 60°; but for lateral struts and small members it is allowable to take the angle somewhat less.

88. The last sentence is to prevent a single channel from being overstrained at the end before the channel is united to the other channel by the batten, after which the strain runs through the section as a whole. The radius of gyration for a single channel is less than for the whole section, and the allowed unit strain in the jaw would therefore be less than for the whole member. Hence there should be an excess of section until the strain has reached the main member proper.

90. The cover plate of a top chord unites the two channels forming its webs, but it would seem doubtful whether it takes its share of the strain, as it does not directly touch the pin, while the webs do. Therefore Mr. Cooper keeps the proportion of cover plate to total section as small as is consistent with a firm union of the two channels.

91. This is a special case of § 62.

92. This is to get the full value out of the abutting top chord joints.

94. This is to insure that the pin-hole in one channel of a built member shall come directly opposite that in the other.

96. This is to provide against rust and weathering. Most specifications make the limit one square inch.

97. The attachment of the lateral system to the chords shall be thoroughly efficient. If connected to suspended floor beams, the latter shall be stayed against all motion.

98. Preference will be given for a stiff angle iron lateral system between the chords on the level of the floor.

99. All through bridges with top lateral bracing shall have wrought-iron latticed portals of approved design at each end of the span, connected rigidly to the end-posts. They shall be as deep as the specified head-room will allow. (§ 38.) **Transverse Diagonal Bracing.**

100. When the height of the trusses exceeds 25 feet, an approved system of overhead diagonal bracings shall be attached to each post and to the top lateral struts.

101. Pony trusses and through plate, or lattice, girders, shall be stayed by knee braces or gusset plates attached to the top chords at the ends, and at intermediate points, not more than ten feet, or a panel length, apart, and attached below to the cross floor beams or to the transverse struts.

102. All deck girders shall have transverse braces at the end. All deck bridges shall have transverse bracing at each panel point. This bracing shall be proportioned to resist the unequal loading of the trusses. The transverse bracing at the ends shall be of the same equivalent strength as the end top lateral bracing.

103. All bed-plates must be of such dimensions that the greatest pressure upon the masonry shall not exceed 250 lbs. per square inch. **Bed-Plates.**

104. All bridges over 75 feet span shall have at one end nests of turned friction rollers, formed of wrought iron or steel, running between planed surfaces. The rollers shall not be less than two inches diameter, and shall be so proportioned that the pressure per lineal inch of rollers shall not exceed the product of the square root of the diameter of the roller in inches multiplied by 500 pounds ($500\sqrt{d}$). **Friction Rollers.**

105. Bridges less than 75 feet span shall be secured at one end to the masonry, and the other end shall be free to move upon planed surfaces.

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97. This latter may be done by running a rod parallel to the truss, and checking into the floor beam web by a nut. The better plan, however, is to arrange the laterals so that they will cause no motion in the floor beam in the first place.

98. Such a system will better take the unknown strains caused by vibration, as well as the known ones due to wind.

99. There are a great variety of designs for these portals, but those in which the iron is curved are no longer "approved design." The weight of the portal bracing and its arrangement depend more on the width of the bridge than anything else.

100. If there is sufficient head-room, deep bracing, with either a stiff lattice system or rods, is generally used, preferably the former, as in this case knee-braces can also be used, running to a panel point of the lattice. If the head-room does not allow a deep strut, a shallow one with knee-braces is used.

102. In deck-plate girder spans which are long enough, it is good practice to put in, besides the end transverse bracing, intermediate transverse cross-braces about every 16 feet apart. The transverse bracing must be figured to transmit the panel wind load to the lower chord, and, if on curves, for unequal loading and the centrifugal force. The transverse bracing between the inclined end-posts of deck spans should carry the entire wind load which may come to the abutment through the top chord.

103. The quality of masonry is apt to vary considerably. If it is known that the masonry is particularly good, 300 lbs. per square inch will not be too great a pressure.

104. The rollers are kept in position by straps uniting their centres. They roll thus together *en masse* on the planed surface of the bed-plate. Angle iron checks riveted to the bed-plate keep them from rolling too far. These checks should allow for change of length of the truss due to temperature.

105. Such short spans will slide on the bed-plate, and rollers are unnecessary. The holes for the foundation bolts are oblong at the expansion ends, thus leaving room for play.

106. Where two spans rest upon the same masonry, a continuous wrought-iron plate, not less than $\frac{3}{8}$ inch thick, shall extend under the two adjacent bearings.

107. All the bed-plates and bearings under fixed and movable ends must be fox-bolted to the masonry; for trusses, these bolts must not be less than $1\frac{1}{4}$ inches diameter; for plate and other girders, not less than $\frac{7}{8}$ inch diameter. The contractor must furnish all bolts, drill all holes, and set bolts to place with sulphur.

108. While the roller ends of all trusses must be free to move longitudinally under changes of temperature, they shall be anchored against lifting or moving sideways.

Camber. 109. All bridges with parallel chords shall be given a camber by making the panel lengths of the top chord longer than those of the bottom chord, in the proportion of $\frac{1}{8}$ of an inch to every ten feet.

Bolts. 110. All bolts must be of neat lengths, and shall have a washer under the heads and nuts where in contact with wood.

111. The lower struts in trestle towers shall be securely anchored to intermediate masonry piers when the magnitude of the structure, in the opinion of the engineer, requires it; these struts shall always have ample stiffness to move the tower columns under the effects of changes of temperature, and prevent the slacking of the diagonal brace rods.

Bed-Plates. 112. Tower footings and bed-plates must be planed on all sliding surfaces; and the holes for anchor bolts slotted to allow for the proper amount of movement. (§ 27.)

113. All joints in the tower columns shall be fully spliced for all possible tension strains, and to hold the parts firmly in position.

114. The connection of all the diagonal tension members with the columns shall, preferably, be made by means of pins passing through the column's axis.

115. The tension diagonals shall be adjustable, but must have check nuts at all adjustable points; and shall be supported and clamped at suitable intervals to prevent sagging and rattling.

Workmanship. 116. All workmanship shall be first-class in every particular.

117. Whenever necessary for the protection of the thread, provision shall be made for the use of pilot nuts in erection.

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106. This is to keep the ends of the spans at the same level.

107. Foundation bolts are "swedged" or "fox-bolted," as shown page 394. Melted sulphur run into the holes when the bolts are in place, cools, expands, and hardens, and grips the bolt firmly.

108. For this it is necessary to run the foundation bolts through the pedestal plate in oblong holes, as well as through the bed-plate.

109. We have treated camber fully, page 411.

110. The object of the washer is to secure a greater bearing surface.

111. If these longitudinal struts were not able to move the tower columns, the longitudinal diagonal rods would slacken, owing to the movement of the girders resting on the top of the columns. For, as these expand or contract, the columns are pulled out of the vertical.

113. Sometimes the wind may cause tension in the windward columns.

114. This is to cause the lines of strain from the diagonals to intersect on the centre line of the column. If possible, the line of the diagonals should intersect at the centre of gravity of the column section.

117. A pilot nut is a rounded cap screwed on the thread of the pin, so that when the pin is driven into place, the thread is protected. The pilot is then removed, and the pin nut screwed on. The pilot nut has a hole in the end, so that by putting a rod through, it may be screwed and unscrewed.

Use of Steel.

118. Medium steel (§ 139) may be used for tension members, plate girders, rolled beams, and top chord sections, with an allowance of 20 per cent. increase above allowed working strains on wrought iron; and for all posts by use of the following formulæ, in place of those given for wrought iron (§ 33): Medium Steel.

$$P = 8{,}500 - 55\frac{l}{r} \text{ for live-load strains.}$$

$$P = 17{,}000 - 110\frac{l}{r} \text{ for dead-load strains.}$$

$$P = 13{,}000 - 85\frac{l}{r} \text{ for wind strains.}$$

Provided that, in addition to the previous details of construction,

119. All sheared edges of plates and angles be planed off to a depth of $\frac{1}{4}$ inch. All punched holes be reamed to a diameter of $\frac{1}{8}$ inch larger, so as to remove all the sheared surface of the metal.

120. No sharp or unfilleted re-entrant corners be allowed.

121. All rivets to be of steel.

122. Any piece which has been partially heated or bent cold, be afterwards wholly annealed.

123. Soft steel (§ 141) may be used under the same conditions as wrought iron for all *riveted* work. Soft Steel.

Provided, that

124. Any rivet-hole punched, as in ordinary practice (§§ 52 and 53), will stand drifting to a diameter 25 per cent. greater than the original hole without cracking, either in the periphery of the hole or on the external edges of the piece, whether they be sheared or rolled.

QUALITY OF MATERIAL.

Iron.

125. All wrought iron must be tough, fibrous, and uniform in character. It shall have a limit of elasticity of not less than 26,000 pounds per square inch. Iron.

Finished bars must be thoroughly welded during the rolling, and be free from injurious seams, blisters, buckles, cinder spots, or imperfect edges; all iron for eye-bars or other forgings, and for bent plates, must be capable of being worked at a proper heat without injury.

126. For all tension members high test bars must be used, capable of standing the following tests: Tension Tests.

127. Full-sized pieces of flat, round, or square iron, not over $4\frac{1}{2}$ inches in

125. The limit of elasticity is that strain in pounds per square inch, below which the iron returns after strain, practically to its original length, without observable set.

The puddle bars are cut in small pieces, piled together, these piles reheated, and then put through the rolls. They should be put through repeatedly, until thoroughly welded.

127. A small flaw or blister in a large bar has not the same effect as in a small one. Also, rusting and weathering have less proportional effect.

sectional area, shall have an ultimate strength of 50,000 pounds per square inch, and stretch 12½ per cent. in the whole length of the body of the bars.

Bars of a larger sectional area than 4½ square inches will be allowed a reduction of 1,000 pounds per square inch for each additional square inch of section, down to a minimum of 46,000 pounds per square inch, and 10 per cent. stretch in the whole length of the body of the bars.

128. When tested in specimens of uniform sectional area of at least ½ square inch for a distance of 10 inches, taken from tension members which have been rolled to a section not more than 4½ square inches, the iron shall show an ultimate strength of 52,000 pounds per square inch, and stretch 18 per cent. in a distance of 8 inches.

Specimens taken from bars of a larger cross section than 4½ inches will be allowed a reduction of 500 pounds for each additional square inch of section, down to a minimum of 48,000 pounds, and 15 per cent. stretch.

129. The same-sized specimens taken from *angle* and other *shaped* iron shall have an ultimate strength of 48,000 pounds per square inch, and elongate 15 per cent. in 8 inches.

130. The same-sized specimens taken from *plates* less than 24 inches in width, shall have an ultimate strength of 48,000 pounds, and elongate 15 per cent. in 8 inches.

131. The same-sized specimens taken from *plates* exceeding 24 inches in width, shall have an ultimate strength of 46,000 pounds, and elongate 10 per cent.

Bending Tests.

132. All iron for tension members must bend cold for about 90 degrees, to a curve whose diameter is not over twice the thickness of the piece, without cracking. At least one sample in three must bend 180 degrees to this curve without cracking. When nicked on one side, and bent by a blow from a sledge, the fracture must be nearly all fibrous, showing but few crystalline specks.

133. Specimens from *angle*, *plate* less than 24 inches in width (130), and *shaped* iron must stand bending cold through 90 degrees, and to a curve whose diameter is not over three times its thickness, without cracking.

Specimens from *plates* wider than 24 inches must stand bending cold through 90 degrees, and to a curve whose diameter is not over six times its thickness, without cracking.

When nicked and bent the fracture must be mostly fibrous.

134. If any of the above material, under the tests, shows a decrease of stretch, accompanied by an increase of tensile strength, amounting to 500 pounds per square inch for each one per cent. change of stretch, this decrease of stretch will not be cause for rejection, if the material still satisfies the bending tests.

135. If the tests of the tension bars (127 and 128) show a decrease of the required tensile strength, accompanied with an increase of stretch, the decrease in the required tensile strength will not be cause for rejection, if the tensile strength does not fall below the stated minimum requirements, and the

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128. Size and form of test specimens have considerable influence on the results of the test, and this is allowed for in the specifications.

132. These bending tests insure the ductility and malleability of the iron.

135. That is, down to a certain point loss of tensile strength may be compensated for by increased ductility. Great strength may exist with brittleness.

increase of stretch amounts to an additional per cent. for each 500 pounds decrease in the tensile strength.

136. Rivets shall be made from the best refined iron, and must be capable of being bent cold until the sides are in close contact, without sign of fracture on the convex side. **Rivet Iron.**

Steel.

137. The steel must be uniform in character for each specified kind. The finished bars, plates, and shapes must be free from cracks on the faces or corners, and have a clean, smooth finish. No work shall be put upon any steel at or near the blue temperature or between that of boiling water and of ignition of hard-wood sawdust.

138. All tests shall be made by samples cut from the finished material after rolling. The samples to be at least 12 inches long, and to have a uniform sectional area not less than ½ square inch. All broken samples must show uniform fine-grained fractures of a blue steel-gray color, entirely free from fiery lustre or a blackish cast.

139. MEDIUM STEEL shall have an ultimate strength, when tested in samples of the dimensions above stated, of 62,000 to 68,000 pounds per square inch, an elastic limit of not less than 33,000 pounds per square inch, and a minimum elongation of 20 per cent. in 8 inches. **Medium Steel**

140. Before or after heating to a low cherry red and cooling in water at 82 degrees Fahrenheit, this steel must stand bending to a curve whose inner radius is one and a-half times the thickness of the sample, without cracking.

141. SOFT STEEL shall have an ultimate strength, on same-sized samples, of 54,000 to 62,000 pounds per square inch, an elastic limit not less than 30,000 pounds per square inch, and a minimum elongation of 25 per cent. in 8 inches. **Soft Steel.**

142. Before or after heating to a light yellow heat and quenching in cold water, this steel must stand bending 180 degrees, to a curve whose inner radius is equal to the thickness of the sample, without sign of fracture.

143. All rivets will be made of soft steel, and the steel for rivets must, under the above bending test, stand closing solidly together without sign of fracture.

Cast Iron.

144. Except where chilled iron is required, all castings must be of tough gray iron, free from cold shuts or injurious blow holes, true to form and thickness, and of a workmanlike finish. Sample pieces, 1 inch square, cast from the same heat of metal in sand moulds, shall be capable of sustaining, on a clear span of 4 feet 6 inches, a central load of 500 pounds when tested in the rough bar. A blow from a hammer shall produce an indentation on a rectangular edge of the casting without flaking the metal. **Cast Iron.**

Timber.

145. The timber shall be strictly first-class white pine, southern yellow pine, or white oak bridge timber; sawed true, and out of wind, full size, free from wind shakes, large or loose knots, decayed or sap wood, worm holes, or other defects impairing its strength or durability. It will be subject to the inspection and acceptance of the engineer. **Timber.**

INSPECTION.

Inspection.

146. All facilities for inspection of the materials and workmanship shall be furnished by the contractor. He shall furnish without charge such specimens (prepared) of the several kinds of iron or steel to be used, as may be required to determine their character.

147. The contractor must furnish the use of a testing machine capable of testing the above specimens at all mills where the iron or steel may be manufactured, free of cost.

148. Full-sized parts of the structure may be tested at the option of the engineer of the railroad company, but if tested to destruction, such material shall be paid for at cost, less its scrap value to the contractor, if it proves satisfactory. If it does not stand the specified tests, it will be considered rejected material, and be solely at the cost of the contractor.

PAINTING.

Painting.

149. All iron-work before leaving the shop shall be thoroughly cleansed from all loose scale and rust, and be given one good coating of pure raw linseed oil, well worked into all joints and open spaces.

150. In riveted work the surfaces coming in contact shall each be painted before being riveted together. Bottoms of bed-plates, bearing-plates, and any parts which are not accessible for painting after erection, shall have two coats of paint; the paint shall be a good quality of iron ore paint, subject to approval of the engineer.

151. After the structure is erected, the iron-work shall be thoroughly and evenly painted with two additional coats of paint, mixed with pure linseed oil, of such color as may be directed. All recesses which will retain water, or through which water can enter, must be filled with thick paint or some waterproof cement before receiving the final painting.

152. Pins, bored pin-holes, and turned friction rollers shall be coated with white lead and tallow before being shipped from the shop.

ERECTION.

Erection.

153. The contractor shall furnish all staging and false work, shall erect and adjust all the iron-work, and put in place all floor timbers, guards, etc., complete, ready for the rails.

154. The contractor shall so conduct all his operations as not to impede the operations of the road, interfere with the work of other contractors, or close any thoroughfare by land or water.

155. The contractor shall assume all risks of accidents to men or material prior to the acceptance of the finished structure by the railroad company.

The contractor must also remove all false work, piling, and other obstructions, or unsightly material produced by his operations.

FINAL TEST.

156. Before the final acceptance the engineer may make a thorough test by passing over each structure the specified loads, or their equivalent, at a speed not exceeding 30 miles an hour, and bringing them to a post at any

point by means of the air or other brakes, or by resting the maximum load upon the structure for 12 hours.

After such tests the structures must return to their original positions without showing any permanent change in any of their parts.

SUPPLEMENTARY.

The following special clauses shall apply, in addition to previous general clauses, to the special work included in the attached contract:

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Proposals for building and erecting complete, ready for the ______________

a bridge over ______________ near ______________

______________, on the ______________ Division

______________ Railroad, in accordance with the

attached specifications and accompanying profile, will be received up to ______________

______. The live load to be adopted for this bridge will be Class ______________

paragraph 23.

LIST OF THE DIFFERENT MEMBERS IN A BRIDGE.—From a paper read by Prof. J. A. S. Waddell, before the "*Pi Eta*" Scientific Society, Rensselaer Polytechnic Inst., Troy, N. Y., we extract the following complete lists of the different members which go to make up a bridge.

I. HIGHWAY BRIDGE.—COMBINATION OF WOOD AND IRON.

WOOD.

Top Chords.
Batter Braces.
Vertical Posts.
End Tie Beams.
End Diagonals.
Floor Beams.
Flooring.
Batter-Brace Stiffeners.
Lateral Braces.
Joints.
Hand-rail Cap.
Hub Plank.
Hand-rail Post.
Felloe Plank.
Corbels.
False Caps.
Wall Plates.
Cover Boards for Chords and Batter Braces.
Lath for same.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.

WROUGHT IRON.

Main Portions.

Main Diagonals.
Counters.
Hip Verticals.
Upper Lateral Rods.
Lower Lateral Rods.
Bottom-Chord Bars.
End Lateral Struts.
Batter-Brace Ties.
Star Iron Side Braces.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.
*Floor Beams.
Beam Truss Rods.

DETAILS.

BOLTS.
- Chord Bolts.
- Batter-Brace Bolts.
- Post Bolts.
- Bracket Bolts.
- Hand-rail Post Bolts.
- Name-Plate Bolts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plate.
- Lower Lateral-Rod Bolts.
- Drift Bolts.
- Floor-Beam Packing Bolts.

Beam Hangers.
Beam-Hanger Plates.
Hip Vertical Plates on Castings.
Lacing on Hip Verticals.
Side-Brace Connections to Chord Pins.
Side-Brace Connections to Floor Beams.
Lateral-Rod Connections to Floor Beams.
Rollers and Roller Frames.
Jaws on End Struts.
Dowels for Upper Laterals.
Fillers for Pins.

SPECIAL WROUGHT-IRON DETAILS.

Hip-Joint Boxes.
Upper-Chord Panel Connections.
Lower Post Sockets.
Pedestals.
Bed Plates.

CORRUGATED OR GALVANIZED IRON.

Cover for Top Chords and Batter Braces.

CAST IRON.

Bed Plates.
Hip-Joint Boxes or Hoods.
Pedestals.
Upper Post Sockets.
Upper-Chord Panel Connection.
Lower Post Sockets.
Lateral Angle Blocks.
Name Plates.
Brackets.
Washer Plates for Main Diagonals and Counters.

WASHERS.
- Chord-Bolt Washers.
- Batter-Brace Bolt Washers.
- Post-Bolt Washers.
- Upper Lateral-Rod Washers.
- Lower Lateral-Rod Washers.
- Beam-Hanger Washers.
- Name-Plate Bolt Washers.
- Bracket-Bolt Washers.
- Hand-rail Post Bolt Washers.
- Bed-Plate Bolt Washers.
- Bevel Washers.
- Floor-Beam Bolt Washers.

PACKING WASHERS.
- Chord-Bolt Packing Washers.
- Lateral-Rod Packing Washers.
- Batter-Brace Bolt Packing Washers.
- Tie-Bar Packing Washers in Batter Braces.
- Post-Bolt Packing Washers.
- Bracket-Bolt Packing Washers.
- Floor-Beam Bolt Packing Washers.

* For details of built floor beams, see list of members in Iron Highway Bridge.

II. HIGHWAY BRIDGE.

WROUGHT IRON.

Main Portions.

- CHANNEL BARS.
  - Top Chords.
  - Batter Braces.
  - Posts.
  - Lateral Struts.
  - Portal Braces.
- PLATE.
  - Top Chords.
  - Batter Braces.
- BARS.
  - Main Diagonals.
  - Counters.
  - Hip Verticals.
  - Upper Lateral Rods.
  - Lower Lateral Rods.
  - Cross Diagonals on Batter Braces.
  - Cross Diagonals on Posts.
  - Lower Chord Bars.

T IRON. Lower Lateral Struts.

- I BEAMS.
  - Floor Beams.
  - Intermediate Struts.
  - Upper Lateral Struts.
  - Lower Lateral Struts.
  - Top Chords.
  - Batter Braces.
- STAR IRON.
  - Side Bracing.
  - Hip Verticals.

IRON HAND-RAILING.
FLOOR BEAMS.
BEAM TRUSS RODS.

DETAILS.

- STAY PLATES.
  - Top Chords.
  - Ends of Posts.
  - Middle of Posts.
  - Ends of Lateral Struts.
  - Batter Braces.
  - Portal Braces.
- FILLING PLATES.
  - At Panel Points of Top Chord.
  - Floor Beams.
- COVER PLATES.
  - Shoe.
  - Hip Joint.
  - Intermediate Panel Points Top Chord.
- REINFORCING PLATES.
  - Hip Inside.
  - Hip Outside.
  - Top Chord, Intermediate Panel Points Inside.
  - Top Chord, Intermediate Panel Points Outside.
  - Bottom Chord, Intermediate Panel Points Inside and Outside for Channel Bottom Chords.
  - Shoe Inside.
  - Shoe Outside.
  - Lower Ends of Posts Inside.
  - Lower Ends of Posts Outside.
  - Middle of Posts Inside.
  - Middle of Posts Outside.
  - Floor Beam at holes for Beam Hangers.
  - Floor Beam Lateral Connections.
- CONNECTING PLATES.
  - Batter Brace to Top Chord.
  - Post to Top Chord.
  - Lateral Struts to Top Chord.
  - Intermediate Struts to Posts.
  - Portal Braces to Batter Braces.
- OTHER PLATES.
  - Shoe Under Lateral Connection to Floor Beams.
  - Roller Plates. Name Plates.
  - Beam Hanger Plates. Top Plate in Floor Beam.

LACING OR LATTICING.
- Top Chord Upper.
- Top Chord Lower.
- Batter Brace Upper.
- Batter Brace Lower.
- Posts.
- Lateral Struts.
- Portal Braces.

TRUSSING.
- Posts.
- Lateral Struts.
- Portal Braces.

PINS.
- Bottom Chord.
- Top Chord.
- Middle of Posts.
- Upper Lateral Connection.
- Lower Lateral Connection.
- Cross Diagonal Connection.

BOLTS.
- Bracket Bolts.
- Name-Plate Bolts.
- Cross Diagonal Bolts in Batter Braces.
- Cross Diagonal Bolts in Posts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plate.
- Upper Lateral-Rod Connection to Chords.
- Lower Lateral-Rod Connection to Floor Beam.
- Hand-rail Post Bolts.
- Lateral Struts Connection to Chord.
- T-Iron Brace Bolts.

BRACKETS FOR PORTALS, INCLUDING ORNAMENTAL WORK.

T-IRON BRACES.
- Posts to Lateral Struts.
- Stiffeners in Built Floor Beams.

BEAM HANGERS.
TURN BUCKLES.
FILLERS FOR PINS.
ROLLER FRAMES.
EXPANSION ROLLERS.
SLEEVE NUTS.

JAWS.
- Upper Lateral Struts.
- Intermediate Lateral Struts.
- Lower Lateral Struts.

PIECES OF CHANNELS.
- Upper Lateral Strut Connection.
- Lower Lateral Strut Connection.
- Batter-Brace Channel Connection to Shoe under Plates.

ANGLE IRON.
- Intermediate Struts to Posts.
- Upper Lateral Struts to Chord.
- Lower Lateral Struts to Pedestal.
- Lower Lateral Struts to Chord (Channel Lower Chords).
- Batter Braces to Shoe Under Plates.
- Side and End Angles for Roller Plates.
- Angles in Built Beams.

WASHERS FOR HAND-RAIL POST BOLTS.

RIVET HEADS.
- Top Plate to Chord and Batter-Brace Channels.
- Latticing or Lacing to Channels in Top Chords, Posts and Struts.
- Stay Plates to Channels.
- Reinforcing Plates to Channels.
- Cover Plates to Channels
- Connecting Plates to Channels.
- Lateral Connection to Floor Beam.
- Trussing to Channels on Bars.
- Ornamental Work in Brackets.
- T-Iron Braces to Posts and Struts.
- Jaws to Lateral Struts.
- The Various Angle Irons to the Parts which they Connect.
- The Various Pieces of Channels to the Parts which they Connect.

DETAILS OF BUILT BEAMS
- Web.
- Top Plate.
- Upper Angles.
- Lower Angles.
- Stiffening Angles.
- T Stiffeners.
- Filling Plates.
- Lateral-Rod Connections.
- Reinforcing Plates at Beam Hanger Holes.
- Rivet Heads.

TIMBER.
- Joist.
- Flooring.
- Hand-rail Cap Pieces.
- Hand-rail Posts.
- Hub Plank.
- Felloe Plank.

III. WOODEN HOWE TRUSS RAILROAD BRIDGE.

WOOD.

Lower Chords.
Clamps and Keys in same.
Upper Chords and Keys for same.
Upper Lateral Braces.
Lower Lateral Braces.
Cross Diagonal Braces in Deck Bridge.
Batter Braces and Keys for same.
Main Braces.
Counter Braces.
Tie Beams at Ends of Top Chords.
Spreaders at Ends of Bottom Chord.
End Diagonals at Portals.
Track Stringers and Packing.
Batter-Brace Stiffeners.
Floor Beams.
Guard Rails.
Corbels.
Wall Plates.
Keys—Corbels to Wall Plates.
Track Ties.

WROUGHT IRON.

Truss Rods.
Upper Lateral Rods.
Lower Lateral Rods.
Batter-Brace Ties.
Camp Bars.
Rods for Batter-Brace Stiffeners.

Dowels for Lateral Braces.
SPIKES. { Ties to Stringers. Guard Rails to Ties.
Truss-Rod Plates at Top and Bottom.

BOLTS. {
Upper Chord Bolts.
Batter-Brace Bolts.
Lower Chord Bolts.
Intersectional Bolts.
Track Stringer Bolts.
Floor Beams to Chords.
Track-Stringers to Floor Beams.
Guard Rails to Ties.
Corbels to Chords.
Brackets to Tie Beams and Batter Braces.
Name-Plate Bolts.
Anchor Bolts.
Drift Bolts.

CAST IRON.

Top-Chord Angle Blocks.
Bottom-Chord Angle Blocks.
End-Chord Angle Blocks.
Top-Chord Lateral Angle Blocks.
Bottom-Chord Lateral Angle Blocks.

WASHERS. {
Upper Lateral-Rod Washers
Lower Lateral-Rod Washers.
Chord Bolt Washers.
Intersectional Bolt Washers.
Track-Stringer Bolt Washers.
Batter-Brace Bolt Washers.
Bracket-Bolt Washers.
Batter-Brace Stiffening Rod Washers.
Name-Plate Bolt Washers,
Corbel or Anchor-Bolt Washers.
Guard-Rail Bolt Washers.

Brackets.
Name Plates.
Clamp Heads.
Lower Chord Keys.

PACKING WASHERS. {
Chord-Bolt Packing Washers.
Batter-Brace Bolt Packing Washers.
Lateral-Rod Packing Washers.
Track-Stringer Bolt Packing Washers.
Bracket-Bolt Packing Washers.
Tie-Bar Packing Washers in Batter Braces.

IV. COMBINATION PRATT TRUSS RAILROAD BRIDGE.

WOOD.

Top Chords.
Batter Braces.
Lateral Braces.
Vertical Posts.
End Tie Beams.
End Diagonals on Batter Braces.
Floor Beams.
Track Stringers.
Track Stringer Packers.
Ties.
Guard Rails.
Chord and Batter-Brace Covering.
Lath for Same.
Batter-Brace Stiffeners.
Corbels.
False Caps.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.

WROUGHT IRON.

Main Portions.

Main Diagonals.
Counters.
Hip Verticals.
Upper Lateral Rods.
Lower Lateral Rods.
Bottom Chord Bars.
Bottom Chord Channels for Stiffened End Panels.
End Lateral Struts.
Batter-Brace Ties.
Cross Diagonals in Deck Bridge.
Lower Lateral Struts in Deck Bridge.
*Floor Beams.
Track Stringers.
Side Braces in Pony Trusses.
Batter-Brace Stiffening Rods.
End-Post Bracing Ties.
Beam Truss Rods.

DETAILS.

BOLTS.
- Chord Bolts.
- Batter-Brace Bolts.
- Post Bolts.
- Bracket Bolts.
- Name-Plate Bolts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plate.
- Lower Lateral-Rod Bolts.
- Stringer Packing Bolts.
- Joint Boxes to Top Chord.
- Guard Rail to Ties.
- Side Brace Bolts.
- Drift Bolts.
- Floor-Beam Packing Bolts.
- Track Stringers to Floor Beams.
- Corbels to Foundations.

Beam Hangers.
Beam-Hanger Plates.
Hip Vert. Plates on Castings.
Lacing on Hip Verts. in Pony Trusses.
Side-Brace Connection to Chord.
Side-Brace Connection to Floor Beams.
Lateral-Rod Connection to Floor Beams.
Pins.
Rollers and Roller Frames.
Jaws on End Struts.
Dowels for Upper Laterals.
Rods for Trussing Beams.
Boat Spikes.
Lacing or Latticing, Stay Plates, Reinforcing Plates and Rivets for Bottom Chord Channels.
Fillers for Pins.
Turn-buckles.
Sleeve-nuts.

SPECIAL WROUGHT-IRON DETAILS.

Hip-Joint Boxes.
Upper Chord Panel Connection.
Lower Post Sockets.
Pedestals.
Bed Plates.
Jaws for Lower Lateral Struts.

CORRUGATED OR GALVANIZED IRON.

Cover for Top Chords and Batter Braces.

CAST IRON.

Bed Plates.
Hip-Joint Boxes or Hoods.
Pedestals.
Upper Post Sockets.
Upper Chord Panel Connection.
Lower Post Connection.
Castings for Trussing Wooden Beams.
Lateral Angle Blocks.
Name Plates.
Brackets.

WASHERS.
- Chord-Bolt Washers.
- Batter-Brace Bolt Washers.
- Post-Bolt Washers.
- Upper Lateral-Rod Washers.
- Lower Lateral-Rod Washers.
- Beam-Hanger Washers.
- Name-Plate Bolt Washers,
- Bracket-Bolt Washers.
- Track-Stringer Bolt Washers.
- Bed-Plate Bolt Washers.
- Joint-Box Bolt Washers.
- Guard-Rail Bolt Washers.
- Side-Brace Bolt Washers.
- Batter-Brace Stiffening-Rod Washers.
- Floor-Beam Bolt Washers.

PACKING WASHERS.
- Chord-Bolt Packing Washers.
- Lateral-Rod Packing Washers.
- Batter-Brace Bolt Packing Washers.
- Tie-Bar Packing Washers in Batter Braces.
- Post-Bolt Packing Washers.
- Bracket-Bolt Packing Washers, in Batter Braces.
- Stringer-Bolt Packing Washers.
- Floor-Beam Bolt Packing Washers.

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* For details of built floor beams, see list of members in Iron Highway Bridge.

V. WROUGHT-IRON RAILWAY BRIDGE.

MAIN PORTIONS.

- CHANNEL BARS.
  - Top Chords.
  - Batter Braces.
  - Posts.
  - Lateral Struts.
  - Portal Braces.
  - Bottom Chords.
  - Track-Stringer Bracing Struts.
- I BEAMS.
  - Floor Beams.
  - Intermediate Struts.
  - Upper Lateral Struts.
  - Lower Lateral Struts.
  - Top Chords.
  - Batter Braces.
  - Track-Stringer Bracing Struts.
- PLATE.
  - Top Chords.
  - Batter Braces.
- BARS.
  - Main Diagonals.
  - Counters.
  - Hip Verticals.
  - Upper Lateral Rods.
  - Lower Lateral Rods.
  - Portal Bracing Diagonals.
  - Track-Stringer Bracing Diagonals.
  - Vibration Rods.
  - Lower Chord Bars.
- T IRON.
  - Lower Lateral Struts.
  - Side Bracing.
  - Hip Verts.
  - Track-Stringer Bracing Struts.
- FLOOR BEAMS.
- TRACK STRINGERS.
- RAILS.

DETAILS.

- PLATES.
  - STAY PLATES.
    - Top Chords.
    - Ends of Posts.
    - Middle of Posts.
    - Ends of Lateral Struts.
    - Batter Braces.
    - Portal Braces.
    - Stiffened Bottom Chords.
  - REINFORCING PLATES.
    - Hip Inside.
    - Hip Outside.
    - Top Chord Intermediate Panel Points Inside.
    - Top Chord Intermediate Panel Points Outside.
    - Bottom Chord Intermediate Panel Points Inside and Outside for Channel Bottom Chords.
    - Shoe Inside.
    - Shoe Outside.
    - Lower Ends of Posts Inside.
    - Lower Ends of Posts Outside.
    - Middle of Posts Inside.
    - Middle of Posts Outside.
    - Floor Beam at Holes for Beam Hangers.
    - Floor Beam Lateral Connection.
- PLATES.
  - FILLING PLATES.
    - At Panel Points of Top Chord.
    - At Panel Points of Stiffened Bottom Chords.
    - Floor Beams.
  - COVER PLATES.
    - Shoe.
    - Hip Joint.
    - Intermediate Panel Points Top Chords.
  - CONNECTING PLATES.
    - Batter Brace to Top Chord.
    - Posts to Top Chord.
    - Lateral Struts to Top Chord.
    - Intermediate Struts to Top Chord.
    - Portal Braces to Batter Braces.
    - Track-Stringer Splice Plates on Web.
    - Track Stringer Splice Plates on Flanges,
    - Iron Stringer Connection to Floor Beams.
    - Wooden Stringer Connection to Floor Beams.
    - Track-Stringer Bracing Connection to Stringers.
  - Pedestal Plates.
  - Roller Plates.
  - Beam Hanger Plates.
  - Lateral Connection to Floor Beam.
  - Name Plates.
  - Top Plate in Floor Beam.
  - Bottom Plate in Floor Beam.
  - Top Plate in Track Stringer.
  - Bottom Plate in Track Stringer.
  - Bed Plates for Track Stringers.

LACING OR LATTICING.
- Top Chord Upper.
- Top Chord Lower.
- Bottom Chord Upper.
- Bottom Chord Lower.
- Batter Brace Upper.
- Batter Brace Lower.
- Posts.
- Lateral Struts.
- Portal Baces.
- Track-Stringer Bracing Struts.

TRUSSING | Verts in Pony Trusses.

PINS.
- Bottom Chord.
- Top Chord.
- Middle of Posts.
- Upper Lateral Connection.
- Lower Lateral Connection.
- Vibration Diagonal Connection.
- Track-Stringer Bracing Diagonal Connection.

BOLTS.
- Bracket Bolts.
- Name-Plate Bolts.
- Vibration Diagonal Bolts in Batter Braces.
- Vibration Diagonal Bolts in Posts.
- Bed-Plate Bolts.
- Expansion Pedestal Fastening to Bed Plates.
- Upper Lateral-Rod Connection to Chords.
- Lower Lateral. Rod Connection to Floor Beams.
- Lateral Strut Connection to Chords.
- T-Iron Brace Bolts.
- Track-Stringer Bracing Connection.
- Rail Splice Bolts.
- Track-Stringer Packing Bolts.
- Guard Rails to Ties and Track Stringers.
- Shim Bolts.

BRACKET CONNECTION FOR POSTS TO FLOOR BEAMS IN PONY TRUSSES.
BRACKETS ATTACHING IRON TRACK-STRINGERS TO BEAMS.
BRACKETS FOR PORTALS, INCLUDING ORNAMENTAL WORK.

T-IRON BRACES.
- Posts to Lateral Struts.
- Stiffeners in Built Floor Beams and Track Stringers.

BEAM HANGERS.
EXPANSION ROLLERS.
ROLLER FRAMES.

FILLERS FOR PINS.
SPLICE PLATES FOR RAILS.
SPIKES FOR TIES AND GUARD-RAIL FACING.

JAWS
- Upper Lateral Struts.
- Intermediate Lateral Struts.
- Lower Lateral Struts.
- Track-Stringer Bracing Struts.

PIECES OF CHANNELS.
- Upper Lateral Strut Connection.
- Lower Lateral Strut Connection.
- Batter-Brace Channel Connection to Pedestal Plates.

WASHERS FOR STRINGER BOLTS.

ANGLE IRON.
- Intermediate Struts to Posts.
- Upper Lateral Struts to Chords.
- Lower Lateral Struts to Pedestals.
- Lower Lateral Struts to Chords (Channel Lower Chords).
- Batter Braces to Pedestal Plates.
- Side and End Angles for Roller Plates.
- Angles in Built Beams and Track Stringers.
- Wooden Track. Stringer Side Fastening to Beams.
- Wooden Track-Stringer Supporting Angles.
- Iron Track. Stringer Supporting Angles.
- Facing on Guard Rails.

RIVET HEADS.
- Top Plate to Chord and Batter-Brace Channels.
- Latticing or Lacing to Channels in Chords, Posts and Struts.
- Intersection of Lattice.
- The Various Stay Plates to Channels.
- The Various Reinforcing Plates to Channels.
- Cover Plates to Channels.
- Connecting Plates to Channels, etc.
- Lateral Connection to Floor Beams.
- Trussing to Channels, Bars, or T iron.
- Ornamental Work in Brackets.
- T-iron Braces to Posts and Struts.
- Jaws to Lateral Struts.
- The Various Angle Irons to the parts which they connect.
- The Various Pieces of Channels to the parts which they connect.
- Brackets to Floor Beams, Track Stringers and Posts.
- Track-Stringer Splice Plates to Stringers.
- Iron Stringers to Floor Beams.
- Floor Beams to Posts.

DETAILS OF BUILT BEAMS.

- Web.
- Top Plate.
- Bottom Plate.
- Upper Flange Angles.
- Lower Flange Angles.
- Stiffening Angles.
- T Stiffeners.
- Filling Plates.
- Lateral-Rod Connections.
- Reinforcing Plates at Beam Hanger Holes.
- Rivet Heads.
- Stringer Supports.
- Stringer Side Connection.

DETAILS OF BUILT TRACK STRINGERS.

- Web.
- Top Plate.
- Bottom Plate.
- Upper Flange Angles.
- Lower Flange Angles.
- Stiffening Angles.
- T Stiffeners.
- Filling Plates.
- Connection for Bracing.
- Connection to Floor Beams.
- Rivet Heads.

LUMBER.

SHIMS FOR TRACK STRINGERS.
TRACK STRINGERS AND PACKING.
GUARD RAILS.
TIES.

LIST OF MEMBERS IN A DECK PLATE GIRDER BRIDGE.

Webs,
Top Plates,
Bottom Plates,
Upper Flange Angles,
Lower Flange Angles,
Vertical Stiffening Angles,
Inclined Stiffening Angles,
Filling Plates,
Bed Plates,
Web Splice Plates,
Anchor Bolts with Nuts,
Cross Frames at ends,
Intermediate Cross Frames,
Connecting Plates for same,
Rivets,
Tie Bolts,
Spikes for rails.
Guard Rail Angles,
Washers for Tie Bolts.

In plates 19, 20, and 21, will be found illustrations of most of the members included in the preceding lists, so that the student need be at no loss to understand precisely what the terms used signify.

CHAPTER X.

COMPLETE DESIGN FOR AN IRON RAILWAY BRIDGE.

In the first part of this work we have shown how to find the strains, in the second part how to design the various members to resist these strains. The student is now prepared to learn the art of designing.

We have given at the end of this work the working drawings for an actual bridge, as furnished by the bridge company that designed and erected it. We shall now give the figuring necessary to design this bridge on the basis of Cooper's specifications, except that we shall assume for our live load the system of our diagram, page 89. As the bridge was actually designed according to other specifications and live load, we shall not get precisely similar results. This is not our object. But by comparison it will be seen what differences in design we obtain. Then, by careful study of the working drawings given, the student should be able to make his own working drawings to suit the new results.

Finally, he can obtain, at slight expense, blue prints of working drawings from some of our leading bridge companies, and can check, by actual calculation, the design, according to the specifications and live load adopted. He can obtain such drawings in great variety, for plate girder spans, square and skew, as well as for swing spans, highway bridges, etc. It is therefore unnecessary to multiply illustrations here. Having brought the student to this point, his further progress must be left largely to himself.

We shall, therefore, only give in detail the calculations for this single example.

REQUIRED TO DESIGN A SINGLE TRACK THROUGH SPAN PRATT TRUSS BRIDGE, 153 FT. C. TO C. OF END PINS; 9 PANELS; DEPTH, 26 FT. C. TO C. OF PINS; WIDTH, 16 FT. 3 INCHES C. TO C.; STRINGERS, 7 FT. 6 INCHES C. TO C., RIVETED BETWEEN FLOOR BEAMS. FLOOR BEAMS RIVETED BETWEEN POSTS. COOPER'S SPECIFICATIONS AND LIVE LOAD ACCORDING TO OUR DIAGRAM, PAGE 89. TRACK, 400 LBS. PER FT.

We first proceed to design the floor system by itself, and commence with the stringers.

STRINGERS.—The end stringers which rest on the masonry are longer than the intermediate stringers. These latter are 17 ft. "o. a.," over all, and this length is also effective. The end stringers we shall take as 21 ft. o. a. and 18 ft. effective, from c. to c. of bearing.

By Cooper's specifications (§ 36),* we must take a depth of not *less* than $\frac{1}{10}$ of the span, or 20 inches. By our table, page 425, the least weight depth over all is 29 inches, and about $\frac{8}{10}$ of this gives for the effective depth 23 inches, for least cost. We shall take 22 inches effective and 24 inches o. a.

From our table, page 425, the weight for a live load similar to Cooper's "Class A" is 1,634 lbs. For our assumed live load we add 18 per cent., and have 1,928 lbs. This gives for weight per ft. $\frac{1928}{17} = 115$ lbs. nearly, for each stringer. The track is 400 lbs., or 200 lbs. per ft. for one stringer, and the dead load per ft. is, therefore, 315 lbs.

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* We shall hereafter always refer to Cooper's specifications by giving the clause number in this manner.

Our live load concentrates 128,000 lbs. in 17 ft., which is 32,000 lbs. at end of each stringer. The end shear therefore is $32000 + 2677 = 34677$ lbs.

The maximum moment due to the dead load is $\frac{wl^2}{8} = \frac{315 \times 17^2}{8} = 11380$ ft. lbs. The maximum moment due to the live load is when the centre of gravity of the loading is as far on one side of centre as a driver is on the other side, and as much load as possible is on (page 218). We find for this position, second driver at $1\frac{1}{16}$ ft. on left of centre, and maximum moment 225,300 ft. lbs. Half of this for one stringer gives 112,650 ft. lbs. The maximum moment then is $112650 + 11380 = 124030$ ft. lbs.

This gives for the chord strain $\dfrac{124030}{\frac{22}{12}} = 67650$ lbs. (§ 42.) For the lower flange this calls (§ 30) for $\frac{67650}{7000} = 9.66$ square inches *net*.

The web (§ 43) must not be less than $\frac{34677}{4000} = 8.67$ square inches.

We shall take our web plate, $24'' \times \frac{3}{8} = 9$ square inches. This weighs 30 lbs. per foot. (NOTE.—*Area in square inches multiplied by* 10 *and divided by* 3 *gives weight per foot for iron. For steel add* 2 *per cent.*) For 17 feet long, we have weight of web plate 510 lbs.

We take, for the lower flange, two angles each $6'' \times 4''$, 18 lbs. per foot. This gives a thickness of about $\frac{9}{16}''$ (*Carnegie*, page 107). The area of each angle is then 5.4, or for both, 10.8 square inches *gross*. For $\frac{7}{8}''$ rivets we have rivet-hole $1''$. (§ 58.) Deduct two rivet-holes, $2 \times 1 \times \frac{9}{16} = 1.13$ square inches, and we have 9.67 square inches net. (§ 58.)

We take the same top angles as bottom. (§ 35.) The weight of top angles is $2 \times 18 \times 17 = 610$ lbs., and bottom the same.

We must have fillers at the ends, two at each end, or four in all, which fit in between the flange angles, so that the connecting angles which fasten the stringer to the floor beams can be riveted on. They must have same thickness as the flange angles, or about $\frac{9}{16}''$. Taking them $6''$, their area is about 3 square inches, or 10 lbs. per foot. The weight of four is 40 lbs.

We have four connecting angles, two at each end, each 2 feet long. Taking them $6'' \times 4''$, 12.5 lbs., they weigh 25 lbs. apiece, or 100 lbs.

The allowable shear (§ 44), since $H = 64$, is 5,074 lbs. per square inch. As the web at ends is safe for 4,000 lbs. unit strain, no intermediate stiffeners are required.

If we pitch the rivets at $3''$ throughout the top flange, and $6''$ for centre $8\frac{1}{2}$ feet of bottom flange, and $3''$ at ends, we have 140 rivets. Weight from *Carnegie*, page 63, 43.1 lbs. per 100. Hence, rivets weigh 60 lbs.

We have then, for one intermediate stringer,

1 web plate $24'' \times \frac{3}{8}''$, area 9 square inches	510 lbs.	
2 top angles $6'' \times 4'' \times \frac{9}{16}''$, 18 lbs., 10.8 sq. in. gross	610 "	
2 bottom angles $6'' \times 4'' \times \frac{9}{16}''$, 18 lbs., 9.67 sq. in. net	610 "	
4 end fillers $6'' \times \frac{9}{16}''$	40 "	
4 end angles $6'' \times 4'' \times 12.5$ lbs.	100 "	
140 $\frac{7}{8}''$ rivets	60 "	
	1,930 "	assumed 1,928 lbs.

There are to be fourteen of these intermediate stringers, hence their weight is $1930 \times 14 = 27020$ lbs.

For the end stringers the effective length is 18 feet. The depth is the same as for

the intermediate, viz., 24″ over all, and 22″ effective. We take the weight a little larger than for the intermediate, say 120 lbs. per foot. The track makes the total dead-load 320 lbs. per foot.

Our live-load gives end shear 33,780 lbs., and dead-load 2,880 lbs., total end shear = 36,660 lbs.

The maximum moment for live-load is 249,555 ft. lbs., for dead-load 12,960 ft. lbs., total 137,760 ft. lbs.

The chord strain is then $\frac{137760}{\frac{22}{12}} = 75140$ lbs., and hence for the lower flanges, at 7,000 lbs. per square inch (§ 30), we have 10.73 square inches, net. The area of web plate should not be less then $\frac{36660}{4000} = 9.16$ square inches. (§ 43.) This is so close to 9 square inches that we take web plate as before, viz., 24″ × ⅜″ = 9 square inches, 30 lbs. per foot, or 630 lbs. in all.

For the lower flange we take two angles 6″ × 4″, 20 lbs. per foot. This gives a thickness of about ⅝″ (*Carnegie*, page 107). The area is then 12 square inches gross. Deduct for rivets 2 × 1 × ⅝ = 1.25 square inch, and we have 10.75 square inches, net. (§ 58.)

Taking same top angles as bottom (§ 35), we have weight of top angles 2 × 20 × 21 = 840 lbs., and bottom the same.

At the cross-girder end we have two end fillers 1 foot long, 6″ × ⅝″ = 3.75 square inches, or 12.5 lbs. per foot, weight 25 lbs. We have also two end connecting angles 2 feet long, 6″ × 4″, 12.5 lbs. per foot, weight 50 lbs. At the masonry end we take four end fillers 1 foot long, 3″ × ⅝″ = 1.87 square inches, or 6.25 lbs. per foot, weight 25 lbs., and four end angles 2 feet long, 3½″ × 3″, 7⅝ lbs. per foot, weight 60 lbs.

No intermediate stiffeners are necessary.

If we pitch the rivets as before, we have 180 ⅞″ rivets, weight 43.1 lbs. per 100, or 80 lbs. (*Carnegie*, page 163).

In addition we have a foundation or wall plate, say 24″ × 6½″ × ¾″, weight 30 lbs., and two foundation bolts 1″ diameter and 10″ long, weight 10 lbs.

We have, then, for end stringer,

1 web plate 24″ × ⅜″, area 9 square inches	630 lbs.
2 top angles 6″ × 4″ × ⅝″, 20 lbs., 12 square inches gross	840 "
2 bottom angles 6″ × 4″ × ⅝″, 20 lbs., 10.75 square inches net	840 "
2 end fillers 6″ × ⅝″	25 "
2 end angles 6″ × 4″, 12.5 lbs	50 "
4 end fillers 3″ × ⅝″	25 "
4 end angles 3½″ × 3″, 7⅝ lbs	60 "
180 ⅞″ rivets	80 "
1 wall plate 24″ × 6½″ × ¾″	30 "
2 foundation bolts 1″, 10″ long	10 "
	2,590

Weight per foot $\frac{2590}{21} = 123$ lbs., assumed 120 lbs.

There are four of these end stringers, and their weight is 2590 × 4 = 10360.

Finally we have, at the masonry ends, between end stringers, two sets of end cross-frames, as shown in Plate 27 at end of this work, at 140 lbs. per set, weight 280 lbs.

CROSS-GIRDERS.—The width of bridge c. to c. is 16′ 3″. Allowing for posts, we take

for the floor beams or cross-girders a length of 15′ 6″ o. a. and effective. From our Table page 428, we see that the depth is about 34″. But the stringers have been taken at 24″. In order that they may be riveted to the floor-beam webs without interference of the angles, we take the depth of floor beams at 36″ o. a., or 34″ effective. From Table, page 428, the weight is 1,725 for live load similar to "Class A," Cooper's specifications. For our assumed live load add 18 per cent., and we have for weight of a cross-girder 2,035 lbs. This gives for weight per ft. $\frac{2035}{15.5} = 130$ lbs., nearly.

The half weight of an intermediate stringer is 965 lbs., of an end stringer, 1,295. Hence, load concentrated on floor beam at points where stringers are attached, taking in the track, is $965 + 1295 + 200 \times 17 = 5660$ lbs. The concentration at each of these points due to the assumed live load is 46,680 lbs. Total, 52,340 lbs. The half weight is 1,018 lbs., and hence end shear is $52340 + 1018 = 53360$ lbs., nearly.

The stringers are attached 4 ft. from ends, hence the moment due to external loading is $52340 \times 4 = 209360$ ft. lbs., and due to own weight of girder, $\frac{130 \times 15.5^2}{8} = 3900$ ft. lbs., nearly. Total bending moment $= 209360 + 3900 = 213260$ ft. lbs.

The chord strain is then $\frac{213260}{\frac{34}{12}} = 75270$ lbs., and hence, for the area of lower flanges at 8,000 lbs. (§ 30), we have 9.4 sq. in. net. The area of web plate should not be less than $\frac{53360}{4000} = 13.34$ sq. in. (§ 43.) We take web plate $36'' \times \frac{3}{8}''$, area 13.5 sq. ins., weight 45 lbs. per ft., or $15.5 \times 45 = 700$ lbs., nearly.

For the lower flange we take two angles, $6'' \times 4''$, $17\frac{5}{8}$ lbs. per ft. This gives a thickness of about $\frac{9}{16}''$ (*Carnegie*, page 107). The area is 10.6 sq. ins. gross. Deduct for rivets, $2 \times 1 \times \frac{9}{16} = 1.13$, and we have 9.47 sq. ins. net. (§ 58.)

We take the same top angles as bottom (§ 35), 10.6 sq. ins. gross, or $35\frac{1}{3}$ lbs. per ft. Weight of top angles, $35\frac{1}{3} \times 15.5 = 550$ lbs., nearly, and bottom flanges the same.

At each end we have two end fillers, $6'' \times \frac{9}{16}''$, area, 3.375 sq. ins., and weight, 11.25 lbs. per ft. Each of these is 2 feet long, and weighs 22.5 lbs. Weight of the four, 90 lbs.

We also have four connecting angles, $6'' \times 4''$, 45 lbs. per ft., or weight $= 15 \times 3 \times 4 = 180$ lbs.

If we pitch the rivets 6″ for 7 feet in centre, and 3″ at ends, and allow for rivets in stringer connecting angles,* we have 130 rivets. Weight at 43.1 lbs. per 100 (*Carnegie*, page 163), about 60 lbs.

We have, then, for one cross-girder,

1 web plate, $36'' \times \frac{3}{8}''$; area 13.5 sq. ins.	700 lbs.
2 top angles, $6'' \times 4'' \times \frac{9}{16}''$, area 10.6 sq. ins., gross	550 "
2 bottom angles, $6'' \times 4'' \times \frac{9}{16}''$, area 9.47 sq. ins., net	550 "
4 end fillers, $6'' \times \frac{9}{16}''$	90 "
4 end angles, $6'' \times 4''$, 15 lbs.	180 "
130 $\frac{7}{8}''$ rivets	60 "
	2,130 lbs.

* Value of $\frac{7}{8}''$ rivet in double shear, 2,256 lbs. (Table I., page 386). Hence, $\frac{52340}{2256} = 24$ rivets, stringer to floor beam, $\frac{53360}{2256} = 24$ rivets, floor beam to post.

This gives for weight per ft., $\frac{2130}{15.5} = 138$ lbs., assumed 130 lbs. There are eight of these floor beams, and their weight is $2130 \times 8 = 17040$ lbs.

In Fig. 206, Plate 8, page 345, we have illustrated the connection of floor beam to post, and stringers to floor beam.

It will be seen that there are plates, or "diaphragms," between the post channels, just as though the web of the floor beam ran straight through the inside channel. These plates are fastened by angles on inside of post channels. We consider these diaphragms as continuation of the floor-beam web, and hence consider them with their angles as part of the floor system.

We take the diaphragms, each $8'' \times \frac{3}{8}'' \times 36''$, weight 30 lbs. Also four angles, $5'' \times 3'' \times 36''$, $8\frac{1}{2}$ lbs. per ft., or 100 lbs. And including rivets for floor beam to post, 80 $\frac{7}{8}''$ rivets, weight 40 lbs.

Hence diaphragm, angles, and rivets weigh 170 lbs. There are sixteen of these, or $170 \times 16 = 2720$ lbs.

We can now recapitulate the results for the floor.

FLOOR.

14 Intermediate stringers @ 1930 lbs.	27020 lbs.
4 End stringers @ 2590 lbs.	10360 "
2 Sets of end cross frames @ 140 lbs.	280 "
8 Floor beams @ 2130 lbs.	17040 "
16 Diaphragms @ 170 lbs.	2720 "
Total for floor	57420 lbs.

Weight per ft. of floor, $\frac{57420}{153} = 376$ lbs.

These results are entirely independent of length of span, and can be obtained for given width and panel length and live load, without reference to any special span. In the office, such designs are numerous, and in any special case a floor system can generally be found to suit, already estimated, so that this portion of the design need take but little time, especially if a close estimate of weight for a bid is all that is needed.

We now proceed to design the main trusses. We have the track 400 lbs. per ft., and the floor, as just found, about 380 lbs. per ft.

We must estimate the weight of trusses and laterals. This we can do as illustrated in the example, page 451, according to any of the methods there given. For the case in hand, we have there found the total weight of iron 1,400 lbs. per ft. We have just found the floor about 380 lbs. per ft., and if we subtract this from 1,400, we have 1,020 lbs. for trusses and laterals.

We have, then,

$$\text{Dead load,} \begin{cases} \text{Track,} & 400 \text{ lbs. per ft.} \\ \text{Floor,} & 380 \quad " \quad " \\ \text{Trusses, etc.,} & \underline{1020} \quad " \quad " \\ & 1800 \end{cases}$$

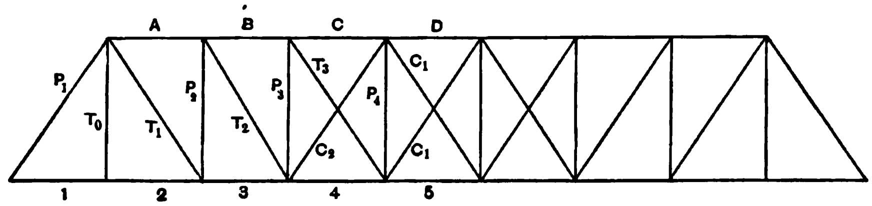
About half of the weight of trusses, etc., is taken as acting on the unloaded chord, or, in this case, the upper. The rest on the loaded, or lower, chord. We have thus 500 lbs. per ft. for upper chords, and 1,300 lbs. per ft. for lower chords. We must take one-half of these for one truss, or 4,250 lbs. upper apex dead load, and 11,050 lower apex dead load, per truss.

We can now find the strains for dead load and for live load, by use of our diagram, as illustrated on page 222.

We give these strains here, and the student should check them. A comparison with those given on Plate 22, at the end of this work, will show the differences caused by our live load and specifications, from that of the Bridge Company. The results of Plate 22 represent the practice of several years ago. We have changed the notation of Plate 22 to one which seems more convenient.

We allow, for estimating, 3 ft. additional length for chord bars and ties, in order to make the eye-bar heads. This makes length of chord bars, for estimate, 20 ft., and of ties, 34 ft. The length of posts over all is taken at 27 ft., of inclined end-posts, at 32.5 ft.

For the hip vertical T_0 we add 1.5 feet for length over all. The end upper panel, A, we take 17.5 feet long, the rest 17 feet.



Strains.		Unit Strains (§ 30).	Area □″	Total area required.	Sizes.	Area □″	Total length.	Weight.
T_1	Live, 135250	8000	16.91	20.33	4 Bars 5″ × 1″	20	136 ft.	9070 lbs.
	Dead, 54840	16000	3.42					
T_2	Live, 100620	8000	12.58	14.86	2 Bars 5″ × 1½″	15	136 ft.	6800 "
	Dead, 36560	16000	2.28					
T_3	Live, 70500	8000	8.81	9.95	2 Bars 5″ × 1″	10	136 ft.	4530 "
	Dead, 18280	16000	1.14					
C_1	Live, 45380	8000	5.67	5.67	2 Bars 1¾″ square	6.12	144 ft.	2940 "
	Dead, 0	16000	0					
C_2	Live, 8340	8000	1.04	1.04	1 Bar 1⅛″ square	1.27	144 ft.	610 "
	Dead, 0	16000	0					
1	Live, 95760	8000	11.97	14.47	2 Bars 6″ × 1$\frac{3}{16}$″	14.26	80 ft.	3800 "
	Dead, 40000	16000	2.50					
2	Same as 1							3800 "
3	Live, 163680	8000	20.46	24.84	2 Bars 6″ × 2$\frac{1}{16}$″	24.75	80 ft.	6600 "
	Dead, 70040	16000	4.38					
4	Live, 207940	8000	26.0	31.63	4 Bars 6″ × 1$\frac{5}{16}$″	31.52	80 ft.	8400 "
	Dead, 90040	16000	5.63					
5	Live, 233580	8000	29.20	35.45	4 Bars 6″ × 1½″	36	40 ft.	4800 "
	Dead, 100040	16000	6.25					
T_0	Live, 46680	7500	6.22	6.96 net	Two 12″ channels, 20 lbs. per ft., 12 sq. inches gross. Deduct for rivets 4 × 1″ × $\frac{5}{16}$″ = 1.25 and 2 × 1″ × ⅜″ = 0.75 . . 10 net.		110 ft.	4400 "
	Dead, 11050	15000	0.74					

The lightest 20 lbs. 12″ channels we can have are used.

In designing the built sections for posts and upper chords, we shall make use of "Osborn's Tables" (*Tables of Moments of Inertia*, etc., by Frank C. Osborn, Engineering

News Publishing Co., New York, 1889). These are readily obtained by the student, and are, together with *Carnegie*, necessary in checking our results. Bridge companies have, of course, their own tables of built sections. We take built sections because they can be made, at present prices, cheaper than rolled.

Thus, for P_1 we have live load strain, 174,970 lbs.; dead load, 73,120 lbs., $l = 372$ inches. Taking $r = 6.2$,* we have (§ 33) for the unit strains allowable for live load, 4,600 lbs.; for dead, 9,200 lbs. Hence area $= 38.03 + 7.94 = 45.97$. From Osborn's *Tables*, page 61, we see that No. 106 very nearly fills the requirements. If we make the top plate $20'' \times \frac{1}{2}''$, the area will be 46.04 sq. ins. As the eccentricity is 1.25, this will add to the moment of inertia $1 \times (8 - 1.25)^2 = 45.56$ inch lbs. We have, then, $I = 1770.56$, and $r^2 = \dfrac{1770.56}{46.04} = 38.46$, or $r = 6.2$, which agrees with what we assumed.

In this way we get the following results:

	Strains.		Unit Strains (§ 33).	Area required.					
P_1	Live, 174970	$l = 372''$	4600	38.03	45.97	1 Cover Plate $20'' \times \frac{1}{2}''$,	10.0 sq. ins.	130 ft.	19950 lbs.
	Dead, 73120	$r = 6.2$	9200	7.94		2 Webs $16'' \times \frac{11}{16}''$,	22		
						2 Angles $3'' \times 3''$, 9.6 lbs.	5.76		
						2 " $3'' \times 4''$, 13.8 lbs.	8.28		
							46.04		
A	Live, 163680	$l = 204$	7060	23.18	28.13	1 Cover Plate $20'' \times \frac{3}{8}''$,	7.5	70 ft.	6615 lbs.
	Dead, 70040	$r = 6.5$	14120	4.95		2 Webs $16'' \times \frac{5}{16}''$,	10.0		
						2 Angles $3'' \times 3''$, 6.8 lbs.	4 0		
						2 " $3'' \times 4''$, 11.6 lbs.	6.9		
							28.4		
B	Live, 207940	$l = 204$	7060	29.45	35.8	1 Cover Plate $20'' \times \frac{1}{2}''$, area	10.0	68 ft.	8160 lbs.
	Dead, 90040	$r = 6.5$	14120	6.35		2 Webs $16'' \times \frac{7}{16}$,	14.0		
						2 Angles $3'' \times 3''$, 6.4 lbs.	3.84		
						2 " $3'' \times 4''$, 13.6 lbs.	8.16		
							36		
C	Live, 233580	$l = 204$	7013	33.30	40.43	1 Cover Plate $20'' \times \frac{1}{2}''$, area	10.0	68 ft.	9290 lbs.
	Dead, 100040	$r = 6.2$	14026	7.13		2 Webs $16'' \times \frac{5}{8}$;	20.0		
						2 Angles $3'' \times 3''$, 6.3 lbs.	3.78		
						2 " $3'' \times 4''$, 12 lbs.	7.2		
							40.98		
D	Same as for C							34 ft.	4645 lbs.
P_2	Live, 84200	$l = 312$	4056	20.76	25.06	Two 12" channels, $41\frac{2}{3}$ lbs., 25 sq. ins.		108 ft.	9010 lbs.
	Dead, 34850	$r = 4.24$	8112	4.30					
P_3	Live, 59000	$l = 312$	4144	14.24	16.60	Two 12" channels, $27\frac{2}{3}$ lbs., 16.6 sq. ins.		108 ft.	5980 lbs.
	Dead, 19550	$r = 4.4$	8288	2.36					
P_4	Live, 37980	$l = 312$	4200	9.04	9.54	Two 12" channels, 20 lbs., 12 sq. ins.		110 ft.	4400 lbs.
	Dead, 4250	$r = 4.46$	8400	0.5					

These are the lightest 12" channels we can take.

Total weight of trusses .... 123800 lbs.

We take for the pins.

8 End Pins, $5\frac{1}{16}''$, 14 ft. .... 940 lbs.
28 Intermediate, $4\frac{9}{16}$, 44 feet .... 2400 "
Nuts for same .... 400 "

3740 lbs.

Total weight of trusses and pins .... 127540 lbs.

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* An approximate rule for assuming r, is to take r, $\frac{4}{10}$ of the depth of web desired. In this case, for 16" web, we have $r = 6.4$. With this to guide us we use the Table.

LATERALS AND DETAILS.—In the example, page 398, we have already calculated the strains in the lower lateral ties for this case of 153 feet span. Taking the unit strain, 15,000 lbs. (§ 30), we have the following sizes, referring for notation to the figure, page 398. The areas and weights of rods for different diameters are given in *Carnegie*, page 192. The length of a panel diagonal is about 23 feet. But we shall attach the lateral rods at bottom by clevises, so that the length of each rod is only about 20 feet. As we have two rods in each panel, one for wind on one side and one for wind on the other side, we shall want 40 feet of rod in each panel, on *each side of centre*, and 40 feet in centre panel.

We have then:

	Strain.					
End panel (1),	44,280 lbs.,	2.95 sq. ins.,	1 rod 2″	diam.,	80′ long,	840 lbs.
" (2),	34,028 "	2.27 " "	1 " $1\frac{3}{4}$″	"	80′ "	640 "
" (3),	24,600 "	1.64 " "	1 " $1\frac{1}{2}$″	"	80′ "	470 "
" (4),	16,000 "	1.07 " "	1 " $1\frac{1}{4}$″	"	80′ "	320 "
Centre panel (5),	8,200 "	0.55 " "	1 " $1\frac{1}{8}$″	"	40′ long,	130 "
	*36 clevises for these rods					500 "
	Pin plates					700 "
	Bolts					200 "
						3,800 lbs.

The clevises are attached by a bolt passing through a pin plate riveted to the bottom flange of the cross girder. There are, therefore, no bottom lateral struts except at the ends between end-posts.

We make these struts of two angles each, 6″ × 4″, $13\frac{1}{3}$ lbs. per foot, or 434 lbs. the pair, adding angle rests and rivets, 450 lbs. each strut, or, for both struts, 900 lbs.

For the top laterals we take all rods, $1\frac{1}{8}$″ diameter, 3.313 lbs. per foot, and 350 feet of rod gives 1,160 lbs. There are twenty-eight angle brackets for these rods, at 20 lbs. each, making 560 lbs., or, total, 1,720 lbs.

For the top intermediate struts we take two angles, 3″ × $2\frac{1}{2}$″, $4\frac{1}{3}$ lbs. per foot, and a plate 4″ × $\frac{3}{8}$″, as represented in Plate 11, Fig. 221, page 348. Weight of plate 5 lbs. per foot. The struts are 16 feet c. to c. of flange angles; weight of angles and plate, 218 lbs. Taking 64 rivets at 43 lbs. per 100, we have 27 lbs. for rivets. Total weight of strut about 250 lbs. There are six of these struts, and weight = 250 × 6 = 1,500 lbs.

For the portal struts, we take four angles $3\frac{1}{2}$″ × 3″, $7\frac{2}{3}$ lbs. per foot, latticed; weight, including lattice bars and rivets, 600 lbs. Two of these make 1,200 lbs.

We have twelve knee braces, each 2 angles, 3″ × $2\frac{1}{2}$″, $4\frac{1}{3}$ lbs. per foot, each weighing 75 lbs., or 900 lbs. for all.

Also, four portal knee braces, consisting of 2 angles, $3\frac{1}{2}$″ × 3″, $7\frac{2}{3}$ lbs. per foot, at 150 lbs. apiece, or 600 lbs. for all.

Total for laterals:

Lower lateral ties	3,800 lbs.
2 lower end struts @ 450	900 "
Top lateral ties with brackets	1,720 "
6 top intermediate struts @ 250	1,500 "
2 portal struts @ 600	1,200 "
12 intermediate knee braces @ 75	900 "
4 portal knee braces @ 150	600 "
Total weight of laterals	10,620 "

* For weight of clevises see page 415.

DETAILS.—*Of top chord.*

2400 $\frac{7}{8}''$ Rivets	1,070 lbs.
12 Intermediate web splices, $9'' \times \frac{3}{8}'' \times 12''$	130 "
6 " cover splices, $20'' \times \frac{3}{8}'' \times 21''$	270 "
14 Bottom splice and battens, $24'' \times \frac{5}{16}'' \times 24''$	700 "
* $3'' \times \frac{3}{8}''$ Lattice	480 "
Pin plates at hip	150 "
2 Hood plates, $20'' \times \frac{7}{16}'' \times 21''$	100 "
	2,870 × 2....5,740 lbs.

DETAILS.—*Of inclined end-posts.*

560 $\frac{7}{8}''$ Rivets	250 lbs.
2 Battens, $24'' \times \frac{5}{16}'' \times 24''$	100 "
$3'' \times \frac{3}{8}''$ Lattice	160 "
Pin plates	300 "
	810 × 4.....3,240 lbs.

DETAILS.—*Of intermediate vertical posts and suspenders.*

120 $\frac{7}{8}''$ Rivets	60 lbs.
4 Battens, $14'' \times \frac{5}{16}'' \times 15''$	70 "
$2\frac{1}{4}'' \times \frac{3}{8}''$ Lattice	300 "
Jaw plates	200 "
	630 × 16..10,080 lbs.

Total for laterals and details.................. 29,680 lbs.

Subtract from this 900 for lower end struts, and we have 28,780 lbs. We have then already found,

Floor	57,420 lbs.
Laterals and details	28,780 "
Trusses and pins	127,540 "

$$213{,}740 \text{ lbs., or, } \frac{213740}{153} = 1397 \text{ lbs. per foot.}$$

We assumed for our calculation 1,400 lbs. per foot.

MASONRY MEMBERS.—For the pedestals, we have, from Table I., page 377, for the lineal bearing on pin $5\frac{1}{16}''$, 0.031 inches per ton. The end shear is 146,420 live, 61,200 dead, total, 207,620 lbs., and hence lineal bearing is $\frac{207620}{2000} \times .03 = 3.1$ inches. At 250 lbs. per square inch, we require $\frac{207620}{250} = 830$ square inches of wall plate.

We take, for the pedestal,

2 $12'' \times \frac{3}{8}''$ Webs, 32" long	150 lbs.	
2 $6'' \times 6''$ angles, 29 lbs., 32" long	150 "	
2 $6'' \times \frac{3}{4}''$ fillers, 28" long	60 "	
For fixed pedestal, 1 base plate, $30'' \times \frac{7}{8}'' \times 32''$.	240 "	
For roller pedestal, 1 base plate, $30'' \times \frac{3}{4}'' \times 32''$.		200
	600	560

* For weight of lattice see page 354.

We have then,

2 Fixed pedestals @ 600	1,200 lbs.
2 Roller " " 560	1,120 "
2 Sets of rollers " 520	1.040 "
2 Roller wall plates, 30″ × $\frac{7}{8}$″ × 32″	520 "
8 Foundation bolts, $1\frac{1}{4}$″, 18″	70 "
	3,950 lbs.

Our total weight is then as follows:

Masonry members	3,950 lbs.
Laterals and details and end struts	29,680 "
Floor	57,420 "
Trusses and pins	127,540 "
Total net weight of span	218,590
Add 3%	6,560
Gross weight of span	225,150 lbs.

The excess of this weight over that given for same span at end of this work is due to the very heavy live load, and to the proportions, as well as to the specifications adopted.

As we have seen, page 451, by taking 5 or 6 panels instead of 9, and a depth of about 32 feet instead of 26 feet, we could reduce the weight to 1,300 lbs. instead of 1,400 lbs. per foot. This shows the use of our formula for weight, page 443.

The allowance of 3 per cent. is to cover waste, corners of plates clipped off, holes punched out, etc.

ESTIMATE OF COST.—We can now estimate the cost of the bridge, somewhat after the following manner:

Iron, say	2.1¢ per lb.
Labor	1.1¢ "
Freight—for a haul of 100 miles	0.1¢ "
Engineering	0.3¢ "
Profit	0.4¢ "
Erection (varies according to local circumstances)	1.0¢ "
	5 cts. per lb.

For a plate girder span labor would be less, say 0.7 cent per lb. Erection varies more widely than any of the other items. Local freight rates can always be ascertained. The cost of the iron, " f. o. b.," that is, " free on board," or loaded on cars ready for shipment, would be, in the above case, 3.9 cents per lb., after deducting freight and erection.

The total cost of our span would now be 225150 × .05 = $11257.50, and on this basis a bid can be made, offering to deliver and erect the bridge for so much, the masonry, of course, to be supplied by other parties. Accompanying this, a strain diagram is furnished, which consists of a skeleton outline of the truss, with the live load, dead load, and all other data on it, and also all the sections. In short, all the results we have just figured out, similar to Plate 22, at the end of this work.

THE MEMORANDUM. CAMBERED LENGTHS, AND SKETCHES OF DETAILS.—Before the working drawings can be made, and the work put into the shop, the actual length of

the various members must be carefully figured as detailed in the Example, page 412. After these lengths are found, the engineer must carefully sketch the details at each joint, and get the data so arranged that the draughtsmen can commence on the shop drawings.

All these data and results should be noted by the engineer, and constitute the "MEMORANDUM."

In our case, taking $E = 26000000$ lbs., we have, page 411, for the length of lower chord bars, taking panel 5,

$$e = \frac{100040}{26000000\left[\frac{100040}{16000} + \frac{233580}{8000}\right]} = 0.000108, \quad \text{and}$$

$$\textit{length of lower chord bars c. to c.} = 204'' - 0.022 - 0.025 = 16 \text{ feet } 11\tfrac{15}{16} \text{ inches.}$$

For the other panels we would get the same result, but as no difference is ever made in the lengths of chords, or posts, we take, in applying our method, the heaviest member of each kind, and find the cambered length for it, and make the others the same.

Thus, for the posts, we have, taking P_2,

$$e = \frac{34850}{26000000\left[\frac{34850}{8112} + \frac{84200}{4056}\right]} = 0.000053, \quad \text{and}$$

$$\textit{length of post c. to c.} = 312'' + 0.016 + .025 = 36 \text{ feet } 0\tfrac{1}{32} \text{ inch.}$$

For the upper chord panels we have, taking D,

$$u' = 8307, \qquad u = 9411, \qquad i = 0.000908, \qquad e = 0.000096.$$

We have, then, for A,

$$\textit{length of } A = 210'' + 0.19 + 0.02 = 17 \text{ feet } 6\tfrac{7}{32} \text{ inches.}$$

For the other panels,

$$\textit{length} = 204'' + 0.19 + 0.02 = 17 \text{ feet } 0\tfrac{7}{32} \text{ inch.}$$

For the inclined ties we have, for T_1,

$$i = 0.000908, \qquad e = 0.000103, \qquad p + \frac{ip}{2} = 204.0926, \qquad l = 372.82,$$

$$\text{length of ties c. to c.} = 372.82'' - 0.028 - 0.025 = 31 \text{ feet } 0\tfrac{3}{4} \text{ inch.}$$

Sketches of the details for top and bottom chord packing at every joint, giving the exact distances, clearances, thickness of pin plates, width of jaws, arrangement of top chord splices, etc., should now be made. Also list of all the eye bars, with data for ordering the same. The pins can now be refigured exactly, to see that they are not overstrained (page 374). This completes the memorandum.

CHAPTER XI.

SHOP DRAWINGS.

By MORGAN WALCOTT, C. E.

To make a shop drawing well requires some little skill and practice. The constant aim should be to make everything clear and plain for the men in the shops. All necessary dimensions should be plainly marked on the drawings in shop units, that is, in feet, inches, and halves, quarters, eighths, sixteenths, and thirty-seconds of an inch; this latter being the smallest measurement used in bridge engineering. Unnecessary dimensions should be avoided. End views, or sections, should be placed at the ends which they represent. For the sake of clearness, any brackets or other details on one end of a piece, which would show in a true mechanical drawing or projection of the other end, are nevertheless not shown in this projection; but a special view of their end is made, on which they are shown.

With beginners, the drawings should first be made with pencil on paper, as there will probably be alterations which can more readily be made on paper than on tracing linen. Experienced draughtsmen, however, generally make simple drawings directly on the tracing linen. In order to "take" the ink, the surface of the tracing linen must be perfectly clean. To secure this, rub the surface thoroughly with a clean towel, and if this does not answer, rub a very little powdered chalk on it. If it becomes necessary to erase, and afterwards to draw over the spot, the ink will probably blot, unless the spot has been rubbed with soapstone. When the work to be erased is of any magnitude, nothing but a prepared rubber ink eraser should be used. Small points or short lines can often be picked out with the sharp point of a penknife or ink scratcher.

It is usual to use the dull or unglazed side of the tracing linen. The advantage of using the smooth or glazed side, is that ink lines are more easily erased than on the dull side. The advantages of the dull side are: (1) If it is desired to make pencil sketches on the finished drawings, the pencil marks will show better on this side. (2) If the ink lines are on the dull side of the cloth, the drawings will lie flat, while, if they are on the glazed, the drawings will curl, or roll up. The reason of this is, that the preparation on the glazed side, and the ink lines, both tend to shrink the sides that they are on, and thus make the drawing roll up. If the glazing and the ink lines are on opposite sides of the cloth, their tendencies to roll the cloth up neutralize each other.

All drawings should be made in black ink; red ink is rarely used even for dimensions. Black ink is preferred because it takes better blue prints than any other color. Outlines are made heavy, and the dimension lines fine.

A good scale for the shop drawings is one inch to the foot; sometimes a scale of three-quarters of an inch to the foot, and sometimes a scale of an inch and a half to the foot, may be used advantageously. The drawings should be on sheets of tracing linen usually about 3 feet long by 20 inches wide. Frequently long posts and other sections can be shortened up by omitting the central portions, and indicating the length by some such

device as "10 Panels @ 2′—0″ each = 20′—0″." If there are any brackets or pin-holes in the centre portion of the piece, it may be impossible to indicate the length in this manner. Or, it may be possible by making two breaks in the piece instead of one. It is well to make the drawings to scale, as this serves as a check in designing. The exact scale, however, is not of such importance as it might seem at first sight, as every needed dimension should be clearly marked on the drawing, and the men in the shops are not allowed to scale distances. If any dimension is lacking, it must be supplied by the draughtsman who made the drawing. Some of the general data which should go on every shop drawing are: sizes of rivets, sizes of open holes, number of pieces wanted and their mark, title, scale, date, and initials of draughtsman.

The rivets on one drawing are quite apt to be all of the same size, so that a general remark, such as "All rivets $\frac{7}{8}$″○," will often be all that is needed. In like manner the sizes of the open holes can generally be covered by some such remark as "All open holes $\frac{15}{16}$″○, unless marked otherwise." If there are any pin-holes or bolt-holes of a different size, their size is then specially marked near them on the drawing, with an arrow running to them.

In giving the number of pieces wanted, and their marks, they can be given thus: "2 pcs. wanted, mark P_1R.'

The only title necessary is something of the following nature:

INCLINED END-POSTS

FOR

1—153′—0″ S. Tr. Thro' Span,

FOR

SHEFFIELD SCIENTIFIC SCHOOL.

It is a waste of time to print titles for such work. They should be legibly written in a large, plain hand. Script writing should be avoided, however. Each letter should be distinct, and separate from the others. The scale, date, and initials of the draughtsman should be written in small letters in the extreme lower right-hand corner of the drawing. It is often customary, after the word "scale," to put a dash, and omit giving the scale on the drawing. Writing the word "scale" shows that the draughtsman has not forgotten it, while the dash after it warns any one not to take distances from the drawing by scale. When the two halves of a member are alike, it is only necessary to show one-half in full, and, at most, the general outlines of the other half, placing on the drawing some such note as "This half exactly like other half." Or, if the two halves differ slightly, the note would be something like this: "All dimensions on this half, not marked otherwise, same as for other half."

Wherever it is possible to make two pieces alike, or only differing in right and left, it should always be done, as then the punching of the two pieces is alike, and a complete set of templets is saved. Having the pieces alike may also facilitate erection. In drawing lattice bars, it is only necessary to draw their centre lines, except for one or two at the ends, which should be drawn in full. If there is reason to fear rough handling of the iron in transit, it may be necessary to ship pieces loose, which could otherwise be shipped fast, but the more loose pieces the more field riveting, and field riveting is expensive, not so good as shop riveting, and delays erection.

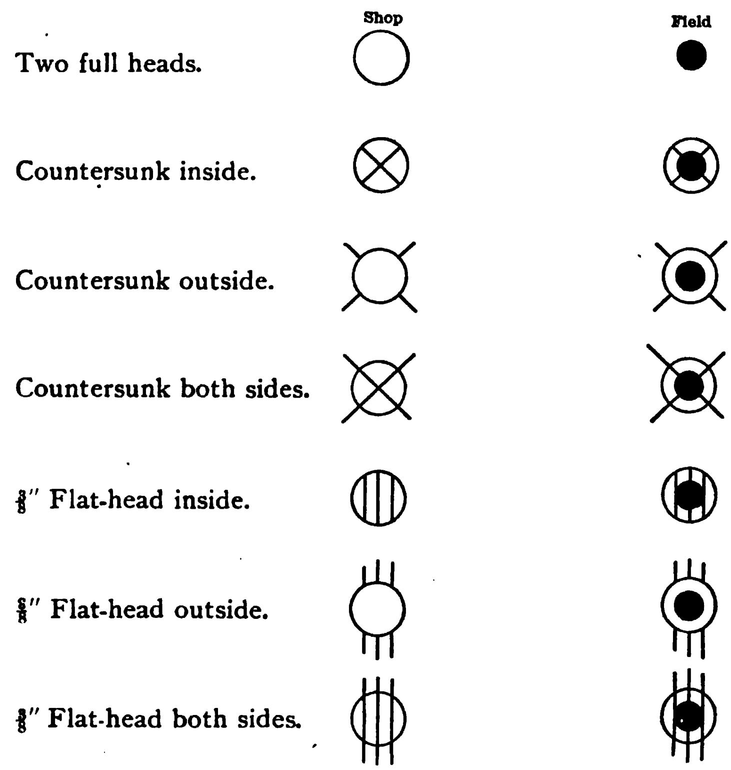
Rivets are denoted either by a cross or by a circle of the same size as the head. The latter method is about as quick and easy as the first, and shows more clearly what it is intended to represent.

Open holes through which rivets are to go in the field, are denoted by a blackened hole of the same size as the rivet.

A countersunk rivet is one which has either one or both of its heads flush with the plate. A flat-head rivet has either one or both of its heads flat, generally $\frac{3}{8}''$ high. Countersunk rivets are used only when it is necessary to get sufficient clearance, or in the bottom of a plate which rests on masonry, or another plate. If it is possible to substitute a $\frac{3}{8}''$ flat-head for a countersunk rivet it should always be done.

Pin-holes are too large to blacken, and should be hatched, to indicate that they are open.

The following Table gives Osborn's notation for rivets. This notation has now been very generally adopted by all the large bridge companies:



The foundation of the system is the diagonal cross to represent a countersink, the blackened circle for a field rivet, and the vertical stroke to represent a flattened head. The position of the cross with respect to the circle (inside, outside, or both sides) indicates the location of the countersink, and the number and position of the vertical strokes indicates the height and position of the flattened head. Any combination of field, countersunk, and flat-head rivets, liable to occur, may be readily indicated by the proper combination of these signs.

A point which comes up in the notation for rivets is, "Which side of the piece is inside and which outside?" About as good a way as any other is to let the outside be the near side, or side shown in the view in question; and to let the inside be the far side, or the side not shown in the view in question.

After laying out a complete system of rivets for any member, the draughtsman may check his addition by seeing that the sum of the rivet spaces and end distances are equal to the length of the member.

Allowing the rivets in the webs of girders, posts, chords, etc., to come opposite the rivets in the flanges should be carefully avoided. First, because it may necessitate hand driving the rivets; and, secondly, because if the member is in tension it will take out more section in a given line than if the rivets were staggered. When there are more than two consecutive rivet spacings alike, instead of giving them separately they should be given thus: "9 spaces @ 3″ each = 2′ 3″." This also applies to panels of lattice bars, which may be given thus: "11 panels @ 17″ = 15′ 7″."

Instead of giving the exact sizes of the pin-holes, it is preferable to give the sizes of the pins which are to go through them, thus: "Bored for $4\frac{1}{8}$″○ turned pin."

When angles are turned off, they should be given in feet and inches, not in degrees. This is done by giving the slope; that is, so many feet horizontal to so many vertical. Thus, a 53° angle may be given by a distance of 1′ $11\frac{1}{2}$″ horizontal to 2′ $7\frac{3}{16}$″ vertical. The templet makers can then lay the angle off directly from measurements. In some cases it is permissible to give the angle 45° in degrees. Thus, when there is a projecting corner, it may be ordered "clipped at 45°." But in all other cases angles should be given by their slopes in feet and inches.

In giving the sizes of pin-plates, battens, and other small plates, it is better to give these sizes in the nearest clear space to the plate, and draw an arrow to the plate, rather than to put the size directly on the plate, if in so doing it is necessary to crowd it in with rivet spacing and other data. The size of a plate should always be given thus, "10″ × $\frac{7}{16}$″ pl., $14\frac{1}{2}$″ lg.," the first being the width of the plate, that is, the direction at right angles to the fibres. A plate may thus have a greater width than length, as "24″ × $\frac{3}{8}$″ pl., 22″ lg." When lattice bars are in a position where they will be seen, they should have rounded ends. In giving their lengths, give them both from centre to centre of rivet holes and over all, thus, "All lattice 3″ × $\frac{3}{8}$″, $18\frac{13}{32}$″ c. to c., $22\frac{3}{32}$″ o. a." If the lattice is in a position where it will not be seen, the ends may be cut bevel.

In splicing the top chord the splice plates should be so arranged that all the field-driven rivets do not come in the same member.

For the sake of appearance, projecting corners of gussets and brackets should be clipped off.

When cover plates or stiff lateral bracing is used on plate-girder spans, care must be taken that the rivets in the flange do not come opposite those in the web, and also that the flange rivets do not come opposite the leg of a stiffener, otherwise the stiffener must be clipped or the rivet countersunk. To countersink one or two holes in a long plate requires a special handling of that plate, and is expensive. To clip the leg of the stiffener would be cheaper, but looks badly.

The number of views necessary to show a piece depends upon the piece and the amount of detail there is on it.

Generally a top view, an elevation, a sectional plan, and a couple of end views or sections will be all that is necessary.

The reason that a sectional plan is preferred to a bottom view is that, in the sectional plan, details are shown in the same relative position as in the top view. When the member has a system of lattice on both sides, in the top view the top lattice should be shown with full, and the bottom lattice with broken, lines. The bottom lattice should be shown in the top view, as the relative position of the two systems of lattice is thus clearly shown.

The following is a list of the shop drawings which would probably be required for an ordinary 150 foot pin-connected through span:

1. Inclined end-posts.
2. End chord sections.

3. Intermediate chord sections.
4. Intermediate posts.
5. Vertical suspenders, if other than forged eye-bars.
6. Portal bracing.
7. Intermediate upper bracing.
8. End lower struts.
9. Intermediate stringers.
10. End stringers.
11. Floor beams.
12. Pedestals.
13. Roller frames and rollers.
14. Wall plates.
15. Castings, such as filling rings.
16. Pilots for pins.

The last four items are often only sketched on the lists ordering the iron, instead of making a regular drawing.

The eye-bars, counters, lateral rods, pins, and blacksmith work are also in general only sketched on the order lists. For longer bridges, the number of drawings will increase, as several sheets will be necessary for the chords and posts. If the bridge is on a skew, the number of shop drawings needed will be nearly doubled.

To sum up: Make all drawings clear and distinct. Do not crowd any of the dimensions or data so they cannot be read. Do not waste time on fancy lettering. Make figures, especially, plain; words may be guessed at, but not figures. Aim to simplify the work of the men in the shops.

NOTE.—The drawings at the end of this work can now be carefully inspected, referring to the data of the strain sheet. Then, with the data already made out in the preceding chapter, the student will be prepared to make his own working drawings for the span figured out. By application to our large bridge companies, he can also undoubtedly obtain blue print copies of working drawings in great variety. A careful study of these will do much for the student.

CHAPTER XII.

THE ORDER BOOK, SHIPPING, AND INSPECTING.

AFTER the shop drawings have been made, or while they are in course of preparation, all the iron needed must be ordered of proper dimensions for the shop. The orders are grouped according to some convenient system on sheets properly ruled and headed, and these sheets when bound together constitute the "Order Book." The Order Book thus contains a list of every piece of iron which goes into the bridge. The forms for the Order Book are various. Without professing to give the actual practice of any company, we shall give in this chapter a series of forms which will illustrate sufficiently well how such a book may be made, and the information it should contain.

The different forms may be classified as follows:

Form A. Castings.
Form B. Built members.
Form C. Eye-bars and upset rods.
Form D. Pins, pin-nuts, and pilots.
Form E. Bolts and small forgings.
Form F. List of shop rivets.
Form G. List of field rivets.

As an example of Form A, we give the following:

FORM A.

CASTINGS.

FOR 117′ 6″ D. TR. TRO. SKEW SPAN, *for*...

	NO. OF PCS.	MARK.	DESCRIPTION.	FACINGS.	PIN-HOLE BORED FOR.	DIA. OF TENON TURNED.	PATTERN NO.	DRAWING NO.	SHIPPER'S NO. OF PCS.	GEN'L MARK.
1	2	C_1 R	Check washers.	Rough.	Core	$2\frac{13}{16}''$	New.	5000	2	C_1 R
2	8	L P_1	$7\frac{1}{2}'' \times 2\frac{1}{4}''$ collars.	Rough.	Core	6″	New.	5000	8	L P_1
3	1	W D_5	Wall plates, $13\frac{3}{8}''$ between lugs.	Rough.	$15\frac{1}{8}'' \times 1\frac{31}{32}''$	× 18″	New.	5010	1	W D_5
4	2	28 N	Bed plates, 29″ between lugs.	1	$37'' \times 1\frac{7}{8}''$	× 30″	New.	5011	2	28 N
5										
6										
7										
8										
9										
10										

The span is 117′ 6″, double track, through skew.

In the first column is the number of the item. The sheet may be of any length, to accommodate any desired number of items, as for instance 30, on a page. In the second column the number of pieces wanted is given, and in the third the mark which is to be put on each. In the fourth is a description, and when necessary sketches may be made in it. In the fifth is given the number of facings. In the sixth the size of pin-hole, if the hole is bored. In the seventh the diameter of any turned tenons which the casting may

have. In the eighth the pattern number, and in the ninth the drawing number, which enables the working drawing for the piece to be found.

In the tenth column is given the shipper's number of pieces, which will generally be the same as the number of pieces in the second column; but if two pieces cast separately are bolted together for shipment, the shipper's number would be different. In the last column the general mark of the piece is given, which may differ from that in the third column for the same reason.

We have filled in a few items for illustration merely. The first item is two check washers which are not faced smooth, and are therefore marked "rough." As they have no bored pin-hole nor a tenon, it is simply noted that the core is $2\frac{7}{8}''$ diameter. The patterns being new, shows that there are no old patterns of the size required.

The next item is 8 collars, $7\frac{1}{2}''$ outside diameter and $2\frac{1}{4}''$ thick, with a core 6″ diameter, no facings, bored pin-holes, or tenons.

The next item is a wall plate $13\frac{3}{8}''$ between the "lugs" or projections for confining the rollers, faced on one side.

All the castings required are thus entered, one by one, and the iron required can be furnished, put into the shop, and finished according to the drawing for each piece.

FORM B.

BUILT MEMBERS.

FOR 117′ 6″ D. TR. TRO. SKEW SPAN, *for*..

	NO. OF PCS.	MARK.	SHAPE AND SIZE.	TOTAL LENGTH.	HOW CUT.	MAY VARY.	BEVEL.	TOTAL NO. AND DESCRIPTION OF FINISHED PIECES.	DRAWING NO.	NO. OF PCS.	GEN'L MARK.
			″ ″	′ ″		″					
1	8		12 × $\frac{5}{16}$ Pls. Web.	26 0	Sq.	± $\frac{1}{4}$					
2	8		$11\frac{1}{4}$ × $\frac{5}{16}$ Pls. Pin.	23	Sq.						
3	8		$11\frac{1}{4}$ × $\frac{5}{16}$ Pls. Pin.	2 1	Sq.					2	P_2 R
4	8		6 × $\frac{5}{16}$ Fillers.	23	Sq.			Riveted up into 4 Int. Posts, 26′ $0\frac{1}{4}$″ lg. o. a., 25′ $0\frac{1}{32}$″ c. to c. of pin-holes, $12\frac{1}{4}$″ o. to o. of angles.			
5	8		6 × $\frac{5}{16}$ Fillers.	2 1	Sq.				7081		
6	16		$17\frac{1}{2}$ × $\frac{5}{16}$ Pls. Battens.	21	Sq.					2	P_2 L
7	208		$2\frac{1}{4}$ × $\frac{3}{8}$ Pl. Lattice Bars.	$20\frac{1}{4}$	Temp.	$17\frac{1}{2}$ c. to c.					
8	16		3 × 3 angles, 18 p. y.	26 0	Sq.	± $\frac{1}{4}$					
9	4		5 × 3 angles, 28 p. y.	12	Sq.						
10	4		$3\frac{1}{2}$ × 3 angles, 23 p. y.	$11\frac{1}{4}$	Sq.						
11	4		3 × $\frac{5}{16}$ Fillers.	6	Sq.						
12											
13											
14	12		36 × $\frac{3}{8}$ Sh. Pl. S. S. Web.	23 5	Sq.	± $\frac{1}{8}$	2 corners clipped, 4 kinds.			3	I D_1
15	48		6 × $\frac{5}{8}$ Pl. Fillers.	2 3	Sq.			Riveted up into 12 Int. Track Stringers, 23′ $5\frac{1}{2}$″ back to back of end stiffs., 36″ deep.	9158	3	I D_2
16	24		6 × 4 angles, 68 p. y. Top Fl.	23 $3\frac{1}{4}$	Sq.	± $\frac{1}{8}$	4 kinds.				
17	24		6 × 4 angles, 62 p. y. Bot. Fl.	23 5	Sq.	± $\frac{1}{8}$	All alike.			3	I D_3
18	48		6 × 4 angles, 39 p. y. End Stiff.	2 $10\frac{1}{4}$	Temp.	R.& L.				3	I D_4
19	96		3 × $2\frac{1}{2}$ angles, 13 p. y. Int. Stiff.	2 $10\frac{1}{4}$	after	bend'g					

FORM B. BUILT MEMBERS.—Under the heading "Total Length," is given the length over all. Under "How Cut," square denotes that the ends are cut perpendicular to the length of the piece, and can therefore be sheared off without a templet. A templet is made of wood of the exact shape desired, and is laid on the iron, its ends and edges marked, and the iron is then cut by these marks. Under the heading "May Vary," the margin of variation of length is put. Thus ± ¼" means that the piece must not be longer or shorter than the required length by more than ¼", while − ¼" would indicate that it must not be shorter than ¼", and must not be longer than order.

In the 7th item, for lattice bars, two lengths are given, 20¼" length over all, and 17½" length c. to c. of rivet-holes.

There is no "bevel" in the items given. A piece whose ends are cut slanting to its length, is beveled, and the bevel given is the length of the projection, in the direction of the length, of the slant side. It is the base of the right triangle, of which the slant side is the hypothenuse, and the width of piece the other side.

In item 14, "Sh. Pl." stands for "Sheared Plate." This directs the mill to supply a sheared plate instead of a universal rolled plate. The letters "S. S." stand for "Strain Shear." This indicates the character of strain the plate is subjected to, and may influence the manner in which the iron is piled for rolling in the mill. The intermediate stiffeners, which are given in the last item, are bent around the flange angles, to avoid putting fillers underneath the stiffeners. The length of these stiffeners is given 2' 10¼" *after bending*. A sketch can be made to illustrate.

FORM C.

EYE BARS AND UPSET RODS.

FOR 117' 6" D. TR. TRO. SKEW SPAN, *for*..................

	NO. OF PCS.	MARK.	SHAPE AND SIZE.	ROUGH LENGTH FOR M. O.	FINISHED LENGTH. O. A.	FINISHED LENGTH. C TO C. C. TO END.	PIN-HOLE BORED FOR	SIZE OF RING OR UPSET.	NO. OF DIE.	SHIPPERS NO. OF PCS.	GEN'L MARK.
				' "		' "	"	" "			
1	2	L T_2	6" × 1¾" eye bar.	38 6		35 5 21/32	5 11/16 4 15/16	14 × 1⅞ 14 × 1⅞	179 179	2	L T_2
2											
3	4	S_2	5" × 1½" eye bar.	26 6		23 5 15/16	4 15/16	12 × 1⅝	93	4	S_2
4											
5	4	S C	2⅛" sq. eye and upset rod.	33 3		29 11⅜	4 15/16	9 × 3	Rt. Th'd.		
6	4	S C	2⅛" sq. eye and upset rod.	7 9		4 0	4 15/16	9 × 3	Left Th'd.	4	S C
7	4	S C	Cleveland Turn buckles.	2d length	9" clear		3" th'd	R & L			
8											
9											
10											
11											
12											

FORM C. EYE BARS AND UPSET RODS.—In the illustration given, item 5 is a counter, made of square bar iron in two pieces, these pieces united, before shipment, by a Cleveland Turn buckle. As drawings are seldom made for counters, a sketch of counter can be made to give any dimensions not provided for in the columns. The lengths 29' 11⅜" and 4' 0" are from the centre of pin-hole to end of rod, not from c. to c., as is the case for eye bars. In item 1, the pin-holes at each end are of different sizes, as they fit different-sized pins. In item 2, both ends go on same-sized pins.

FORM D.

PINS, PIN-NUTS, AND PILOTS.

FOR 117' 6" D. TR. THRO. SKEW SPAN, *for*..

	NO. OF PCS.	MARK.	SHAPE AND SIZE.		TOTAL LENGTH.	SKETCH.	DESCRIPTION.	SHIPPER'S NO. OF PCS.	GEN'L MARK.
			ROUGH.	FINISHED.					
1	4	L E	6"	$5\frac{11}{16}$"	2' $2\frac{3}{4}$"			4	L E
2									
3									
4									
5	1	L E P	6"	$5\frac{11}{16}$"	7	Pilot with $4\frac{5}{8}$" th'd to	fit on L E	1	L E P
6									
7									
8									
etc.									

MALLEABLE NUTS FOR ABOVE PINS.

	NO. OF PIECES.	MARK.	HOLE.	DIA. OF THREAD.	SHORT DIAM.	THICKNESS.	THREAD.	RECESS.
24	32	P_{10}	$3\frac{3}{4}$"	$3\frac{7}{8}$"	$6\frac{1}{4}$"	$1\frac{1}{2}$"	1"	$\frac{1}{2}$"
25	8	P_{13}	$4\frac{1}{2}$"	$4\frac{5}{8}$"	7"	$1\frac{1}{2}$"	2"	$\frac{1}{2}$"
26								
27								
28								
etc.								

FORM D. PINS, PIN-NUTS, AND PILOTS.—In the columns headed "Shape and Size," the rough diameter gives the size of iron as rolled, the finished diameter is that to which the pin is turned down. The pilot protects the thread of the pin while it is being driven. The pin-nuts have a recess on inside, so as to fit over the head of the pin. Item 1 should have a sketch, giving length of pin between shoulders, length of pin over all, length of threaded ends, and diameter of thread.

FORM E.

BOLTS AND SMALL FORGINGS.

FOR 117' 6" D. TR. THRO. SKEW SPAN, *for*________________________.

	NO. OF PIECES.	MARK.	SHAPE AND SIZE.	LENGTH FOR M. O.	FINISHED LENGTH.	SKETCH.	SHIPPER'S NO. OF PIECES.	NOTE FOR SHIPPER.	GEN'L MARK.
1	14	①	$1\frac{1}{4}$" Foundation Bolts.		18"		14		①
2	14	①	Stand. Hex. Nuts for	$1\frac{1}{4}$"	th'd	$1\frac{1}{4}$" thick.		Fast on.	①
3									
4									
5									
6									
7									
8									
9									
etc.									

FORM E. BOLTS AND SMALL FORGINGS.—We can use this form for bolts and blacksmith work, such as loop swivels, clevises, and other small forgings. The example given is for a swedged foundation bolt (page 394). A sketch should be made, giving length over all, length of thread on end, distances between indentations or shoulders, etc.

FORM F.

LIST OF SHOP RIVETS.

FOR 117′ 6″ D. TR. THRO. SKEW SPAN, *for*..............................

	NO. OF PIECES.	MARK OF MEMBER.	SIZE OF RIVET.	LENGTH UNDER HEAD.	LOCATION.	DRAWING NO.
1	408	1 Pc.	$\frac{7}{8}$″	$2\frac{3}{8}$″	button. Cov. Pl. + angles, also Top Fl. angles + Web. Also Splice Pl. + Web.	
2		S C				
		and				
3	12	1 Pc.	$\frac{7}{8}$″	$3\frac{5}{8}$″	countersunk. Ins. Pl. + Web + Filler + Outs. Pl. + Jaw Pl.	
4		S A				
5						
6						
7						
8						
9						
10						
etc.						

FORM F. LIST OF SHOP RIVETS.—We have given an illustration of round-head rivet, and also of a countersunk rivet. The length of round-head and flat-head rivets should be given from underneath the head, the other head is made when the rivet is put in. The length of a rivet with one round and one countersunk head should be given from underneath the round head, the countersunk head being made when the rivet is put in. A rivet with two countersunk heads should have its length over all given. Sketches should be inserted for items 1 and 3, showing the rivet with length marked.

The following Table gives the additional length for making head, to be added to length of metal passed through.

DIAMETER OF RIVET.	ADDITIONAL LENGTH REQUIRED TO FORM ONE HEAD IN PASSING THROUGH THE FOLLOWING THICKNESSES OF METAL.			
	$\frac{3}{4}$″ and below.	$1\frac{1}{2}$″ and below to $\frac{3}{4}$″.	$2\frac{1}{2}$″ and below to $1\frac{1}{2}$″.	above $2\frac{1}{2}$″
″	″	″	″	″
$\frac{1}{2}$	$\frac{7}{8}$	1	1	1
$\frac{5}{8}$	1	$1\frac{1}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$
$\frac{3}{4}$	$1\frac{1}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$
$\frac{7}{8}$	$1\frac{1}{4}$	$1\frac{3}{8}$	$1\frac{1}{2}$	$1\frac{5}{8}$

No percentage for waste need be added to the number of shop rivets ordered.

FORM G.

LIST OF FIELD RIVETS.

FOR 117′ 6″ D. TR. THRO. SKEW SPAN *for*..............................

	NO. OF PIECES.	SIZE OF RIVETS.	LENGTH.	LOCATION.	SHIPPER'S NO. OF PCS.
		″	″		
1	50	$\frac{3}{4}$	$2\frac{3}{8}$ button.	Int. knee brace + Gusset, also Int. knee brace + Bracket.	50
2					
3	100	$\frac{7}{8}$	$3\frac{1}{4}$ "	Hood + Pl. on Portal + Flange End Post,	100
4				also Stiff. Bracing + Loose Pl. + Stringer Flange.	
5					
6					
etc.					

FORM G. LIST OF FIELD RIVETS.—Anywhere from 5 % to 25 % should be added for waste, due to burning of rivets in the field, etc. The greater the number of short rivets the less the percentage allowed, and the greater the number of long rivets the greater the percentage allowed, because a short rivet can be made from a long one, if the short rivets run out. It is a help to the erectors to order each item separately, even if the rivets are the same size and length, as they can thus see the number required for any particular joint.

SHIPPING.—Every piece of iron shipped, except rivets and bolts, should have its mark for the erectors.

Rivets and bolts are shipped in boxes, and have, in the case of rivets, their size marked on the outside. In the case of bolts the bolt mark is on the outside, if the bolt has a mark, if not, their size is given on outside of box.

The mark of a member should not consist of more than three figures or letters if possible, and it is well to have the marks have some meaning so far as may be, as P, R, for intermediate post, " right." A member is right or left when it can only be used on one side, and is not reversible, so as to be used on the other. Two members, as posts, may be exactly alike in all respects, except that the addition of a bracket, or some similar addition, on one side, may prevent it from being used on the other truss, as in that case the brackets would come on the wrong side.

A list of all the iron ordered can be sent to the erectors, and with an erection plan, consisting of a skeleton outline of the truss, with the mark and location of every piece, they can erect the bridge without the shop drawings.

In general a piece over ten or twelve feet high cannot be shipped as a whole, but must be spliced in the field. Girders seldom exceed this height. They can be shipped on two or three flat cars coupled together, properly braced by wooden braces.

Chord sections, posts, eye bars, etc., can be shipped in one piece. A deep portal, over, say, twelve feet high, will have to be shipped loose and riveted up in the field, as it cannot be taken on its side, or vertical.

A short deck girder span would have the girders shipped separately, as, if the transverse bracing were riveted on in the shop, and the whole shipped together, it would be clumsy to manage in erection, and the transverse bracing liable to injury.

INSPECTING.—When, as is sometimes the case, iron bridges have been standing for years, which were originally designed for much lighter loads than those in present use, a careful examination is necessary. A judicious strengthening of such a structure, based on such examination, may prolong its life for many years. A neglect of such examination may result in disaster and loss of life. The fact that a structure has fulfilled its duty for many years is no evidence of its present efficiency, and sometimes is quite the contrary.

The examination should consist in a careful external inspection for external evidence of weakness, and calculations of the strains to which each member is subjected, based upon the present traffic and the actual dimensions, as given by the working drawings or by actual measurement. Both of these investigations are necessary, as a bridge may be weak and give no external evidence of its condition; and, on the other hand, there may be defects of construction, material, manufacture, or injuries, which can only be discovered by actual inspection. All bridges should have such a field examination at least once a year.

As rolling-stock increases in weight and heavier locomotives are built, many iron bridges carry daily loads in excess of those assumed in the original design. Such structures, however, are often sufficiently strong to serve their purpose for years, or may be made so by proper strengthening. Others possibly require immediate removal. Constant and thorough examination thus becomes more imperative every year.

Of equal importance is the inspection of structures in process of construction, to

insure that the requirements of the specifications are complied with. (See Cooper's specifications under the head of "Inspection.")

The following paragraphs are from the Atlantic Coast Line's specifications: "For wrought iron a set of specimens shall include one specimen tensile, transverse or compressive test, and one specimen bending tests. A set of specimens for channels and beams shall be understood to include one set, as above specified, from the web, and one set from the flange. The test specimens and the pieces from which they are taken shall be marked with the same stamp, so that these pieces can be found if they prove defective. The test specimens shall be prepared from pieces selected by the inspector, and shall be sufficient in number to fairly represent, in his judgment, the material furnished, not to exceed the following, however, at the option of the inspector:

"On any contract for wrought iron a minimum number of ten sets of specimens shall be tested, and when the contract is for an amount exceeding 100,000 pounds, one set of specimens shall be tested for each additional 20,000 pounds, provided that each order is completed at one rolling; when this is not the case, the requirements of the preceding sentence may be applied to each rolling.

"To determine the strength of the eyes, full-sized eye-bars and rods with eyes may be tested to destruction. Notice will be given in advance of the number and size required, so that the material can be rolled at the same time as that for the structure.

"The following tests shall be made at the option of the inspector:

"One full-sized bar or rod for each 25 bars or rods of wrought iron, unless a lot contains less than that number, in which case a like number may be required for each lot. Any lot of bars or rods from which full-sized members are tested shall be accepted, provided:

"First. That the bar or rod tested does not break in the head or neck.

"Second. That its quality is not inferior to that required by the specifications.

"All full-sized built members taken for tests, and which prove to be good and acceptable material, shall be paid for by the railroad company, at the net cost less its scrap value; but no payment shall be made for any material, workmanship, or testing of any member which proves defective.

"Test specimens from universal mill plates shall not be taken from the edge of the plate. No greater deficiency than 2½ per cent will be allowed between the estimated and the actual weight of any piece of material.

"The acceptance of any material by the inspector or his assistants shall not prevent its subsequent rejection, if found defective, after delivery; and such material shall be replaced by and at the expense of the contractor."

The above gives some idea of the number of tests required, and of the responsible duties of the inspector. It is also his duty to detect all shop errors before shipment. Thus, through error, track stringers might be made too long, so as not to fit between posts, or the posts may be riveted up so that the rivet-holes for the floor beams come in the outside web. Such errors affect the erection. Other errors may even endanger the structure, as when a plate girder should have a ¾" cover plate on the top flange, and a ½" cover plate on the bottom flange, and the plates get reversed in the shop. Mistakes affecting erection, when not corrected in the shop, cause delay in the field, and are an indirect expense to the purchaser. Even if the manufacturer is the erector also, there is annoyance and expense to the purchaser. Correction in the field of shop errors is also liable to be done hurriedly and incompletely, to the detriment of the structure. Members having errors which reduce strength but do not affect erection are not so likely to be sent back for correction.

These facts show the importance, to the purchaser, of having an inspector not only for the mill, but also for the shop.

The specifications quoted show that the railroad companies as purchasers appreciate this importance. As guardians of the public safety it is in some cases perhaps to be regretted that they do not seem to equally appreciate the importance of thorough and regular inspection of their bridges after erection.

CHAPTER XIII.

THE ERECTION OF ENGINEERING STRUCTURES.

BY JOHN STERLING DEANS, M. AM. SOC. C. E., ENGINEER THE PHŒNIX BRIDGE COMPANY.

WITHIN the past few years the subject of the final "Erection of Engineering Structures" has become a much more important branch of engineering work, and this department, which has until lately been somewhat slighted by most construction companies, has been found to demand the same careful supervision and attention as are called for in the designing of the permanent structures themselves.

In the past it was not so much what was an economical "false work" and "traveller," as what was "strong enough," and the competition amongst contractors, and the cost of materials *then*, demanded nothing different; now these conditions have changed, and the margins upon which contracts are secured or lost are daily becoming less.

These temporary structures, therefore, must be of the most economical design, and their principal members designed and proportioned for the exact loads which will come upon them. In view of these facts, it seems eminently proper to give this subject of "Erection" a place in the text-books on "The Strains in Framed Structures," that students and others interested may become more familiar with this important branch of engineering work.

To some the present short chapter on this subject may seem to be written too much in detail, and contain points which are so well and generally known as to hardly warrant insertion in such an article; but it must be borne in mind that this is written primarily for students, and those who have not, as yet, been engaged in the active work of the profession.

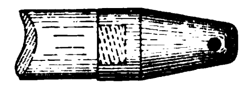
POINTS TO BE CONSIDERED IN DESIGNING PERMANENT STRUCTURES.—From holding the final erection in view does the present type of "American Pin-connected Truss" owe its design, as much as to any other single fact. This truss requires the least amount of field work; its joints being pin-connected there is no field riveting except that for the floor system and minor details, and field rivets should be as few as possible, since, owing to the fewer facilities for doing the work, they cannot be driven so well, nor as cheaply, as in the shops; most specifications therefore require as high as 20 per cent. excess for field-driven rivets. In many instances, where rivets are hard to drive, it is much better to use turned bolts in drilled or reamed holes. In a large bridge lately built in the West, the contractors used turned bolts for all the floor connections, believing there was a saving in so doing.

Another important matter to consider, especially when the structure is to span a stream subject to sudden rises, is to so design the connections that the trusses may be swung and be self-supporting in the least possible time, leaving the floor beams, stringers, outer chord bars, and most of the bracing to be put in later, without the liability of being washed out.

Aside from the economy in the shops, the number of pieces composing a structure should be as few as possible, by making long panels and concentrating the metal, since it requires about as much time and power to handle and connect a large piece as a small one.

Allow plenty of clearance, at least $\frac{1}{4}''$, after due allowance for packing of plates and rivet heads, between the jaws of built members, and also in packing members between the jaws of built sections; to keep a whole gang waiting in the field, while chipping is being done, is a very expensive piece of experience.

Always furnish pilot nuts for each size of pin, with an easy draught, to facilitate the centring and connection of members at panel points. These pilots should have a draught of at least $1\frac{1}{2}$ inches, as in the rough assembling of panels the bars and other members comprising the joint are often over 1 inch short, and the pilot on the end of the pin will catch and centre these members.



Carefully mark the individual pieces composing each riveted joint, which have been assembled, faced, and reamed, or drilled together, in the shops, with some distinguishing letter, so that the same pieces may be put together in the field, saving all unnecessary fitting.

Anchor bolt holes should be so arranged that drilling of masonry can be done after the span has been connected and swung, and its exact location on the supports established.

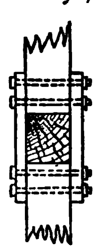
Have as few adjustable members as possible, since such members work loose, and the adjustment is usually allotted to those who fail to realize its importance in the proper working of the structure under load. Many other points which should be borne in mind to facilitate erection might be mentioned, but those indicated are the ones which most frequently occur, and which should be especially considered in the designing of details.

MATERIALS AND TOOLS USED IN ERECTION.—The principal timber used in these temporary structures is "Long leaf Southern Yellow Pine," owing to its more uniform strength and reliability.

Where lightness is an item to be considered, and where it is necessary to do considerable framing, "White Pine" is used. "Oak" is used rarely, owing to its weight and expense. "Hemlock" should be discarded except for the most insignificant work, owing to its unreliability. "Spruce" is better than hemlock, but rarely used.

For "piling" yellow pine is most generally used, and these piles can be had in perfect straight lengths up to 70 feet. In localities where yellow pine is scarce and expensive, chestnut, oak, beech, hickory, or any of the hard woods may be used; the latter, however, being harder to remove after the work is finished, when it is only necessary to break off the piles. No pile should be used less than 8 inches full diameter at the small end, and it should be straight throughout its length.

It should never be left out of sight that false works are simply temporary structures, and only intended to answer as a support for a very limited period, and therefore the material used should be of suitable quality to answer such a purpose, with due regard to safety; and the least possible expense should be expended upon it in framing, etc. In most cases good round timber may be used for legs instead of square stuff; and where it is necessary to make trestles of two or more stories, it is rarely necessary to "dap" the legs into the caps; abutting the ends of legs or abutting the legs against the cap and splicing the joint with a piece on each side, answering every purpose. This same idea should be held uppermost in the framing of all joints, only expending the amount of work on each which the actual safety of the structure demands—nothing more.



For stringers and other members subject to bending, the strain on the extreme fibres for good yellow pine may be taken as high as 1,600 lbs. per square inch; and upon this assumption the following table is figured, which table shows the capacity in bending moments (foot-pounds) for 1,600 lbs. strain on the extreme fibres:

TABLE I.

Width Inches.	Depth of Beam in Inches.											
	6	7	8	9	10	12	14	16	18	20	22	24
3	2400	3267	4266	5397	6666	9600	13066	17066	21600	26666	32266	38300
4	3200	4355	5688	7200	8888	12800	17421	22755	28800	35555	43020	51200
5	4000	5444	7101	9000	11112	16000	21778	28441	36000	44444	53770	64000
6	4800	6533	8533	10800	13333	19200	26133	34133	43200	53333	64533	76800
7	5600	7623	9956	12600	15623	22400	30489	39756	50400	62223	75289	89600
8	6400	8711	11378	14400	17777	25600	34844	45511	57600	71110	86044	102400
9	7200	9800	12800	16200	20000	28800	39200	51200	64800	80000	96800	115200
10	8000	10889	14222	18000	22222	32000	43556	56890	72000	88889	107556	128000
12	9600	13066	17067	21600	26667	38400	52266	68266	86400	106666	129066	153600

TIMBER COLUMNS.—In members subject to compression good yellow pine may be strained as high as 1,100 lbs. per square inch for columns when the ratio of least side to length does not exceed 20; for columns over this length the unit strains should be reduced by the following formula, $U = 1500 - 18\frac{l}{d}$, where U equals unit strain, l equals length in inches, and d equals the least side in inches. White pine should be strained about 30 per cent. less than above. No column should be used longer than fifty times its least width. The following table has been figured for timber columns, using the formula given:

TABLE II.

$\frac{l}{d}$	Yellow Pine. $1500 - 18\frac{l}{d}$	White Pine. $1000 - 18\frac{l}{d}$	$\frac{l}{d}$	Yellow Pine. $1500 - 18\frac{l}{d}$	White Pine. $1000 - 18\frac{l}{d}$
20	1140	640	32	924	424
22	1104	604	34	888	388
24	1068	568	36	852	352
26	1032	532	38	816	316
28	996	496	40	780	280
30	960	460	42	744	244

For structures where traffic is carried during the erection, the strains per square inch in those members supporting the live load should be reduced by 20 per cent. from the unit strains derived by using Tables I. and II.

PILING.—In the driving of piles of sizes usually used in false work, viz., 8″ to 10″ diameter at small end and 60 feet long, a hammer should be used weighing about 2,400 pounds, which should have a final fall of 30 feet. After driving piles on this plan and into the ground 15 to 18 feet, if the pile does not penetrate more than two inches under the last blow, a load of 18 tons may safely be placed upon it. Formulæ for obtaining the safe bearing values of piles are numerous, but none are entirely satisfactory, nor can they be relied upon, as so much depends upon the particular soil into which they are driven, and other attendant circumstances. The formula proposed by Sanders will answer for most cases, and by using a factor of 10 a safe result is obtained,

$$T = \frac{wh}{S};$$

where T = ultimate bearing value; w = weight of hammer in pounds, h = height of last fall in inches, S = penetration of pile under last blow.

PRICES AND SPECIFICATIONS.—Good long leaf Southern yellow pine can be bought in the northern market of suitable sizes for $25 per M., and at the mills in the South as low as $11 per M. The following specification answers for false work lumber:

"To be long leaf southern yellow pine, cut from sound, untapped trees; to be free from large or loose knots and other material defects; straight, well manufactured, true and full to sizes given."

Good oak costs from $15 to $30 per M., according to location. Specification as follows:

"To be cut from sound, live trees, straight-grained, free from knots, wind shakes, and other imperfections."

The specifications for material for permanent structures of course are more severe; generally calling for three corners to show heart lumber throughout the length of the piece, and the remaining corner may show sap wood for $\frac{1}{10}$ the width of its face. Yellow-pine piling costs at the site from 6 to 10 cents per lineal foot.

"Piles must be cut from sound trees, not less than 8 inches in diameter at small end, and straight throughout their length."

ROPE.—For hoisting and rigging purposes the best manilla rope should be used. It is sold in coils containing from 950 feet in the larger sizes to 1,100 feet in the smaller sizes. The following table shows the weight per foot and the ultimate strength of the usual sizes. The "working strain" is usually taken at $\frac{1}{4}$ of the ultimate. Present price of rope, $12\frac{1}{2}$ cents per pound.

TABLE III.

DIAMETER.	WEIGHT PER FOOT.	ULTIMATE STRENGTH.	DIAMETER.	WEIGHT PER FOOT.	ULTIMATE STRENGTH.
$\frac{3}{4}$"	.17	3900 lbs.	$1\frac{1}{4}$"	.47	10600 lbs.
$\frac{7}{8}$"	.25	5700 "	$1\frac{1}{2}$"	.75	16900 "
1"	.30	6750 "	2"	1.30	29300 "

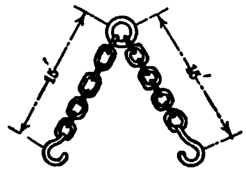
Wire rope is also used for guys, etc., and occasionally for hoisting purposes, but for general practice the manilla is better and cheaper. Only when the wire rope is used for hoisting and running rapidly, making it liable to become heated, is it necessary to use a wire centre; in all other cases the wire rope should be laid with a hemp centre. When it is necessary to use metal, it is generally better to select steel rope, as it is much lighter and stronger than the iron. The following table gives the weight per foot, ultimate strength, and cost of the usual sizes. Working strain taken at $\frac{1}{4}$ ultimate.

TABLE IV.

DIAMETER.	WEIGHT PER FOOT, *Iron.*	ULTIMATE IN TONS, 2000.	WEIGHT PER FOOT, *Steel.*	ULTIMATE IN TONS, 2000.	PRICE.
$\frac{3}{4}$"	.88	8.8	.88	17.	$1\frac{1}{2}$" Steel, 40 cts. per ft.
$\frac{7}{8}$"	1.12	12.3	1.12	22.	$\frac{3}{4}$" " 19 " "
1"	1.50	16.0	1.50	30.	
$1\frac{1}{4}$"	2.28	25.0	2.28	44.	$1\frac{1}{2}$" Iron, 30 cts. per ft.
$1\frac{1}{2}$"	3.37	36.0	3.37	62.	$\frac{3}{4}$" " 9 " "

CHAIN.—Nothing but the best material should be used in chains, and they should be of the most approved manufacture and nothing smaller than $\frac{3}{4}$" diameter of iron in links should be allowed. These chains are used principally in hoisting, and made usually about 25 or 30 feet long, with a hook at each end and a large ring in the centre.

CRABS.—For hand power hoisting "A" crabs are used (see Sketch 1) having a drum around which the rope is wound. In some cases, as derricks, or when attached to the legs of the traveller, a "square-framed" crab is more convenient.



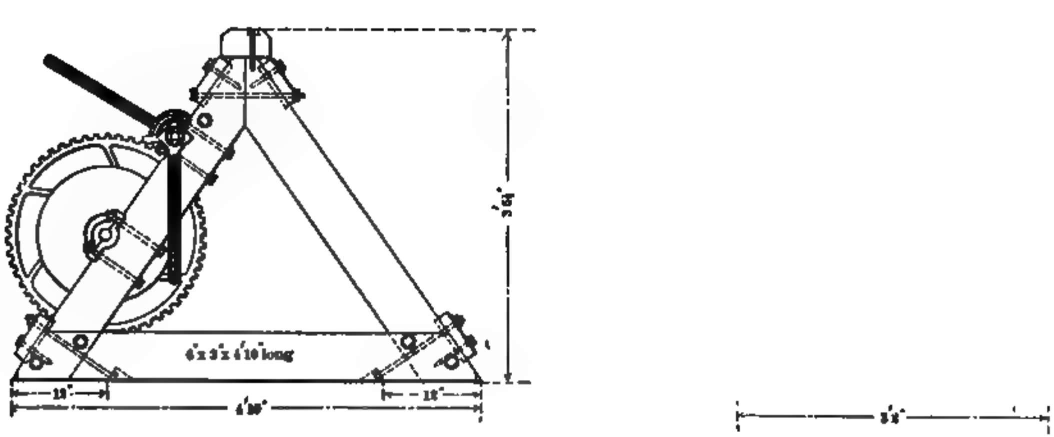
BLOCKS.—All blocks should be of the most approved pattern, extra heavy strapped, and metalline bushed sheaves (see Sketch 2).

HYDRAULIC AND SCREW JACKS.—Jacks are indispensable where it is necessary to raise or lower heavy weights, and are especially useful when ready to swing the permanent truss free of the false work, in raising the truss to relieve the pressure on the blocking, so that it can be removed, and then lowering the span (see Sketch 3).

ENGINES.—For hoisting, the 4-spool engine is usually used (see Sketch 4), the hoisting-rope being wrapped around the spool. For pile driving the double-drum engine is used (see Sketch 5).

LIST OF TOOLS.—The preceding are the main materials and tools used in the erection of structures; the following will be found an average list of the tools required for the erection of particular classes of work:

SPANS UP TO 100' AND WORKING 30 MEN.	SPANS 100' TO 300' AND WORKING 75 MEN.	SPANS 300' TO 600' AND WORKING 200 MEN.
4 Sets 10" Double Blocks.	4 Sets 16" Triple Blocks.	20 Sets 16" Triple Blocks.
4 Sets 8" Double Blocks.	6 Sets 14" Double Blocks.	20 Sets 14" Double Blocks.
2 10" Single Blocks.	6 Sets 12" Double Blocks.	20 Sets 12" Double Blocks.
4 10" Snatch Blocks.	8 Sets 8" Double Blocks.	10 Sets 10" Double Blocks.
	8 12" Single Blocks.	20 Sets 8" Double Blocks.
4 1" Lines, 160' each.	8 12" Snatch Blocks.	16 14" Snatch Blocks.
4 ¾" Lines, 160' each.	8 1¼" Lines, 300 feet each.	10 12" Snatch Blocks.
6 1" Hand Lines, 40' each.	6 1¼" Lines, 200 feet each.	20 12" x 14" Single Blocks.
20 Lashings, 30' each.	10 1¼" Hand Lines, 60 feet each.	10 Coils 1½" Rope.
8 Rope Slings.	40 1¼" Lashings, 40 feet each.	15 Coils 1¼" Rope.
	20 Slings.	10 Coils 1" Rope.
4 "A" Crabs.	2 Coils ⅞" Rope.	10 Coils ⅞" Rope.
1 Axe.	2 Coils 1" Rope.	20 1¼" Hand Lines, 80'.
1 Cross-cut Saw.	1 Derrick.	40 1¼" Lashings.
1 Man Saw.	4 "A" Crabs.	40 Slings.
4 ⅞" Crank Augers.	2 Square Crabs.	4 Derricks.
2 Iron Crowbars.	1 4-spool Hoisting Engine.	10 "A" Crabs.
2 Steel Crowbars.	2 Complete Riveting Kits.	4 Square Crabs.
4 Steel Connecting Bars.	1 Blacksmith Kit.	5 Hoisting Engines.
4 8-lb. Sledges.	2 Riveting Kits.	1 Pile Engine and Driver.
	1 Dolly Car.	1 Blacksmith Kit.
1 Complete Riveting Kit.	6 10-lb. Sledges.	4 Dolly Cars.
3 Flat Chisels.	6 8-lb. Sledges.	15 10-lb. Sledges.
3 Round-nose Chisels.	4 Axes.	10 8-lb. Sledges.
3 Cold Cutters.	4 Cross-cut Saws.	10 Axes.
2 Chipping Hammers.	4 Man Saws.	15 Cross-cut and Man Saws.
2 Timber Jacks.	10 ⅞" Crank Augers.	20 ⅞' Crank Augers.
2 Fork Wrenches for ¾" Bolts.	6 Iron Crowbars.	6 Iron Crowbars.
2 Fork Wrenches for ⅞" Bolts.	6 Steel Crowbars.	10 Steel Crowbars.
2 Monkey Wrenches, 18" long.	6 Steel Connecting Bars.	15 Steel Connecting Bars.
	12 Flat and Round-nose Chisels.	4 Complete Riveting Kits.
2 Stone Drills.	6 Cold Cutters.	20 Chisels and Cold Cutters.
8 $\frac{13}{16}$" Drift Pins.	4 Chipping Hammers.	6 Chipping Hammers.
2 Button Sets, ¾" rivets.	6 Timber Jacks.	15 Timber Jacks.
2 Button Sets, ⅞" rivets.	8 Fork Wrenches, ¾", ⅞", 1½" and 2."	15 Fork Wrenches, ¾" to 3".
	4 Monkey Wrenches, 18" and 21" long.	4 Key Wrenches.
	2 Stone Drills.	6 Monkey Wrenches.
50 ¾" Fitting-up Bolts, 3" long.	12 $\frac{13}{16}$" and $\frac{15}{16}$" Drift Pins.	15 Cant Hooks.
50 ⅞" Fitting-up Bolts, 3" long.	10 Button Sets, ¾" and ⅞".	4 Stone Drills.
	6 Cant Hooks.	20 $\frac{13}{16}$" and $\frac{15}{16}$" Drift Pins.
150 Washers.	200 ¾" Fitting-up Bolts, 3½".	20 Button Sets, ¾" and ⅞".
	200 ⅞" Fitting-up Bolts, 4".	400 ¾" Fitting-up Bolts.
	500 Wrought Washers.	400 ⅞" Fitting-up Bolts.
		1,000 Wrought Washers.



"A" CRAB.
SCALE, ¼ INCH TO A FOOT.

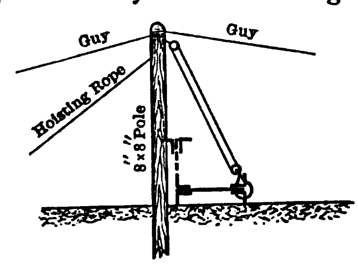
Having reviewed the points which should be borne in mind when designing the details of permanent structures, the materials required, and the tools used for the erection of the temporary structures, we will now proceed to give the methods and plans pursued in the erection of various sizes and types of engineering structures. In all cases where railway spans are considered, they are assumed to be for "single track," as the same plans would hold good for "double-track" spans, differing only in the width and strength of the false work, and the heavier rigging necessary to support and handle the increased weight.

SPANS UP TO 25′.

Girders for spans of this length are usually made of double I beams for the shorter lengths, and plate girders for those approaching 25′. These girders are usually connected with stiff lateral bracing. Girders of this length can be handled directly from the car to the bridge seat, by skidding down on rails to a temporary wooden stringer thrown across the opening, and then pulled directly over the final supports and up-ended. This is assuming the road is a new one; if it is necessary to provide for traffic, only slight interruption to which is allowed, it would be better to put the whole span together near the site, tear up the track between trains, and put it in position by sliding it on to the bridge seats from the side, or skidding it from a dolly-car at the end of the track—see Plate I. A span like this should be put in position and finished completely by ten men in two days; the heaviest single piece to handle being 4,500 pounds.

PLATE OR LATTICE GIRDERS, 25′ TO 85′.

The same plan is to be pursued as in the case of the shorter spans, but owing to the increased lengths, one or more bents of the false work should be put in to support the girders as they are being launched out from the car; bents made of 2 — 8″ × 8″ legs spaced 15′ apart longitudinally, and supporting 4 lines of 8″ × 12″ stringers, would be ample when no traffic is to be carried. These large girders should be launched from the car lying on the side, to prevent accident from upsetting, and after being set directly over the bridge seat, turned up to a vertical position by means of a pole rigged to one side; or by placing a temporary wooden frame over the girders and attaching hoisting blocks to it.



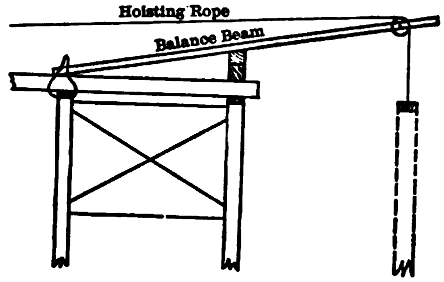
The longer span lattice girders are often shipped in two or more pieces to facilitate handling, in which case great care must be exercised to see that the girder is in perfect line and has a proper camber before riveting is commenced. The same care should be used in all cases, to have the girders in perfect line before riveting the lateral bracing.

Girders are also often put in position from the track by being supported from the side of the car, run out over the bridge seats, and lowered into position; or if the seat is directly under the rail, lower girders to one side, remove the track, and slide them into position. (See Plate II.) This is probably the cheapest plan for placing girders where such a method can be used, and a span should be finished completely by fifteen men in three days; the heaviest piece to handle being 15,000 pounds.

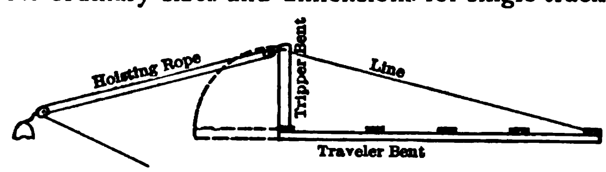
As the spans approach the longer lengths, and become very heavy, it is better to erect two or three bents, depending on the length of girder of the upper false work; so that the girder can be picked directly off of the car and the car run from under it, and the girder then lowered into position. (See Plate III.)

THROUGH SPANS, 85′ TO 150′.

For the erection of single track through spans, the false work is usually made in bents of 3 legs each, spaced about 20 feet c. to c., and capped. On these bents are placed 4 lines of stringers. The sizes given in Plate IV. are for ordinary height and weight of span.



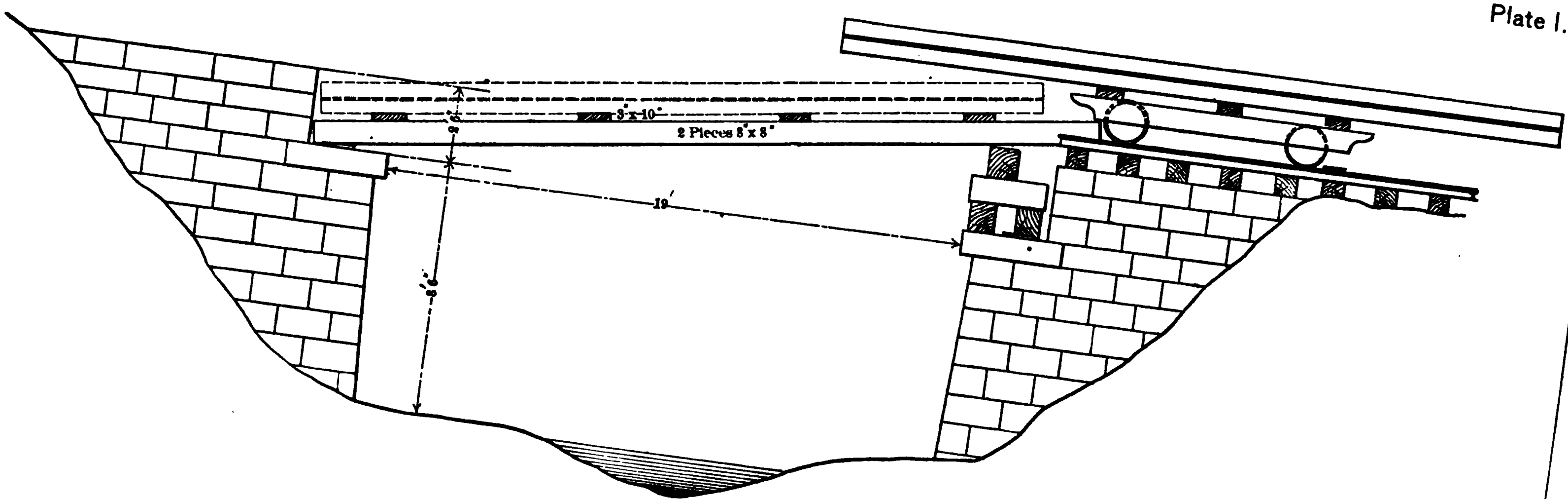
These bents of false work are usually framed and put together on shore, and floated to position and upended in place by the means of balance beams; or if it is not practicable to float the whole bent out, bolted together, it is put in place by the same means piece by piece. The top of the false work is so designed as to be at least 12″ below the lowest iron to be erected, so that there may be plenty of room to block up. After the false work is ready, the "traveller," or top movable staging, is put up; this traveller runs on rails spaced sufficiently far apart to allow it to span the new truss. (See Plate IV. for ordinary sizes and dimensions for single track spans.) The bents of the traveller are framed complete and bolted together lying down on the false work, and raised to a vertical position on the sills by means of a "tripper bent," and the two bents are then braced together. On the stringers on top of the traveller, 4 "A" crabs are placed, one near each corner, if the hoisting is to be done by man power.



After the false work and traveller are ready, the next proceeding is to lay out the longitudinal centre line of the trusses on the false work and locate the position of the panel points; at each of these points a sufficient amount of blocking is placed, upon which the iron rests, to give the new truss, when first placed in position and before swinging clear of the false work, an increased camber, varying from about 3″ for 100′ spans to 9″ for 550′ spans; this increased camber is put on to facilitate the connection of the new trusses by shortening the distance, as it does, between the diagonal panel points. The end wall-plates and shoes are then placed in position, beginning with the fixed end, and on centres furnished by the engineer; the lower chord bars are then distributed at their proper panels, as well as the pins and washers.

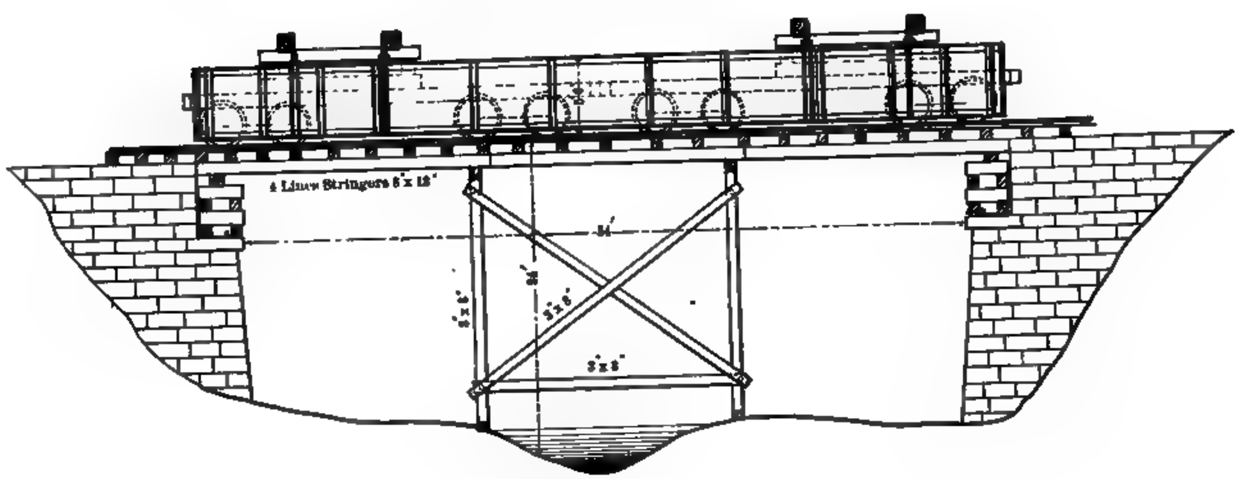
The erection of the trusses begins at the centre panel, as at this point we find diagonal members running in each direction, and the panel is therefore held both ways. The section of upper chord of the centre panel is first hoisted slightly above its final position and lashed to the traveller; this latter relieves the "A" crabs, and the two interior posts are taken hold of and hoisted into position, and the lower pins, connecting the lower chord bars, tie bars, and interior posts, are then driven; only sufficient chord bars, however, being put on at this time to complete the connection, as those on the outside of the post can be put on later. The section of the upper chord is then lowered slightly into its position over the interior posts, and, the upper ends of the diagonal bars and counters being hoisted into position, the upper pins are driven, thus completing one panel of the truss. The same plan is pursued simultaneously with the centre panel of the opposite truss, and the upper lateral and transverse bracing joining the trusses is put in place. Great care must be exercised in the adjustment of this first panel of the bracing to see that the panel points are exactly opposite and square with the centre line of the bridge, and that the interior posts are perpendicular. The facility of final connection depends very much on the careful adjustment of the first panel; if it is started square and true, and the others adjusted to it as they are erected, the chances are there will be little trouble at the end. During the

Plate I.



SPANS UP TO 25 FEET.

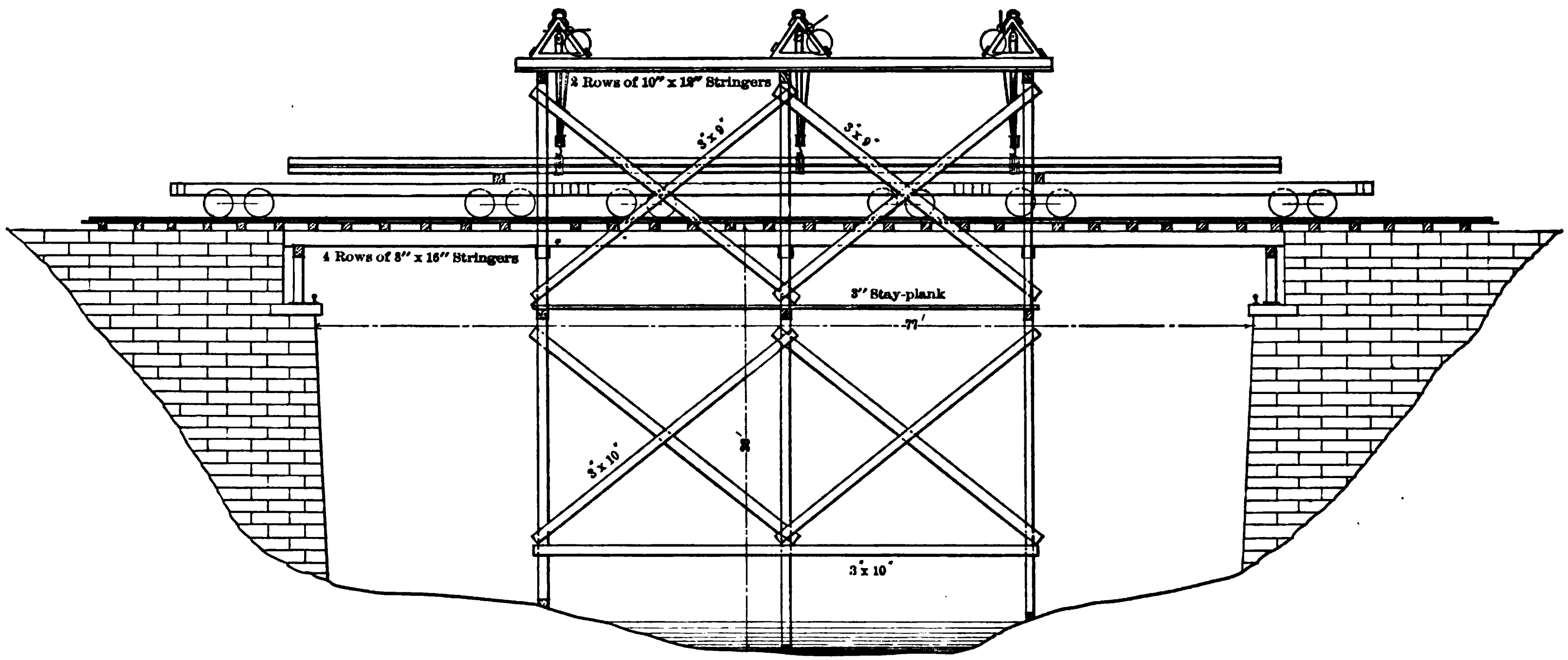
Plate II.



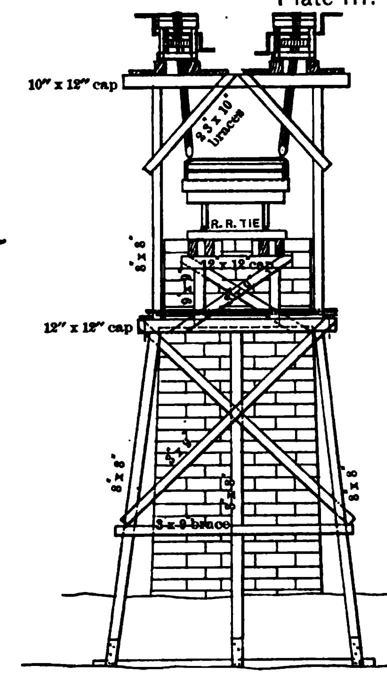
SECTION.

SPANS 25 FEET TO 60 FEET.

Plate III.



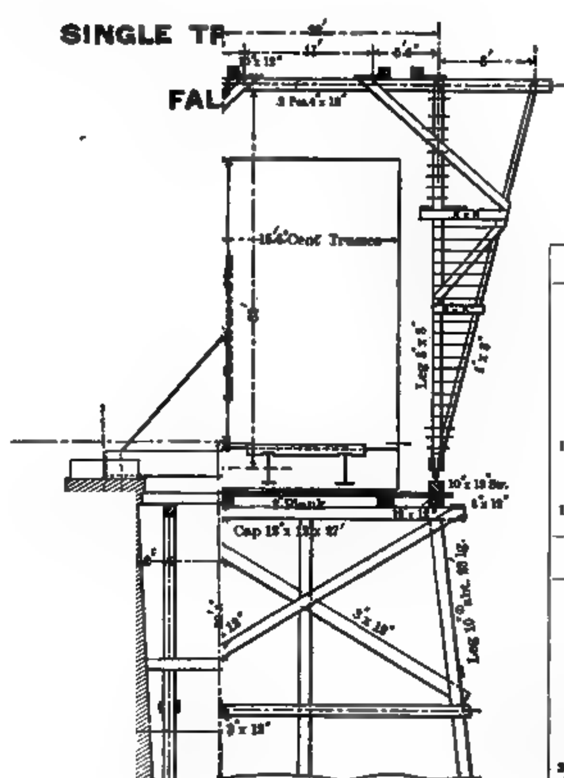
SPANS 60 FEET TO 80 FEET.



SECTION.

SCALE, 16 FT. TO 1 IN.

Plate IV.

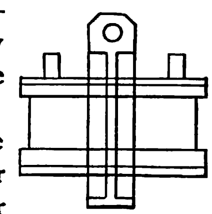


BILL OF MATERIAL.

FALSEWORK.			
6	Caps,	12 x 12 in.	[illegible] ft.
18	Legs, 10 in. small end di.		[illegible] "
12	Braces,	3 x 12 in.	[illegible] "
12	"	" "	[illegible] "
8	"	" "	[illegible] "
12	"	" "	[illegible] "
12	Stringers,	12 x 12 "	[illegible] "
8	"	" "	[illegible] "
[illegible]	"	10 x 12 "	[illegible] "
6	"	" [illegible] ft.	10 in.
[illegible]	Floor plank, 3 in. thick,		[illegible] ft.
150	Oak Wedges.		
30	Bolts,	3-4 in. diam.	17 in.
15	"	" "	[illegible] "
40	"	" "	[illegible] "
50	Drift bolts,	" "	[illegible] "
165	Washers.		
TRAVELLER.			
4	Caps,	4 x 10 in.	[illegible] ft.
4	Legs,	8 x 8 "	24 "
4	Braces,	4 x 8 "	33 "
4	"	3 x 8 "	17 "
[illegible]	"	" "	9 "
4	"	2 x 6 "	5 "
4	"	4 x 8 "	[illegible] "
4	"	" "	[illegible] "
[illegible]	"	3 x 8 "	27 "
4	Sills,	4 x 12 "	[illegible] "
4	Stringers,	10 x 12 "	[illegible] "
12	Floor plank,	3 x 12 "	[illegible] "
8	Cleats,	3 x 8 "	[illegible] "
[illegible]	Lin. ft. 1 1-4 in. x 4 ft.		
32	Bolts,	3-4 in. diam.	19 in.
12	"	" "	16 "
24	"	" "	16 "
8	"	" "	19 "
4	"	" "	27 "
6	"	" "	17 "
4	Wheels with Journals.		

erection the false work should be watched closely, and if it settles—especially when it settles unevenly—the blocking should be increased at such points, so that the relative elevations of the centre line of the lower chord pins above a level line are kept as intended, and in a regular, increased camber curve.

After the centre panel is complete the traveller is moved one panel toward the "fixed" end of the span, and this panel is put in position and connected, pursuing exactly the same course as with the centre, and so on to the end; the traveller is then run back to the panel beyond the centre and toward the "roller" end, and these panels are put up in order. The last pin to be driven is usually the pin at the top of the leaning end-post. It is often necessary to raise or lower new iron to make connections, especially the final ones, and for this purpose good, powerful hydraulic jacks should be kept at hand. In applying these jacks to move any point place them under the bars or pins, but close to the support in the jaws of the post; otherwise, through the liability of unequal loading, members would probably get seriously bent. After the span is all connected and all splices well filled with good bolts, we should begin swinging it clear of the false work by starting at the highest point, which point is shown plainly by the excessive buckling of the diagonal members; and lowering this point until the buckling is nearly taken out, and then lower the panel points adjacent, working toward each end, and lowering each point about 1″ to $1\frac{1}{2}$″, and taking the greatest care not to overload any point by permitting it to remain high and those adjacent too low; this undue loading is plainly seen, as before stated, by the diagonal members buckling. Before starting to "swing" the span, the counters should be slackened thoroughly, and after the span is swung and complete, including ties and rails and any other dead load it is to carry, they should be tightened, and brought simply to a good square bearing on the pins.



If the floor is so designed that it must be connected at the same time as the lower chord bars and web members—that is, if the floor beams have riveted end-hangers through which the pin passes—the floor beams are placed directly on the blocking first, the stringers put in between them, and then the remainder of the erection proceeds exactly as outlined above.

DECK SPANS, 85′ TO 150′.

For the erection of "Deck Structures" up to 150′ in length the same character of false work is used as for "Through" spans of the same length and weight; and when the false work is only run up to the lower chord of the iron truss, the erection of the latter is proceeded with in precisely the same manner as for "Through" spans. At times, however, it is advisable to continue the false work up to a short distance below the upper chord. (See Plate V.) In this plate the false work is also shown arranged to carry traffic and to remove the old truss. The iron in this case is run out on a derrick car and *lowered* into position, the upper chord being first lined out, starting from the fixed end and from points given on the masonry by the engineer. At about the level of the lower chord the bracing on the false work is arranged with some additional planking (see plate), so as to support temporarily the lower chord and the lower ends of the interior posts, until the connections are made by driving the upper chord pins first and then those for the lower chord, at the centre panel, and working from here toward the "fixed" end, and then from the centre toward the "roller" end of the span. The same remarks apply to the adjusting of counters, the thorough bolting of joints before swinging off, and the care to be exercised in the adjustment of bracing, and squaring and lining up of trusses, particularly of the first panel.

SPANS, 150′ TO 350′.

In considering the longer spans we will at the same time assume that it is necessary to use high false works, at least 60′ high. At this height it is advisable to increase the panel lengths, making the stringers correspondingly heavier, thereby reducing the number of bents required, the cost "in place" of these bents increasing rapidly as this height is approached. For ordinary structures 12″ × 12″ stringers can be used for panel lengths of 20′; over this 12″ × 16″ up to 24′ panels, and if necessary or advisable to increase the panels to 25′ or even 30′, the stringers can be trussed simply. (See Plate VI.) This plate also shows false work arranged to carry traffic and to remove the old structure. These long panels are put in where the crossing is over streams subject to sudden freshets, accompanied with a heavy run of drift, in order to leave as much open water-way as possible. The braced towers in such cases are made less in width, say 12′ to 16′, according to the most economical panel division. When the false work is 50′ high or more, it is always best to use steam power for hoisting, as man power is too slow for the necessary long lifts.

The hoisting engine is either placed at some convenient place on the bank and a lead-line run from there to sheaves in the ends of balance beams, or the engine is set up on a travelling crane or derrick traveller (see Plate XIV.), and the false work is lowered or raised in place from a swinging boom, which boom is long enough to command the position of all legs of the bent ahead of the last one erected. In our discussion so far we have assumed that false work legs could be set directly on the bottom, the formation being rock or other material sufficiently hard to preclude the possibility of scour or settlement under the loads which are to come upon them. When the bottom is of a medium solid character, it is advisable to use a sill at the foot of the trestle legs running transversely and receiving all the legs of the bent; and if the bottom is very soft, in addition to this sill short mud-sills must be placed at right angles, to still further distribute the pressure. If, with even these additions, it is probable settlement or scour will take place, piles must be driven. Piles should be selected so that they may be well driven, and sawed off at some distance above the ordinary stage of water. Piles should be straight throughout, and before driving are pointed, and the butts roughly dressed square. They are driven at the panel points of the trestle, and as near as possible in line transversely and longitudinally. After all the piles in one bent are driven, they are sawed off, pulled still further into line by being braced to the previously finished bent, and then capped and braced. The false work is then put on this cap, in the same manner as previously outlined for ordinary cases, where the false work bent is placed directly on the bottom. In ordinary material piles are usually driven 12′ to 18′ with a 2,600-lb. hammer, dropping at the last blow 40′, and the penetration of the pile under this blow should not be more than 2″. Piles driven in such a manner can sustain safely a load of 18 tons. Forty piles should be driven each day of ten hours, under ordinary conditions, with one driver. (See Plate VII., showing driver, boat, and engine complete.)

The travellers for these longer spans are usually made of three bents, as the panels of trusses are generally made at least 25′ long, and the traveller must extend at least 2′ 6″ beyond the centre of the panel, making it 28′ centre to centre of the end legs (see Plate VIII.), or the traveller may be designed with two regular end braced bents and a centre cap supporting the upper stringers, which cap is supported by two leaning posts in the same plane as the inner legs of the bents. (See Plate VI.) This last traveller is probably the cheaper design, containing less lumber and framing, and yet answering the purpose. Great care must be exercised in the framing and raising of these large travellers, to see that every bent is square and plumb, and kept so when hoisting; and if necessary to insure this, good guys should be used. We have thus briefly outlined the false work and travellers required

Plate V.

FALSE WORK FOR DECK SPAN,

ARRANGED TO REMOVE OLD STRUCTURE AND CARRY TRAFFIC.

SCALE, 15⅓ FT. TO 1 IN.

MATERIAL FOR TRAVELLER.		
4 Caps,	3 x 12 in.	36 ft.
4 Legs,	8 x 8 "	24 "
4 Braces,	4 x 8 "	25 "
4 "	3 x 10 "	15 "
4 "	3 x 8 "	7 "
4 Sills,	3 x 12 "	21 "
4 "	12 x 12 "	21 "
12 G Planks,	3 x 10 "	21 "
12 Bolts,	3-4 in. diam.	13 in.
24 "	" "	14 "
16 "	" "	15 "
4 "	" "	18 "
112 Washers.		

FALSEWORK.		
16 Caps,	12 x 12 in.	24 ft.
16 Legs,	" "	17 "
16 "	" "	19 "
16 "	10 x 12 "	19 "
16 "	" "	17 "
12 Stringers,	12 x 12 "	16 "
8 "	" "	15 "
8 "	" "	18 "
6 "	10 x 12 "	16 "
4 "	" "	15 "
4 "	" "	18 "
16 Braces,	3 x 12 "	12 "
16 "	" "	14 "
16 "	" "	27 "
16 "	" "	22 "
8 "	" "	8 "
40 "	" "	18 "
30 Floor Plank,		14 "
40 Blocking,		8 "

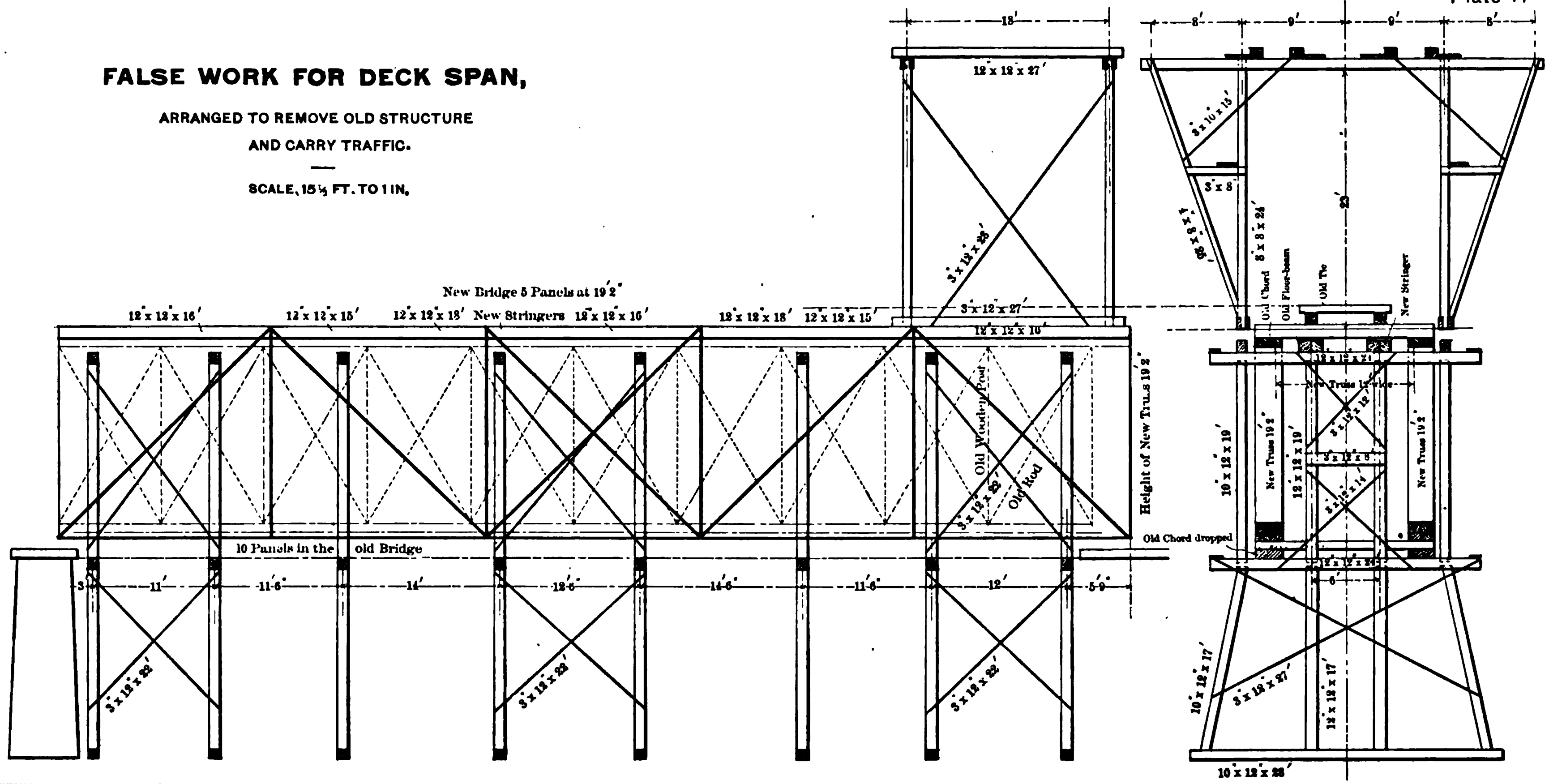
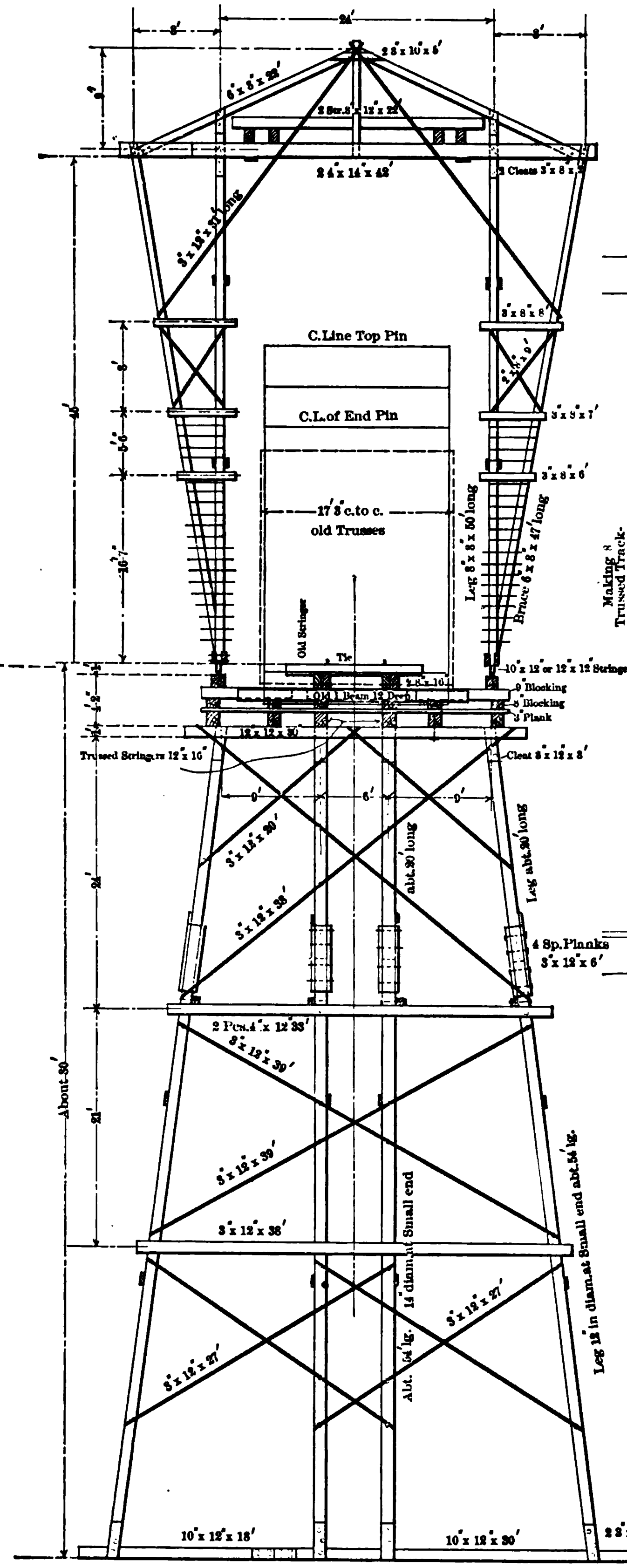


Plate VI.



BILL OF MATERIAL.

FALSEWORK.

		Inches.	Feet.	
	10 Caps,	12 x 12	30	
	20 Legs, 12 in. diam. small end,		56	
	20 " 12 " " "		20	
	20 " 14 " " "		56	
	20 " 14 " " "		20	
	30 Braces,	3 x 12	38	
	20 "	"	39	
	40 "	"	27	
	30 "	3 x 10	19	
	60 Tie pcs.	"	14	
	16 " "	6 x 8	26	
	32 Braces,	3 x 8	12	
	20 False Caps,	4 x 12	33	
	10 Sills,	10 x 12	30	
	10 "	"	18	
	160 Splice Planks,	3 x 12	6	
	160 Cleats,	"	3	
Making 8 Trussed Track-stringers.	6 Stringers,	12 x 16	27	6"
	2 "	"	22	
	12 1¼ in. Upset Rods,		28	10¼"
	4 " " "		22	6¼"
	6 Cast Bearings,		L.T.	
	2 " "		S.T.	
	16 Washers,	8 x 1	14	6"
	8 Posts,	10 x 12	3	
	16 Cleats,	2 x 10	2	6"
	12 Stringers,	12 x 16	27	
	4 "	12 x 12	21	
	2 "	"	19	
	6 "	"	18	
	10 "	"	14	
	6 "	"	24	3½"
	2 "	10 x 12	20	
	7 "	"	18	
	8 "	"	14	
	6 "	"	11	7"
	40 Floor Plank,	3 x 12	27	
	100 Pcs. Blocking,	9 x 10	6	
	300 Oak Wedges.			

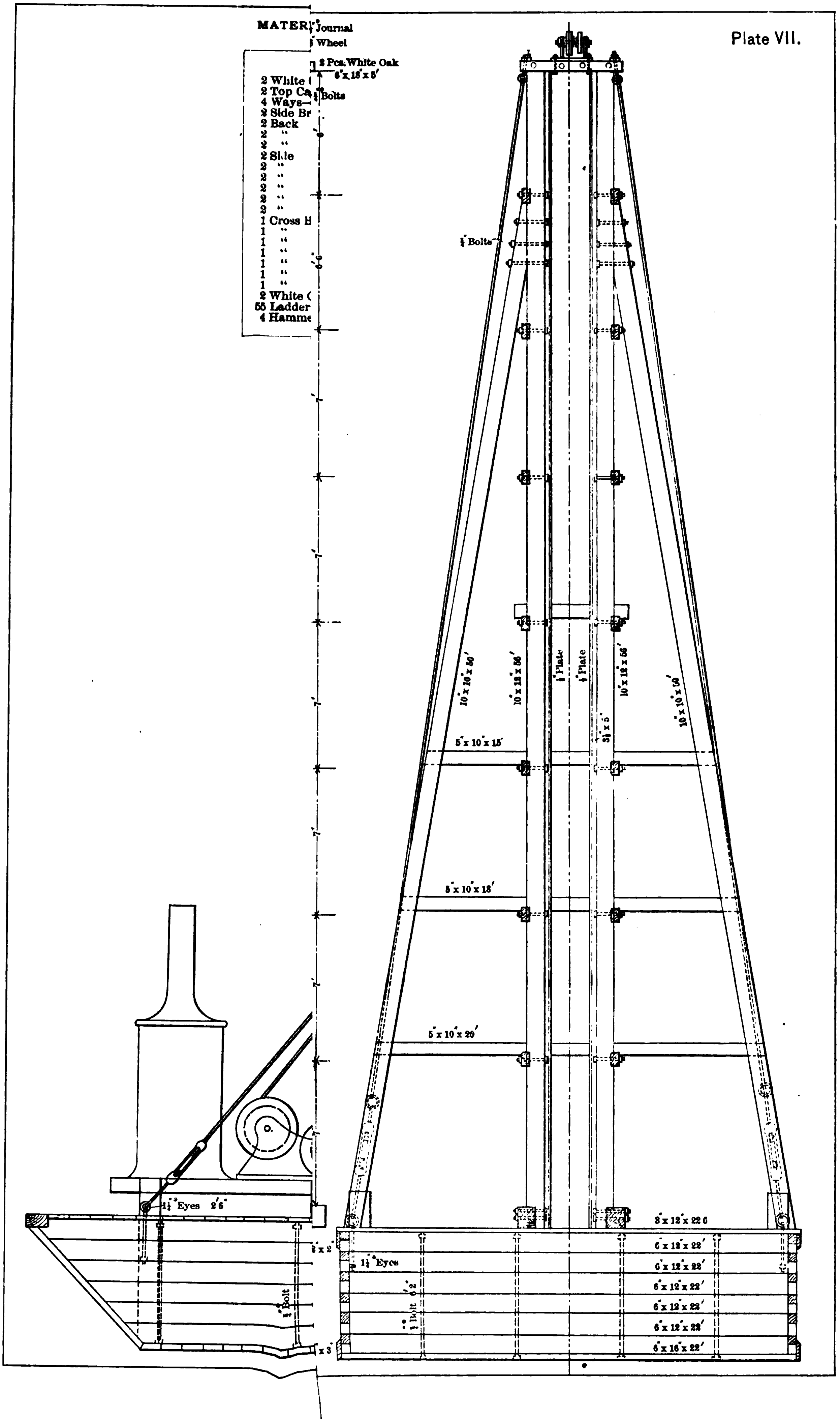
		Inches.
40 Bolts,	¾ in. diam.	30
45 "	" "	25
36 "	" "	24
50 "	" "	23
330 "	" "	22
125 "	" "	21
560 "	" "	20
200 "	" "	19
40 "	" "	13
2900 Washers.		

TRAVELLER.

	Inches.	Feet.
4 Legs,	8 x 8	50
4 Braces,	6 x 8	48
4 "	"	47
4 Caps,	4 x 14	42
1 "	8 x 16	26
4 Sills,	4 x 14	33
4 Braces,	6 x 8	22
2 "	"	12
4 "	3 x 12	31
4 "	3 x 8	8
4 "	"	7
16 "	"	6
8 "	2 x 8	9
4 "	3 x 8	14
8 Tie Planks,	"	32
2 Caps,	8 x 8	3
4 Stringers,	8 x 16	33
2 "	8 x 12	22
12 Floor Plank,	2 x 12	32
8 Cleats,	3 x 8	3
8 "	"	2
4 "	3 x 10	5
4 Ladder pcs.	2 x 6	26
300 Lin. ft.	1½ x 4 strips.	

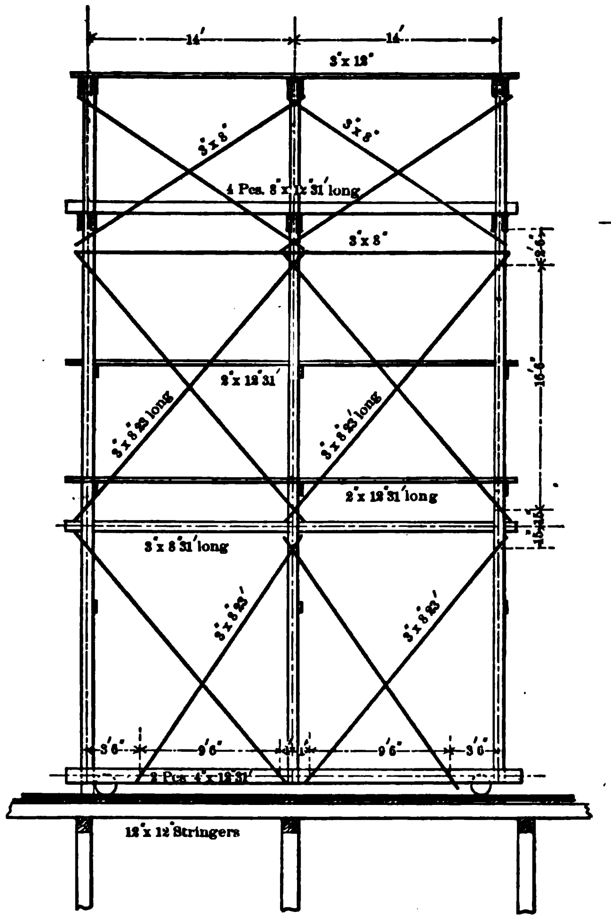
		Inches.
12 Bolts,	¾ in. diam.	8
24 "	" "	11
10 "	" "	12
35 "	" "	13
20 "	" "	14
10 "	" "	15
70 "	" "	16
6 "	" "	20
2 "	" "	29
6 "	" "	37
400 Washers.		

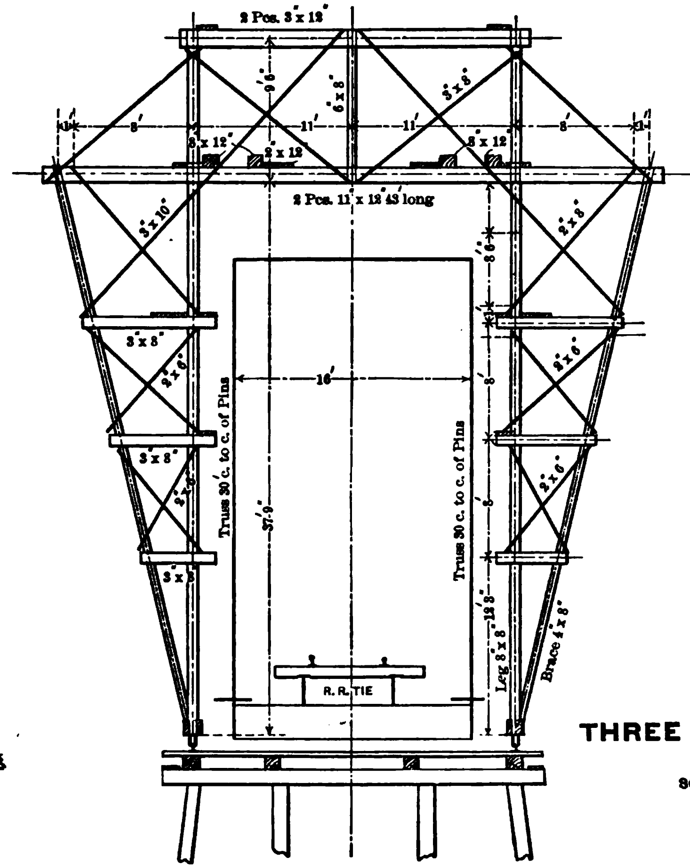
Plate VII.



Journal
Wheel
2 Pcs. White Oak 6" x 18" x 5'
Bolts
2 White
2 Top Ca
4 Ways
2 Side Br
2 Back
2 Side
1 Cross B
2 White
55 Ladder
4 Hamme
Bolts
10" x 10" x 50'
10" x 12" x 56'
Plate
Plate
10" x 12" x 56'
10" x 10" x 50'
5" x 10" x 15'
3½" x 5"
5" x 10" x 18'
5" x 10" x 20'
1¼" Eyes 2'6"
8" x 12" x 22 6
6" x 18" x 22'
1¼" Eyes
6" x 12" x 22'
6" x 12" x 22'
6" x 12" x 22'
6" x 12" x 22'
6" x 16" x 22'
Bolt

Plate VIII.





BILL OF MATERIAL.

		Inches.	Feet.
6	Caps,	4 x 12	43
6	Legs,	8 x 8	49
6	Braces,	4 x 8	40
6	"	3 x 10	26
6	"	2 x 8	14
6	"	3 x 8	9
6	"	2 x 6	9
6	"	2 x 6	10
6	"	3 x 8	7
6	"	2 x 6	11
6	"	2 x 6	9
16	"	3 x 8	23
4	"	2 x 8	31
4	Stringers,	8 x 12	31
4	Sills,	4 x 12	31
4	Ladder pcs.	2 x 6	30
16	Floor Plank,	2 x 12	31
200	Lin. ft.	1 1-4 x 4 strips.	
6	Caps,	3 x 12	24
8	Braces,	3 x 8	18
12	"	3 x 8	14

66 Bolts, 3-4 in. di. 12 in. long.
55 " " " 13 "
8 " " " 14 "
20 " " " 17 "
16 " " " 18 "
12 " " " 19 "
8 " " " 21 "

THREE BENT TRAVELER,

SCALE, 13 FT. TO 1 IN.

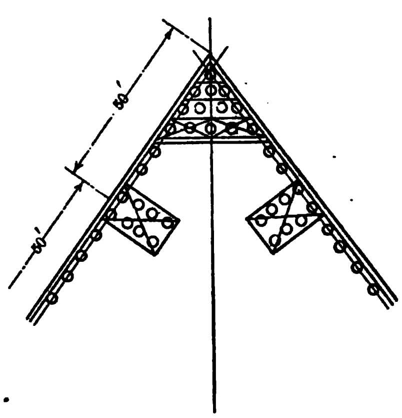
to raise and handle these long spans; the erection of the spans themselves does not differ, in plan, from that described under the head of smaller spans; erection being begun at the centre panel and working first toward the "fixed" end and finishing with the "roller" end.

SPANS, 300′ TO 600′ LONG.

For the very longest spans, where the trusses run up to 85′ and more in depth, the traveller becomes a very important and expensive part of the erection plant. Usually it is of four bents, with its upper bracing a Howe truss 14′ deep (see Plate IX.). Such a traveller, when fully rigged for work, has four hoisting engines, of the type shown in Sketch 4, with two boilers; lines from these engines run to snatch blocks fastened to the lower platforms, and from there to the top of the traveller, passing through sheaves in twin beams resting on the stringers which bear directly on the Howe trusses, and directly over the new trusses. Fifty sets of blocks are required, varying in size from the heaviest 16″ triple to 8″ single blocks; some 35 coils of rope are needed, varying from $1\frac{1}{4}''$ to $\frac{3}{4}''$. Such a traveller contains 75,000 feet B. M. of lumber, and requires twelve days to frame and erect, and three days to rig complete.

The false work for these extremely long spans is similar to those already described, where the same height and character of bottom is found; these spans, however, are generally designed for the crossing of important streams, often those subject to sudden and heavy rises, and for such cases it is advisable to still further increase the unbraced open spans, and keep the braced towers comparatively narrow. A usual division is to make towers of about 11′ and adjoining openings 50′. (See Plate X.)

These towers are formed first of piles, driven as previously described, and capped and braced; upon these the bents of the false work are erected and braced, and on top of the caps the necessary trusses or stringers, the long spans being Howe trusses about 9′ deep. (See Plate XI., showing a design of heavy Howe truss.) These trusses for false work weigh probably six tons, and can be framed and connected together and launched into position by an "upper" traveller with overhanging boom rigged for the purpose. This traveller should also be provided with a boom sufficiently long to reach over the tower in advance, so that it can be erected by hoisting directly from the traveller. In case there is not sufficient material over the rock in which piles may be driven, it is necessary to build a crib of rough timbers and fill the same with stones until it rests firmly on the bottom; and on this temporary pier erect the towers to carry spans. (See also Plate X.) In those localities where the stream is not only subject to sudden rises, but also to a heavy run of drift, the false work is not only liable to be carried away by the heavy pressure against it, but is subject to the greater danger of being scoured out, or literally washed out, by having drift accumulate against it; and after packing nearly to the bottom of the stream, the rush of water underneath scours out the false work until it is so undermined and weakened that it fails.



To prevent this, a "protection" or boom should be placed above the false work, of V shape, and having the point of the V at least the full length of the span above the centre of the opening, giving the sides thereby a good slope. This protection is best made of piles, well driven, with centres about 4′ apart, and of sufficient length to be above water during a heavy rise; these piles are connected on the outside by horizontal pieces of 6″ × 8″, with spaces between each row of about 6 inches. At points about 50′ apart a group of 8 or 10

piles should extend on the inside of the V and be thoroughly braced together, giving great strength to the protection to resist the pressure of water and drift. Such a protection, in plan, would appear like the sketch on preceeding page.

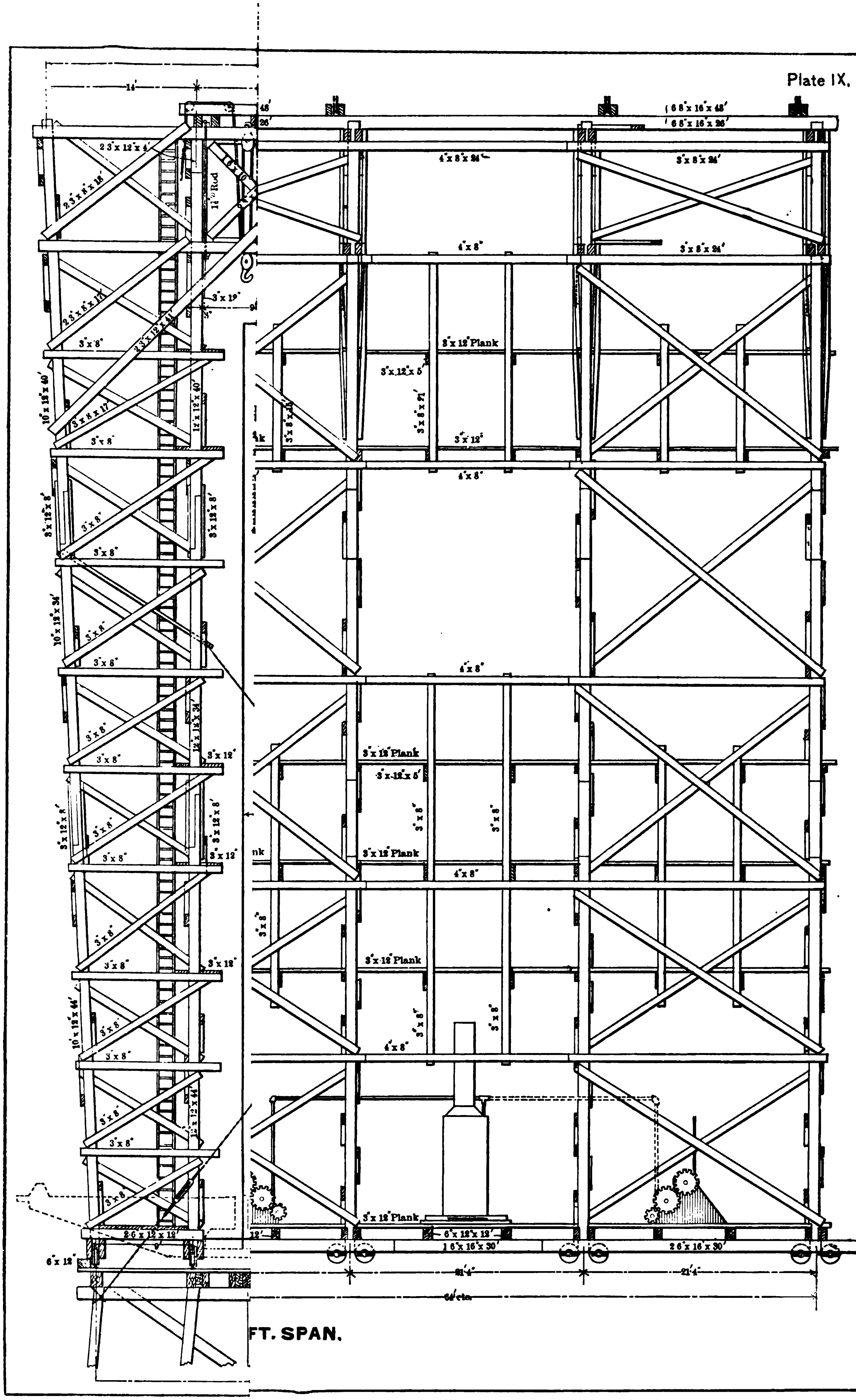
The false work of the centre span of the Ohio River bridge at Cincinnati, built in 1888, and erected on piles driven into the bottom 18′ to 20′, and the whole work being of the most substantial character, sustained successfully a flood and depth of water of 45′, and a most unusual run of heavy drift lasting for three days, the drift having accumulated during this time in a solid triangle above and against the false work, extending to over 500′ above the bridge. This drift was nearly solid to the bottom of river. Even with this tremendous pressure the false work showed no signs of yielding, and not until the upper line of piles was completely undermined by scouring out, as was afterwards plainly shown, to a depth of 12′, did the false work yield and collapse, as it did on August 26, 1888, falling *up stream*. When the false work was again put up, as was done immediately, it was protected in the manner above described, and this protection withstood without sign of weakness two floods of 48′ of water, and while the run of drift was not very heavy, it is believed, however, it would have stood equally well the heaviest drift, as the sloping sides would not allow it to accumulate and start scouring. It is not necessary to further discuss the erection of the iron-work of these long spans, as the plan pursued is precisely similar to that described for smaller spans, as given earlier in this chapter; the long spans simply demanding extra-heavy rigging, and care in the handling of the enormous pieces required in their construction, some of the members weighing 40,000 pounds.

DRAW SPANS.

The erection of draw spans presents no new problems, as far as the false work and travellers are concerned, the same conditions of height, length of span, etc., calling for the same method of erection.

Still greater accuracy, however, is demanded in the masonry adjustment and alignment of the span. The upper surface of the lower track, upon which the draw revolves, should be set perfectly level. This is usually secured by imbedding track segments in a composition of iron filings and sal-ammoniac (32 parts filings, 1 part sal-ammoniac), which composition soon sets into a very hard and compact mass. With this track set perfectly level and at the exact elevation below grade, the further erection of the turn-table should give no trouble, as it is all machine work and " iron to iron." Especial care must be taken to set the pinion so that it does not gear too deeply into the gear segments on the track, or hard turning, caused by binding, will be the result. Most of the draws are now designed to carry all the dead load to the centre; therefore, when simply carrying its own weight, the rollers or wedges under the ends of the draw should be so adjusted that they simply come to a bearing. The " locking gear " and shaft operating these rollers or wedges also demands the most careful attention, to see that it is in perfect alignment and adjustment, that there may be no binding. Draws are usually erected upon the " rest " piers, that is, open; the iron-work at the centre is raised first, and from these each way to the ends. All pin-truss draws have an " open joint" in one of the lower chord panels near and on each side of the centre; shimming plates varying in thickness from $\frac{1}{8}''$ to $\frac{3}{4}''$ are provided for insertion in this joint, as the case demands. It is impossible to estimate exactly the deflection of the ends of draw trusses, owing to the inaccuracy of workmanship and other causes, and this deflection is found exactly during the erection, and just sufficient plates put in this " open joint " to raise or lower the end to its proper elevation. In the case of lattice girder draws or plate girders, where there is no open joint, and the girders are shipped in pieces, the ends should be blocked up *above* their final position from 1″ to 2,″

Plate IX.



FT. SPAN.

FOR 550 FT. SINGLE TRACK THROUGH SPANS

SCALE OF FEET

Plate XI.

BILL OF MATERIAL.

4 pcs.	10 x 12″		38′
12 "	"		32′
8 "	"		30′
8 "	"		22′
4 "	"		14′
12 "	8 x 10″		6′
24 Splice Clamps.			
96 Packing Blocks.			

16 Braces,	10 x 12″		18′
24 "	8 x 12″		13′
20 "	8 x 8″		18′
8 "	8 x 10″		10′
4 Blocks,	No. 1,	holes	1¼″
4 "	No. 4,	"	"
12 "	No. 3,	"	2¼″
4 "	No. 2,	"	3″
8 "	"	"	2¼″
8 "	"	"	2¼″

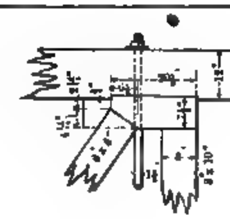
4 Blocks,	No. 2,	holes 1¼″			
8 Upset Rods,		1¾″ di.		18′	6″
4 "	"	"	"	18′	8″
6 "	"	1¾″	"	18′	8″
8 "	"	2¼″	"	18′	9″
8 "	"	2¼″	"	18′	10″
8 "	"	2¼″	"	18′	10″
16 Straps, 6 x ½ x 34″		holes		3¼″ di.	
16 "	" "	"		2¼″	"
12 "	" "	"		1¾″	"

150 Bolts,	⅞″ diam.		38″
24 "	"		24″
20 "	"		36″
390 Washers,	1″ hole,		2¼ x ⅜″
34 Steel I-beams,	15″–150		44′
6 "	"	"	"

FLOORBEAM SUSPENDERS.

12 Rods,	1¼″ diam	3′ 4″
24 Washers,	6″ square.	

10 spaces at 8′ each



according to length of span, before riveting is commenced, to allow for the deflection when the temporary support is removed.

VIADUCTS.

Under the head of Viaducts we class those structures composed of short spans resting upon bents or towers. These towers are erected by two principal methods: first, by means of gin-poles; second, by a traveller on top, with long, projecting boom. Following the first plan, a gin-pole, about 10 feet longer than the height of each story of the tower, is placed at each corner, and near the position of a vertical column.

These gin-poles are thoroughly guyed, and have a set of blocks lashed to the top. The columns of the tower are hooked to this set of "falls," and hoisted into position; the transverse and longitudinal bracing is put in place between the columns, and the first story is complete.

The next proceeding is to raise all the gin-poles sufficiently, so that the second-story columns and bracing can be hoisted and placed in position; this is done by raising poles to a sufficient height, and clamping them thoroughly to the columns of the story last finished; the second story is then erected, and so on to the top. This plan was used very successfully on the famous Kinzua Ravine Viaduct, near Bradford, Pa., the highest in the world, being 306 feet from the bed of the small stream to the base of rail; this viaduct was designed and erected by Clarke, Reeves & Co., now The Phœnix Bridge Company. The viaduct, as before stated, is 306 feet high, 2,052 feet long, and is composed of spans of 60 and 40 feet, each tower containing 4 columns. The structure in course of erection is shown in Plate XII.

If it is proposed to use the *second* plan, by using the upper traveller, both for setting the girders and raising the towers, the traveller must be designed with a horizontal stiff boom sufficiently long to reach from the completed portion of the structure directly over the tower to be raised. This second plan is probably the cheapest and best plan for most cases. The only special care to be exercised in the erection of the towers is in the adjustment of the bracing, to see that all columns are carried up plumb and square; this requires the aid of the transit, for high towers, to see that it is accurately done. All the adjustment should be complete before the final riveting of the bracing or joints is done. After the towers are raised, the girders supporting the track are usually brought in from one end on a "dolly" car, and run out to the traveller at the end of the structure just finished, where they are taken hold of by a set of falls, fastened to the upper overhanging boom of the traveller, and lowered into position.

In some rare cases it may be better to hoist the girders from the bottom of the structure to the top of the tower, and place them on their seats on the columns.

ELEVATED RAILWAYS.

Elevated roads similar to those in New York and Brooklyn come practically under the head of viaducts, as just discussed; but as most of these roads traverse crowded thoroughfares, it is absolutely essential that the streets be obstructed as little as possible; the traveller, therefore, is placed on top, and arranged with booms sufficiently long to reach out and set a column of the bent ahead with its transverse girders and bracing, and then hoist and place the longitudinal girders in place. If, however, the streets are not so important, and can be more or less blocked, a traveller designed to run on sills on the ground and spanning the structure is probably the more economical plan to pursue, and no

doubt the iron can be thus placed in position more rapidly. The traveller, with its engine, boiler, and rigging complete, is necessarily an expensive piece of machinery, and it should only be employed in raising and placing the larger and more important members of the structure, only sufficient bracing being put in to enable the traveller to be run out safely on the completed structure, or permit it to move ahead, if it is designed to run on the ground. (See Plate XIII. for ground traveller.) The remaining part of the work, such as the bracing, ties, guards, and rails for the track, can be raised with a much simpler arrangement; even a common gin-pole, with a set of blocks lashed to the top, answering the purpose. At least 150 feet of elevated railway structure should be erected complete each day of ten hours, raising on the above plans, where the spans are about 40 feet, and the columns not over one story, or 20 feet high. (See Plate XIV. for viaduct top traveller.)

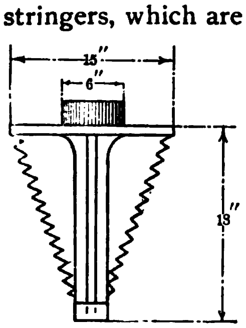
TRAIN SHEDS, ROOFS, ETC.

The erection of roof trusses is a much more simple problem than that of railroad structures, owing principally to the fact that they contain very much lighter pieces to handle and connect. For roofs up to 50-feet span the trusses are usually riveted together on the ground, and after the columns supporting the same have been placed in position at the proper points, the truss is hoisted bodily, and placed on columns by means of a pole placed on the centre line of the building, and extending 10 or 15 feet above the highest point of the truss; the top of the pole being well guyed in four directions, and having a set of falls fastened to it.

As the trusses increase in length they soon become so heavy that it is best to use two poles, one for each side. Only assemble one-half of each truss on the ground, hoist the same into position, supporting each half with poles until the centre connections are made, thus forming one complete truss. When the span reaches 100′, or more, it is generally found better to design the roof trusses with pin-connections, in which case it is necessary to put a couple of bents of false work in, to temporarily support the trusses while connections are being made.

OCEAN PIERS.

The building of ocean piers at the popular seaside resorts is becoming quite general; these piers are for promenading and pleasure, as well as for commercial purposes. We will therefore briefly outline the mode of erection of these structures, only those made of iron being considered. Piers are usually made in spans of about 20′ each, the supporting columns being braced at every panel transversely, and longitudinally braced together in pairs, forming towers; on these columns transverse girders are placed, and upon these the stringers, which are usually of wood.



There are several plans used for the sinking of these columns; that most generally used, and, where the material through which the column is sunk will allow it, by far the cheapest, is by means of a water jet, either supplied by a force-pump or by the ordinary pressure of the city mains, if such are convenient. Generally the columns are made hollow, in which case the 2″ pipe, ending in a long metal nozzle, conveying water, is placed in the centre and run down to the point of the column. As soon as the water is forced through it loosens the sand, and the column sinks of its own weight to the proper depth. When the water-pipe is removed, the sand sets immediately, and the column is fixed in place.

A depth of 10 feet is sufficient to sink columns in ordinary sand. In case it is probable obstructions will be met with, it is advisable to design the columns with a cast-iron shoe,

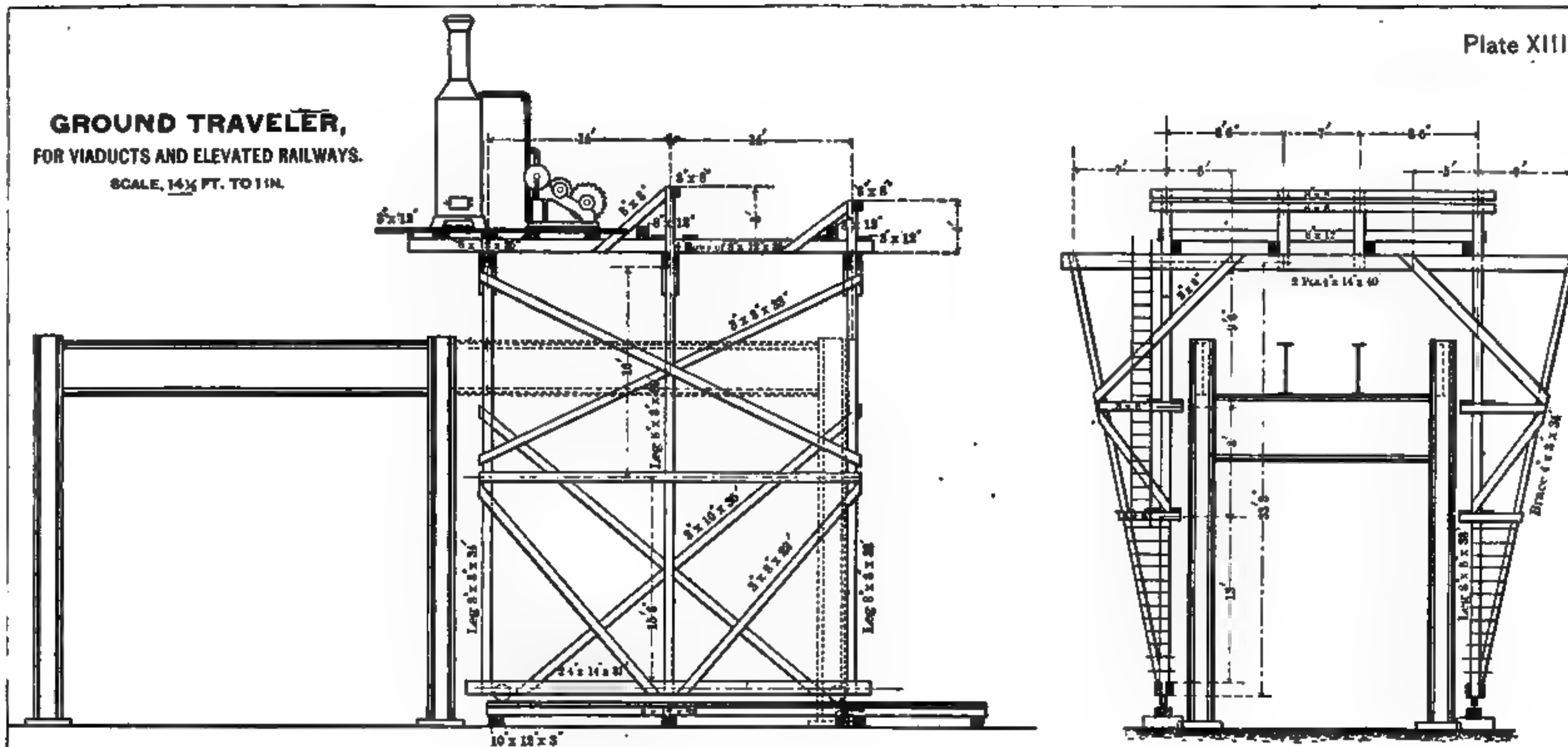
Plate XII

Plate XIII

GROUND TRAVELER,

FOR VIADUCTS AND ELEVATED RAILWAYS.

SCALE, 14½ FT. TO 1 IN.



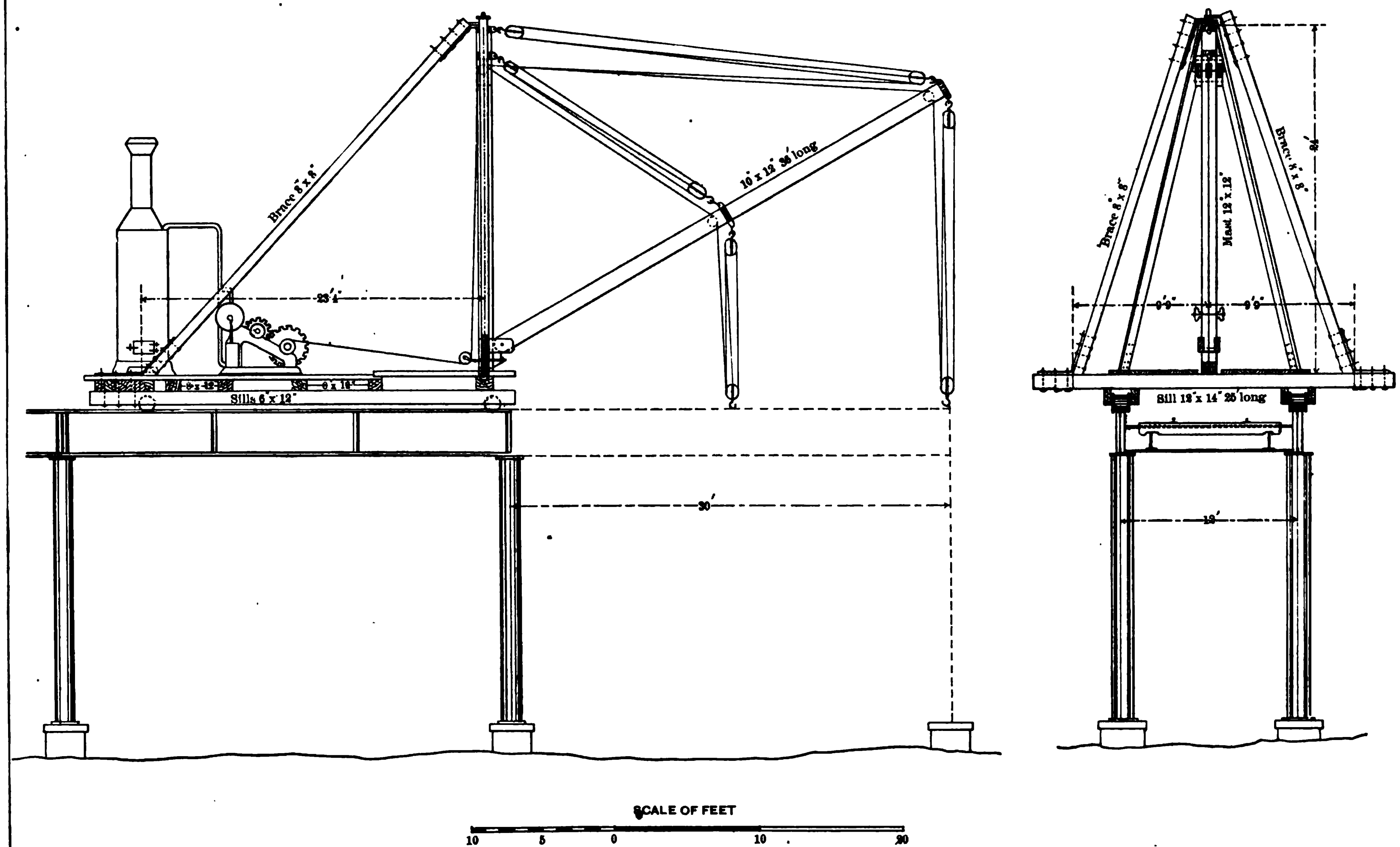
BILL OF MATERIAL.

		Inches.	Feet.			Inches.	Feet.
6	Caps,	4 x 14	40	4	Stringers,	6 x 12	25
2	Legs,	8 x 8	39	2	"	8 x 12	35
2	"	"	36	4	"	"	28
2	"	"	34	1	"	3 x 6	25
2	Caps,	"	20	4	Fl. Plank,	3 x 10	25
6	Braces,	4 x 8	34	16	"	3 x 12	19
6	"	3 x 8	16	8	"	2 x 12	30
6	"	"	7	12	Pcs. Blkg.	10 x 12	3
6	"	"	10	4	Rails,		28
6	"	"	8				Inches.
4	"	"	33	60	Bolts,	¾	13
2	"	"	20	24	"	"	14
4	"	"	35	10	"	"	15
4	"	"	22	68	"	"	16
4	"	6 x 8	8	20	"	"	19
4	"	"	7	12	"	"	20
2	Posts,	"	7	8	"	"	26
2	"	"	6	8	"	"	30
4	Sills,	4 x 14	31	410	Washers,	"	
4	Stringers,	6 x 12	35				

Plate XIV

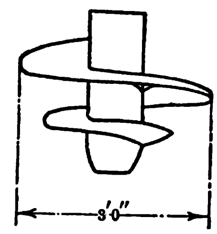
TOP TRAVELER,

FOR VIADUCTS AND ELEVATED RAILWAYS



having ribs with cutting edges; and, as the soil is loosened by the water, the column is turned, and assists the sinking by cutting the harder portions. If the column is to be sunk into a very loose soil, giving little supporting friction to the column, the end of the column is furnished with a very large disk and screw; this is forced down by turning the column, and gives much additional bearing surface.

The columns are turned by securing a square wooden frame to the top, of a diameter sufficient to give the leverage required, and the power applied, usually by men, although an engine can be attached, if necessary. It is also the habit, in some localities, and was the plan used by the government engineer in charge of the pier just finished at Old Point Comfort, Va., to first drive what is known as a "pilot" pile, usually creosoted, some 16′ or 18′ into the bottom by means of a pile-driver, and cut off some 12′ above the bottom. Over this pile, as a guide, the cast or wrought iron column—hollow, of course—is placed, and, being furnished with a screw, it is forced down by revolving. By pursuing this plan a more regular position of column spacing is secured.



There is nothing peculiar to describe in the placing of the transverse girders and wooden stringers, etc., balance beams reaching out over the panel being all the rigging necessary to raise the material and place it in position.

CANTILEVERS.

We finally come to the consideration of the erection of cantilever trusses, that style of structure which next to the suspension bridge requires the least amount of false work for its erection. Cantilever spans may be divided into two general classes: first, "Through" truss cantilevers, or those in which the live load is carried on the floor between the upper and lower systems of lateral bracing; second, "Deck" truss cantilevers, or those in which the live load is applied to the floor above or in the same plane as the upper lateral bracing. As notable examples of the latter may be mentioned the Niagara bridge of the Michigan Central Railway, and the very fine structure just finished spanning the Hudson River at Poughkeepsie, N. Y. As examples of the former may be mentioned the bridge over the Ohio River between Louisville and New Albany, Ind., and the longest span in this country, now in course of erection at "Red Rock," California, over the Colorado River, on the line of the Atlantic and Pacific Railway, it being 660 feet centre to centre of piers. The renowned "Forth Bridge" in Scotland is also a cantilever structure of this style, but the magnitude of the work (span being over 1,700 feet) demanded such treatment, being literally built piece by piece in place, that it cannot be cited or described as illustrating any general or practical mode of erection.

The "Deck" cantilever structures, presenting as they do the least difficulties in erection, will first be considered. The false work to temporarily support the anchor arm of the structure is first put in place, using the same plan and methods, according to the various conditions of height, water, and character of bottom, as have been given under the head of ordinary truss spans, previously considered. The pedestals and feet are then set on the pier with the greatest care, both as regards elevation and lateral position, all being done under the immediate supervision of the engineer, and set to his marks and centres. From the centre of the pin in this foot as a starting-point, the lower chord is lined out and connected to bars in the anchor pin; it is assumed that the anchorage, including the necessary eye-bars to connect with the trusses, has been put in place during the construction of the pier. The traveller to erect the trusses is of a pattern described earlier in this chapter, runs on the regular track, and is so designed that the overhanging portion projects nearly two panels ahead of that part of the structure connected. After the chord has

been lined out from the pier to the anchorage, the pieces joined together by the end anchorage pin are then hoisted and held in place from the traveller while the pin is driven; the bars and members thus connected are hung to the traveller, while the first panel of the upper chord is lowered into position over the posts, and the upper ends of the bars just connected in the lower chord are hoisted to their proper upper panel point and the pins driven. This completes one panel of the truss. The floor beams, usually set directly upon the chord at the panel points, are then put on and the first panel of the stringers placed in position, the horizontal and transverse laterals connected, sufficient wooden ties and rails put on the stringers, and the traveller can then be run ahead one panel and the erection proceeded with in exactly the same manner, panel by panel, to the pier. It may be necessary to support the loose ends of partly connected members to the traveller while it is moved ahead; this can be done by a proper arrangement of the supports. It will also be noticed that it is necessary to design the details so that the chord splices occur near the panel points, but on the side away from the traveller; otherwise the traveller would have to reach out nearly three panels. After the anchor arm is complete the erection of the lever arm continues, panel by panel, in the same manner, only there is no false work under it, none being necessary, as it, with the suspended span, is held up in position by the dead weight of the anchor arm and the masonry of the anchorage. The erection of this lever arm presents no new problems or difficulties until we reach the panel between the lever arm and the suspended span. In this panel large and powerful vertical wedges are placed on line of both upper and lower chords. These wedges are for two purposes: first, to raise or lower the centre of the suspended span to facilitate the final connections; second, to shorten or lengthen the distance between the centre of the suspended span and the end pin of the lever arm, or, in other words, to increase or to diminish the length of iron-work between piers. This is found to be absolutely necessary, as, even after exercising the greatest care in the triangulation of the span lengths and in the placing of the pedestals and shoes on the piers, the distance c. to c. is liable to considerable variation. It will also be seen upon reflection how necessary it is to have the adjustment vertically, as, with the traveller, tools, and men on the centre of the suspended span, it would be impossible to figure the deflection to the nicety required to make the final connections. These wedges work against frames built in the members composing the upper and lower chords, and are worked by means of heavy, powerful screws, passing directly through the wedges vertically, the nuts of which screws bear on an independent frame. The pins connecting the sections of these chord panels at the wedges pass through oblong holes in the main members, permitting the lengthening or shortening of the panel.

Upon examination of the diagram it will be seen that the province of the upper wedge is simply to raise or lower the centre of the span, as it cannot possibly lengthen or shorten the distance between the end pins; while the movement of the wedge in the lower chord panels affects both the elevation of the centre and the length between the end pins. As previously stated, the erection of the lever arm is proceeded with precisely as for the anchor arm, each panel being supported by that portion of the structure previously erected. This also applies to the panels connecting the lever arm and the suspended span and the suspended span itself, and with the elevation and length of span in our control, to cover any discrepancy in the figured length or deflection, no difficulty should be experienced in the final connection. After this connection is made, the wedges

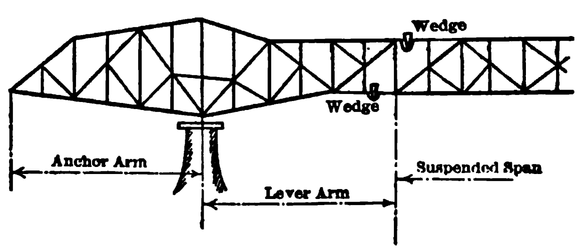


Plate XV.

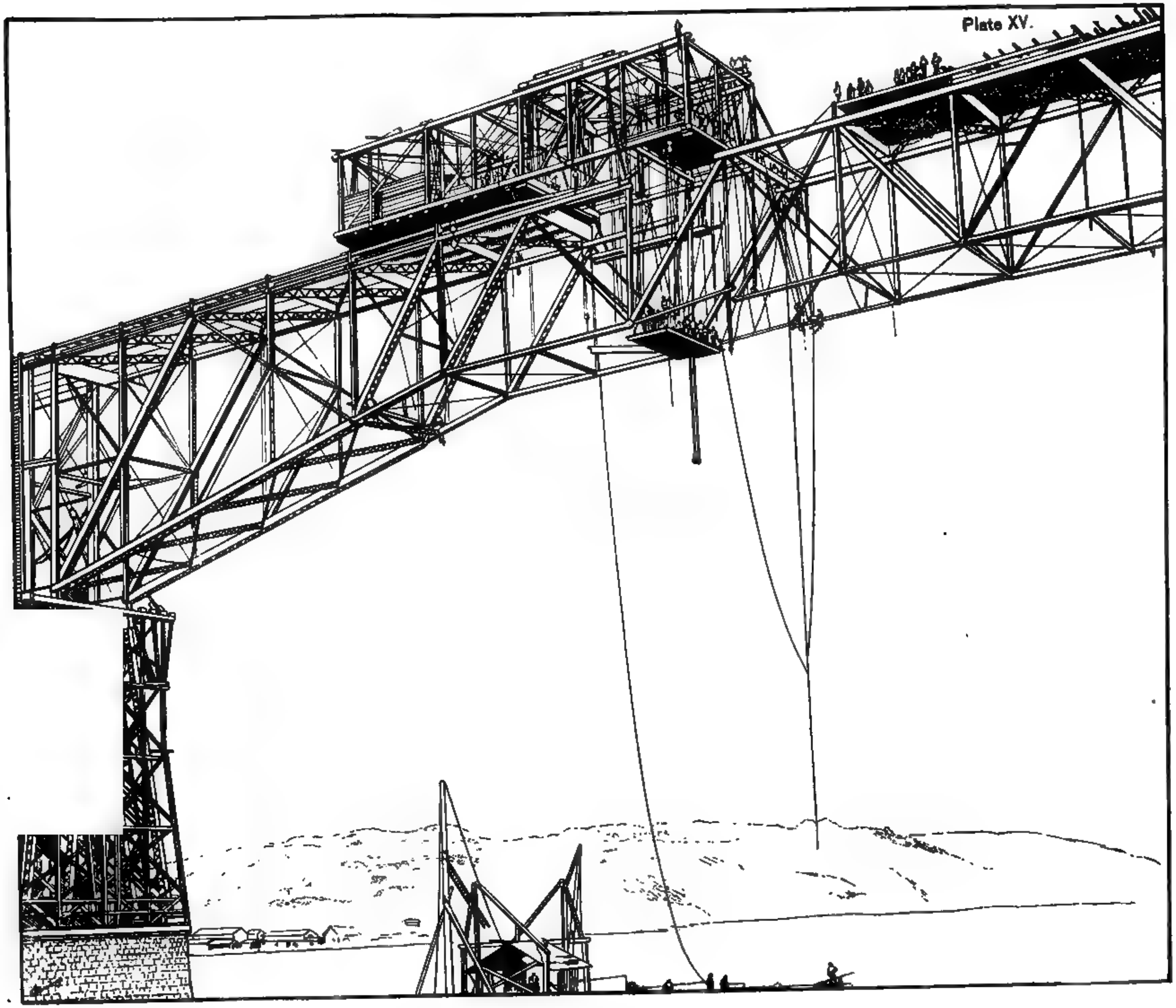
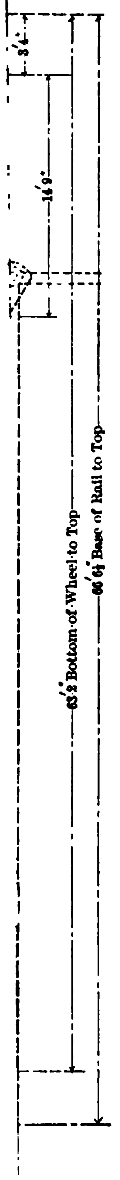
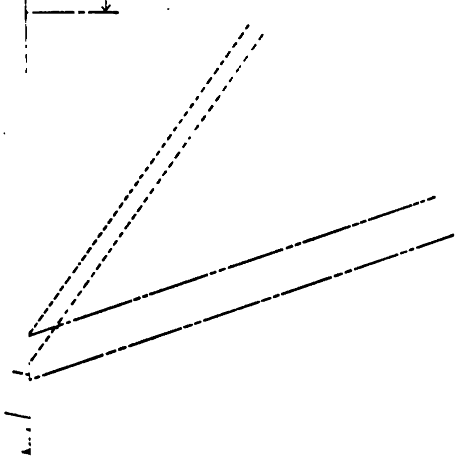


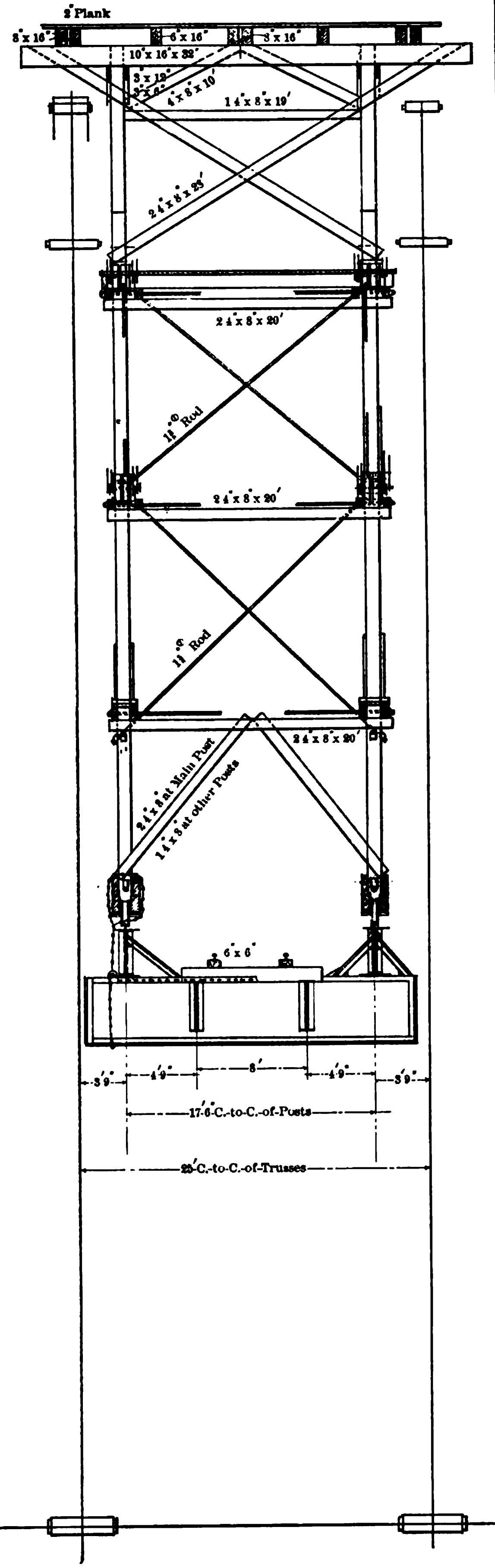
Plate XVI.

BILL OF TIMBER.

		Inches.	Feet.
18	Top Str.	8 x 16	30
6	" "	6 x 16	30
8	Longl. Str.	8 x 12	30
4	" "	3 x 12	28
46	" "	3 x 6	2
14	Top Sill.	10 x 16	32
8	Long Struts,	4 x 8	33
4	" "	"	28
16	" "	"	20
6	" "	"	15
4	" "	"	10
5	Trans. Str.	"	19
26	" "	"	21
20	Top Braces,	"	28
10	" "	6 x 8	6
10	" "	4 x 8	10
4	Sills,	7 x 14	25
4	"	"	33
2	"	"	52
2	"	"	5
2	Posts,	12 x 12	50
2	"	"	48
2	"	"	39
2	"	6 x 8	17
2	"	"	32
2	"	"	47
2	"	"	16
2	"	"	28
2	"	"	38
2	"	"	18
2	"	"	46
2	"	"	31
4	"	"	14
30	Joists,	4 x 12	21
9	"	6 x 12	21
2	"	10 x 12	3
2	Booms,	"	30
2	Masts,	8 x 8	15
2	Brackets,	"	7
4	Splice,	6 x 12	7
4	"	8 x 8	7
2	"	"	4
4	Check Pl.	3 x 14	5
4	Blocking,	4¼ x 16	5
4	"	6 x 6	8
6	Sill Spl.	3 x 16	5
2	" "	4 x 8	4
6	Blocks,	4 x 12	4
2	"	9 x 12	5
12	"	2 x 6	2
4	"	6 x 8	2
2	"	8 x 8	1
8	Braces,	3 x 10	7
2	"	4 x 10	15
7	Ties.	8 x 10	21
2	Braces,	3 x 6	20
4	Blocks,	3 x 12	2
16	Braces,	4 x 8	15
8	Ladders,	2 x 6	21
80	Steps,	1 x 4	2
400	Planks,	2 x 10	16
8	Blocks,	3 x 12	2¼ in.
60	Rail Blocks,	6 x 6	10 ft.
62	Checks,	6 x 9	1¼ "







:LER FOR THROUGH CANTILEVERS,

SCALE OF FEET

0 10 20 30

should be removed, allowing the suspended span to hang by the vertical bars to the lever arm, and free to lengthen or shorten, by means of the oblong holes around the pins, for variations of temperature and loading. Before the erection of the suspended span is begun, it is advisable to so adjust the wedges that no raising of the centre will be necessary, as it is a much easier matter to lower than to raise.

There are no peculiar points to watch during the erection, not already covered in this discussion; of course, the foreman will see that his traveller is well anchored down when lifting heavy pieces, or when it has great weight hanging to it; he will also watch particularly the alignment of his trusses as the erection proceeds, and have ready means at hand for lashing the traveller down in case of high winds, as it is in a peculiarly exposed position, and without the convenience of false work to guy to. If the structure is so situated, it is better, and at times cheaper, to hoist the iron directly from boats below and place it in position; if this is not possible, it will be run out, on top, from the bank on a separate car, to the traveller, where it can be reached with a set of "falls" and lowered into position. Swinging platforms are hung from the traveller at convenient points for the men to work at the connections, driving pins, etc. Plate XV. shows the Poughkeepsie Bridge in course of erection, including the traveller, wedges, etc., and when ready to connect the centre and last panel. It is advisable to place the hoisting engine and boiler directly on the traveller, near the rear end, as it is convenient for hoisting material, and the weight of machinery, etc., is just what is needed to help counterbalance the loaded overhanging portion of the traveller.

THROUGH CANTILEVERS.

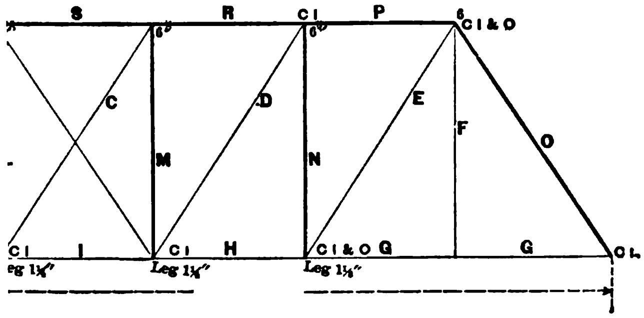
As has been stated, "Through Cantilevers" present more difficulties in erection than "Deck" structures, necessitating greater care and attention. First, the traveller must run on the track and on the inside of the trusses, as there is no means of support outside; this necessitates a very narrow traveller. Second, the traveller being on the inside and extending above the trusses, it is not possible to put in the transverse and upper lateral bracing until the whole traveller has passed the panel point; this necessitates the omission of bracing, above the floor, at two panel points back of the panel being erected, and demands the closest attention of the foreman to see that the bracing is put in at the earliest possible moment, and that he is not caught napping by high winds. This design of structure also demands the extra handling of iron; part, as will be seen upon examination of Plate XVI., must be hoisted and placed in position above, part must be lowered into place below; while all must be brought out over the track to the traveller and swung out from the centre until it clears, leaving the overhanging boom, where it is taken hold of by other sets of "falls" and passed back alongside of the traveller, and directly over the centre of the trusses to the proper point, and lowered or raised to its position. The erection of "Through Cantilevers" is begun and proceeded with in the same manner as described under the head of "Deck Cantilevers;" the wedges are provided in the panels connecting the lever arm and the suspended span, and operate in the same manner.

The Plate XVI. shows "Through Cantilever" traveller in position to raise the lever arm, and by studying it closely the above description can be more intelligently followed. If the permanent structure details will permit it, outside temporary brackets at the panel points, with stringers to carry the traveller, could be provided, greatly simplifying the erection and reducing danger, as the traveller would then be outside of all, and the bracing could be put in immediately. Of course, it would only be necessary to provide brackets and stringers for the number of panels covered by the traveller, as they can be moved ahead as the erection proceeds.

Plate XVII. shows the traveller, shown in detail in Plate XVI., raising the Red Rock Cantilever.

INDEX.

STRAIN SHEET.



Width, c to c 16'.3". Width in clear, 15'.0".
.10 lbs.

2 Lines Iron Stringers 7'.6" c to c.
Distance between Channels of Posts 8¼".

	Total Cross Section □".	Max. strain per □" Lbs.
....	.785	5,642
....	4.810	7,670
....	7.5	8,826
....	11.25	9,124
....	15.625	9,085
....net.	5.469	7,209
....	11.5	8,751
....	19.	9,090
....	25.	8,739
....	27.5	8,875
;" × 36" long 4 Pin plates 6½" × 7/16" × 12" long | Cross plates 12" × ½"....	6.	5,663
· × 36 " 4 " " 8½ × 7/16 × 12 " | Lattice bars 2 × ½" double.	9.6	6,093
r × 36 " 4 " " 8½ × 9/16 × 12 "	13.2	6,742
't of end posts 0–⅝" thick inside..............	26.62	6,907
" " 0–4 plates 9½" × 9/16. Coupling plates 9½" × ⅜"	24.75	6,978
over all posts 16½" × ½" × 18" long..............	29.05	7,530
rs 15" × ½ × 15" long..............	31.5	7,748
¼". Lattice bars 2" × 5/16" double..............	31.5	7,748
....	....	......
9/16"; Area, 9".27 gross. Bottom angles, 5" × 3½ × 9/16; Area 8".29 net...	T. Gross 9.27 B. Net 8.29	7,153 7,999
end. Stringer bracket angles 3 × 9/16 × 12 long..............	....	......
× ½" × 12" long..............		
' × 9/16"; Area 7".79 gross. Bottom Angles, 4½" × 3 × 9/16; Area, 7".0 net	T. Gross 7.79 B. Net 7.	6,431 7,157
at each end. Coupling plate across beams, 9½" × ½" × 23" long....	....	......
' × ⅜" × 27" long. Cross frames 2 Angles 3" × 5/16", gusset plates 5/16" thick.	....	......
ring. Roller plate, 20" × 1¼" × 24" long. Angles, 4" × ½"	....	......
ong. Anchor bolts in main bed plates 1¼" 0, in Stringer bed plates 1"–0....	....	......
× 1."..............	....	......
....	....	......
....	....	......
....	....	......
....	....	......
7/16". Lattice bars, 4" × 5/16" and 2 × 5/16"..............	....	......
Angles, 2½" × 5/16"..............	....	......
....	....	......
....	....	......
....	....	......
to 8½"..............	....	......
bolted to each 2d tie with ⅝" bolts..............	....	......
....	....	......
....	....	......

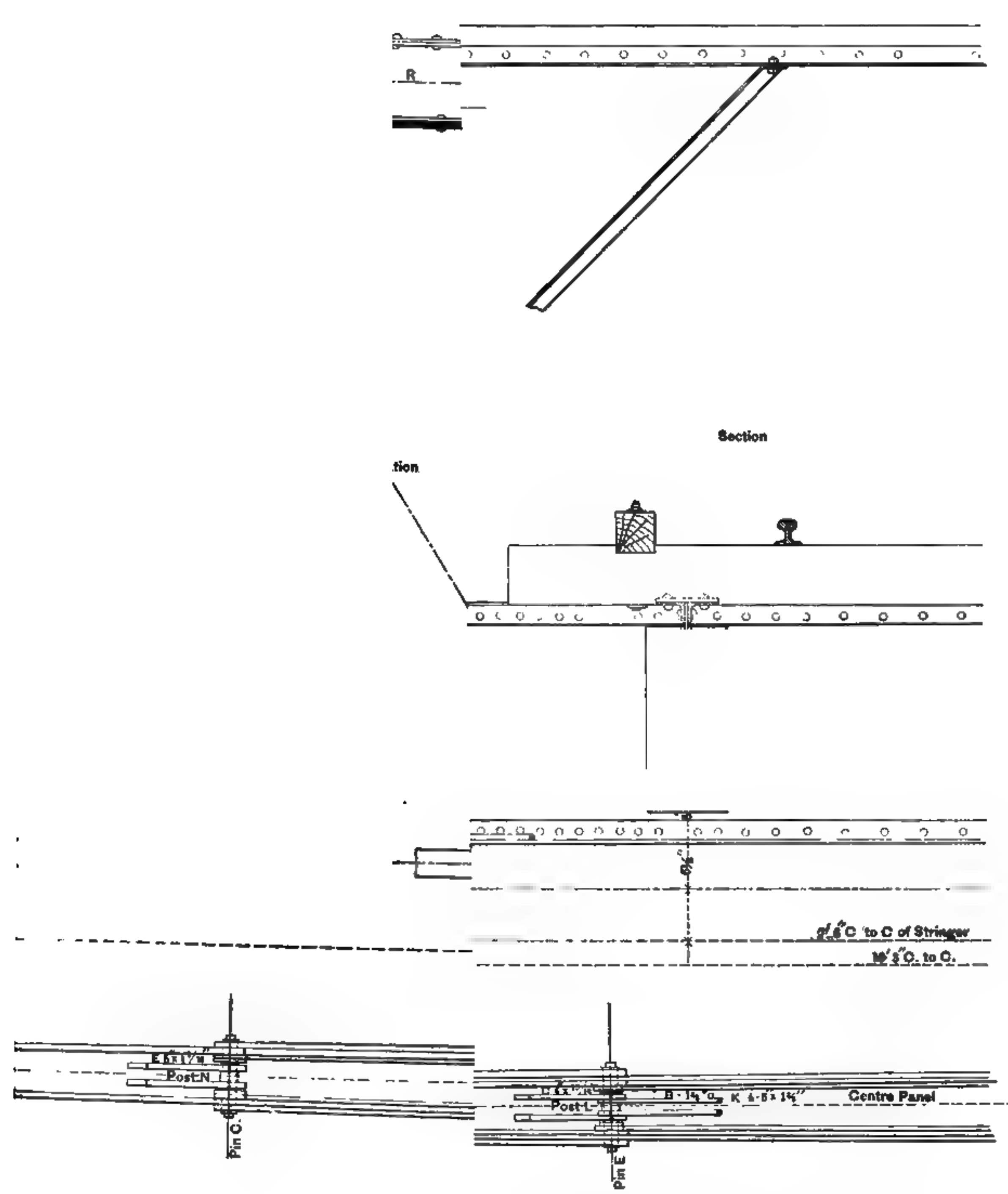
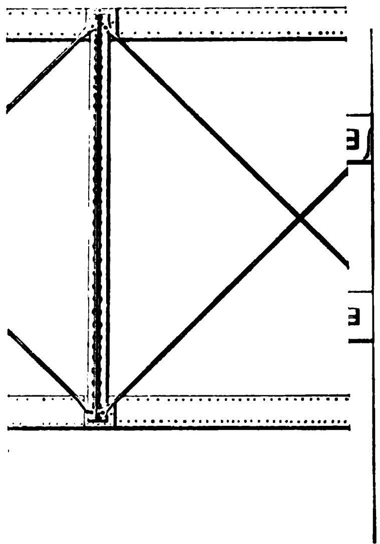
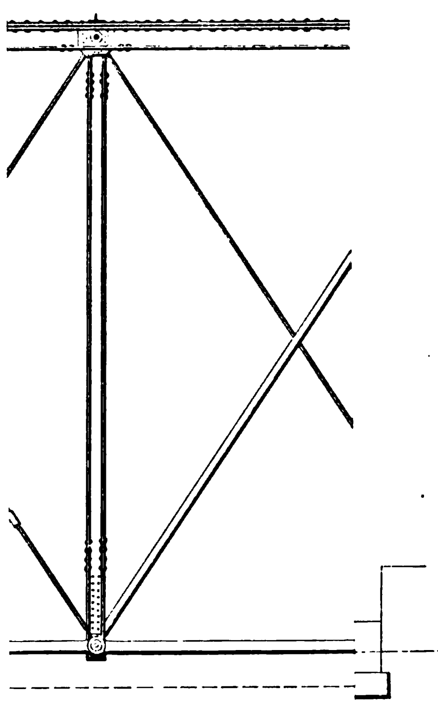
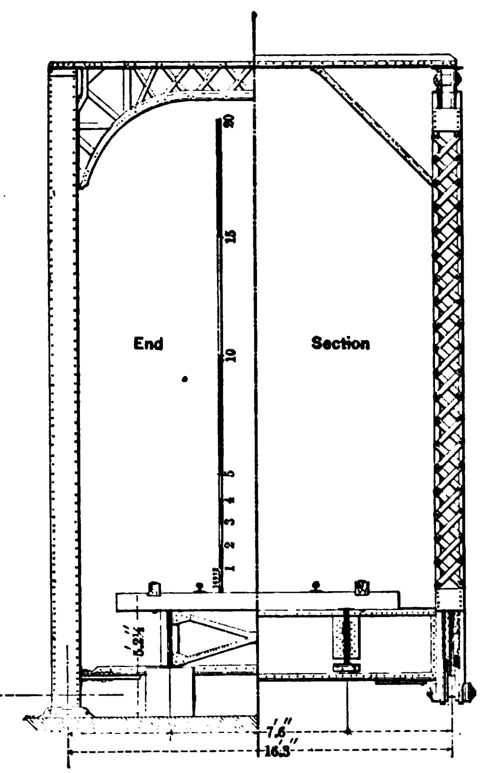
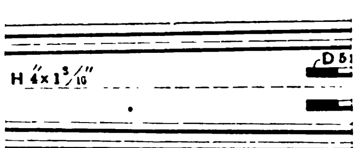

R
Section
tion
C to C of Stringer
C. to C.
Post N
Pin C.
Centre Panel
Post L
Pin E

PLATE 24.









ıwer Chord

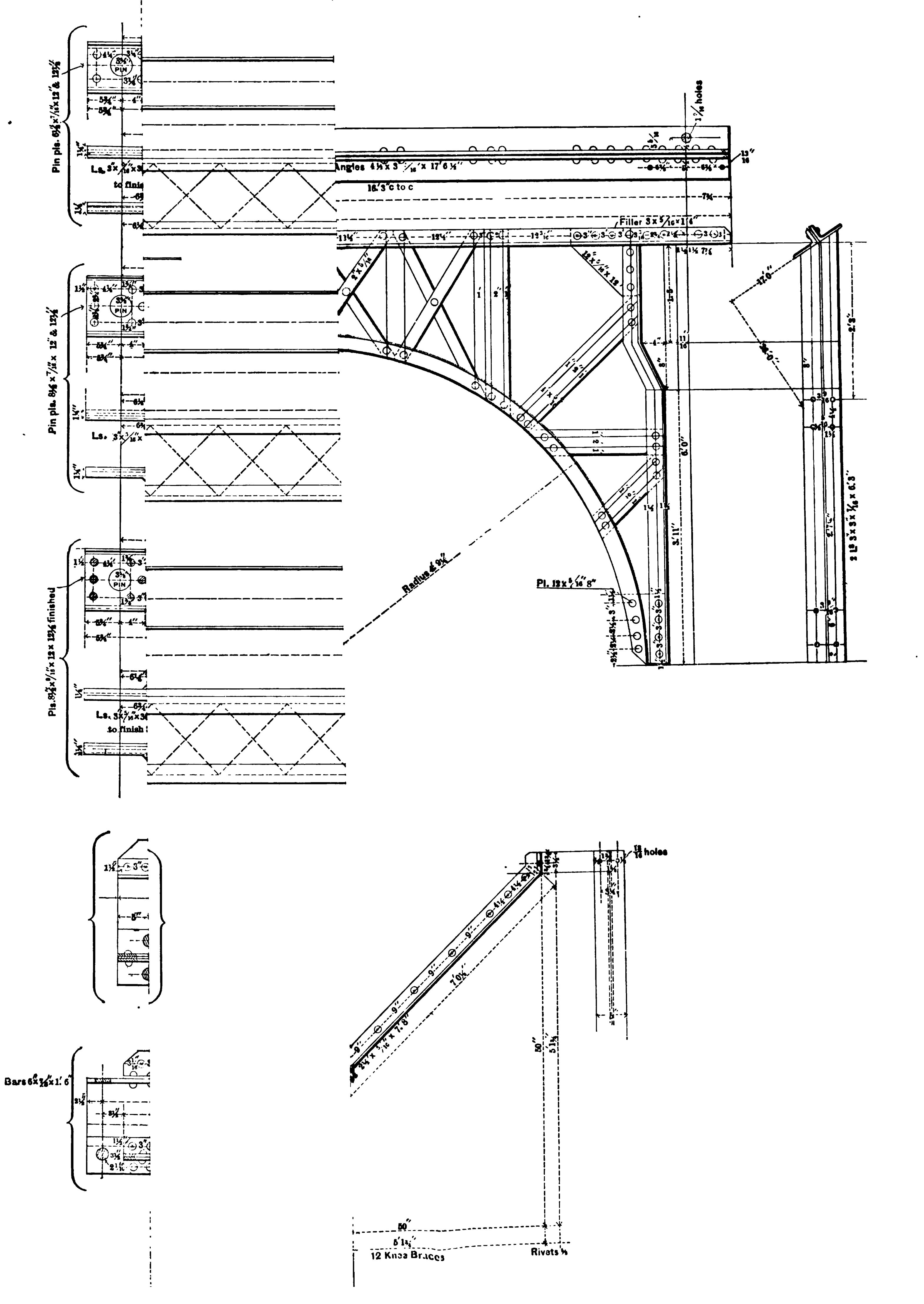

16'3" c to c
Radius 4'9½"
12 Knee Braces

PLATE 2C

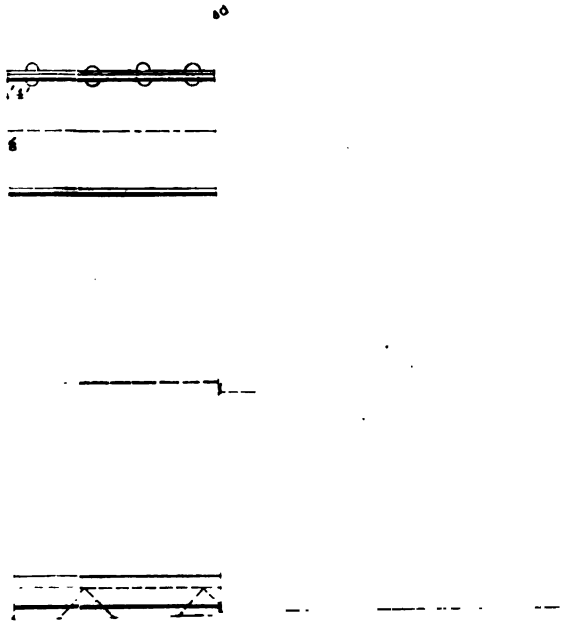
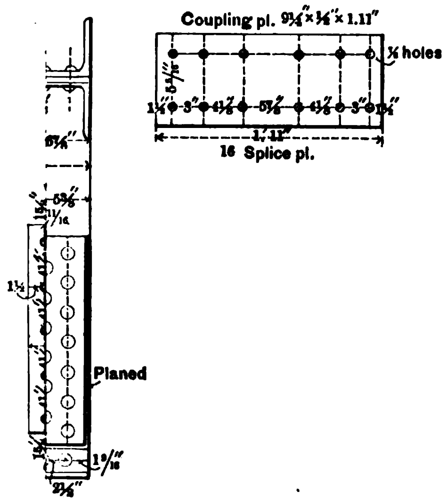
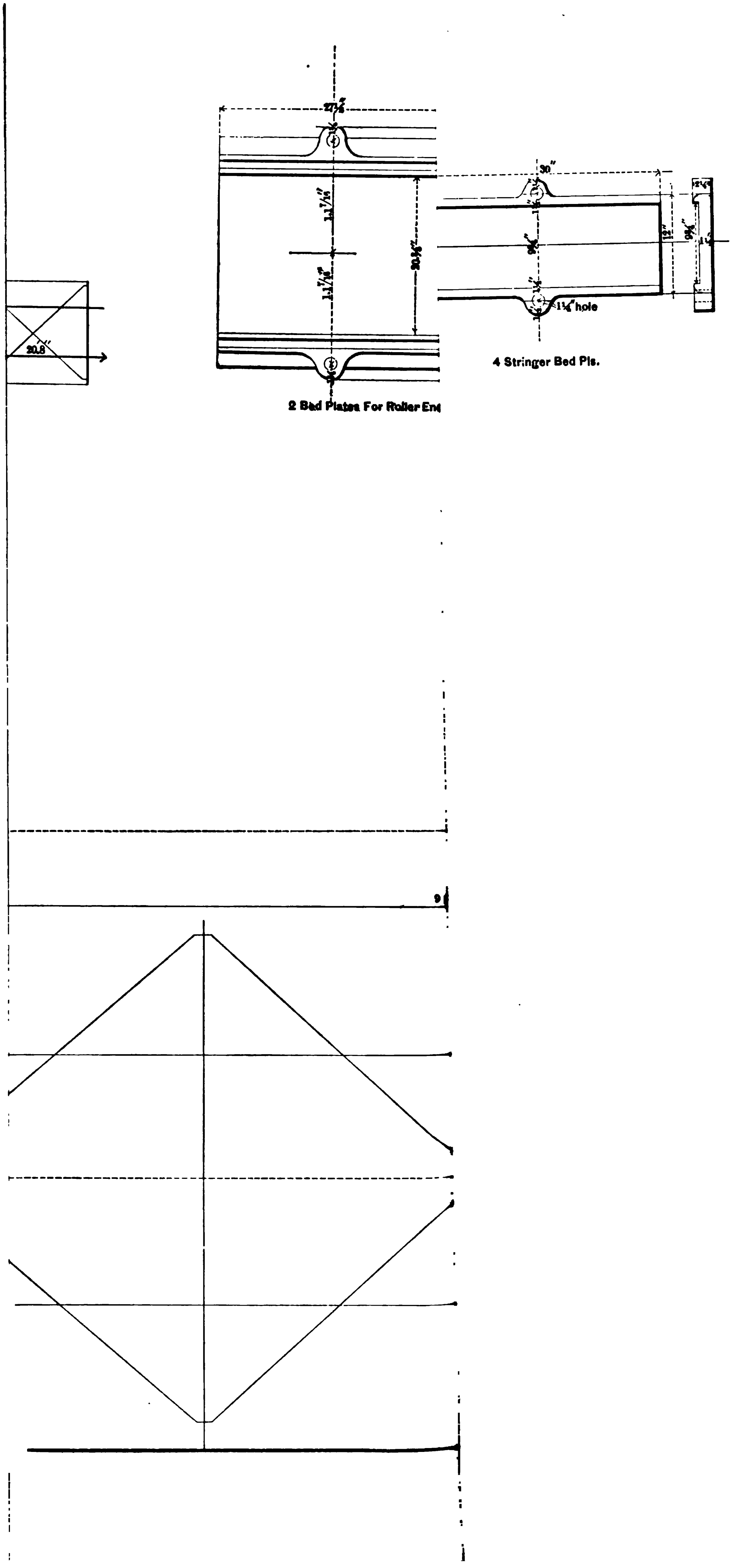


PLATE 27.





2 Bed Plates For Roller End

4 Stringer Bed Pls.

M. P. R. R.

Bridge No. 44.

SCALE OF FEET

0 1 2

KELLOGG & MAURICE,
Athens. Pa.